Semantics of Projective Locative Expressions: An Empirical Evaluation of Geometrical Conditions

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Abstract

This thesis presents a method for evaluating semantic theories of projective locative expressions such as 'X is above Y' and 'X is to the right of Y'. The method is implemented for semantic theories that represent meaning of projective locative expressions in terms of geometrical constraints in two-dimensional space.

A set of semantic theories is defined according to proposals from the literature. These theories predict precise geometrical constraints for projective locative expressions. Furthermore, a formalism is proposed which is used to combine these theories in order to generate new semantic theories that are capable of handling vagueness of projective locative expressions.

The empirical basis of the evaluation is a set of expressions that subjects of a 'map task' experiment (Anderson et al., 1991) have used to describe spatial relations in two-dimensional space. Each expression refers to a specific map of which two-dimensional geometrical representations are derived.

The semantic theories are tested with these data by checking whether the geometrical constraints predicted for an expression are satisfied by the corresponding geometrical representation.

The evaluations show good results for most theories which have been proposed in the literature. The results are systematically improved by the corresponding theories that handle vagueness.

A more detailed **summary** can be found on page 167.

Eine ausführliche **Zusammenfassung** in Deutsch befindet sich auf Seite 173.

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Chapter 1

Introduction

This thesis investigates geoemtrical aspects of the meaning of projective locative expressions. Projective locative expressions describe the location of an object (*located object* or LO) relative to another object (*reference object* or RO) by means of a direction. Direction is expressed by projective prepositions (Herskovits, 1986) such as *above*, *below*, *to the right of*, and *to the left of*. Projective locative expressions may contain modifiers, e.g. *directly*, *slightly*, *straight*, and *1 inch*, which modify the interpretation of the direction or add constraints on the distance between the located object and the reference object (Zwarts, 1997). The following two sentences contain examples of projective locative expressions with and without modification:

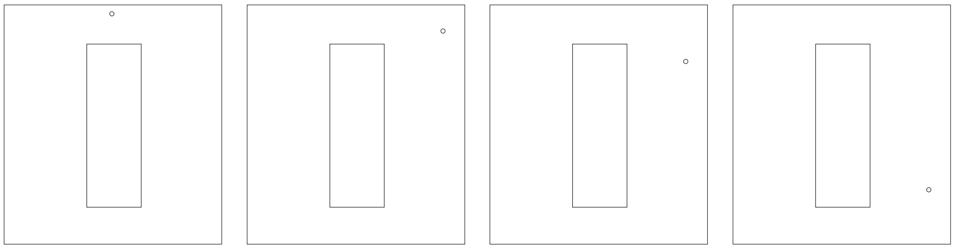
(1) The circle is above the rectangle.

(2) The circle is directly above the rectangle.

Projective locative expressions can be used in positive statements or in their negations. I will use the terms *positive use* for positive statements and *negative use* for negations of positive statements. *Positive uses* of projective locative expressions convey that the relation denoted by the expression applies to the given spatial configuration. The previous two sentences are examples of positive uses. *Negative uses* of projective locative expressions convey that the relation denoted by the expression does not apply to the corresponding spatial configuration. The following sentence is a negative use corresponding to (1):

(3) The circle is not above the rectangle.

I will focus on two distinct but related aspects of the meaning of projective locative expressions. The first aspect concerns the truth value of an expression: What are the truth conditions that determine whether the statement of a projective locative expression is true or false with respect to a spatial configuration? And to what extent is the meaning of these expressions vague? – so that the question of the truth conditions of an expression is not simply a question of it being either true or false, but



(a) (b) (c) (d)

Figure 1.1: A circle in different positions relative to a rectangle.

one of being *definitely true*, *definitely false* or *indefinite*, where indefiniteness means that we cannot unequivocally determine the truth value, so that the statement might be either true or false. Bearing in mind vagueness the domain of a projective locative expression is divided into three parts. One part consists of all spatial configurations which unequivocally make the expression true (*definitely true*), another part consists of spatial configurations with respect to which the expression could be either true or false (*indefinite*), and the remaining part consists of all spatial configurations which unequivocally make the expression false (*definitely false*).

The second aspect concerns the goodness of fit between a description and a situation: To what extent do a projective locative expression and a spatial configuration match? Answers to this question are given in terms of functions from spatial configurations to values on a linear scale expressing *degrees of applicability*. Let us call such functions *applicability functions* and their range *applicability scale*. Degrees of applicability allow for two ways of interpretation. On the one hand, they provide a way of comparing and ranking alternative expressions according to their goodness of fit. On the other hand, they can be interpreted as the degrees in which spatial configurations instantiate a given projective locative expression. Logan & Sadler (1996), Regier & Carlson (2001), and Gapp (1995) report rating experiments that exhibit varying degrees of applicability for projective locative expressions with respect to systematically varied spatial configurations. Figure 1.2 shows the results of a rating experiment reported in (Regier & Carlson, 2001). Subjects rated sentence (1) ("*The circle is above the rectangle*") with respect to different spatial configurations like the ones shown in Figure 1.1 on a scale from 0 to 9, where a rating of 0 means that the description is *not acceptable at all* and a rating of 9 means that the description is *perfectly acceptable*. The matrix in Figure 1.2 shows average ratings for 56 different locations of the circle. Sentence (1) receives the following degrees of applicability with respect to the configurations shown in 1.1: (a) 8.9, (b) 7.1, (c) 4.1, and (d) 0.6. These figures clearly rank the spatial configurations with respect to their extent of instantiating sentence (1). The configuration (a) is ranked highest, (b) and (c) receive intermediate degrees, and (d) is ranked lowest. Intuitively, these degrees of applicability also say something about the truth of sentence (1). Its rating relative to (a) is 8.9 which is close to 9 (i.e. close to

7.2	7.5	8.4	8.9	8.2	7.7	7.0
6.7	6.5	8.4	8.9	8.3	7.2	7.1
3.0	3.0				3.1	4.1
1.6	1.4				1.5	1.4
1.0	1.3				1.1	1.4
0.9	0.6				0.9	0.7
0.5	0.2				0.5	0.6
0.5	0.4				0.6	0.2
0.3	0.2				0.3	0.6
0.2	0.2	0.1	0.3	0.6	0.3	0.2
0.5	0.3	0.3	0.4	0.4	0.4	0.1

Figure 1.2: Average ratings of the sentence “The circle is above the rectangle.” with respect to 56 different locations of a small circle like in Figure 1.1 reported by (Regier & Carlson, 2001). The ratings lie between 0 for *not at all acceptable* and 9 for *perfectly acceptable*.

“perfectly acceptable”) and thus suggests that the sentence is true with respect to (a). However, the fact that this degree is not identical to the maximum of the applicability scale raises the question whether the sentence can be false relative to (a) in some respects. If so, a degree of applicability of 8.9 would indicate that the spatial configuration belongs to the part of the domain that is associated with *indefiniteness* regarding the truth of sentence (1). If not, a degree of 8.9 would indicate that the sentence is *definitely true*. We can ask the same question with respect to all other spatial configurations, and also for the corresponding negative use (3) with respect to all four spatial configurations. In essence, we ask for the relation between degrees of applicability and the partition of the domain into *definitely true*, *indefinite*, and *definitely false*.

The relation between degrees of applicability and truth values can be established via partitions which divide the applicability scale into intervals, each of which is associated with one of the three truth values (*definitely true* / *indefinite* / *definitely false*). A natural way of partitioning of the applicability scale, at least for *unmodified* projective prepositions, is given by a division of the scale into three intervals. The upper interval comprises all spatial configurations with a high degree of applicability, and it is interpreted unequivocally as being associated with the truth value *true* – corresponding expressions are *definitely true*. The interval in the middle is the area of *indefiniteness*, i.e. both truth values are possible. And the lower interval comprises all spatial configurations where the description scores a low degree of applicability – low enough for an expression to count as *definitely false*.

Adverbial modifiers that are combined with projective prepositions, such as *directly*, *slightly*, *just*, and *sort of*, change the interpretation of the modified term (see

Zwarts (1997) and Lakoff (1973)). We have two principal options for analysing such modification operations. On the one hand, modifiers could be taken to change the underlying applicability function, while the partitions defined by the modified terms remain the same. On the other hand, modifiers might redefine the division of the applicability scale and leave the underlying applicability function unchanged. Whatever analysis of modification we choose, we can treat both modified and unmodified projective locative expressions as single non-decomposable units which divide the domain into three sets associated with *definitely true*, *indefinite*, and *definitely false*.

It should be noted that the question of truth and falsity of an expression can be treated independently of the question of its degree of applicability. In fact, any discrete approach that divides the domain into three such sets would give us a suitable amount of truth values. However, in order to gain complete understanding of previous work on projective locative expressions, it is necessary to integrate discrete and continuously graded approaches, since later work on spatial language has made great advances in developing methods for abstracting over spatial properties by means of continuously graded degrees of applicability, e.g. (Kelleher, 2003), (Regier & Carlson, 2001), (Matsakis et al., 2001).

Apart from work reported in (Abella, 1995; Abella & Kender, 1994), the question of truth and falsity of projective locative expressions with respect to spatial configurations has not been investigated on an empirical basis. It is therefore an open question how people really use projective locative expressions when they talk about spatial configurations. Can we confirm the truth conditions that are specified by previous approaches with data taken from actual conversations? In particular, do those truth conditions correctly predict the positive and negative uses that people produce?

The following three factors play a considerable part in determining the degree of applicability and the truth value of projective locative expressions: (i) the *frame of reference* – in a concrete spatial setting a *frame of reference* defines reference directions relative to which the directions referred to by projective prepositions (e.g. *above*) are aligned; (ii) *geometrical* relations and properties; and (iii) *functional* relations between objects, i.e., relations which are determined by ontological, conventional and intentional aspects of the objects.

Since this work is concerned with the influence of the second factor (*viz* geometrical properties and relations) the other two factors need to be controlled, so that distinct degrees of applicability and truth values are caused by differences in geometrical conditions, but neither by distinct frames of reference nor distinct functional relations.

The *HCRC Map Task* corpus (Anderson et al., 1991; Isard, 2001) is the empirical basis for the work described in this thesis. It is a collection of route description dialogues produced by people trying to accomplish a *map task*. The map task described in (Anderson et al., 1991) engages two participants in a conversation about a route that is printed on a map. Each of the participants in this task has a schematic map containing line drawings of objects, so called *landmarks*. While the map of one participant – the *instruction giver* – has a path drawn in it, this path does not appear on the map of the other participant – the *route follower*. Their joint task is to replicate that path on the map of the route follower. Communication between the participants is restricted to natural language communication – the participants cannot see the other participants' maps and they are not supposed to use gestures nor to show their maps to each other.

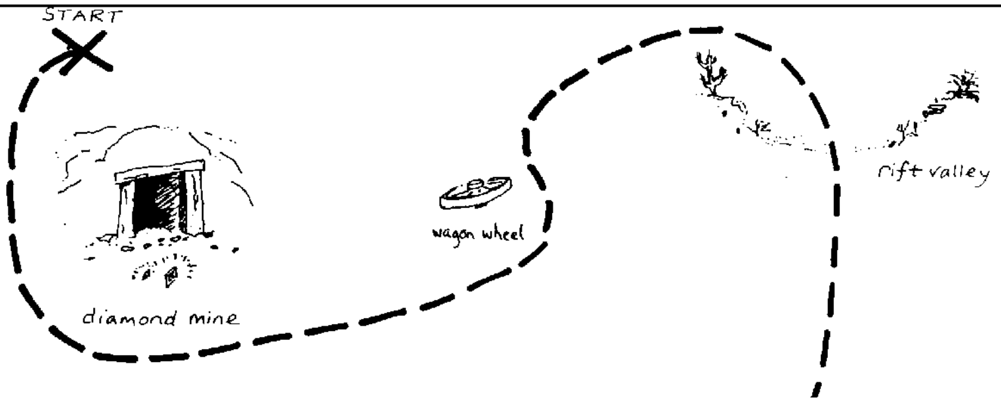


Figure 1.3: Upper part of an instruction giver's map. It contains landmarks with the labels *start*, *diamond mine*, *waggon wheel*, and *rift valley*.

Figure 1.3 illustrates the top part of an instruction giver's map. The maps used in the map task experiments are schematic maps containing line drawings that serve as landmarks. Each landmark is associated with a textual label. An additional difficulty was introduced to the map task by having maps in one experiment that were slightly different. More precisely, in every experiment the map of the instruction giver and the map of the route follower slightly deviate from each other as the landmarks on both maps do not match exactly. Nonetheless, they are supposed to represent the same situation. There are three kinds of differences: (i) landmarks appearing on one map are not printed on the other map; (ii) landmarks occurring on both maps at the same position are associated with different textual labels; and (iii), different landmarks, i.e. distinct line drawing and distinct label, appear at the same location on the corresponding maps. These mismatches cause the participants to align their information about the existence and the location of landmarks before they use them in the route description task. The instruction giver needs information about the route follower's map in order to describe the route in a way that the route follower can understand easily. And the route follower needs information about the instruction giver's map in order to understand all route directions by the instruction giver. The speakers achieve alignment by either identifying a label from the other participant's map with a landmark from their own map or by constructing spatial extension and position of a missing landmark according to the description provided by the other participant. In either case, the participants typically use locative expressions to describe the location of a landmark. In some cases, location is described relative to the entire map, in some cases it is given relative to the current position of the path, and in the cases which are relevant for this study, the location of a landmark is specified relative to another landmark.

All occurrences of the latter kind of locative expressions have been extracted from the corpus, i.e., expressions describing the location of a landmark relative to another landmark, in order to investigate which truth conditions speakers associate with projective locative expressions.

The general assumption concerning the data is that speakers describe spatial configurations on their map with expressions that are true with respect to their map. Thus, positive uses of projective locative expressions in application to given spatial configurations provide empirical evidence for the truth of the expressions with respect to those spatial configurations. And negative uses of projective locative expressions provide empirical evidence for the falsity of the expressions with respect to the corresponding spatial configurations. In terms of the truth conditions of a projective locative expression this means that spatial configurations which are described by positive uses should belong to *definitely true* or the *indefinite* part of the extension of the expression, and those ones that are described by negative uses should belong to the *definitely false* or the *indefinite* part of the extension. In this way, the data can be used to evaluate all theories of the semantics of projective locative expressions that provide a partition of the domain into two parts (*true* / *false*) or three parts (*definitely true* / *indefinite* / *definitely false*).

Outline

Chapters 2 and 3 introduce theoretical notions relevant for the semantics of projective locative expressions and give an overview of the state of the art. Chapter 2 introduces the geometric notion of *locative direction relations*. Locative direction relations are binary relations which convey information about the location of an object relative to another object by means of a direction. This notion is a mathematical notion, and thus independent of questions of cognition or linguistics. A connection to natural language is established in Chapter 3, which provides an overview of semantic theories of projective locative expressions modelling meaning in terms of locative direction relations. In order to capture the vague nature of these meanings, underspecified representations and a procedure called *biased valuation* are suggested. **Chapter 4** describes a novel way of interpreting the formal semantic framework. The approach is implemented as a computational procedure that automatically applies semantic theories to locative projective expressions and determines their truth value with respect to geometric representations, i.e., the spatial configurations those expressions refer to. **Chapter 5** presents the preparation of the data, a set of utterances and representations of the situations those utterances refer to. The natural language data have been manually extracted from a corpus of route description dialogues and annotated with information. The corresponding spatial data have been translated into a geometric representation. The approach described in Chapter 4 is applied to the data (Chapter 5) in order to analyse different semantic theories (Chapter 2 and 3). The results are presented in **Chapter 6** and they are used to determine new semantic theories by a data-driven algorithm. More specifically, the algorithm determines semantic theories for unmodified projective locative expressions and expressions which are modified by *directly* and *slightly*, respectively. **Chapter 7** and **Chapter 8** provide summary and conclusions in English and German, respectively.

Contributions of the thesis

This thesis provides contributions to empirical linguistic research, linguistic resources, semantic theory, and qualitative spatial reasoning:

linguistic research This thesis empirically evaluates prevalent semantic theories of projective prepositions, and proposes improved and empirically motivated semantic theories for unmodified projective prepositions and prepositions modified by *slightly* and *directly*.

linguistic resources The data has been derived from the *HCRC Map Task Corpus* (Anderson et al., 1991) for which an additional annotation layer has been made publicly available. It marks all projective locative expressions which are used to describe the location of a landmark on a map relative to another landmark. The annotation layer contains semantic and pragmatic information, and, most importantly, it contains reference pointers to the landmarks the expressions refer to. Geometric representations of each map complement the annotation layer.

semantic theory This thesis brings together formal semantics and automatic interpretation of semantic representations. It introduces a formalisation of the relation between geometric aspects of actual situations and canonical models of formal semantics. A proof of concept is provided by the implementation of an algorithm that generates finite first-order logic models from geometric data.

qualitative spatial reasoning This thesis gives a comprehensive overview of locative direction relations from the disciplines linguistics, cognitive science, logic, and computer science.

Chapter 2

Locative Direction Relations

This chapter gives an overview of locative direction relations. The literature provides various techniques to define locative direction relations which will be presented in a systematical fashion and finally used to specify schemata that define locative direction relations. The relations of these systems will provide the basis for representing the semantics of projective locative expressions in Chapter 3. They are applied to geometric data in order to generate first-order logic models for such semantic representations in Chapters 4 and 5.

As I will point out in the introduction (Section 2.1), locative direction relations are defined in terms of three ingredients. First, there is a level of representation which represents the *relative position* of an object with respect to another object. Second, there are *prototypical directions*; and third, there is a classification algorithm that categorises the representations of relative position according to those prototypical directions. The sections from 2.2 to 2.5 summarise the techniques of deriving representations of relative position that we find in the literature. More specifically, Section 2.2 presents ways to represent the location of objects. Section 2.3 introduces topological relations and describes in which ways they are used in the literature to restrict the domain of locative direction relations. Section 2.4 discusses ways to determine the orientation of prototypical directions with respect to particular spatial configurations. And Section 2.5 integrates the preceding sections and describes in detail techniques from the literature to represent the location of an object relative to another object. Two different approaches to classify such representations are presented in Sections 2.6 and 2.7. The former presents continuously graded degrees of class membership and the latter discrete categories. In Section 2.8 I describe my own proposal, which combines the techniques from the literature presented in the previous sections. I present a set of relation schemata which define systems of locative direction relations. Most relations defined in the literature can be matched with one or more of these relations. Finally, the last part of this chapter (Section 2.9) describes in detail all studies about locative direction relations used in this chapter.

2.1 Introduction

Locative direction relations provide classifications of relative position in terms of prototypical directions. They specify the location of an object, the *located object* or *LO*, relative to another object, the *reference object* or *RO*, by means of a predicate that is associated with a prototypical direction. For example, let a and b be spatial objects. And let the relation symbols $ABOVE$ and $RIGHT$ be associated with the prototypical directions *up* and *right*, respectively, see Figure 2.1(a). The following predications are examples of locative direction relations:

(1) $ABOVE(a, b)$
$RIGHT(a, b)$

These predications express that a is located in direction *up* and in direction *right* from b, respectively.

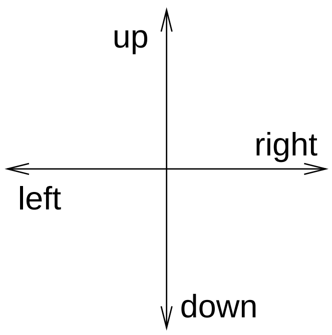
The question whether an object is located in a particular prototypical direction is a problem of classification. On the one hand, there are representations that represent the relative position of LO with respect to RO; and on the other hand, there are sets of prototypical directions. Representations and prototypical directions are set into relation by classification conditions which associate those representations with none, one or more prototypical directions.

Representations of relative position abstract particular spatial properties, and they are designed to explicitly represent features that specify directional information of LO being located with respect to RO. A very intuitive example of this level of representation can be given for a pair of spatial points. Suppose, a and b are arbitrary spatial points which do not coincide. The relative position of a with respect to b can be represented by a vector $\vec{r} = \overrightarrow{ba}$ starting in b and ending in a. Vector $\vec{r}$ abstracts over the absolute coordinates of a and b, but the directional component of $\vec{r}$ precisely specifies the direction in which a is located relative to b.[1]

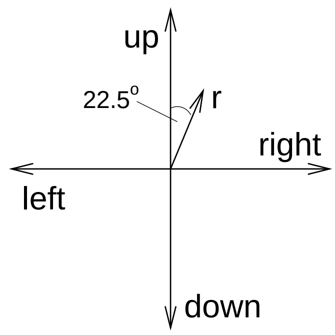
Common sets of prototypical directions are the main cardinal directions *north*, *south*, *west*, and *east*, or the set of directions *up*, *down*, *right*, *left*, *front*, *back*. Figure 2.1(a) illustrates a subset of the latter set where all four directions *up*, *down*, *right* and *left* lie in one plane.

