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Substructured Model Order Reduction for Simple Exchange of Subsystems

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This contribution deals with substrucutured model order reduction for simple exchange of substructures. It is shown, that the Krylov-Subspace-Method is well suited in this context and a numerical example is presented.

Substructured model order reduction offers the possibility, to exchange various substructures independently and to reuse the other reduced substructures. In the course of this, different substructures, for example components, are reduced independently.

The equation of motion of an undamped linear described subsystem reads $\boldsymbol{M}_k\ddot{\boldsymbol{q}}_k + \boldsymbol{K}_k\boldsymbol{q}_k = \boldsymbol{B}_k\boldsymbol{u}_k$ with the vector of nodal displacements $\boldsymbol{q}_k$, the mass matrix $\boldsymbol{M}_k$ and the stiffness matrix $\boldsymbol{K}_k$. The term $\boldsymbol{B}_k\boldsymbol{u}_k$ describes the forces, that are applied on the substructure with the input matrix $\boldsymbol{B}_k$ and the input $\boldsymbol{u}_k$. A set of important displacements can be selected as outputs $\boldsymbol{y}_k = \boldsymbol{C}_k\boldsymbol{q}_k$ with the output matrix $\boldsymbol{C}_k$. The substructures can be coupled using constraint equations. The motion of the assembled system with K subsystems can be described as

$$\begin{bmatrix}\boldsymbol{M} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}\begin{bmatrix}\ddot{\boldsymbol{q}}\\ \ddot{\boldsymbol{\lambda}}\end{bmatrix} + \begin{bmatrix}\boldsymbol{K} & \boldsymbol{J}^T\\ \boldsymbol{J} & \boldsymbol{0}\end{bmatrix}\begin{bmatrix}\boldsymbol{q}\\ \boldsymbol{\lambda}\end{bmatrix} = \begin{bmatrix}\boldsymbol{B}\\ \boldsymbol{0}\end{bmatrix}\boldsymbol{u} \tag{1}$$

with $\boldsymbol{q} = [\boldsymbol{q}_1 \dots \boldsymbol{q}_K]^T, \boldsymbol{M} = \mathrm{diag}(\boldsymbol{M}_1, \dots \boldsymbol{M}_K), \boldsymbol{K} = \mathrm{diag}(\boldsymbol{K}_1, \dots \boldsymbol{K}_K), \boldsymbol{B} = \mathrm{diag}(\boldsymbol{B}_1, \dots \boldsymbol{B}_K), \boldsymbol{u} = [\boldsymbol{u}_1 \dots \boldsymbol{u}_K]^T$, the Lagrange multipliers $\boldsymbol{\lambda}$ and the Jacobian matrix of the constraint equations $\boldsymbol{J}$, compare [1].

The subsystems are reduced by projecting the nodal displacement vector on a suitable subspace $\mathcal{V}_k$, spanned by a matrix $\boldsymbol{V}_k$. The reduced vector is then $\widetilde{\boldsymbol{q}}_k = \boldsymbol{V}_k\boldsymbol{q}_k$. With $\boldsymbol{V} = \mathrm{diag}(\boldsymbol{V}_1, \dots \boldsymbol{V}_K)$ a reduction matrix

$$\begin{bmatrix}\widetilde{\boldsymbol{q}}\\ \boldsymbol{\lambda}\end{bmatrix} = \begin{bmatrix}\boldsymbol{V} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}\begin{bmatrix}\boldsymbol{q}\\ \boldsymbol{\lambda}\end{bmatrix} \tag{2}$$

can be constructed, that can be applied on the assembled system in equation (1). This is equivalent to the coupling of the independently reduced substructures, compare [2]. In [1] it is discussed, that moment matching is a well suited method for the independent reduction of substructures, that are coupled after the reduction. For systems, that are coupled using bushing elements, it is shown, that moment matching on subsystem level is retained after the coupling. In the following this is extended for systems that are coupled via constraint equations.