We further need a *classification condition* – let us use the binary predicate $classify(\cdot, \cdot)$ – that indicates whether or not a representation of relative position of a with respect to b – here let us simply use pair $\langle a, b\rangle$ – belongs to the category defined by a prototypical direction. Locative direction relations $REL(a, b)$ between two objects a and b are then defined by the condition that $\langle a, b\rangle$ belongs to the category defined by the prototypical direction ρ which is associated with the predicate REL:

[1] The scope of this kind of representation is restricted to points that do not coincide, because, otherwise, the resulting vector $\overrightarrow{ba}$ could be the null vector $\vec{o}$ which does *not* have a unique directional component. Other reasons to restrict the scope of a relation will be pointed out in Section 2.3.



(a) The directions *up*, *down*, *left*, and *right*.



(b) The smallest absolute angles between a vector r and prototypical directions. The angle between r and *up* is 22.5°

Figure 2.1: An example of prototypical directions in two-dimensional space and a vector $\vec{r}$.

(2) $\forall a, b : REL(a, b) \equiv classify(\langle a, b\rangle, \rho)$

Consequently, UP, $RIGHT$, $LEFT$, and $DOWN$ are defined in general terms as follows:

(3) a. $\forall a, b : UP(a, b) \equiv classify(\langle a, b\rangle, up)$
b. $\forall a, b : RIGHT(a, b) \equiv classify(\langle a, b\rangle, right)$
c. $\forall a, b : LEFT(a, b) \equiv classify(\langle a, b\rangle, left)$
d. $\forall a, b : DOWN(a, b) \equiv classify(\langle a, b\rangle, down)$

Above we have already seen an example of a representation of relative position between points, namely, a single vector from RO to LO; and we have also seen a set of prototypical directions in two-dimensional space. Let us now look at an example of a classification predicate, $classify_\theta(\vec{r}, \rho)$, that indicates whether a vector $\vec{r}$ belongs to the category defined by a prototypical direction ρ. Let the predicate be true if the angle between $\vec{r}$ and ρ is below an arbitrary threshold θ, say, $\theta = 80°$:

(4) $classify_{80}(\vec{r}, \rho) \equiv \angle(\vec{r}, \rho) < 80°$

Figure 2.1(b) shows a vector $\vec{r}$ among four prototypical directions. The angle between $\vec{r}$ and *up* is $22.5°$. The angle between $\vec{r}$ and *right* is $67.5°$. With a threshold of $\theta = 80°$ the relative position represented by $\vec{r}$ is categorised as both *UP* and *RIGHT*, i.e., a is in direction *up* and in direction *right* from b. It is not categorised as *LEFT* and *DOWN* since the angles between $\vec{r}$ and *left* and between $\vec{r}$ and *down* are greater than $\theta = 80°$, namely $112.5°$ and $157.5°$, respectively.

Range levels The classification conditions denoted by $classify(\cdot, \cdot)$ determine the *range* of locative direction relations. The *range* of a locative direction relation is defined as the set of all pairs of objects from all possible situations that satisfy the relation. Ranges are particularly interesting for comparing locative direction relations that are associated with the same prototypical direction. In some cases the range of a locative direction relation REL_1 will be completely subsumed by the range of another locative direction relation REL_2. We will then say that the range of REL_1 is *narrower* than the range of REL_2, and conversely, the range of REL_2 is *wider* than the range of REL_1. We will also say, that REL_1 is on a lower *range level* than REL_2, and conversely, that REL_2 is on a higher *range level* than REL_1. The notion of subsumption is defined by the following implication:

(5) REL_1 is subsumed by REL_2 iff $\forall a, b : REL_1(a,b) \implies REL_2(a,b)$

In order to illustrate range levels of relations, let us define relations UP_n, $DOWN_n$, $LEFT_n$ and $RIGHT_n$ for $n \in \{1,2,3,4\}$ in terms of the classification conditions given in (3). For $n = 1$ we assume a threshold of $\theta = 10°$, for $n = 2$ the threshold is $\theta = 45°$, for $n = 3$ let $\theta = 90°$, and for $n = 4$ let $\theta = 135°$. The index n indicates the range level of the relation. It is easy to see that relations from lower range levels are subsumed by corresponding relations from a higher range levels. For example,

UP_n is subsumed by UP_m with $m, n \in \{1,2,3,4\}$ and $n \leq m$.

On the first *range level* ($n = 1$, $\theta = 10°$) the vector $\vec{r}$ in Figure 2.1(b) is not categorised at all. On the second *range level* ($n = 2$, $\theta = 45°$) the vector $\vec{r}$ is categorised as UP_2, because the angle between $\vec{r}$ and *up* is 22.5° and lower than the threshold of $45°$. On the third *range level* ($n = 3$, $\theta = 90°$) the vector $\vec{r}$ is categorised as UP_3 and $RIGHT_3$, because the angles between between $\vec{r}$ and *up* and between $\vec{r}$ and *right* are lower than the threshold of $90°$. Finally, on the fourth *range level* ($n = 4$, $\theta = 135°$) the vector $\vec{r}$ is categorised as UP_4, $RIGHT_4$, and $LEFT_4$.

In these examples, different range levels are defined by the parameter θ. Other classification conditions are parametrised in different ways, as we will see in Section 2.8. The effect, however, will be similar; locative direction relations from lower range levels imply the corresponding locative direction relations from higher range levels.

Complements and disjointness Different classification conditions and different sets of prototypical directions determine different sets of locative direction relations which we call *systems of locative direction relations*. Each such system is associated with a particular range level. Other properties of those systems that are relevant for this study are *complements* of relations and *disjointness* of relations.

This study will be limited to sets of four prototypical directions which correspond to the axes of an orthogonal coordinate system in the Euclidean plane as shown in Figure 2.1(a). Therefore, there is always exactly one inverse direction for each direction of the set. Based on this observation we can formulate the *complement property* for any system of relations used in this study:

(6) A relation REL' is the *complement* of a locative direction relation REL if and only if REL and REL' are both members of the same system of relations and the direction ρ, which is associated with REL, is inverse to the direction ρ', which is associated with REL'.

Since every direction has exactly one inverse direction, which is also in the set of prototypical directions, every locative direction relation has exactly one complement relation within a system of relations. Examples for complement pairs are $RIGHT_1$ and $LEFT_1$, and UP_3 and $DOWN_3$.

Apart from this complement property we will look at disjointness properties of systems of locative direction relations: *pairwise disjointness* or *mutual exclusion* and *complement disjointness* or *mutual exclusion of complements*.

(7) Relations from a system of relations are *pairwise disjoint* if and only if for every pair of the domain, there is at most one relation that applies to that pair.

This disjointness property is very strong since it does not allow for any overlap between the relations of a system. For example, suppose UP_3 and $LEFT_3$ are relations from a system that satisfies pairwise disjointness. In that case, there is no pair of objects a and b such that the relation between them may be characterized as UP_3 and, at the same time, as $LEFT_3$. A weaker property is *complement disjointness*:

(8) Relations from a system of binary relations are *disjoint with their complement* if and only if for every relation REL and its complement relation REL' and for every pair of the domain there is at most one relation among REL and REL' that applies to that pair.

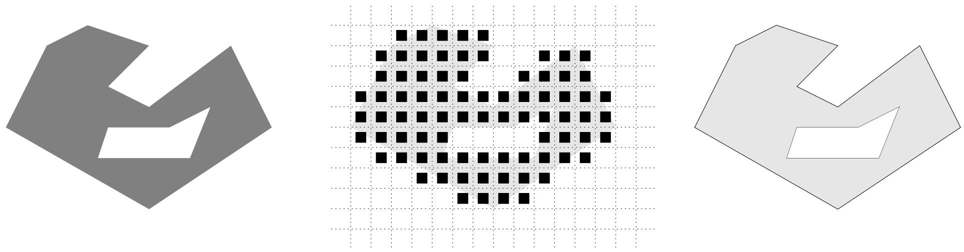
For example, let UP_1 and $DOWN_1$ be relations from a system that satisfies complement disjointness. Then there are no two objects a and b such that the relation between them may be characterized as UP_1 and, at the same time, as $DOWN_1$.

2.2 Location

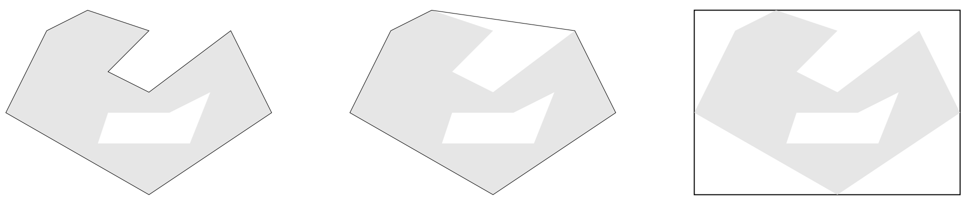
The location of an object is defined by the set of spatial points which it occupies. Let x be an object and let $\mathcal{R} \times \mathcal{R}$ denote the total set of points in two-dimensional space. The function loc yields the set of points occupied by x.

(9) $loc(x) \subset \mathcal{R} \times \mathcal{R}$

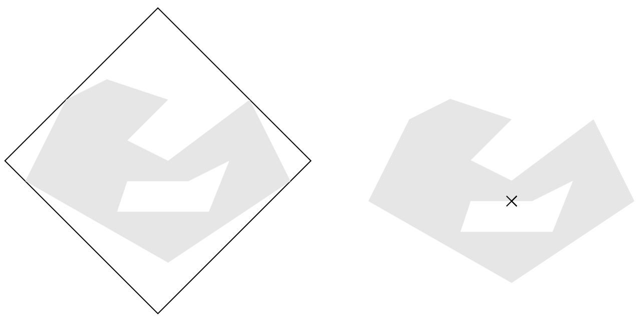
Computational theories of direction relations approximate an exact notion of location by using functions which yield a finite set of features to represent the (approximate) location of their arguments. This section gives an overview of the functions that are used in the literature.



(a) The source object, an infinite set of points, $loc(x)$.
(b) Finite set of points, $loc_{pix}(x)$.
(c) Geometric description, $loc_g(x)$



(d) Hull, $loc_{hull}(x)$.
(e) Convex hull, $loc_{convex}(x)$.
(f) Bounding box. $loc_b(x)$



(g) Diagonally aligned bounding box.
(h) Centroid. loc_{cent}

Figure 2.2: Different ways of representing location. The source object is displayed in the background of each representation in light grey.

The first panel of Figure 2.2 shows the border of some object x, a region with a partly concave boundary and a hole. The other panels illustrate representations that approximate the location of x. They correspond to the following functions described here in more detail: (i) raster scan function, (ii) complete geometric description, (iii) hull, (iv) convex hull, (v) bounding boxes, and (vi) centroids.

The location function loc_{pix} yields for each object (or region) a finite set of points where each point represents a pixel of a raster scan of that object. Panel (b) illustrates the representation $loc_{pix}(x)$ for x being the object of panel (a); each black square represents a 'pixel' that represents a part of the object. Such functions are used by (Miyajima & Ralescu, 1994) and (Matsakis & Wendling, 1999). They are also used to produce simple grid representations of spatial scenes (Varges, 2005). The location function loc_g yields a complete geometric description such that all points of x lie within the geometric object and all points that are not part of x do not lie within it. Complete geometric descriptions in two-dimensional space specify the area of an object including holes and discontinuities and preserve all geometric properties of the objects. Complete geometric descriptions are used in (Matsakis & Wendling, 1999) and (Regier & Carlson, 2001). An example of a complete geometric description is given in Figure 2.2(c). It is a complex polygon that describes all outer and inner boundaries of the object. The representation most commonly used in the literature is given by the function loc_{hull} which yields a geometric description of the object's hull or its outer boundary, see Figure 2.2(d). The hull encloses all points of x, but in contrast to complete geometric descriptions, it also encloses points that are not in x, namely, points that are part of holes in x. To be precise, $loc_{hull}(x)$ yields a geometric object which encloses all points which are either points of x or from which it is not possible to find a path to the outside of the convex hull – the notion of convex hull is specified below – without intersecting x. The following studies use hulls: (Kelleher, 2003), (Schmidtke, 2001), (Goyal, 2000), (Gapp, 1994a), (Schirra, 1993), (Wazinski, 1992), (André et al., 1987), (Herskovits, 1986). Hulls can be approximated by simple polygons with arbitrary accuracy. Closed polygons are described by a single, closed boundary which consists of straight line segments between the polygon's vertices. Closed *simple* polygons are closed polygons with no line-segments intersecting each other.

A geometric description of the object's convex hull (loc_{convex}) contains all points located on straight lines between any two points of x and nothing more. Like hulls in general, convex hulls can also be approximated by simple polygons. Zwarts & Winter (2000) represent the location of an object by convex spaces which can additionally contain points in the vicinity of an object which do not belong to the convex hull. The convex hull of the example object is depicted in Figure 2.2(e).

Another common representation format are bounding boxes (loc_b, see Figure 2.2(f)). Bounding boxes are minimal rectangles which contain all points of x such that there are no other rectangles aligned in the same way which contain all points of x and have an area smaller than the area of the bounding box. Bounding boxes are usually aligned with the vertical and horizontal axis of the coordinate system, see (Hernandez, 1994), (O'Keefe, 1996), (Papadias & Sellis, 1994), (Rajagopalan, 1993), (Topaloglou, 1994). Bounding boxes can also be aligned to axes that are determined on intrinsic features of an object, for example, Abella & Kender (1993); Fuhr et al. (1995) use bounding boxes that are aligned to the reference objects' own principal axes. In most of such cases the bounding box is not aligned with the coordinate system. Figure 2.2(g) gives an example of a bounding box that is aligned with respect to diagonal axes.

Finally, the most abstract way of approximating location is to represent it by a single point. A common method to determine such a point is to compute the centroid of an object (Hernandez, 1994), see Figure 2.2(h). Let us call the corresponding location function loc_{cent}. The (geometric) centroid coincides with the center of mass of an object under the assumption that the density of that object is homogeneous. Formulas for computing the centroid of basic two-dimensional geometric shapes, such as polygons, can be found, for instance, in (Heckbert, 1994).

2.3 Domain restrictions

Locative direction relations do not cover a domain exhaustively, there is at least one pair of objects in any domain for which there is no locative direction relation that holds between them: two objects (or regions) whose locations coincide. It would be unintuitive to define direction relations as reflexive relations, so that for example, every object is *left* of itself, and also *right*, *below* etc. It can be argued whether direction relations should be exhaustive for any two non-identical objects. But it seems to be easy to find situations where two distinct objects pose a problem similar to the previous one. For example, think of a circle that is in the center of a ring (see Figure 2.3). What is the direction of the ring with respect to the circle? Either we cannot determine a direction at all, or it lies in all relevant directions because it surrounds the circle. Most accounts of direction relations avoid such problematic cases by restricting the domain of locative direction relations to cases which seem to be unproblematic. Another rather practical reason for introducing domain restrictions is the observation that different ways of defining locative direction relations yield a different quality of results depending on the distance between located object and reference object. Some definitions might yield good results for objects that are relatively far away from each other, but their results get worse with decreasing distance. For example, (Hernandez, 1994, p.49) discusses three different types of definitions of locative direction relations, each of them is restricted to a domain defined by a condition on the distance between the located object and the reference object.

Most definitions of locative direction relations are associated with domain restrictions that specify preconditions on pairs of objects in order to determine whether a direction relation can be computed at all. Domain restrictions are specified by means of conditions on topological relations and on the distance between the located object and the reference object.

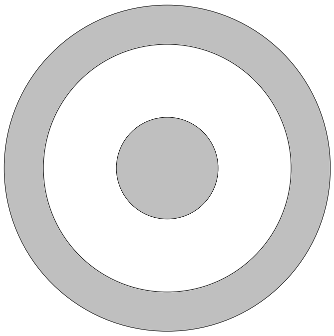


Figure 2.3: A problematic case for determining a direction relation.

Topological relations Topological relations express the quality of spatial *connectedness* between two objects. Two regions x and y are connected, $C(x, y)$, if they share at least one point p:

(10) $C(x, y) \Leftrightarrow \exists p : p \in x \wedge p \in y$

Based on this predicate of connectedness, Randell et al. (1992) define a set of topological relations on the domain of regions, henceforth, *RCC relations*. They comprise the following relations: DC(x,y) "*x is disconnected from y*", P(x,y) "*x is part of y*", PP(x,y) "*x is proper part of y*", EQ(x,y) "*x is identical with y*", DR(x,y) "*x is discrete from y*", O(x,y) "*x overlaps y*", PO(x,y) "*x partially overlaps with y*", EC(x,y) "*x is externally connected with y*", TPP(x,y) "*x is tangential proper part of y*", NTPP(x,y) "*x is nontangential proper part of y*", and the inverse relations P^{-1}(x,y), PP^{-1}(x,y), TPP^{-1}(x,y), and NTPP^{-1}(x,y). The relations are defined in the following way:

(11) Let x, y, and z be regions.

a. $DC(x, y) \Leftrightarrow \neg C(x, y)$
b. $P(x, y) \Leftrightarrow \forall z[C(z, x) \rightarrow C(z, y)]$
c. $PP(x, y) \Leftrightarrow P(x, y) \wedge \neg P(y, x)$
d. $EQ(x, y) \Leftrightarrow x = y \Leftrightarrow P(x, y) \wedge P(y, x)$
e. $O(x, y) \Leftrightarrow \exists z[P(z, x) \wedge P(z, y)]$
f. $PO(x, y) \Leftrightarrow O(x, y) \wedge \neg P(x, y) \wedge \neg P(y, x)$
g. $DR(x, y) \Leftrightarrow \neg O(x, y)$
h. $EC(x, y) \Leftrightarrow C(x, y) \wedge \neg O(x, y)$
i. $TPP(x, y) \Leftrightarrow PP(x, y) \wedge \exists z[EC(z, x) \wedge EC(z, y)]$
j. $NTPP(x, y) \Leftrightarrow PP(x, y) \wedge \neg\exists z[EC(z, x) \wedge EC(z, y)]$
k. $P^{-1}(x, y) \Leftrightarrow P(y, x)$
l. $PP^{-1}(x, y) \Leftrightarrow PP(y, x)$
m. $TPP^{-1}(x, y) \Leftrightarrow TPP(y, x)$
n. $NTPP^{-1}(x, y) \Leftrightarrow NTPP(y, x)$

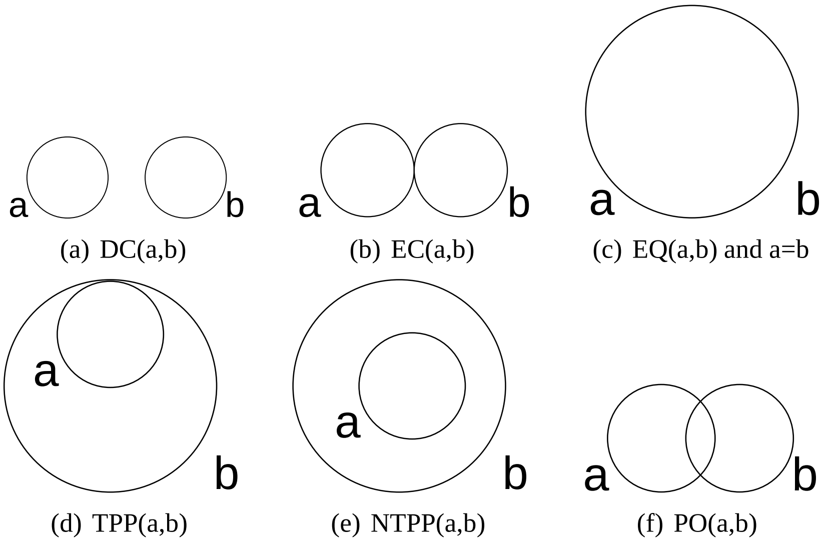


Figure 2.4: Some RCC relations.