Using moment matching by the Krylov-subspace-method for an expansion point at σ_0, the reduction matrix for subsystem k reads $\boldsymbol{V}_k = (\sigma_0^2\boldsymbol{M}_k + \boldsymbol{K}_k)^{-1}[\boldsymbol{B}_k\ \boldsymbol{J}_k^T]$, where $\boldsymbol{J}_k$ is the assigned submatrix of $\boldsymbol{J}$. For the sake of readability, the following calculations are performed for a system consisting of two subsystems, but they can be easily extended to more subsystems. A reduction matrix according to equation (2) reads

$$\boldsymbol{V}_{\text{assembled}} = \begin{bmatrix}(\sigma_0^2\boldsymbol{M}_1+\boldsymbol{K}_1)^{-1}\boldsymbol{B}_1 & (\sigma_0^2\boldsymbol{M}_1+\boldsymbol{K}_1)^{-1}\boldsymbol{J}_1^T & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & (\sigma_0^2\boldsymbol{M}_2+\boldsymbol{K}_2)^{-1}\boldsymbol{B}_2 & (\sigma_0^2\boldsymbol{M}_2+\boldsymbol{K}_2)^{-1}\boldsymbol{J}_2^T & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}. \tag{3}$$

If the Krylov-subspace method is directly applied to the assembled system, the reduction matrix reads

$$\boldsymbol{V}_{\text{kry}} = \begin{bmatrix}\sigma_0^2\boldsymbol{M}_1+\boldsymbol{K}_1 & \boldsymbol{0} & \boldsymbol{J}_1^T\\ \boldsymbol{0} & \sigma_0^2\boldsymbol{M}_2+\boldsymbol{K}_2 & \boldsymbol{J}_2^T\\ \boldsymbol{J}_1 & \boldsymbol{J}_2 & \boldsymbol{0}\end{bmatrix}^{-1}\begin{bmatrix}\boldsymbol{B}_1 & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{B}_2\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}. \tag{4}$$

Rewriting this and applying the Kailath-Variant $(\boldsymbol{A}+\boldsymbol{B}\boldsymbol{C})^{-1} = \boldsymbol{A}^{-1} - \boldsymbol{A}^{-1}\boldsymbol{B}(\boldsymbol{I}+\boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{B})^{-1}\boldsymbol{C}\boldsymbol{A}^{-1}$ one gets

$$\begin{aligned}\boldsymbol{V}_{\text{kry}} &= \left(\begin{bmatrix}\sigma_0^2\boldsymbol{M}_1+\boldsymbol{K}_1 & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \sigma_0^2\boldsymbol{M}_2+\boldsymbol{K}_2 & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix} + \begin{bmatrix}\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{J}_1^T\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{J}_2^T\\ \boldsymbol{J}_1 & \boldsymbol{J}_2 & -\boldsymbol{I}\end{bmatrix}\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}\right)^{-1}\begin{bmatrix}\boldsymbol{B}_1 & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{B}_2\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}\\ &= \begin{bmatrix}(\sigma_0^2\boldsymbol{M}_1+\boldsymbol{K}_1)^{-1}\boldsymbol{B}_1 - (\sigma_0^2\boldsymbol{M}_1+\boldsymbol{K}_1)^{-1}\boldsymbol{J}_1^T\boldsymbol{\Gamma}_1 & (\sigma_0^2\boldsymbol{M}_1+\boldsymbol{K}_1)^{-1}\boldsymbol{J}_1^T\boldsymbol{\Gamma}_2\\ (\sigma_0^2\boldsymbol{M}_2+\boldsymbol{K}_2)^{-1}\boldsymbol{J}_2^T\boldsymbol{\Gamma}_1 & (\sigma_0^2\boldsymbol{M}_2+\boldsymbol{K}_2)^{-1}\boldsymbol{B}_2 - (\sigma_0^2\boldsymbol{M}_2+\boldsymbol{K}_2)^{-1}\boldsymbol{J}_2^T\boldsymbol{\Gamma}_2\\ \boldsymbol{\Gamma}_3 & \boldsymbol{\Gamma}_4\end{bmatrix}.\end{aligned} \tag{5}$$

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Comparing the second line of equation (5) with equation (3), one finds, that $\boldsymbol{V}_{\text{kry}} \subseteq \boldsymbol{V}_{\text{assembled}}$ and, therefore, moment matching is retained after coupling of reduced subsystems.