Figure 2.4 presents six fundamental topological configurations and the corresponding RCC relations. All other RCC relations can be defined from these using the operations *union* and *inverse*. For example, the relation *x is part of y* is true if either x is tangential proper part of y, x is nontangential proper part of y, or x is identical with y, see (12-a). And the relation *x overlaps with y* is true if either x partially overlaps with y, x is part of y, or y is part of x (12-b):

(12) a. $P(x, y)$ iff $EQ(x, y) \vee NTPP(x, y) \vee TPP(x, y)$.
b. $O(x, y)$ iff $PO(x, y) \vee P(x, y) \vee P^{-1}(x, y)$.

There is a great amount of literature about topological relations and their applications, for an overview I refer the reader to Cohn & Hazarika (2001).

Domain Restrictions The majority of locative direction relations are defined on objects which are disconnected, see for example (Schmidtke, 2001, p422). Some kinds of definitions additionally presume that the objects' representations derived by location functions (see Section 2.2) are disconnected, as for example in (Zwarts & Winter, 2000), (André et al., 1987), (Wazinski, 1992), (Gapp, 1994a), and (Abella & Kender, 1993). Matsakis et al. (2001) and Papadias & Sellis (1994) explicitly discuss the effect of overlapping objects on the computation of locative direction relations. But only (Herskovits, 1986) and (Wazinski, 1992) provide an extra treatment of cases in two-dimensional space where the located object is part of the reference object. An example of such a situation is a spatial configuration where the located object is part of a picture, and its location is described with respect to that picture, like in "*The bird is at the top of the picture.*"

Even if the objects are required to be disconnected, the corresponding representations might overlap. Papadias & Theodoridis (1997) represent the location of objects by bounding boxes, and they discuss the application of locative direction relations for disconnected objects only, but the definitions of locative direction relations are also defined for overlapping bounding boxes. As mentioned earlier, (Hernandez, 1994, p49) defines three different kinds of locative direction relations with different domain restrictions. The first kind of relation is used if the objects overlap. The second kind is used if the centroid of the located object lies inside the circle with a certain radius around the reference object. Otherwise the third kind of relations is used.

2.4 Frames of reference

In the beginning of this chapter we introduced sets of prototypical directions which were needed for defining locative direction relations. This section describes common strategies to determine such sets of prototypical directions. The strategies and the corresponding set of directions are commonly called *frames of reference* or *reference frames*. The choice of a frame of reference determines the description of a spatial relation – where is an object LO located in relation to another object RO? It is now generally accepted that we need to distinguish between three types of frames of reference (Levinson, 2003): (i) *absolute*, (ii) *relative*, and (iii) *intrinsic*. In addition, the situation in which a spatial relation is described may require a three-dimensional or a two-dimensional frame. To both the two-dimensional and the three-dimensional case the distinction between the three reference frames is applicable.

The standard case of a 3-dimensional frame of reference that guides the choice of the descriptions of spatial relations between objects on earth is that where one of the three axes of the frame is the *vertical*, an axis whose direction is determined by gravity, and which is usually represented as pointing in the direction that is opposite to that of the gravitational force. The other two axes – both orthogonal to the vertical – form the horizontal plane. It is the choice of these axes that the difference between the three frames of reference shows itself. In addition to the directions of its axes a coordinate system also needs an origin. In connection with descriptions of spatial relations the origin is always centered on the reference object, or more precisely, on some point determined by the reference object, for example, its center of gravity.

In the **absolute frame of reference** directions are determined by features of the environment that contains LO and RO. For instance, the horizontal axes might be chosen according to some geographical convention such that they are aligned with the cardinal directions *north* and *east*. Cardinal directions are used in Schmidtke (2001), Papadias & Sellis (1994), Topaloglou (1994), Yamada et al. (1988).

The absolute frame of reference can also be applied to directions other than the cardinal directions if the space that contains LO and RO uniquely determines some directions. For example, in Olivier & Tsujii (1994) and Hernandez (1994) rooms have intrinsic sides such as a front, a back, a left-hand side, and a right-hand side. Spatial configurations that are located within such rooms can be described in the 'absolute' frame of reference specifying the prototypical directions *front*, *back left*, and *right* corresponding to the intrinsic sides of the room, respectively.

The **intrinsic frame of reference** is applicable if the reference object has intrinsic sides such as *bottom*, *top*, *back* and *front*, and consequently we can derive the sides *left* and *right*. These intrinsic sides are determined by features of the reference object such as shape, function, conventional use, characteristic motion, and canonical orientation. For example, the spatial scene depicted in Figure 2.5 shows a ball and a van. Since the van has an intrinsic front, the situation can be described as (13) in the intrinsic frame of reference:

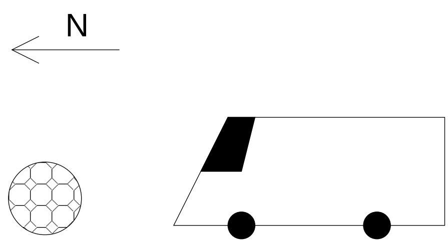


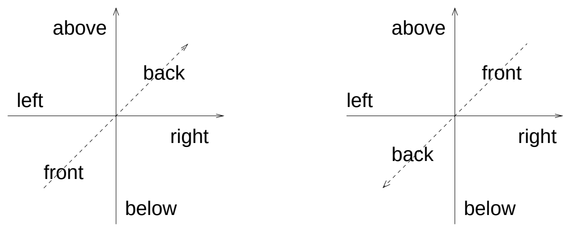
Figure 2.5: Depending on the frame of reference the ball is in front of, to the left of, or north of the van.

(13) The ball is *in front of* the van.

The meaning of the sentence is independent of the location of an observer. The intrinsic frame of reference is used in Kelleher (2003), Gapp (1994a), Olivier & Tsujii (1994), Hernandez (1994), Schirra (1993), André et al. (1987), Herskovits (1986).

The **relative frame of reference** is the result of the projection – it is a reflection to be precise – of the intrinsic orientation system of the observer as he is facing the reference object. The reference frame's direction *front* is the inverse direction with respect to the observer's direction *front*, i.e. the direction from the reference object to the observer. The other directions, *left*, *right*, *up*, and *down*, are preserved. For example, if the coordinate system in Figure 2.6(a) represents the orientation system of the observer, then Figure 2.6(b) shows the coordinate system that is imposed onto the reference object. Sentence (14) describes the location of ball in Figure 2.5 in the relative frame of reference from the perspective of the reader of this text.

(14) The ball is *to the left of* the van.



(a) Coordinate system of the observer.

(b) Coordinate system imposed onto the reference object

Figure 2.6: Coordinate systems involved in the relative frame of reference.

In three-dimensional space the relative frame of reference defines the vertical directions *up* and *down*, and the horizontal directions *right* and *left*, and *front* and *back*, see for example Kelleher (2003); Fuhr et al. (1995); Herskovits (1986).

In many situations it is only spatial relations within the horizontal plane that are of interest. In such cases a two-dimensional reference system will be all that is needed. However, since this only involves discarding the vertical axis, the options are basically, the same as above: the difference between absolute, relative and intrinsic frames is as much of an issue here as in the three-dimensional cases.

Nevertheless, the choice of a two-dimensional frame can involve options that do not arise in connection with three-dimensional frames. This concerns primarily relative frames and has to do with the position of the observer. One possibility is that the observer is a virtual observer which is assumed to belong himself to the space spanned by the two axes of the two-dimensional frame of reference. In this case the intrinsic orientation system of the virtual observer is such that the vertical axis is perpendicular to the two-dimensional plane. The projection of the intrinsic orientation system is exactly as it has been described above, but after the projection the vertical axis is discarded. This use of 2-dimensional relative frame of reference is found in Olivier & Tsujii (1994), Hernandez (1994), Schirra (1993), and André et al. (1987).

The other possibility is that the observer is assumed to be outside the space of the reference frame and is looking at this space from outside. The most natural case is that where the observer is thought as looking at the space from above, in the way we look at drawings, diagrams or maps. In this case the two-dimensional plane is best thought of as being aligned with the vertical axis, i.e. the direction of the gravitational force. The intrinsic orientation system of the observer is projected onto the two-dimensional reference object and the axis that is perpendicular to the two-dimensional plane is discarded, i.e. the axis determining the directions *front* and *back*. The direction *up*, *down*, *left*, and *right* are preserved. This use can be found in Matsakis et al. (2001), Logan & Sadler (1996), Abella & Kender (1993), Rajagopalan (1993), and Wazinski (1992).

Frame of reference conventionally used with maps With geographical maps there is yet another factor that plays a role. That is that we have the convention that maps are to be looked at in such a way that the relative vertical coincides with the geographical north. This convention is now so deeply rooted that we think of the relative vertical as pointing to the north not only in actual geographical maps but also in fictional maps such as, for instance, the maps used in the HCRC Maptask experiments which are described in Section 5.1. This makes it possible for the observers of such maps to describe an object LO that is located on the map in relation to a reference object RO as indicated above not only as "above" but also as "to the north of" the reference object.

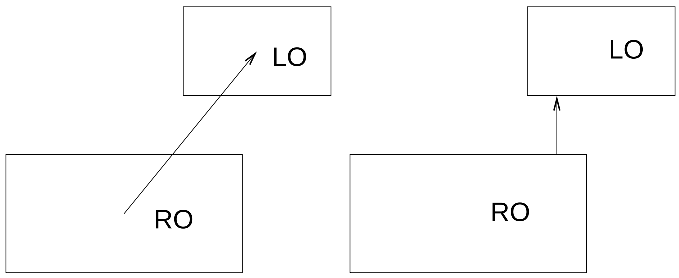
2.5 Representation of relative position

This section gives an overview of different approaches towards representing the location of an object LO relative to a reference object RO. Representations of relative position are derived from pairs ⟨LO,RO⟩ that satisfy given domain restrictions (Section 2.3). They abstract over the particular locations of LO and RO (Section 2.2) and the particular directions specified by a frame of reference (Section 2.4). Locative direction relations will be defined on the representations developed in this Section.

There are two principal kinds of representations that provide abstraction in this sense: *angular* representations and *axial* representations. Angular representations represent relative position by angles and real values associated with them expressing distance or degrees of truth. Axial representations consist of one or more axes and orthogonal projections of LO and RO onto these axes.

Angular Representations Angular representations are based on polar coordinates, which define vectors in two-dimensional space by specifying an angle ϕ and a distance r. The distinction between different representation formats introduced here is a distinction of the number of vectors and differences of the interpretation of the angular and distance components.

The simplest angular representation format used in the literature is a single vector $\vec{v} = \langle \phi, r \rangle$ which determines the position of LO relative to RO (Hernandez, 1994), (Gapp, 1994a), (Olivier & Tsujii, 1994), (Schirra, 1993), (Yamada et al., 1988), and (André et al., 1987). The computation of a vector between two objects is trivial if the objects are points. In that case, there is a unique vector from RO to LO. For locations that are represented by spatially extended representations Regier (1992) describes two basic ways for determining a single vector that represents the relative position between spatially extended objects: *proximal* and *center-of-mass* directions. The *proximal direction* of LO with respect to RO is given by the minimal vector, i.e. the vector with minimal length, from all vectors connecting a point from RO with a point from LO. The *center-of-mass direction* of LO with respect to RO is determined by the vector from the centroid of RO ($loc_{cent}(RO)$) to the centroid of LO ($loc_{cent}(LO)$). Examples are presented in Figure 2.11 (a) and (b).



(a) Center of mass direction. (b) Proximal direction.

Figure 2.7: Vectors between spatially extended objects.

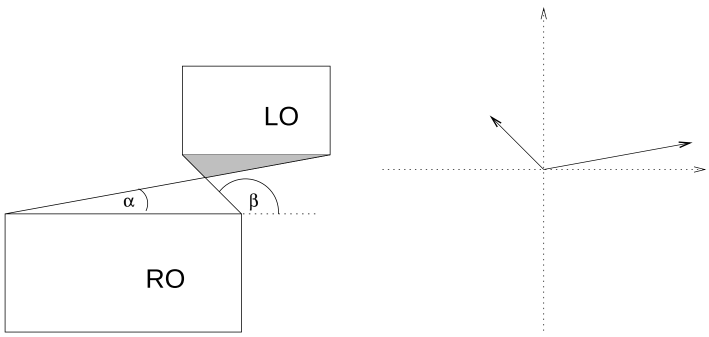
Other representation formats use multiple vectors or multiple angles to represent directions between locations with spatial extension. The representations used in Schmidtke (2001) are equivalent to angle intervals which span over the directions of all possible straight connections from RO to LO. For example, the direction of the rectangle LO in Figure 2.8 with respect to the rectangle RO is specified by the angle interval $[\alpha, \beta]$ measured counter-clockwise from the x-axis of the coordinate system. α is the smallest possible angle between the x-axis and any vector connecting an arbitrary point of RO with an arbitrary point of LO. Similarly, β is the greatest possible angle. The angle interval $[\alpha, \beta]$ contains the directions of all possible vectors from RO to LO. Therefore, an angle interval represent the sector that is occupied by the LO from the perspective of the RO.

Multiple vectors that specify directions and distances are used by (Wazinski, 1992) and (Kelleher, 2003). Wazinski uses a tuple of 9 vectors to represent the centroids of 9 parts of the LO. Kelleher (2003) represents relative position by a list of vectors from the centroid of the RO to each of the vertices of the polygon representing the hull of the LO, see Figure 2.9.

Fuhr et al. (1995) represent relative position by a *grid model* based on angular deviation. *Grid models* partition space into a finite array of cells which is centered on the RO. Each cell is associated with the proportion of LO overlapping with that cell. The proportion of a cell is computed as follows. Let the function $|\cdot|$ return the area (or volume in 3D space) of its argument.

(15)

$$p_{cell}(LO) = \frac{|LO \cap cell|}{|LO|}$$



(a) α is the minimal angle; β is the maximum angle.

(b) The vectors defining the angle interval placed in the coordinate system.

Figure 2.8: Angle interval representing the relative position of LO with respect to RO.

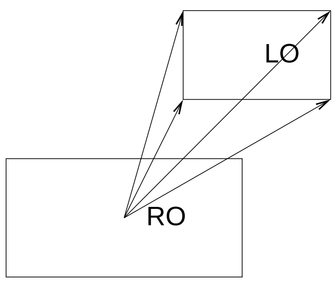
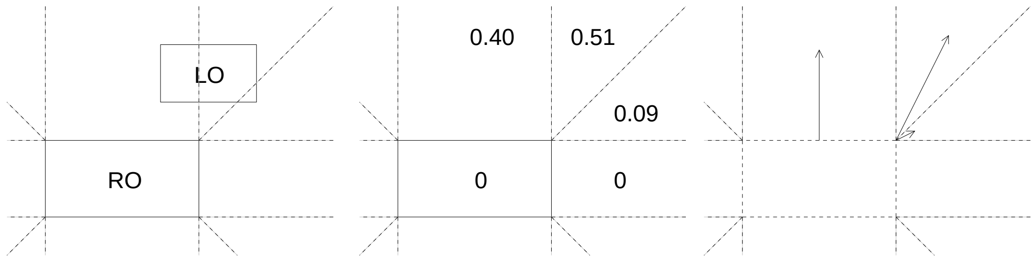


Figure 2.9: Vectors from the centroid of the RO to the vertices of the hull of the LO represent the direction LO and RO.

The numerator is the area (volume) of the LO overlapping with the cell. The denominator is the area (volume) of the whole LO. The proportions of all cells sum up to 1. The angular deviation grid from Fuhr et al. (1995) applied to two-dimensional space partitions space around the bounding box of the RO in rectangles and 45° sectors as shown in Figure 2.10 (a). The proportions according to (15) are shown in (b). Every region (apart from the center region) is associated with a direction representing the average direction of that region. The proportions can be stored as a tuple of vectors combining average direction of the sector and proportion as length. Figure (c) shows three relevant vectors representing (a): $\langle 90°, 0.40\rangle$, $\langle 67.5°, 0.51\rangle$, and $\langle 22.5°, 0.09\rangle$.



(a) LO and RO.

(b) The proportion of LO in each cell.

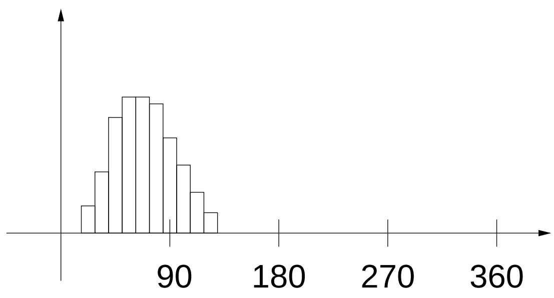
(c) Vector representations consisting of average direction and proportion.

Figure 2.10: Angular deviation grid representing relative position of LO with respect to RO.

Matsakis & Wendling (1999) represent relative position by a histogram of angles[2]. Each bar in the histogram associates an angle α with a certain weight representing the weight of the proposition "LO is in direction α relative to RO". The details of the computation of such an angle histogram from a given spatial configuration are described in Section 2.9.4. The histogram shown in Figure 2.11(a) represents the location of the rectangle LO relative to the rectangle RO from previous examples, e.g. Figure 2.10.

Based on angle histograms Matsakis et al. (2001) further describe a procedure to derive a single vector $\vec{r}$ which represents the direction of LO from the reference object RO *and* relative to some prototypical direction ρ. Matsakis et al. (2001) say that this vector indicates the "*average direction*" of the *angle histogram*, but note, that $\vec{r}$ is dependent on the prototypical direction ρ, that means, that the corresponding vectors $\vec{r}_\rho$ are distinct for ρ=*up*, ρ=*down*, ρ=*left*, and ρ=*right*. The details of the computation of this vector can be found in Section 2.9.4.

[2]Matsakis & Wendling (1999) call it histogram of forces, since the histogram expresses some kind of 'gravitational' force exerted by LO on RO in the direction of each angle.



(a) Angle histogram.

Figure 2.11: Angle histogram representing the direction of LO with respect to RO.

Similarly, Regier & Carlson (2001) present a procedure to derive a single vector, which they call the *attentional vector sum* or short AVS, that represents relative position of LO w.r.t. RO in a way that is dependent on some prototypical direction ρ. Due to this dependency the AVS can be seen as representing the deviation of LO from ρ from vantage point RO. The *attentional vector sum* is a vector that is an average weighted sum of all vectors pointing from each point of the RO to the LO. The details of the computation are described in Section 2.9.2. Important for us at this point is that the computation of the AVS is constrained in the following way: The direction of the AVS always lies between the *center-of-mass* direction, i.e. the direction of the vector from the centroid of the RO to the centroid of the LO, and the direction of a certain vector $\vec{v}$ connecting RO and LO and minimizing the angle between $\vec{v}$ and the prototypical direction in question.

For example, Figure 2.12 illustrates the difference between the AVSs corresponding to two distinct prototypical directions (a) *up* and (b) *right*. Both AVSs are supposed to represent the relative direction of the black circle with respect to the rectangle. The grey sectors mark the range of possible directions of the AVS. For (a) *up* possible directions of the AVS are constrained by the direction *up* and by the *center-of-mass* direction *c*. For (b), *right*, possible directions of the AVS are also constrained by the *center-of-mass* direction *c*. The other constraint, however, is given by the vector *a* from the top left corner of the rectangle to the black circle. The difference between the constraints for *up* and for *right* in this example is obvious. The constraints for *right* subsume the constraints for *up*. AVSs for this example associated with *right* can point (i) up and to the left, (ii) up, or (iii) up and to the right. AVSs associated with *up* can only point (i) up and (ii) to the left and up.

Axial Representations Axial representations reduce the complexity of two- or three-dimensional space to a set of one-dimensional representations. An object is projected onto the axes of the coordinate system that is specified by the frame of reference. For each axis this projection yields an interval which represents the object's extension with respect to the axis' direction. Axial representations are used by Goyal (2000), O'Keefe

(a) Range of possible vectors for the prototypical direction *up*.

(b) Range of possible vectors for the prototypical direction *right*.

Figure 2.12: A single vector represents relative position with respect to a given prototypical direction in the attentional vector sum model (Regier & Carlson, 2001).

(1996), Papadias & Sellis (1994), Topaloglou (1994), Abella & Kender (1993), and Rajagopalan (1993). A related approach is taken by (Mukerjee & Joe, 1990) to represent the relative position between objects with an intrinsic front.

For two-dimensional space, axial representations of the location of an object relative to another object require the representation of four intervals in the set of real numbers $\mathcal{R}$. Let the functions $x(\cdot)$ and $y(\cdot)$ yield intervals that are the result of an orthogonal projection of the argument (a region) onto the horizontal and vertical axis, respectively, and let the functions $x_{min}(\cdot)$, $x_{max}(\cdot)$, $y_{min}(\cdot)$, and $y_{max}(\cdot)$ return the corresponding interval boundaries. An example is shown in Figure 2.13.

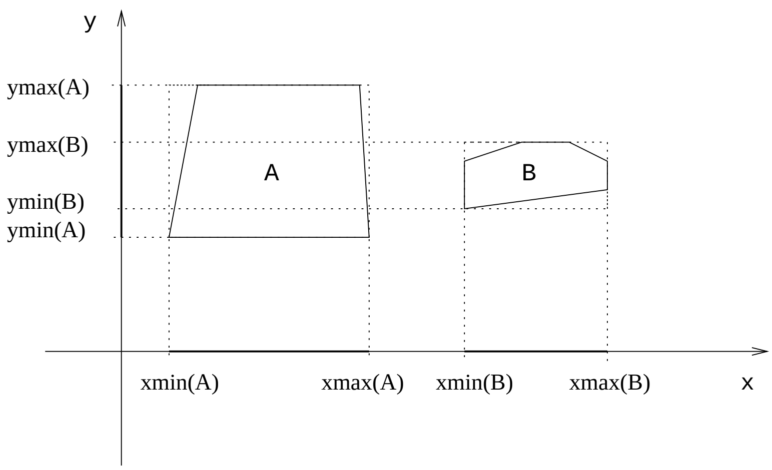


Figure 2.13: Orthogonal projection.

This information provides us with qualitative information about the relation between the intervals and with quantitative information about horizontal and vertical extension of the objects and horizontal and vertical distance between them.

If only qualitative information is needed to compute a direction relation, we can further reduce the representation to a pair of interval relations, as they are defined by Allen (1983). Allen defines a set of 13 relations on intervals over the domain of time. They are illustrated in Figure 2.14. Let t and s be intervals, the indices min and max denote the lower and upper interval boundaries, respectively.

(16)
a. t<s $\equiv t_{max} < s_{min}$
b. t=s $\equiv (t_{min} = s_{min}) \wedge (t_{max} = s_{max})$
c. t overlaps s $\equiv (t_{min} < s_{min}) \wedge (t_{max} > s_{min}) \wedge (t_{max} < s_{max})$
d. t meets s $\equiv t_{max} = s_{min}$
e. t starts s $\equiv (t_{min} = s_{min} \wedge t_{max} < s_{max})$
f. t finishes s $\equiv (t_{min} < s_{min} \wedge t_{max} = s_{max})$
g. t during s $\equiv ((t_{min} > s_{min}) \wedge (t_{max} =< s_{max})) \vee ((t_{min} >= s_{min}) \wedge (t_{max} < s_{max}))$
h. t during^{-1}s $\equiv$ s during t
i. t starts^{-1}s $\equiv$ s starts t
j. t finishes^{-1}s $\equiv$ s finishes t
k. t meets^{-1}s $\equiv$ s meets t
l. t overlaps^{-1}s $\equiv$ s overlaps t
m. t>s $\equiv$ s<t

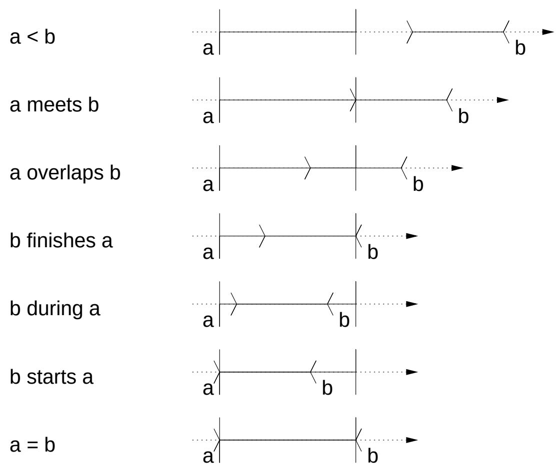


Figure 2.14: Allen's interval relations

The relations are jointly exhaustive and pairwise disjoint. Therefore, we can determine a unique relation between any two intervals.

The interval relations that indicate coincidence of interval boundaries (*viz* '=', 'meets', 'starts', 'finishes', and their inverse relations) are dependent on the conditions that define *spatial coincidence*. In the context of this study, i.e., people using locative expressions to describe a spatial configuration which they see, it is sensible to think of *spatial coincidence* as *approximate spatial coincidence* rather than mathematically precise spatial coincidence. Topaloglou (1994) redefines the identity relation '=' and the precedence relation '$<$' in order to reason with interval relations and a notion of approximate spatial coincidence. The identity relation is replaced by a vicinity predicate which is true for any two points whose distance is below some threshold. The precedence relation is adjusted to the vicinity predicate: it is the truth value of the original '$<$' if the objects are not in each others vicinity, otherwise it is false. The effect of the redefinition can be illustrated graphically by assuming extended interval boundaries for one of the reference intervals. Extended interval boundaries are intervals themselves, and a point p is in the vicinity of an extended interval boundary if p is part of it. Substituting the identity predicate for the vicinity predicate yields approximate interval relations. For example, let ϵ be the distance that is added to and subtracted from the original interval boundary to obtain the boundaries of the extended interval boundary, so that the boundary t_{max} is substituted for the interval $[t_{max} - \epsilon; t_{max} + \epsilon]$ and the condition is changed from identity "=" to the topological *part-of* relation "$P(\cdot, \cdot)$". For example, the definition for the interval relation $meets$ is changed to the following definition:

(17) t meets $s \equiv P(s_{min}, [t_{max} - \epsilon; t_{max} + \epsilon])$

Figure 2.15 shows an example of ***a meets b*** redefined. The interval boundaries of a are substituted for intervals which are represented by grey areas. ***a meets b*** because the lower interval boundary of b is part of the upper extended interval boundary of a.

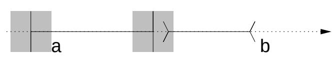


Figure 2.15: Extended interval boundaries for approximate representation.

Orthogonal grid models *Orthogonal grid models*, like angular deviation grid models presented above, divide the space around the RO into cells. Each cell, $cell$, is associated with the proportion $p_{cell}(LO)$ of LO overlapping with that cell. The following definition is a repetition of (15). Let the function $|\cdot|$ return the area of its argument. The proportion $p_{cell}(LO)$ is computed as follows:

(18)

$$p_{cell}(LO) = \frac{|LO \cap cell|}{|LO|}$$

The numerator is the area of the LO overlapping with the cell. The denominator is the area of the whole LO. The values of all cells sum up to the value 1.

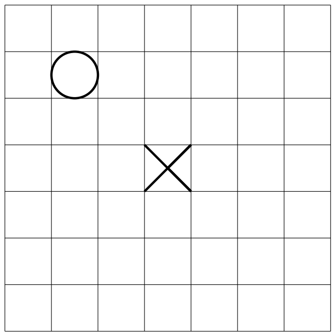


Figure 2.16: A 7×7 grid imposed over a spatial configuration consisting of a cross and a circle. The grid is centered on the cross.

Logan & Sadler (1996) use a 7×7 grid shown in 2.16 which is imposed over a two-dimensional spatial scene centered on the RO. Since Logan & Sadler only consider objects that are part of a single cell, the situation can be represented by a 7×7 matrix where the element representing the cell with the LO has the value one, and all other elements are zero. The spatial scene depicted in the figure is represented by a 7×7 matrix with cell (2,2) set to the value 1:

(19) $$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A treatment of located objects that overlap with more than one cell is proposed by Wazinski (1992) and Goyal (2000) who introduce orthogonal 3×3 grids. The proportions are represented by a 3×3 matrix. The potential of the orthogonal grid model to convey directional information is discussed in great detail by Goyal (2000). Figure 2.17 shows an example of such a grid imposed over a spatial configuration.

The proportions of LO overlapping with each cell is represented by the following matrix:

(20) $$\begin{bmatrix} 0.0 & 0.2 & 0.7 \\ 0.0 & 0.0 & 0.1 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}$$

Let us for convenience label the regions of the 3×3 grid as shown in Figure 2.18. The rectangle in the center is associated with the label RO, because it is the

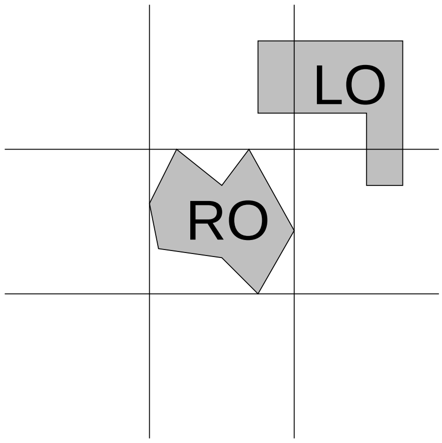


Figure 2.17: 3×3 orthogonal grid defined by the bounding box of the RO.

bounding box of the reference object. The other eight regions are labelled according to the cardinal directions *north* (N), *south* (S), *west* (W), *east* (E), *northwest* (NW), *northeast* (NE), *southwest* (SW), and *southeast* (SE).

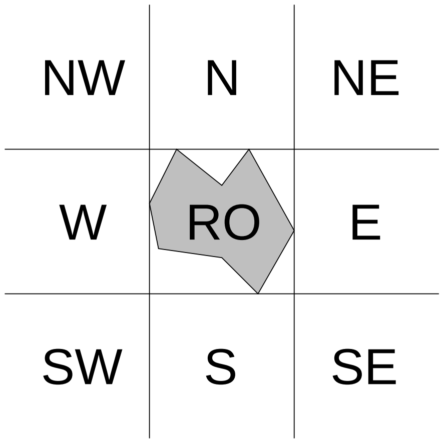


Figure 2.18: 3×3 grid defined by the bounding box of the RO.

Axial representations with two orthogonal axes are equivalent to representations that express relative position by a pair of bounding boxes that are aligned to these axes. The relative position of the LO with respect to the RO can be expressed by topological relations between the LO and the regions of an orthogonal grid defined by extending the sides of the bounding box of the RO to infinite straight lines as shown in Figure 2.18.

Symmetry between locative direction relations We have so far looked at different ways of characterising the location of LO relative to RO. As we said earlier in this chapter, locative direction relations will be defined as conditions which categorise such representations of relative position into categories determined by a given set of prototypical directions. Before we will look at particular classification conditions in the next sections I would like to point out that all locative direction relations which will be defined later are symmetric in a certain way. This symmetry property is derived from the symmetry properties of the sets of prototypical directions which we are using to define the relations. The classical example is the set of the four main cardinal directions *north*, *west*, *south*, and *east* corresponding to the locative direction relations *to the north/west/south/east of*. The set consists of four orthogonal vectors which we may label for convenience N, W, S, E. Suppose that the first relation, *to the north of* is defined in terms of theories for N by a definition D(LO,RO,N) – that is, LO counts as *to the north of* RO iff D(LO,RO,N). Then the three other relations *to the east / south / west of* are defined by D(LO,RO,E), D(LO,RO,S), and D(LO,RO,W), respectively. We use the symmetry property in the following way. Suppose, *to the north of* is given (i.e. we can tell for any pair ⟨LO,RO⟩ whether the relation between LO and RO holds). Then we can determine for any pair ⟨LO,RO⟩ whether LO is, say, *to the east of* RO by rotating the plane (with LO and RO) anticlockwise over 90°, so that the vector E coincides with the old position of N. If the new images LO' and RO' of LO and RO are such that LO' is *to the north of* RO', then and only then is LO *to the east of* RO.

Locative direction relation sets satisfying such symmetry conditions simplify the computation of the relation between LO and RO. We need explicit computation for only one of the relations (e.g. *to the north of*). All other relations can then be computed by carrying out simple transformations to reduce the problem to this one case.

Technically, angular representations are adjusted to a particular prototypical direction by computing the difference between the angles of the representation of relative position and the prototypical direction.

Axial representations are adjusted to a particular prototypical direction by inverting and swapping axes. For example, in order to rotate an axial representation anticlockwise for 90°, the interval of the horizontal axis is mapped onto the vertical axis, and the inverted interval of the vertical axis is mapped onto the horizontal axis:

(21)
$$\begin{aligned} x'_{min}(a) &:= -y_{max}(a), \\ x'_{max}(a) &:= -y_{min}(a), \\ y'_{min}(a) &:= x_{min}(a), \\ y'_{max}(a) &:= x_{max}(a) \end{aligned}$$

For 3×3 grid representation a ninety degree anti-clockwise rotation is defined by the following mapping. The number associated with cell N is mapped onto cell W, and so on:

(22)
$$N \mapsto W', W \mapsto S', S \mapsto E', E \mapsto N', NW \mapsto SW', NE \mapsto NW', SW \mapsto SE', SE \mapsto NE', RO \mapsto RO'$$

2.6 Degree of applicability

There are many ways to compute an answer to the question whether a particular locative direction relation applies to a spatial configuration consisting of a reference object and a located object. A general approach is to compute a real number which specifies a *degree of applicability* of a locative direction relation applied to the LO and the RO. The general notation of the degree of applicability – whatever the actual representation format of relative position may be – is the applicability function $a(\cdot)$ applied to the triple consisting of LO, RO, and the locative direction relation REL which is associated with the prototypical direction ρ:

(23) $a(\langle LO, RO, REL\rangle)$

We may also sometimes use:

(24) $a(\langle LO, RO, \rho\rangle)$

This section introduces basic functions from representations of relative position to real numbers. They are defined on basic parameters of the representations like angles and distances, and they offer different kinds of interpretation: *applicability functions* express a degree to which a relation applies; higher degrees indicate better fit. *Cost functions* yield a degree indicating the deviation of the representation from optimally fitting a particular relation. Lower degrees indicate better fit, higher degrees indicate a greater deviation from optimal fit.

Binary applicability functions *Binary applicability functions* implement a classical set membership function ranging over the set {0,1} defining the extension of a locative direction relation. If a representation is mapped onto the value 1, then the relation holds of the objects being represented. Otherwise, if it is 0, then the relation does not hold. Binary applicability functions are defined via acceptance intervals or acceptance areas A which determine points that belong to the extension of the direction relation. Examples for acceptance intervals for angular representations are [-90,90], [-45,45], [-22.5, 22.5], and [0,0]. For axial representations using the orthogonal 3×3 grid common acceptance areas are the region N and the composite region $N \cup NW \cup NE$.

In case the representation of relative position of $\langle LO, RO\rangle$ is given by a point p, the degree of applicability $a(\langle LO, RO, REL\rangle)$ is 1 if the point p lies within the acceptance interval or acceptance region A that is determined by RO and REL, otherwise it is 0.

(25) $a(p) := \begin{cases} 1; & \text{if } p \in A \\ 0; & \text{otherwise} \end{cases}$

In case relative position of $\langle LO, RO\rangle$ is represented by a region (or interval) R, then the degree of applicability is determined by topological relations such as *part-of* (P) and *overlap* (O) (or corresponding interval relations). Let $Topo$ be such a topological relation, then $a(\langle LO, RO, REL \rangle)$ yields 1, if R is in relation $Topo$ to the acceptance region (or interval) A that is determined by RO and REL, otherwise it is false:

(26) $$a(R) := \begin{cases} 1; & \text{if } Topo(R, A) \\ 0; & \text{otherwise} \end{cases}$$

For example, let us define an applicability function $a_N^O(\cdot)$ that is based on the topological relation *overlap* and the acceptance region N, and another applicability function $a_N^P(\cdot)$ that is based on the topological relation *part-of* and the acceptance region N.

(27) a. $$a_N^O(R) := \begin{cases} 1; & \text{if } O(R, N) \\ 0; & \text{otherwise} \end{cases}$$

b. $$a_N^P(R) := \begin{cases} 1; & \text{if } P(R, N) \\ 0; & \text{otherwise} \end{cases}$$

Figure 2.19 shows two regions LO and RO, their bounding boxes $loc_b(LO)$ and $loc_b(RO)$, and the 3×3-grid determined by the bounding box of the RO. Since the

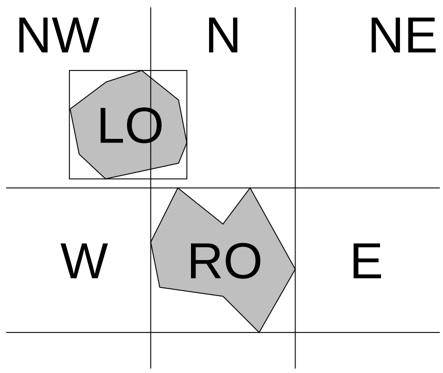


Figure 2.19: The bounding box of LO overlaps with the region N in the 3×3 grid around RO.

bounding box of LO overlaps with the N region, $a_N^O(loc_b(LO))$ yields the value 1. And $a_N^P(LO)$ yields 0, because the bounding box of LO is not part of the N region. Locative direction relations that are defined by means of binary applicability functions are presented in (Schmidtke, 2001), (Zwarts, 1997), (O'Keefe, 1996), (Papadias & Sellis, 1994), (Hernandez, 1994), (Topaloglou, 1994), (Rajagopalan, 1993), and (André et al., 1987).

Graded applicability functions *Graded applicability functions* return a degree of applicability for a given pair of a relation and a representation of relative position. They return values on a scale that may either be discrete or continuous. The lowest value indicates no applicability, the highest value full applicability. Other values from the scale indicate intermediate degrees of applicability. For example, Logan & Sadler

(1996) use functions that yield real numbers between 1 and 9. If the range of the applicability function is restricted to real numbers from the interval $[0, 1]$, then we speak of *fuzzy applicability functions*.

Fuzzy applicability functions based on angular representations determine a degree of applicability with respect to angular deviation α from a prototypical direction ρ. This means that $a(\langle LO, RO, \rho\rangle)$ is defined as a function $a(\alpha)$. A function used by (Kelleher, 2003), (Fuhr et al., 1995), and (Matsakis & Wendling, 1999) is the following triangular fuzzy membership function; the graph is plotted in Figure 2.20:

(28) $$a(\alpha) := \begin{cases} 1 - \frac{|\alpha|}{\alpha_{\max}}; & \text{if } |\alpha| < \alpha_{\max} \\ 0; & \text{otherwise} \end{cases}$$

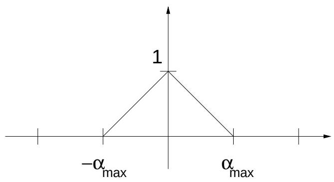


Figure 2.20: triangular fuzzy membership function

This function satisfies the following properties: An angular deviation of $0°$ scores a degree of applicability of 1, and deviations of some maximum deviation α_{max} and higher yield a degree of 0. Fuzzy applicability functions are zero below $-\alpha_{\max}$ and above $\alpha_{\max}$. Their global maximum is at $\alpha = 0$ with a value of 1. They are strictly monotonic increasing over $[-\alpha_{\max}, 0]$ and strictly monotonic decreasing over $[0, \alpha_{\max}]$.

Gapp (1994a) uses a non-linear function that satisfies the properties specified above. A more general formulation of an applicability function is provided in Regier & Carlson (2001): $a(\alpha) = slope * |\alpha| + c$ where α is the angular deviation, and $slope$ and c are free parameters which make it possible to set the function's range arbitrarily.

Fuzzy applicability functions on distances indicate the degree of applicability with respect to a distance d. The degree decreases with increasing distance and is zero if the distance is greater than some maximum distance $d_{\max}$. The following function is found in Kelleher (2003):

(29) $$a(d) := \begin{cases} 1 - \frac{d}{d_{\max}}; & \text{if } d < d_{\max} \\ 0; & \text{otherwise} \end{cases}$$

Gapp (1994a), Schirra & Stopp (1993) and Wazinski (1992) formulate constraints on distance by defining functions similar to the one in (29). The overall degree of applicability is the product of the degrees computed from angle α and distance d derived from the triple $\langle LO, RO, \rho\rangle$:

(30) $$a(\alpha, d) := a(\alpha) \cdot a(d)$$

Cost functions Cost functions range over positive real numbers including zero. The value zero indicates optimal applicability of the relation, increasing values express less good applicability. Yamada et al. (1988) and Olivier & Tsujii (1994) use cost functions to find the optimal location for a new object given a locative expression. The optimal location is specified by the global minimum of the cost functions.