As numerical example the structure of a high rise building is used developed by the SFB 1244. This structure is braced with diagonal tie rods. It has a base excitation and three nodal displacements are selected as outputs. A setup is tested, that allows to exchange the tie rod models by various linear and nonlinear models. To allow the exchange of different tie rod models in the reduced system, the structure is separated into a structure without tie rods and tie rods. The structure without tie rods is then reduced. The unreduced reference system has a dimension of 36728. For the reduction three different methods are used, the Krylov-subspace-method, the Craig-Bampton-method and modal truncation. All reduced systems have a dimension of 912. Figure 1 shows the Frobenius norm of the transfer function of the structure without tie rods of the unreduced reference system and of three different reduced models. One finds that all three reduction methods lead to an appropriate reduced model of the structure without tie rods.

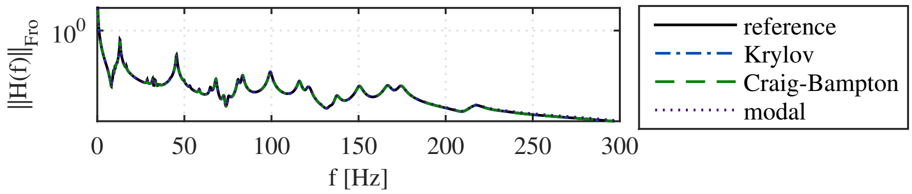

Fig. 1: Frobenius norm of the transfer function of the structure without tie rods.

Different tie rod models are then attached to the reduced structures. In this example, a linear and a nonlinear force element are used as tie rod models. The Frobenius norm of the transfer functions of the structures with linear tie rod models is shown in Figure 2. One finds that the usage of the coupling of the tie rod models to the Krylov-reduced and Craig-Bampton-reduced structure still yields a good approximation result in the assembled system. The coupling of the tie rod models to the modaly reduced structure leads to large difference to the reference. The simulation result for one output as well as a close up are shown for the assembled systems with the linear tie rod models in Figure 3 and with the nonlinear tie rod models in Figure 4. For both tie rod models one finds, that the modaly reduced system yields a bad approximation result in both simulations, compared to the usage of the Krylov-subspace-method and the Craig-Bampton-method. The Craig-Bampton-reduction-basis consists of static correction modes and fixed interface normal modes. Static correction modes lead to moment matching, as well as the Krylov-subspace-method, compare [1]. Thus, the numerical example confirms that moment-matching model order reduction is a good choice for the substructured reduction in the context of exchanging subsystems independently.

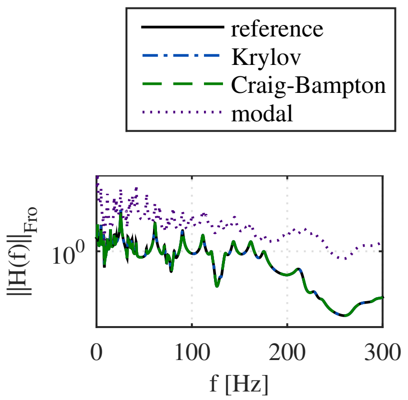

Fig. 2: Frobenius norm of the transfer function of the structure with linear tie rod models.

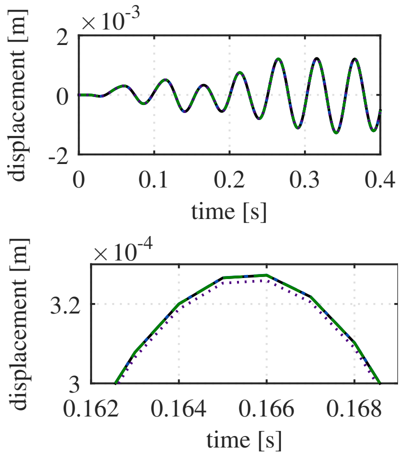

Fig. 3: Simulation result of the structure with linear tie rod models.

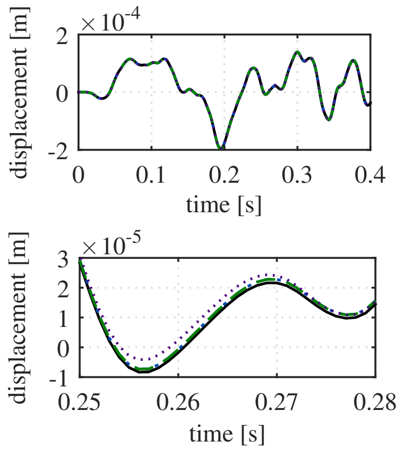

Fig. 4: Simulation result of the structure with nonlinear tie rod models.

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