(Yamada et al., 1988) and (Olivier & Tsujii, 1994) use variants of the following cost function. Let d_{opt} be some arbitrarily chosen optimal distance, and K_d and K_a be arbitrary positive real values. The variable α denotes the angular deviation and d denotes the distance derived from $\langle LO, RO, \rho \rangle$. The overall cost is the sum of the cost of direction and the cost of distance.

(31) $p(\alpha, d) := p(d) + p(\alpha)$

The cost of direction is defined as follows:

(32) $p(\alpha) := \begin{cases} K_a \sin^2(\alpha); & \text{if } |\alpha| \leq 90° \\ undefined; & \text{otherwise} \end{cases}$

The term $p(\alpha)$ is minimal if the direction is aligned with the prototypical direction (i.e., $\alpha = 0$), and it increases monotonically with increasing deviation from the prototypical direction (i.e., $|\alpha| > 0$). The cost is maximal for $\alpha = 90°$.

The cost of distance is defined as follows:

(33) $p(d) := K_d(d - d_{opt})^2$

The term $p(d)$ is minimal if the distance d is equal to some optimal distance d_{opt}. It increases with increasing deviation from that optimal distance.

Degree of applicability for grid models. Grid models partition space into cells. The location of LO relative to RO is represented by means of weights that indicate the proportion of LO overlapping with each of these cells (see Section 2.5). Let p_i be the weight that is associated with cell i. For every grid model there is another grid (with the same cells) which determines a weight w_i for each cell i.[3] Particular assignments of the cells can in principle be determined freely; Logan & Sadler (1996) for example determine the degrees of applicability by empirical studies, Fuhr et al. (1995) use a graded applicability function that is applied to the angle associated with each of the cells in the angular deviation grid. The total degree of applicability for a certain grid model gm determined by $\langle LO, RO, REL \rangle$ is the sum over the products of weight and proportion of all cells:

(34)

$$a(gm) = \sum_i p_i * w_i$$

[3] Logan & Sadler (1996) use the term *spatial template*.

2.7 Classification

This section describes discrete classification of spatial scenes with respect to locative direction relations. It addresses the question whether or not a locative direction relation applies to a spatial configuration consisting of a LO and a RO. Classification of locative direction relations is dependent on the degree of applicability $a(\langle LO, RO, REL\rangle)$ of a relation REL with respect to LO and RO.

Degrees of applicability are real values on a linear scale where higher values imply better applicability. We can define intervals on the applicability scale and associate them with relations. A LO is in relation REL to RO if and only if the corresponding degree of applicability is part of the interval that is associated with REL:

(35) $REL(LO, RO) \Leftrightarrow_{def} a(\langle LO, RO, REL\rangle) \in I_{REL}$

Intervals can be closed intervals $[x, y]$ where the interval boundaries belong to the interval, open intervals $]x, y[$ where the interval boundaries do not belong to the interval, and half-open intervals $[x, y[$ and $]x, y]$. Intervals can also be singleton intervals $[x, x]$ where upper and lower boundary coincide. A relation REL_1 implies a relation REL_2 if and only if the interval associated with the first relation, I_{REL_1}, is part of the interval that is associated with the second relation I_{REL_2}, in other words, every value that lies in I_{REL_1} also lies in I_{REL_2}:

(36) $REL_1(LO, RO) \rightarrow REL_2(LO, RO) \Leftrightarrow_{def} P(I_{REL_1}, I_{REL_2})$

Relations that are based on binary applicability functions can be defined by means of intervals that contain the number 1 but not the number 0. Since these intervals contain the number 1, the above condition yields true if the binary condition of the applicability function is satisfied. Otherwise – if it is 0 – the above condition yields false.

The relations presented in (Abella, 1995) are based on fuzzy applicability functions. They are defined by means of the intervals $[\frac{2}{3}, 1]$ which is associated with the truth value *definitely true* and $[0, \frac{1}{3}]$ which is associated with *definitely false*. Some studies use fuzzy applicability functions to define direction relations in a relative way; either to find the relation which fits a particular pair of LO and RO best (Gapp, 1994a; Wazinski, 1992), or to find an object which best fits a given relation and RO (Kelleher, 2003). I assume that such a relative definition of relations still implies the following absolute properties. A degree of applicability of 0 indicates that the relation is not applicable, and all other values in the half-open interval $]0, 1]$ indicate applicability to some extent. That means, that all values greater than zero indicate that a locative direction relation is applicable to a pair of LO and RO. In cases like this, and in other cases where the applicability scale is simply divided into two intervals, one interval is associated with a direction relation and the other is associated with its negation. Instead of using an interval we can equivalently define these relations by a threshold θ and an equality or inequality condition, like one of the following three definitions:

(37) a. $REL(LO, RO) \Leftrightarrow_{def} a(\langle LO, RO, REL\rangle) = \theta$
b. $REL(LO, RO) \Leftrightarrow_{def} a(\langle LO, RO, REL\rangle) \geq \theta$
c. $REL(LO, RO) \Leftrightarrow_{def} a(\langle LO, RO, REL\rangle) > \theta$

Matsakis et al. (2001) propose an approach to generate modified projective locative expressions consisting of at most one modifier and a projective preposition. Pairs of modifiers and prepositions directly correspond to locative direction relations that are defined in the following way. As said earlier on page 34 they represent relative position by angle histograms. On the basis of such angle histograms, two fuzzy parameters $a_I(\langle LO, RO, REL\rangle)$ and $a_{II}(\langle LO, RO, REL\rangle)$ are computed. Relations are associated with two acceptance intervals A_I and A_{II}, one interval for each parameter. Spatial relations $REL(LO, RO)$ are defined by the following condition:

(38) $REL(LO, RO) \Leftrightarrow_{def}$
$a_I(\langle LO, RO, REL\rangle) \in A_I \wedge a_{II}(\langle LO, RO, REL\rangle) \in A_{II}$

2.8 Relation schemata

This section describes a number of representative relation schemata that define locative direction relations. These relation schemata present an effort to systematically apply the techniques that have been presented in this Chapter so far. Most relation schemata provide locative direction relations which directly correspond to relations defined in the literature. However, some relations from the literature are just modelled approximately, and some are not modelled at all. For each relation schema I will point out which of the relations it defines match with relations from the literature.

Each relation schema defines systems of relations on different range levels (see Section 2.1). Every system of relations consists of four locative direction relations corresponding to the main cardinal directions *north*, *south*, *east*, and *west*. The relation schemata will be constructed in such a way that the relations that are associated with the same prototypical direction are linearly ordered according to their range level: Given two relations R and S that are both associated with the same prototypical direction and that are both defined by the same relation schema, if R is from a lower range level than S then R is subsumed by S:

(39) $\forall x, y[R(x, y) \implies S(x, y)]$

Otherwise, S is on a higher range level than R or on the same level, and then S is subsumed by R:

(40) $\forall x, y[S(x, y) \implies R(x, y)]$

The systems of relations defined by the relation schemata can have the following inferential properties: (i) *pairwise disjointness*, i.e. mutual exclusion of all relations (see (7) on page 21), and (ii) *complement disjointness*, i.e., a relation excludes its complement but not the other relations (see (8) on page 21).

The relation schemata described below will explicitly define relations for *north* only. The other relations corresponding to the directions *south*, *east* and *west* are derived by applying simple rotations described at the end of Section 2.5 on page 39.

2.8.1 Orthogonal projection schemata

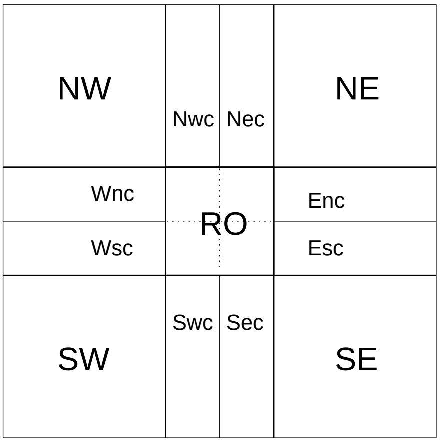
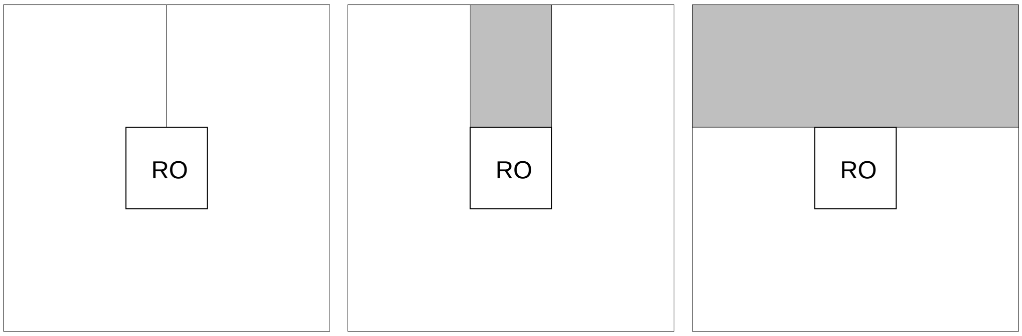


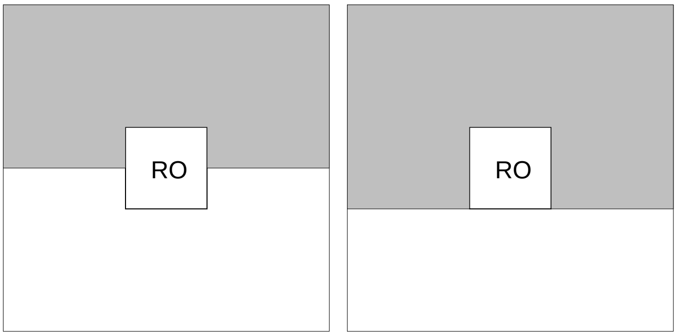
Figure 2.21: An orthogonal projection grid that partitions space into 12 regions.

Orthogonal projection relation schemata are based on axial representations. We specify an orthogonal projection grid by means of the bounding box of the reference object and straight lines through the center of the bounding box as shown in Figure 2.21. The grid refines the 9-region model presented earlier in Section 2.5 Figure 2.18. The four center regions are the region of the RO's bounding box – simply labelled with *RO*. The diagonal regions *NW, SW, SE*, and *NE* are taken from the 9-region model. The regions that are associated with the cardinal directions *N, S, E* and *W* are further split into two halves, for example, *N* is split up into *north-west-center* (*Nwc*) and *north-east-center* (*Nec*). For ease of reference I introduce a few labels for composite regions with the labels N ("north"), N^{strong} ("strong north"), N^{weak} ("weak north"), and $N^{very-weak}$ ("very weak north"). These regions are defined below in (41) and illustrated in Figure 2.22. N is the grey region in Figure 2.22(b), N^{strong} refers to the upper half-plane Figure 2.22(c), N^{weak} is the region of the middle half-plane excluding the area RO, as shown in Figure 2.22(d). And $N^{very-weak}$ is illustrated in Figure 2.22(e), it refers to the composite region consisting of the lowest upper half-plane excluding the area RO.

(41) a. $N := Nwc \cup Nec$
b. $N^{strong} := NW \cup N \cup NE$
c. $N^{weak} := N^{strong} \cup Wnc \cup Enc$



(a) Center *north* line. (b) N (c) N^{strong}



(d) N^{weak} (e) $N^{very-weak}$

Figure 2.22: Regions defined by means of the regions defined by the orthogonal projection grid.

d. $N^{very-weak} := N^{weak} \cup Wsc \cup Esc$

Four relation schemata will be defined by means of this grid: orthogonal projection with *part-of* (OP$_P$), orthogonal projection with *overlap* (OP$_O$), orthogonal projection with *overlap* and *part-of* (OP$_{OP}$), and a grid model (OP$_{Grid}$).

orthogonal projection with *part-of* (OP$_P$) The relation schema OP$_P$ defines locative direction relations in terms of axial representations and binary applicability functions. The conditions of the applicability functions are specified in terms of the topological *part-of* relation (P) and acceptance regions defined above in (41) and Figure 2.21:

All relations defined by this relation schema have a restricted domain. Relations are only defined for spatial configurations with the bounding box (loc_b) of the LO not being part of the bounding box of the RO.

Domain restriction: $\neg P(loc_b(lo), loc_b(ro))$

Overlap of LO's bounding box with the RO's bounding box is tolerated and ignored, i.e., it has no effect. The definitions are specified for the locative direction relation *north*, the other three relations are obtained by adjusting the representation of relative position to the corresponding direction, see last subsection of Section 2.5.

We define 4 range levels. For each range level the relation $north(lo, ro)$ is defined by conditions that relate lo to the composite regions described above.

(42) OP$_{P1}$: $north(lo, ro) \equiv P(loc_b(lo), N \cup loc_b(ro))$
OP$_{P2}$: $north(lo, ro) \equiv P(loc_b(lo), N^{strong} \cup loc_b(ro))$
OP$_{P3}$: $north(lo, ro) \equiv P(loc_b(lo), N^{weak} \cup loc_b(ro))$
OP$_{P4}$: $north(lo, ro) \equiv P(loc_b(lo), N^{very-weak} \cup loc_b(ro))$

This relation schema supports the following inferences. On range level 1 all four locative direction relations ($north^{OP_{P1}}$, $west^{OP_{P1}}$, $south^{OP_{P1}}$, and $east^{OP_{P1}}$,) are mutually exclusive. On range levels 2 and 3, complement direction relations, like for example $north^{OP_{P2}}$ and $south^{OP_{P2}}$, are mutually exclusive.

Figure 2.23 shows an example of a spatial configuration which satisfies the relation $north^{OP_{P2}}(lo, ro)$ and also the $north$ relations of the levels 3 and 4 because they are implied by $north^{OP_{P2}}(lo, ro)$. $north$ from range level 1, $north^{OP_{P1}}(lo, ro)$, is not satisfied. The configuration further satisfies $east^{OP_{P3}}(lo, ro)$ and consequently $east^{OP_{P4}}(lo, ro)$.

OP$_P$ relations are used by O'Keefe (1996), Topaloglou (1994), and to some extent by Abella & Kender (1993).

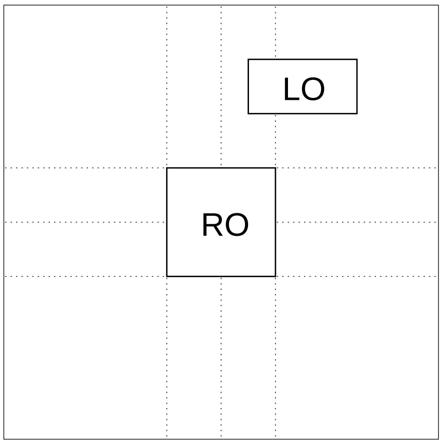


Figure 2.23: Example for orthogonal projection relation schemata OP_P, OP_O, OP_{OP}, and OP_{Grid}.

The following relations defined by the relation schema $\mathbf{OP}_{P2}^{cent}$ are variants of the OP_{P2} relations. They are equivalent to the relations defined by André et al. (1987). They are defined by means of acceptance conditions similar to OP_{P2}, but instead of representing LO by its bounding box LO is represented by its centroid ($loc_{cent}(lo)$) – RO is still represented by its bounding box.

(43) OP_{P2}^{cent}: $north(lo, ro) \equiv P(loc_{cent}(lo), N^{strong} \cup loc_b(ro))$

orthogonal projection with *overlap* (OP_O) The relation schema OP_O defines relation in terms of axial representations and discrete applicability functions based on the topological relation *overlap* between the bounding box of the LO and the regions defined by the orthogonal grid around the RO presented above. All relations defined by this relation schema have a restricted domain. Relations are only defined for spatial configurations where the bounding box of the LO is not part of the bounding box (loc_b) of the RO.

Domain restriction: $\neg P(loc_b(lo), loc_b(ro))$

The relation schema specifies 5 range levels. For each range level the relation $north(lo, ro)$ is defined by means of overlap of the bounding box and the regions defined in (41):

(44) OP_{O1}: $north(lo, ro) \equiv O(loc_b(lo), Nwc) \wedge O(loc_b(lo), Nec)$
OP_{O2}: $north(lo, ro) \equiv O(loc_b(lo), N)$
OP_{O3}: $north(lo, ro) \equiv O(loc_b(lo), N^{strong})$
OP_{O4}: $north(lo, ro) \equiv O(loc_b(lo), N^{weak})$
OP_{O5}: $north(lo, ro) \equiv O(loc_b(lo), N^{very-weak})$

The relation schema does not support any disjointness inferences. We cannot conclude from any relation that another relation does not hold.

The example in Figure 2.23 is not satisfied by $north^{OP_{O1}}(lo, ro)$, but it is satisfied by $north^{OP_{O2}}(lo, ro)$ and higher. Furthermore, it satisfies $east^{OP_{O3}}(lo, ro)$ and higher and $west^{OP_{O5}}(lo, ro)$.

OP$_O$ relations are used by Rajagopalan (1993).

orthogonal projection with *overlap* and *part-of* (OP$_{OP}$) The relation schema OP$_{OP}$ combines the relation schemata OP$_P$ and OP$_O$ in order to provide more fine-grained range levels and maintain the inferential properties of OP$_P$. It defines locative direction relations in terms of axial representations and binary applicability functions which are based on the topological relations *part-of* and *overlap*. The same domain restrictions apply as above. The relations are only applicable if the bounding box of the LO is not *part of* the bounding box of the RO.

Domain restriction: $\neg P(loc_b(lo), loc_b(ro))$

The relation schema specifies 9 range levels. Again, the relations are defined along the composite regions defined in (41). The LO is either *part of* one of these regions, or it *overlaps* with one of these regions. In order to maintain inferential properties the *overlap* conditions are restricted by a constraint that requires the LO to be *part of* the next bigger composite region. For each range level the relation $north(lo, ro)$ is defined by means the bounding box of lo (*viz* $loc_b(lo)$) overlapping or being part of the composite regions defined in (41). Note, that RO is used here to denote the central region which is defined as the bounding box $loc_b(ro)$. The relations for *south*, *west*, and *east* are defined in a similar fashion.

(45) OP$_{OP1}$: $north(lo, ro) \equiv O(loc_b(lo), Nwc) \wedge O(loc_b(lo), Nec) \wedge P(loc_b(lo), N \cup RO)$
OP$_{OP2}$: $north(lo, ro) \equiv P(loc_b(lo), N \cup RO)$
OP$_{OP3}$: $north(lo, ro) \equiv O(loc_b(lo), N) \wedge P(loc_b(lo), N^{strong} \cup RO)$
OP$_{OP4}$: $north(lo, ro) \equiv P(loc_b(lo), N^{strong} \cup RO)$
OP$_{OP5}$: $north(lo, ro) \equiv O(loc_b(lo), N^{strong}) \wedge P(loc_b(lo), N^{weak} \cup RO)$
OP$_{OP6}$: $north(lo, ro) \equiv P(loc_b(lo), N^{weak} \cup RO)$
OP$_{OP7}$: $north(lo, ro) \equiv O(loc_b(lo), N^{weak}) \wedge P(loc_b(lo), N^{very-weak} \cup RO)$
OP$_{OP8}$: $north(lo, ro) \equiv P(loc_b(lo), N^{very-weak} \cup RO)$
OP$_{OP9}$: $north(lo, ro) \equiv O(loc_b(lo), N^{very-weak})$

Relations on the levels 1, 2, and 3 are mutually exclusive. On levels 4, 5, and 6 complement relations exclude each other.

The example in Figure 2.23 satisfies the relations $north^{OP_{OP3}}(lo, ro)$ and higher, $east^{OP_{OP4}}(lo, ro)$ and higher, and $west^{OP_{OP9}}(lo, ro)$.

OP$_{OP}$ relations are used by O'Keefe (1996) and Papadias & Sellis (1994).

orthogonal projection grid (OP$_{Grid}$) The relation schema OP$_{Grid}$ defines locative direction relations by means of a grid model based on the orthogonal grid defined above in Figure 2.21. In contrast to previous schemata which were based on binary applicability functions, this relation schema defines relations by means of graded degrees of applicability (see formula (18) on page 37). Relative position of LO with respect to RO is represented by a 4×4 matrix the elements of which contain the proportion of the LO that overlaps with the corresponding cell. Note, that the shape of the LO is not restricted, and the LO is not reduced to its bounding box.

The corresponding 4×4 matrix that contains a weight for each cell is designed corresponding to the composite regions defined in (41). The cells corresponding to the smallest region, N, are assigned a weight of 1.0. Every new part of the next bigger region receives half the weight of the preceding region. That means, the remaining cells of N^{strong} receive the weight 0.5. The upper part of RO and the cells of N^{weak} which are not associated with a weight receive the value 0.25. The remaining cells of the lower part of RO and of $N^{very-weak}$ are associated with the weight 0.125. The rest is set to zero. The cells of the grid are associated with the weights defined by the following matrix. Figure 2.24 illustrates the association between matrix and cells:

(46) $$\begin{bmatrix} 0.500 & 1.000 & 1.000 & 0.500 \\ 0.250 & 0.250 & 0.250 & 0.250 \\ 0.125 & 0.125 & 0.125 & 0.125 \\ 0.000 & 0.000 & 0.000 & 0.000 \end{bmatrix}$$

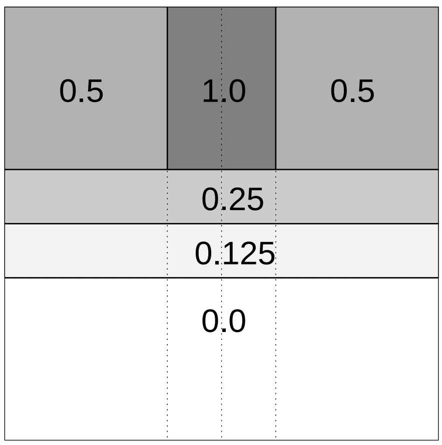


Figure 2.24: Axial representation grid.

The overall degree of applicability is computed according to formula (34) on page 44. It is the sum over the product of weight w_i and proportion p_i of each cell i:

$$a = \sum_i w_i * p_i$$

The grid model yields a degree of applicability between 0 and 1 for all possible combinations of LO and RO. Different range levels are defined by means of different thresholds. Let us define 7 range levels. The thresholds are chosen in such a way that there is a threshold for each value defined by the weight matrix (46) and one threshold corresponding to the means between two consecutive thresholds.

(47) OP$_{Grid1}$: $north(lo, ro) \equiv a_{Grid}(lo, ro) = 1.0$
OP$_{Grid2}$: $north(lo, ro) \equiv a_{Grid}(lo, ro) \geq 0.75$
OP$_{Grid3}$: $north(lo, ro) \equiv a_{Grid}(lo, ro) \geq 0.5$
OP$_{Grid4}$: $north(lo, ro) \equiv a_{Grid}(lo, ro) \geq 0.375$
OP$_{Grid5}$: $north(lo, ro) \equiv a_{Grid}(lo, ro) \geq 0.25$
OP$_{Grid6}$: $north(lo, ro) \equiv a_{Grid}(lo, ro) \geq 0.1875$
OP$_{Grid7}$: $north(lo, ro) \equiv a_{Grid}(lo, ro) \geq 0.125$

A threshold of 1.00 ensures that the whole LO lies within N. With a threshold of 0.75 at least half of the LO lies within N.[4] If LO is completely within N^{strong} then the degree of applicability is greater than 0.5. However, even if parts of the LO do not overlap with N^{strong} the degree of applicability can be greater than 0.5, because these parts can be compensated by other parts of LO which overlap with N in the following way. Let p_x be the proportion of LO overlapping with regions that are associated with a degree of applicability of x; p_1 is associated with N^{strong}, $p_{0.5}$ with NW and NE, and so on. Compensation of parts being in regions with a weight below 0.5 requires satisfaction of the following inequality:

(48) $p_1 + p_{0.5} * 0.5 + p_{0.25} * 0.25 + p_{0.125} * 0.125 \geq 0.5$

The other thresholds can be interpreted in a similar way.

Strictly speaking, however, only the thresholds 0.625 and 0.5 mark the boundaries of pairwise disjointness and complement disjointness, respectively.[5] Therefore, rela-

[4]Let p be the proportion of LO overlapping with N, then $1 - p$ is the proportion of the rest. The maximum degree of applicability that can be achieved by the rest is $(1 - p) * 0.5$. It is easy to see that $1.0 * p + 0.5 * (1 - p) > 0.75$ is equivalent to the condition $p > 0.5$. Therefore, every score above a threshold of 0.75 implies that p is greater than 0.5.

[5]In order to proove this we have to show that for any proportion matrix, if the degree of applicability $\sum_i w_i * p_i$ is greater than the corresponding threshold θ (*viz* 0.625 / 0.5), then there is no (other/complement) relation which scores a higher degree of applicability. Let q_j denote the proportion of cell j in the proportion matrix of the (other/complement) relation, then we have to show:

(i) $\sum_i w_i * p_i > \theta \geq \sum_j w_j * q_j$

We compute θ by determining the proportion distribution of p_i and q_j which maximise the following term:

(ii) $\sum_i w_i * p_i + \sum_j w_j * q_j$

According to the symmetry property described on page 39 there is a mapping between the indices i and j so that we can write (ii) as tions on range levels 1 and 2 are pairwise disjoint. And a degree of applicability greater than 0.5 ensures complement disjointness. If we additionally assume that objects cannot be distributed in a way that they are disconnected such that half of the object is in N and the other half in S, but that there have to be some proportions in other regions, then the degrees of applicability will always be smaller than 0.5, so that we can also accept values of 0.5 in order to conclude complement disjointness. Thus, range level 3 provides complement disjointness.

The example in in Figure 2.23 overlaps 25% with N and 75% with NE. Therefore, $north$ relations score a degree of applicability of 0.6275 ($= 0.25 * 1.00 + 0.75 * 0.50$). The relation $north^{OP_{Grid2}}(lo, ro)$ does not apply, but $north^{OPgrid3}(lo, ro)$ and $north$ relations from higher range levels do. Similarly, the score is 0.4375 ($= 0.25 * 0.25 + 0.75*0.50$) for $east$ relations and 0.03125 ($= 0.25*0.125+0.75*0$) for $west$ relations, so $east^{OPgrid5}(lo, ro)$ and higher applies, but no *west* relations.

OP$_{Grid}$ relations are used by Wazinski (1992). They are ways of defining locative direction relations based on the applicability ratings reported in Logan & Sadler (1996) and Regier & Carlson (2001).

2.8.2 Angular deviation schemata

Angular deviation schemata are based on angular representations. Relations will be defined in terms of acceptance intervals defined on angles between -180° and 180°. A natural partition of a full circle that provides complement disjointness is provided by two $180°$ intervals. Pairwise disjointness of four relations is provided by $90°$ intervals that are defined by $45°$ deviation from the prototypical to both sides. In order to consider the possibility of a finer grained distinction that might be necessary for the semantics of projective locative expressions, we use the sectors derived by 8 $45°$ partitions. The relations defined below will be based on intervals that are centered on

---

(iii) $\sum_i (w_i + w_{i'}) * p_i$

This term is maximal if all the proportion is distributed over cells i that are associated with the highest sum of weights:

(iv) $w_i + w_{i'}$

It is easy to see that for pairwise disjointness (iv) is maximal with 1.25. For example, for the relations *north* and *west* the weights of the regions Nwc and Wnc sum up to 1.25. Consequently, (iii) is maximal if all the proportion of LO is distributed only over those cells that are associated with 1.25.

Then we solve the following equation:

(v) $\theta = \sum_i w_i * p_i = \sum_j w_j * q_j$

which comes out as $\theta = 0.625$.

Similarly, for complement disjointness the highest sum of weights is 1.0 obtained, for instance, by the cells of the regions N and S. The threshold determined by equation (iv) is 0.5.

$0°$ and vary in width from $0°$ to 112.5 in steps of $22.5°$. All partitions are depicted in Figure 2.25. The singleton interval $[0, 0]$ spans a single number, it is depicted in Figure 2.26(a). The other intervals are open intervals which do not include the interval boundaries. They are illustrated in Figure 2.26:

(49)	AD0	:=	$[0, 0]$
	AD22.5	:=	$]-22.5, 22.5[$
	AD45	:=	$]-45, 45[$
	AD67.5	:=	$]-67.5, 67.5[$
	AD90	:=	$]-90, 90[$
	AD112.5	:=	$]-112.5, 112.5[$

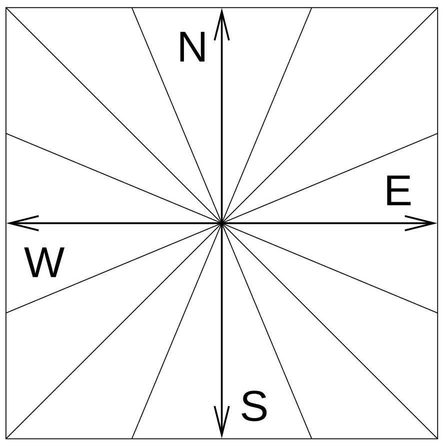
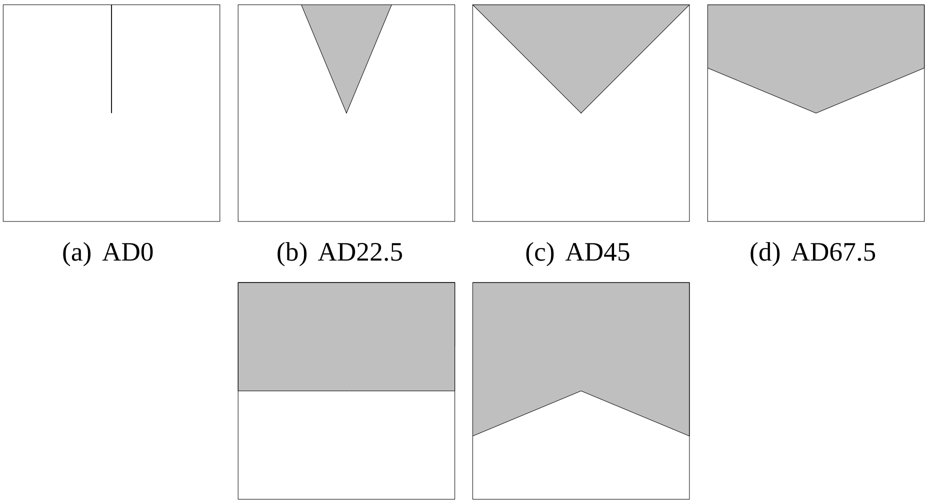


Figure 2.25: Illustration of the division of space into regular 22.5° partitions.

We will define 5 different relation schemata based on angular deviation. The first two, AD^{cent} and AD^{prox}, represent relative position by single angles between center of mass direction or proximal direction, respectively, (see page 31) and the prototypical direction that is associated with a projective expression.

Locative direction relations are defined by the condition that the angle representing relative position is part of the corresponding acceptance interval. Then we will define two relation schemata that represent relative position by means of angle intervals. Similar to the definition of the orthogonal projection relation schemata above, relations are defined by means of the topological relations *overlap* and *part-of* between the representation and the acceptance intervals defined in (49). The relation schema AD$_O^{int}$ is based on *overlap*, and AD$_P^{int}$ is based on the *part-of* relation. The last relation schema, AD^{Grid}, is a grid model based on an angular deviation grid.

angular deviation with centroids (AD^{cent}) The relation schema AD^{cent} defines locative direction relations by means of angular representations and binary applicability functions. Relative position is represented by the angle between the prototypical



(a) AD0 (b) AD22.5 (c) AD45 (d) AD67.5 (e) AD90 (f) AD112.5

Figure 2.26: Acceptance intervals for angular deviation relation schemata.

direction, e.g. *north*, *south*, *east*, and *west*, and the vector starting in the centroid of the RO and ending in the centroid of the LO. Relations are defined by means of the acceptance intervals defined in (49) and shown in Figure 2.26. A relation holds between LO and RO if the angle representing relative position is part of the acceptance interval that is associated with the relation. The relation schema excludes objects from its domain whose centroids (loc_{cent}) coincide:

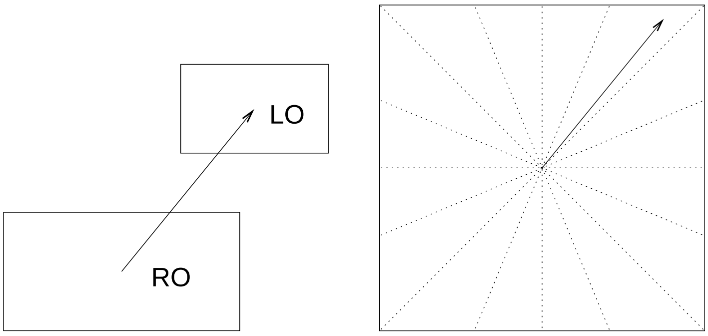
domain restriction: $loc_{cent}(lo) \neq loc_{cent}(ro)$

Let us define 5 range levels. Let α be the angle between the prototypical direction ($north$) and the vector from the centroid of the RO to the centroid of the LO.

(50) AD$_1^{cent}$: $north(lo, ro) \equiv \alpha \in$ AD22.5
AD$_2^{cent}$: $north(lo, ro) \equiv \alpha \in$ AD45
AD$_3^{cent}$: $north(lo, ro) \equiv \alpha \in$ AD67.5
AD$_4^{cent}$: $north(lo, ro) \equiv \alpha \in$ AD90
AD$_5^{cent}$: $north(lo, ro) \equiv \alpha \in$ AD112.5

Range levels 1 and 2 provide pairwise disjointness, range levels 3 and 4 complement disjointness.

The example in Figure 2.27 shows the vector from the centroid of the RO to the centroid of the LO. It can be seen from part (b) that this vector lies in the acceptance interval $]-45°, 45°[$ with respect to the prototypical direction $north$, and in the acceptance interval $]-67.5°, 67.5°[$ with respect to the prototypical direction $east$. Thus, $north^{AD_2^{cent}}(lo, ro)$ and $north$ relations from higher levels are true, and $east^{AD_3^{cent}}(lo, ro)$ and higher are true, too.



(a) Direction between centroids.

(b) The vector placed in the angular deviation grid.

Figure 2.27: Example for relations $north(lo, ro)$ defined by AD^{cent}.

AD^{cent} relations are used by Gapp (1994a), Hernandez (1994), and Yamada et al. (1988). The relations defined in Kelleher (2003) are likewise based on AD^{cent} relations.

angular deviation with proximal direction (AD^{prox}) The relation schema AD^{prox} defines locative direction relations by means of angular representations and binary applicability functions. Relative position is represented by the angle between the prototypical direction and a vector representing the *proximal direction*, that is the angle of the smallest vector connecting RO and LO, see Section 2.5 page 31. Relations are defined in the same way as AD^{cent} relations. A particular relation holds between LO and RO if the corresponding angle is part of the acceptance interval that is associated with that relation. Relations defined by this relation schema are only applicable to objects ro and lo if they are not connected (DC) (see page 26).

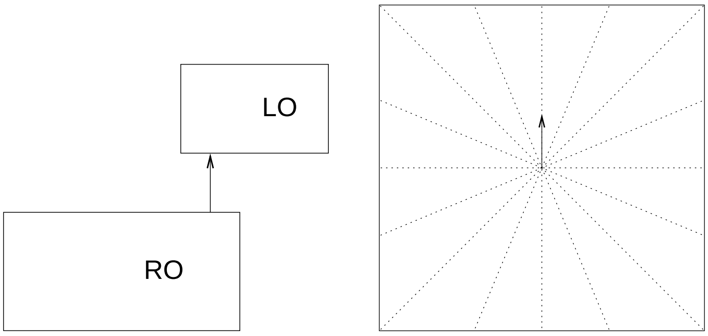
Domain restriction: $DC(lo, ro)$

If they are connected the proximal direction is the $\vec{o}$ vector which has no actual directional component. Definitions and inference properties are identical to AD^{cent}. Let α be the angle between the prototypical direction *north* and the proximal direction.

(51) AD$_1^{prox}$: $north(lo, ro) \equiv \alpha \in$AD22.5
AD$_2^{prox}$: $north(lo, ro) \equiv \alpha \in$AD45
AD$_3^{prox}$: $north(lo, ro) \equiv \alpha \in$AD67.5
AD$_4^{prox}$: $north(lo, ro) \equiv \alpha \in$AD90
AD$_5^{prox}$: $north(lo, ro) \equiv \alpha \in$AD112.5

Range levels 1 and 2 provide pairwise disjointness, range levels 3 and 4 complement disjointness.

The example in Figure 2.28 shows the vector indicating the proximal direction of LO relative to RO. It is aligned with the prototypical direction $north$ and perpendicular



(a) Proximal direction. (b) The vector placed in the angular deviation grid.

Figure 2.28: Example for relations $north(lo, ro)$ defined by AD^{prox}.

to the prototypical directions $east$ and $west$. Thus, $north^{AD_1^{prox}}(lo, ro)$ and $north$ relations on higher levels are true, and $east^{AD_5^{cent}}(lo, ro)$ and $west^{AD_5^{cent}}(lo, ro)$ are true.

The concept of proximal direction is discussed or used in (Regier, 1992), (Schirra, 1993), (Zwarts & Winter, 2000), and (Regier & Carlson, 2001).

angular deviation with angle intervals and *part-of* (AD$_P^{int}$) The relation schema AD$_P^{int}$ defines locative direction relations by means of angular representations and binary applicability functions. Relative position is represented by an *angle interval* that covers all possible directions of vectors connecting RO with LO. Relations are defined by means of the topological relation *part-of* between the angle interval representing relative position and the acceptance intervals specified in (49). Concerning domain restrictions, relations are only defined if lo and ro are not connected (DC) (see page 26).

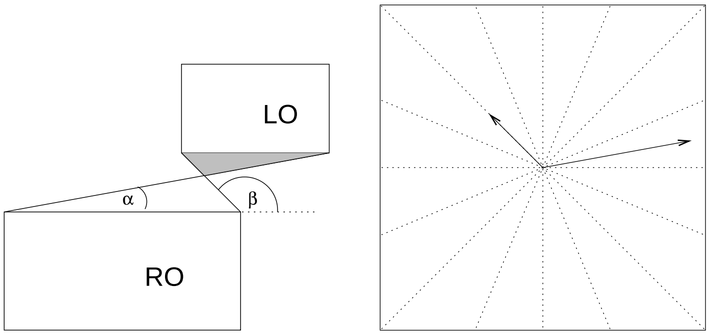
Domain restrictions: $DC(lo, ro)$

We define 5 range levels. Let I be the angle interval representing the relative position of LO with respect to RO:

(52) AD$_{P1}^{int}$: $north(lo, ro) \equiv P(I,$AD22.5$)$
AD$_{P2}^{int}$: $north(lo, ro) \equiv P(I,$AD45$)$
AD$_{P3}^{int}$: $north(lo, ro) \equiv P(I,$AD67.5$)$
AD$_{P4}^{int}$: $north(lo, ro) \equiv P(I,$AD90$)$
AD$_{P5}^{int}$: $north(lo, ro) \equiv P(I,$AD112.5$)$

Like the previous relation schemata, the relations from AD$_P^{int}$ are pairwise disjoint on levels 1 and 2, and complement relations are disjoint on levels 3 and 4.

The example in Figure 2.29 shows the angle interval $[\alpha, \beta]$ representing the relative position of the LO relative to the RO. In (b) the angle interval $[45°, -79°]$ is placed in the angular deviation grid. The relations $north^{AD^{int}_{P4}}(lo, ro)$ and $north^{AD^{int}_{P5}}(lo, ro)$ are the only relations that hold from the relation schema AD^{int}_P.



(a) Angle interval representing the relative position of LO with respect to RO.

(b) The angle interval placed in the angular deviation grid.

Figure 2.29: Example for relations $north(lo, ro)$ defined by AD^{int}.

angular deviation with angle intervals and *overlap* ($\mathbf{AD}^{int}_O$) The relation schema AD^{int}_O defines locative direction relations by means of angular representations and binary applicability functions. Relative position is represented by an *angle interval* that covers all possible directions of vectors connecting RO with LO. Relations are defined by means of the topological relation *overlaps* between the angle interval representing relative position and the acceptance intervals specified in (49). This relation schema provides relations which are applicable only if the objects lo and ro are not connected (see page 26).

Domain restrictions: $DC(lo, ro)$

Let us define 6 range levels. Let I be the angle interval representing the relative position of LO with respect to RO:

(53) AD^{int}_{O1}: $north(lo, ro) \equiv O(I,\mathrm{AD0})$
AD^{int}_{O2}: $north(lo, ro) \equiv O(I,\mathrm{AD22.5})$
AD^{int}_{O3}: $north(lo, ro) \equiv O(I,\mathrm{AD45})$
AD^{int}_{O4}: $north(lo, ro) \equiv O(I,\mathrm{AD67.5})$
AD^{int}_{O5}: $north(lo, ro) \equiv O(I,\mathrm{AD90})$
AD^{int}_{O6}: $north(lo, ro) \equiv O(I,\mathrm{AD112.5})$

AD^{int}_O relations do not provide any disjointness properties.

The example in Figure 2.29 is satisfied by the relations $north^{AD^{int}_{O1}}$ and higher, $west^{AD^{int}_{O4}}$ and higher, and $east^{AD^{int}_{O2}}$ and higher.

The following two sets of relations, AD$^{int}_{cross}$ and AD$^{int/cent}_{O5}$, are variants of the *angle interval* relations. They are defined here for the purpose of having relations that are equivalent to some relations defined in the literature.

- Schmidtke (2001) defines the relations *north*, *south*, *east*, and *west* by means of a disjunction of AD$^{int}_{P4}$ and AD$^{int}_{O1}$ relations:

$$\text{AD}^{int}_{cross} : north(lo, ro) \equiv AD^{int}_{P4}(lo, ro) \vee AD^{int}_{O1}(lo, ro)$$

- The directional component of the relations defined by Kelleher (2003) can be defined by means of a variant of AD$^{int}_{O5}$ relations. For AD$^{int}_{O5}$ relations the angle interval is constructed between the entire RO and the entire LO (compare with Figure 2.8 on page 32). But the angle intervals of the relations corresponding to Kelleher's relations are constructed between the centroid of the RO and the entire LO (compare with Figure 2.9 on the same page). We define a set of relations AD$^{int/cent}_{O5}$ which only differs from AD$^{int}_{O5}$ in that the angle interval I_{cent} is defined from the centroid of RO:

$$\text{AD}^{int/cent}_{O5}: \quad north(lo, ro) \equiv O(I_{cent}, \text{AD90})$$

angular deviation grid (AD^{Grid}) The relation schema AD^{Grid} defines locative direction relations by means of a grid model based on angular deviation. The grid is constructed around the bounding box of the RO. It is based on the 9-region grid described in Section 2.5 and adds 45° partitions to the diagonal cells NW, NE, SW, and SE as shown in Figure 2.30. Each cell is associated with an average angle relative to the corresponding prototypical direction *north*. The N cell is associated with an average angle of $0°$, the upper NW sector with $22.5°$, the lower NW sector with $67.5°$, and so on. On the other side, the upper NE sector is associated with an average angle of $-22.5°$, the lower NE sector with $-67.5°$, and so on.

Relative position is represented by a 13 dimensional vector each element of which reflects the proportion of LO overlapping with the associated cell of the grid.

The weights of the cells are defined as the cosine of the average angle associated with a certain cell. The degree of applicability of the center cell is 0. Thus, cells with an angle between $-90°$ and $90°$ are associated with a positive weight between 0 and 1, and cells with an angle below $-90°$ and above $90°$ are associated with negative weights. In contrast to the grid model OP$_{Grid}$, this grid model can assign negative degrees of applicability. The parts of LO that lie within the cells associated with positive weights must compensate these negative degrees if the pair $\langle$LO,RO$\rangle$ is to qualify as an instance of the relation.[6]

The degree of applicability of a cell i is the product of the weight w_i associated with cell i and the proportion p_i of LO overlapping with RO. The total degree of applicability is the sum of all single degrees.

[6]See Matsakis et al. (2001) for another explicit compensation mechanism.

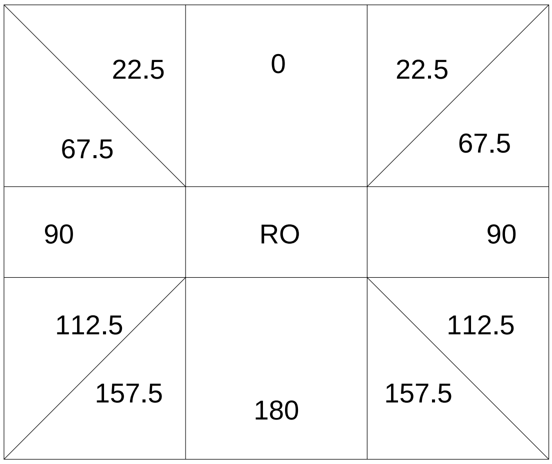


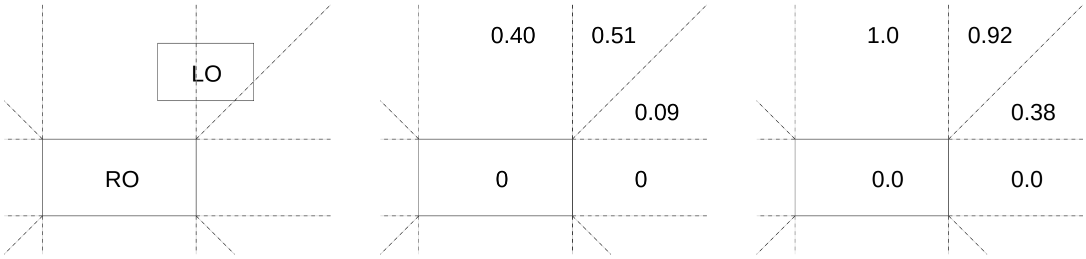
Figure 2.30: Angular deviation grid with average angles of the region with respect to the prototypical direction $north$.

$$a = \sum_i w_i * p_i$$

Relations are defined by means of thresholds for the degree of applicability. Let us define 7 range levels corresponding to approximated cosine values of the angles defining the intervals in (49), namely $0°$, $22.5°$, $45°$, $67.5°$, $90°$, and $112.5°$. Let $app(\cdot,\cdot)$ be the function that yields a degree of applicability:

(54)
AD$_1^{Grid}$: $north(lo, ro) \equiv app(lo, ro) = 1.00$
AD$_2^{Grid}$: $north(lo, ro) \equiv app(lo, ro) > 0.92$
AD$_3^{Grid}$: $north(lo, ro) \equiv app(lo, ro) > 0.7$
AD$_4^{Grid}$: $north(lo, ro) \equiv app(lo, ro) > 0.38$
AD$_5^{Grid}$: $north(lo, ro) \equiv app(lo, ro) > 0.0$
AD$_6^{Grid}$: $north(lo, ro) \equiv app(lo, ro) \geq 0.0$
AD$_7^{Grid}$: $north(lo, ro) \equiv app(lo, ro) > -0.383$

Range levels 1 to 3 provide pairwise disjointness.[7] Relations from range levels 4 and 5 provide mutual exclusion of complements.[8] A degree of applicability of exactly 0.0 indicates that a relation and its complement relation cancel each other out.



(a) LO and RO.

(b) The proportion of LO in each cell.

(c) The relevant weights determined by the cosine of the average angle.

Figure 2.31: Example for relations $north$ defined by AD^{Grid}

The example in Figure 2.31 illustrates the application of $north^{AD^{Grid}}$ relations. Panel (a) shows the LO, the RO and the partition of space into cells. Panel (b) contains the proportion of LO for each cell, and (c) shows the weights associated with each cell. The degree of applicability of $north$ relations is

$$app(lo, ro) = 0.4 * 1.0 + 0.51 * 0.92 + 0.09 * 0.38 \approx 0.90$$

Thus, $north^{AD_3^{Grid}}(lo, ro)$ and corresponding relations on higher range levels apply to this example.

AD$_5^{Grid}$ relations are used by Fuhr et al. (1995).

2.9 Related work

This section presents in detail the literature on locative direction relations. All summaries focus exclusively on the technical side of those studies. In particular, each summary will describe what locative direction relations are defined and how they are

[7]Following the argument in footnote 4 on page 53, the cells that are associated with the highest sum of weights are the cells that correspond to the regions NW and NE. For example, for *north* and *west* these regions are the upper and lower NW sectors summing up to $\cos(22.5) + \cos(76.5) \approx 1.31$. Consequently, the threshold that guarantees pairwise disjointness is 0.65.

[8]Again, following the argument of footnote 4 on page 53, it is easy to see that the corresponding weights of complement relations are inverse to each other, so that they cancel each other out and the following term (which has to be maximised) is zero:

(i) $\sum_i w_i * p_i + \sum_j w_j * q_j = 0$

Consequently, a threshold of 0.0 defines the boundary for complement disjointness.

defined. How do they represent location? Which frames of reference are considered? How do they represent relative position? How do they compute a degree of applicability? And finally, how do they determine binary locative direction relations?

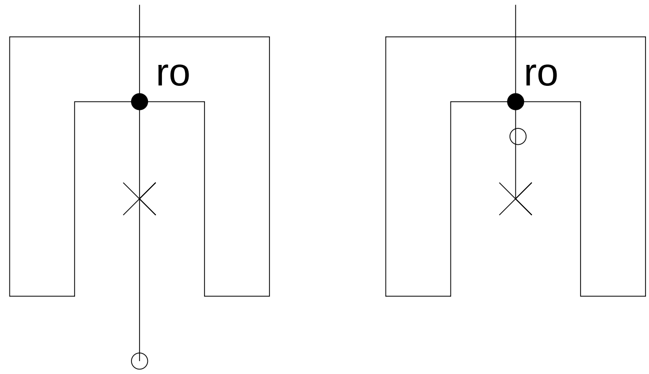
The parts of these studies which describe other aspects of spatial processing, e.g. computation of the frame of reference, topological relations and distance relations, are not considered here. Similarly, little will be said about the spatial domains that the relations are supposed to be applied to, because the main intent of these summaries is to describe the technical inventory that has been used to define locative direction relations. The summaries are ordered according to their publishing date.

2.9.1 Kelleher 2003

Kelleher (2003) presents a system that interprets locative expressions from the perspective of an agent situated in a virtual 3D world. The approach employs angular representations, fuzzy applicability functions, and it considers the influence of perceptual accessibility, namely occlusion of objects from the perspective of the viewer. Locative expressions are interpreted in a combination of the intrinsic and the relative frame of reference. Kelleher (2003) focuses on locative expressions containing the terms *in front of*, *behind*, *to the left of*, and *to the right of* but also discusses the treatment of the prepositions *above* and *below* (e.g. page 263). The interpretation process aims at finding the most suitable candidate in a virtual 3D environment that matches the locative expression.

Relative position The objects in the domain are represented by *meshes*, i.e. 3D polygons. The degree of applicability is computed via proxy points representing the whole object. The proxy points are determined as follows: The location of the RO is represented by a point that is determined by (i) the mesh of the RO, (ii) the point of the viewer, and (iii) the centroid of the RO's bounding box. There are three possibilities: first, if the half-axis from the viewpoint through the centroid of the RO's bounding box intersects with the RO's mesh, the point of intersection is taken to represent the RO, see Figure 2.32(a). Second, if that half-axis does not intersect with the mesh, but the inverted one does – that is, the half-axis from the centroid of the bounding box through the viewpoint – then that point of intersection is taken to represent the RO, see Figure 2.32(b). Third, in the case of no intersection at all, the RO is represented by the centroid of its bounding box. The location of the LO is represented by the vertex of the mesh of the LO that gains the highest degree of applicability. This vertex is determined by applying the applicability function of the given relation to all vertices of the LO's mesh and selecting the vertex with the highest degree as the proxy point representing the LO.

Frames of reference The degree of applicability of locative expressions is computed in the relative (*'viewer-centred'*) frame of reference and, in case the RO has an intrin-



(a) ro is the first intersection of mesh and half-line from the observer (o) through the centroid (x) of the bounding box.

(b) ro is the first intersection of mesh and half-line from the centroid (x) of the bounding box through the observer (o).

Figure 2.32: Determining the proxy point ro for the reference object in (Kelleher, 2003).

sic orientation, in the intrinsic frame of reference. If there are no linguistic cues in the utterance that hint at a particular frame of reference and the intrinsic frame of reference is not aligned with the relative frame of reference, degrees of applicability are computed in both frames and the results are combined in the following way. For each vertex of LO's mesh two degrees of applicability are computed – one for the intrinsic frame of reference and one for the relative frame of reference. The overall degree of applicability of each vertex is the weighted sum of both values normalised with respect to some maximal value. The weights of the weighted sum implement a bias of certain prepositions towards a particular frame of reference. If the prepositions *above* and *below* occur in the relative frame of reference the weight is 2.0, and if the prepositions *to the right*, *to the left*, *in front of*, and *behind* are used in the intrinsic frame of reference a weight of 1.1 is used, see (Kelleher, 2003, p232) and (Kelleher, 2003, Section 10.3) for empirical support of such a bias.

Degree of applicability The underlying function to compute the degree of applicability of a locative expression with respect to a spatial configuration is a fuzzy applicability function defined on angular representations. It assigns a fuzzy score $app(p)$ to a point p given a spatial relation and a reference point ro. The fuzzy score is the product of the scores for direction ($app_{dir}(p)$) and distance ($app_{dist}(p)$).

(55) $app(p) = app_{dir}(p) * app_{dist}(p)$

The degree of applicability for direction (app_{dir}) is dependent on the angular deviation of the vector $\vec{l} = \overrightarrow{ro\,p}$ from reference point ro to point p and the prototypical direction $\vec{\rho}$ that is specified by the projective locative expression and the frame of reference. The angle $\alpha = \angle(\vec{\rho}, \vec{l})$ is the deviation of $\vec{l}$ from $\vec{\rho}$. If it exceeds a certain maximum α_{max} (Kelleher uses $\alpha_{max} = 90°$), then the rating is zero, otherwise it is a real value between zero and one.

(56) $$app_{dir} = \begin{cases} 0; & \alpha \geq \alpha_{max} \\ 1 - \frac{\alpha}{\alpha_{max}}; & \alpha < \alpha_{max} \end{cases}$$

The distance between a point p and reference point ro impacts on the acceptability rating of p by virtue of the fact that if there are two points located at the same angular deviation from the prototypical direction, the point closer to ro will have a higher degree of applicability (Kelleher, 2003, 234). The distance between p and ro is given by the length of the vector $\vec{l}$. If $\vec{l}$ exceeds a certain maximum distance d_{max} the degree of applicability for distance (app_{dist}) will be zero. Otherwise it will be a real value between 0 and 1.

(57) $$app_{dist} = \begin{cases} 0; & |\vec{l}| \geq d_{max} \\ 1 - \frac{|\vec{l}|}{d_{max}}; & |\vec{l}| < d_{max} \end{cases}$$

Discrete conditions: Occlusion If the prepositions *behind* and *in front of* are used in the relative frame of reference, the degree of applicability is additionally influenced by a discrete model based on the occlusion of an object by another object with respect to the viewer. For *behind* it yields a degree of applicability of 1 (i.e. full applicability) if and only if the reference object (partially) occludes the located object, and for *in front of* it yields a degree of applicability of 1 if and only if the located object (partially) occludes the reference object. Otherwise the rating is set to zero. An object A occludes an object B if there is a straight line intersecting the observer, the located object, and the reference object, such that the intersection of the line with A lies between the intersections of the line with B and the observer.

Classification The direction relations defined in Kelleher (2003) are fuzzy. They are used in a task of understanding locative expressions with the purpose of finding the object that matches the locative expression best with respect to a set of objects that are provided by the context. The boundary for binary classification, i.e. applicable and non-applicable, is controlled by the constants for maximum angular deviation (i.e. α_{max}) and maximum distance (i.e. d_{max}). Any fuzzy value greater than zero indicates applicability (to some extent), a fuzzy value of zero indicates that the relation is not applicable.

Comparison with $\mathbf{AD}_{O5}^{int/cent}$ The $\mathrm{AD}_{O5}^{int/cent}$ relations which are defined in Section 2.8 as a variant of the AD_{O5}^{int} relations are equivalent to the truth conditions specified by Kelleher's relations if we assume a threshold of $90°$. Since the LO is represented by the point of the mesh that matches the locative expressions best, truth conditions for the directional component can be expressed by overlap of angle intervals with a

$[-90°, 90°]$ acceptance interval where the angle interval spans over all directions from RO's centroid to points of the LO (compare with Figure 2.9 on page 32).

2.9.2 Regier and Carlson 2001

Regier & Carlson (2001) describe a relation schema defining locative direction relations applicable to two-dimensional spatial configurations. The relations correspond to the projective prepositions *above*, *below*, *to the right of* and *to the left of*.

Relative position The location of objects is represented by their hulls. Relative position is represented with respect to one of the prototypical directions by a vector computed by the *attentional vector sum* (AVS). The *attentional vector sum* is the average sum of weighted vectors pointing from each point of the RO to the LO. Regier & Carlson (2001) describe the procedure only for point-like LOs. The length of each vector is controlled by the *attentional field* specifying the attention each point of RO receives with respect to the computation of the AVS. It is a circular field assigning highest attention to the center of the circle and decreasing attention with increasing distance from the center. In case the focus of attention, i.e. the center of the circle, is not determined due to functional considerations, Regier & Carlson describe it as the starting point of a half-line from the hull of RO through the LO whose direction is closest aligned with the prototypical direction associated with the direction relation in question. Thus, different direction relations evoke different representations of relative position for the same spatial configuration. Figure 2.33 shows examples for *above* and *right*. For *above* the focus point is vertically below the LO, for *right* it is the top left corner of the RO. The attentional vector sum $\vec{s}$ is given by the following sum over the index i iterating over the set of points of RO.

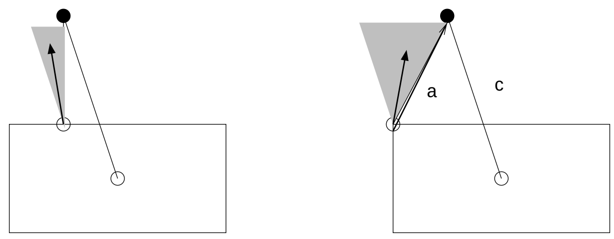
(58) $\vec{s} = \sum_i a_i \vec{c_i}$

The attentional weight a_i is defined by an exponential decay function:

(59) $a_i = exp[-\frac{d_i}{\lambda d_{LO}}]$

The parameter d_i is the distance of point i to the center of the attentional field, λ is a free parameter and d_{LO} the distance between the center of the attentional field and the LO. The latter two parameters, that is the term λd_{LO}, control the width of the attentional field. Narrow width of the attentional field assigns substantially more weight to those points which are closer to the focus of attention; and with a wide attentional field the differences between the weights of the points are very small. If the term λd_{LO} is much bigger than the range of the distance d_i, then for all d_i the weight a_i will be close to 1. Thus, the farther the LO from the RO, or to be precise the farther the LO from the focus of attention, the wider the attentional field will be.

Independent from particular spatial configurations and values of the free parameter λ, the direction of the attentional vector sum will always lie between the direction from the centroid of RO pointing towards the LO and the direction from the center of the attentional field pointing towards the LO. The examples in Figure 2.33 show the line connecting the centroid of the RO with the LO, and the range of possible directions of the AVS which is marked by grey sectors.



(a) Relative position with respect to *above*

(b) Relative position with respect to *right*

Figure 2.33: Attentional vector sum: representation of relative position.

Degree of applicability The degree of applicability app is computed by applying a graded applicability function to the attentional vector sum $\vec{s}$:

(60) $app = g(\vec{s}) * indir(\vec{s})$

The function $g(\cdot)$ is an angular alignment function $g : slope*\alpha+c$ where α denotes the angular deviation of $\vec{s}$ from the prototypical direction $\vec{\delta}$ given by the direction relation in question, $slope$ and c are free parameters. The term $indir(\vec{s})$ indicates whether LO is strictly in direction $\vec{\delta}$ (with a value of 1.0), strictly in inverse direction (with a value of 0.0), or on the same level as RO (with intermediate values). It is calculated by the following equation:

(61) $indir(\vec{s}) = \frac{1}{2}(sig(y - ymax_{RO}, maxgain) + sig(y - ymin_{RO}, 1))$

The parameter $maxgain$ is a free parameter. The variable y stands for the term $proj(\vec{s}, \vec{\delta})$ which denotes the coordinate of the projection of the point LO onto the axis given by the vector $\vec{\delta}$. The variable $ymax_{RO}$ stands for the term $max(proj(RO, \vec{\delta}))$ and $ymin_{RO}$ for $min(proj(RO, \vec{\delta}))$ yielding highest and lowest coordinate of RO in prototypical direction $\vec{\delta}$. The sigmoid function $sig(\cdot, \cdot)$ is defined as follows:

(62) $sig(v, gain) = \frac{1}{1+exp[gain*(-v)]}$

It provides a continuously differentiable function that approximates 0 for negative values of v and 1 for positive values of v. The parameter $gain$ adjusts the abruptness of the change from 0 to 1. Summarising, the relation schema has four free parameters:

the $slope$ and c of the linear-alignment function; λ which controls the width of the attentional field; and the gain of the upper sigmoid in the $indir$ function.

Returning to the applicability function in (60), the first component ($g(\vec{s})$) processes the vector $\vec{s}$ as an angular representation, and the second component ($indir(\vec{s})$) processes $\vec{s}$ as an axial representation. The degree of applicability is 0 if one of the components is 0.

Classification Regier & Carlson (2001) aim at explaining differences in the degree of applicability for cases that are all at least applicable to some extent. Therefore, they do not provide a classification in the sense of Section 2.7. Since the relations defined by Regier & Carlson (2001) are not defined on spatially extended objects, they are not applicable to the spatial domain of this study (*viz* the maps of the HCRC Map Task, see Section 5.1).

Regier & Carlson (2001) introduce a dependency of the representation of relative position on attention given to the different parts of the reference object. Furthermore, they combine axial and angular representations to compute the degree of applicability, thus combining the strengths of both paradigms.

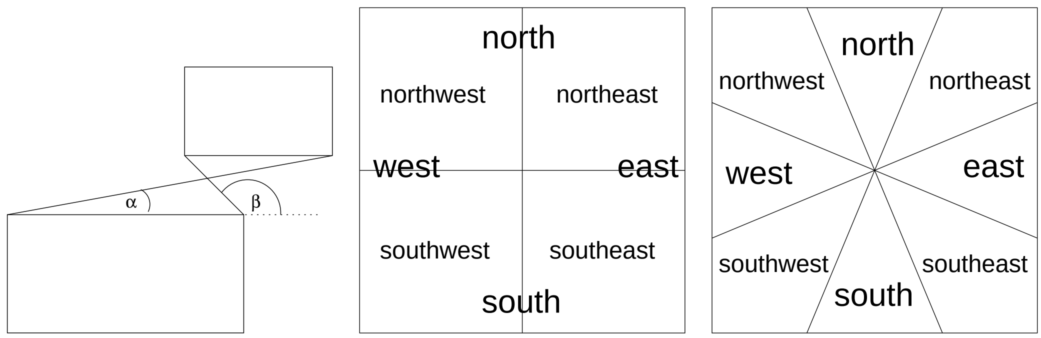
2.9.3 Schmidtke 2001

Schmidtke (2001) describes relation schemata for locative direction relations in two-dimensional space based on cardinal direction in order to provide novel relations for qualitative spatial reasoning. The study covers the direction relations *north*, *south*, *west*, *east*, *northeast*, *southeast*, *northwest*, and *southwest*. The location of an object is either represented by its hull or by a line that is part of the hull and represents a side of that object. A requirement for direction relations is that LO and RO are disconnected (Schmidtke, 2001, p422). Although Schmidtke is not explicit about this, the position of LO relative to RO is in principle represented by an angle interval. The angle interval comprises the angles of all straight lines that connect a point of RO with a point of LO. Figure 2.34(a) shows the line with the minimum angle α and one with the maximum angle β. The angles of all other straight lines connecting RO and LO lie in the interval $[\alpha, \beta]$.

The relation schemata define discrete applicability functions. Sticking to the notion of angle intervals we can define Schmidtke's direction relations in terms of the topological *part-of* relation (P) on angle intervals. Let AD be an angle interval associated with a direction relation dir, and I be the angle interval that represents the relative position of LO with respect to RO. LO is located in direction dir from RO if I is part of AD or if AD is part of I, compare with definition 10 in (Schmidtke, 2001, p426). Two distinct sets of angle intervals are used to define direction relations:

(63) a. $\{[0,0], [0,90], [90,90], [90,180], [180,180], [180,270], [270,270], [270,360]\}$

b. $\{[22.5,67.5], [67.5,112.5], [112.5,157.5], [157.5,202.5], [202.5,247.5], [247.5,292.5], [292.5,337.5], [337.5,382.5]\}$



(a) The angle interval $[\alpha, \beta]$ comprises the angles of all straight lines connecting RO with LO.

(b) Projection based direction relations.

(c) Cone-based direction relations.

Figure 2.34: Relative position and definition of directions in (Schmidtke, 2001).

The first set defines *projection based* direction relations illustrated in Figure 2.34(b). Primary cardinal directions such as *north* are associated with a single angle, that is an interval where start and end coincide. Secondary cardinal directions such as *north-west* are associated with intervals of 90°. The second set defines *cone-based* direction relations. Each direction relation is associated with an interval of 45°, see Figure 2.34(c). Schmidtke notes that in contrast to to cone-based relations, projection-based relations are exhaustive. They are defined for all pairs of non-overlapping objects. However, they are not mutually exclusive!

The projection-based relations for the principal cardinal directions *north*, *south*, *east*, and *west* are equivalent to the set of relations defined by the relation schema $\mathrm{AD}^{int}_{cross}$ which is defined as a disjunction of AD^{int}_{O1} and AD^{int}_{P4}, see page 60.

2.9.4 Matsakis, Wendling, and Keller 2001, 1999

Matsakis et al. (2001) and Matsakis & Wendling (1999) describe a computational system that generates locative descriptions of two-dimensional images. It covers projective prepositions *above*, *below*, *to the right of*, and *to the left of* in combination with the modifiers *perfectly, nearly, mostly, loosely, somewhat, strongly, a little*, and *slightly*. The location of objects is represented by complete geometric descriptions (see Section 2.2). The position of LO relative to RO is represented by a *histogram of* *forces*. A *histogram of forces* is an angular representation that conveys for any direction (expressed by an angle) a force exerted by LO on RO, so that RO tends to move in that direction. Matsakis & Wendling consider two kinds of forces resulting in a histogram of *constant forces* (F_0-histogram)[9] and a histogram of *gravitational forces* (F_2-histogram).

The computation of the histogram of forces is based on three functions: φ_r computes the force between to points, f_r handles the force between two segments on a straight line, and F_r combines the force exerted by multiple segments to longitudinal sections. Let u and v be the coordinates of two points on a straight line, then the force between them is given by $\varphi_r(u, v)$.

(64) $$\varphi_r(u,v) = \begin{cases} \frac{1}{(u-v)^r} & \text{if } u - v > 0 \\ 0 & \text{if } u - v \leq 0 \end{cases}$$

The forces between two line segments on a straight line are computed by $f_r(x, y, z)$ where x is the length of the line segment belonging to LO, z the length of the line segment belonging to RO, and y the distance between them.

(65) $$f_r(x,y,z) = \int_{y+z}^{x+y+z} (\int_0^z \varphi_r(u,v)dv)du$$

$f(x, y, z)$ applies φ_r to all combinations of points from the two segments of LO (from $y + z$ to $x + y + z$) and RO (from 0 to z). In case the intersection of a particular straight line with LO or RO yields more than one line segment, the values of function f_r are summed up for all possible ways of combining line segments of LO with line segments of RO by the function $F_r(\theta, LO^{\theta v}, RO^{\theta v})$ where $LO^{\theta v}$ and $RO^{\theta v}$ denote sets of mutually disjoint line segments of LO and RO on a straight line defined by the angle θ and the point $(0, v)$ on the vertical axis. The variables lo and ro are single line segments iterating over those sets, and the function $\Delta(lo, ro)$ returns the distance between those line segments.

(66)
$$F_r(\theta, LO^{\theta v}, RO^{\theta v}) = \sum_{lo \in LO^{\theta v}, ro \in RO^{\theta v}} f_r(lo, \Delta(lo, ro), ro)$$

The total force exerted by LO on RO in direction θ is computed by integrating over v, that means, over all straight lines defined by the angle θ:

(67)
$$F_r^{LO,RO}(\theta) = \int_{-\infty}^{+\infty} F_r(\theta, LO^{\theta v}, RO^{\theta v})dv$$

The histogram of forces $H(F_r^{RO,LO})$ is a function from angles into positive real numbers including zero with a period of 360°. It is adjusted to a particular direction

[9] F_0-histograms are fundamentally equivalent to histograms of angles (Miyajima & Ralescu, 1994).

relation DIR by shifting all values by $90°$, $180°$, or $270°$, e.g. for the relation *above* $F_r^{RO,LO,above}(\theta) = F_r^{RO,LO}(\theta + 90°)$.

An analysis of the histogram of forces with respect to a particular direction DIR leads to a division of the histogram into *effective*, *contradictory*, and *compensatory* forces. The effective forces are further divided into *optimal* and *sub-optimal* components, and they are used to compute the average direction $\alpha_r(DIR)$, an angle which represents the average direction of optimal and sub-optimal components. *Contradictory* forces are those forces which are associated with an angular deviation of more than 90°. One part of the forces with a deviation of less than 90° is used to compensate the contradictory forces, the other part is called *effective forces*. Below we determine two values θ_- and θ_+ which divide compensatory and effective forces. All forces between θ_- and θ_+ are effective forces. Forces between θ_+ and 90° compensate contradictory forces between 90° and 180°. Forces between -90° and θ_- compensate contradictory forces between -180° and -90°:

(68) Choose θ_+ such that the following condition is met:

$$\begin{cases} \int_{\theta_+}^{180°}(\theta - 90°)F_r^{LO,RO}(\theta)d\theta = 0 & ; \int_{-90°}^{180°}(\theta - 90°)F_r^{LO,RO}(\theta)d\theta \leq 0 \\ \theta_+ = -90° & ; \int_{-90°}^{180°}(\theta - 90°)F_r^{LO,RO}(\theta)d\theta > 0 \end{cases}$$

(69) Choose θ_- such that the following condition is met:

$$\begin{cases} \int_{\theta_-}^{-180°}(\theta + 90°)F_r^{LO,RO}(\theta)d\theta = 0 & ; \int_{90°}^{-180°}(\theta + 90°)F_r^{LO,RO}(\theta)d\theta \geq 0 \\ \theta_- = 90° & ; \int_{90°}^{-180°}(\theta + 90°)F_r^{LO,RO}(\theta)d\theta < 0 \end{cases}$$

There are only effective forces if $\theta_+ > \theta_-$. A threshold τ divides the effective forces into optimal and sub-optimal components. The threshold τ is defined as a weighted average of the effective forces. The weighting function S is a trapezoidal fuzzy membership function on the interval $[-90°, 90°]$:

(70)

$$S(\theta) = \begin{cases} 1; & \text{if } |\theta| < 22.5° \\ 2 - \frac{|\theta|}{22.5°}; & \text{if } 22.5° \leq |\theta| \leq 45° \\ 0; & \text{if } 45° < |\theta| \end{cases}$$

(71)

$$\tau = \frac{\int_{\theta_-}^{\theta_+} S(\theta)F_r^{LO,RO}(\theta)d\theta}{\int_{-90°}^{90°} S(\theta)d\theta}$$

Matsakis et al. (2001) stipulate that optimal components directly support the force that drags RO in direction DIR. Sub-optimal components distract from DIR. The average direction $\alpha_r(DIR)$ is an angle that deviates from the prototypical direction towards the direction given by the sub-optimal forces proportional to the ratio of sub-optimal forces to all effective forces:

(72)

$$\alpha_r(DIR) = \frac{\int_{\theta_+}^{\theta_-} \theta[max(0, F_r^{LO,RO}(\theta) - \tau)]d\theta}{\int_{\theta_+}^{\theta_-} F_r^{LO,RO}(\theta)d\theta}$$

Matsakis et al. (2001) use fuzzy applicability functions on histograms of forces. They are parametrised with the order of the force (i.e. 0 for constant forces and 2 for gravitational forces) with the direction that is associated with a particular direction relation. The term $a_r(DIR)$ expresses the degree of truth of LO being in direction of RO, and $b_r(DIR)$ expresses the proportion of the effective forces relative to all forces exerted by RO on LO.

The proportion of the effective forces with respect to all forces is defined as the ratio of the effective forces (eff_r) to the sum of the effective forces, the compensatory forces ($comp_r$) and the contradictory forces ($cont_r$).

(73) $b_r(DIR) = \frac{eff_r}{eff_r + comp_r + cont_r}$

Let $\mu(x)$ be a triangular membership function of a fuzzy set on $[-180°, 180°]$ as shown in Figure 2.35. The degree of truth of the average direction $\alpha_r(DIR)$ being in direction DIR is computed as follows:

(74) $a_r(DIR) = \mu(\alpha_r(DIR)) * b_r(DIR))$

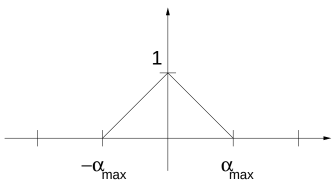


Figure 2.35: Triangular fuzzy set membership function.

Matsakis et al. (2001) propose a complex system for determining a direction relation that consists of two components, namely a modifier and a projective term, e.g. *perfectly-above*. Such combinations are selected dependent on the fuzzy truth value of the parameters $a(DIR)$ and $m(DIR)$ which are derived from the parameters $a_0(DIR)$, $b_0(DIR)$, $a_2(DIR)$, and $b_2(DIR)$, see (73) and (74). The selection algorithm is an attempt to combine the strengths of both histograms of forces, H_0 and H_2, a detailed discussion can be found in (Matsakis et al., 2001, p14).

(75) a. $a(DIR) = \begin{cases} b_0(DIR); & a_2 > b_0 \\ a_0(DIR); & a_0 > b_2 \\ \max(a_0(DIR), a_2(DIR)); & a_2 \leq b_0 \text{ or } a_0 \leq b_2 \end{cases}$

b. $m(DIR) = \min(b_0(DIR), b_2(DIR))$

a(DIR)\m(DIR)	high	medium-high	medium-low
high	perfectly	–	nearly
medium-high	–	nearly	loosely
medium-low	mostly	loosely	loosely

Table 2.1: Determining a modifier for primary direction.

a(DIR)\m(DIR)	high	medium
high	somewhat	strongly
medium	a little	slightly

Table 2.2: Determining a modifier for secondary direction.

Let $a(DIR)$ be the degree of applicability of the proposition "LO is in direction DIR of RO". The value $m(DIR)$ is a measure of the extent both histograms agree on the fact that LO can be considered in direction DIR of RO.

Classification of direction relations is provided by tables relating $a(DIR)$ and $m(DIR)$ and determining a modifier for each combination that is combined with the basic relation associated with DIR. The range of the parameters is divided into sub-ranges by fuzzy membership functions. Table 2.1 is defined for a partitioning into four sub-ranges, namely *high*, *medium-high*, *medium-low*, and *low*. And Table 2.2 for a partitioning into three sub-ranges *high*, *medium*, and *low*. If one of the values is *low*, the relation is not applicable. Otherwise a modifier from the table is selected and combined with the relation associated with DIR. The entry "–" indicates that no modifier should be used. For example, applying high $a(above)$ and high $m(above)$ to the first table yields the relation *perfectly-above*, to the second table the same parameters produce the relation *somewhat-above*. A high $a(above)$ value and a medium-high $m(above)$ gives us the unmodified relation *above*.

Matsakis et al. (2001) use two tables to generate locative expressions describing a primary direction and a secondary direction, for example "LO is mostly to the right of RO but somewhat above." The first part of the locative expression is generated using Table 2.1 and the second part using Table 2.2.

2.9.5 Logan and Sadler 1996

Logan & Sadler (1996) describe spatial relations in terms of the general notion of a *spatial template*. A *spatial template* is a template for a field that determines the degree of applicability to any point in space with respect to the corresponding spatial relation. It is applied to a particular spatial scene by adjusting it to the RO and aligning it with the relevant frame of reference. Logan & Sadler (1996) discuss spatial templates of the following projective prepositions: *above, below, over, under, left of, right of*, and *next to*. They implement spatial templates by a regular, orthogonal 7×7 grid. They report studies where the grid is centered on the RO and the LO is part of one other cell of the grid. The degree of applicability of a direction relation relative to a pair of LO and RO is provided by the value of the cell in the spatial template that is filled by the LO. Logan & Sadler (1996) associate the cells of the spatial templates with the average values of a *relation judgement* experiment where subjects had to rate pairs of LO and RO with respect to a projective preposition on a scale from 1 to 9. They conclude that the spatial templates for direction relations are divided into three distinct regions of acceptability: *good*, *acceptable*, and *not acceptable*. Figure 2.36 illustrates this for *above*. Cells in the dark grey region (*good*) contain the highest degree of applicability. Light-grey regions mark *acceptable* regions, they contain cells with high degrees of applicability. The white region marks positions which are *not acceptable*, it contains cells with low degrees of applicability. While there is a sharp border between *not acceptable* and *acceptable* positions, the *acceptable* and *good* regions blend into one another gradually – the closer the LO is to the *good* region the higher the degree of applicability. For generating locative descriptions, Logan & Sadler (1996) propose to

7.00	7.66	8.10	8.61	8.19	7.32	7.66
6.69	6.56	7.66	8.55	7.13	7.16	6.88
5.63	6.41	7.09	8.53	7.35	6.74	5.53
1.94	2.16	1.88		1.97	1.88	2.00
1.94	1.78	1.66	1.13	1.63	2.41	1.66
1.81	1.94	1.42	1.03	1.50	1.84	1.58
1.44	1.38	1.34	1.19	1.34	2.08	1.44

Figure 2.36: Spatial template for *above*

select the projective preposition whose spatial template produces the highest degree of applicability. If two competing templates fit reasonably well, both prepositions might be produced, e.g., *above and to the right*.

2.9.6 O'Keefe 1996, 2003

O'Keefe (2003, 1996) describes the semantics of spatial prepositions in 3D and 2D space. He proposes meaning definitions for the projective expressions *above, up, over, on top of, below, under, underneath, beneath, beside, behind,* and *beyond*. O'Keefe (2003) considers the modifiers *just* and *far*. Besides specifying truth conditions, a great deal of the discussion attends to the question of degrees of applicability and the semantics of comparative constructions such as “b is *more* under a than c”. The locations of objects are represented by bounding boxes or by points. Direction relations are mainly defined in terms of discrete applicability functions on axial representations.

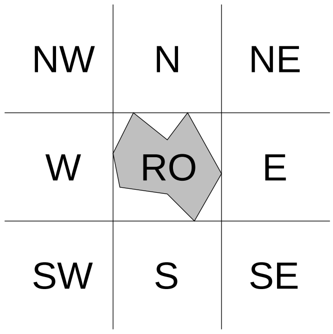


Figure 2.37: 3×3 grid defined by the bounding box of the RO.

The prepositions *above* and *below* denote direction relations defined by the condition that the LO is a part of the half-plane $NW \cup N \cup NE$ or $SW \cup S \cup SE$ relative to RO, respectively, see Figure 2.37. The prepositions *under* and *over* are either synonyms of *above* and *below*, or they overlap with the regions N or S, respectively. The prepositions *on top of*, *beneath*, and *underneath* denote even more specific direction relations. They apply if the LO is part of N or S, respectively. The prepositions *beside* denotes a direction relation that is true if the LO is contained in the W or E region relative to RO. The preposition *beyond* requires an alignment of the frame of reference such that the north-south axis is aligned with the vector from the observer to the RO. The direction relation denoted by *beyond* is then identical to the one of *above*. The next condition deviates from previous conditions by not relying on axial representations. The preposition *behind* denotes a direction relation which is true if the line from the observer to the LO intersects with the RO. The prepositions *up* and *down* are given only path related meanings and are therefore not considered here. In (O’Keefe, 2003) direction relations are refined by fuzzy applicability functions. Most refined relations, however, are identical with the original relations as regards the truth conditions. The refinements augment a direction relation with internal structure in order to facilitate comparing instances with respect to their degree of applicability. The revised definitions of *under* and *beside* differ from previous truth conditions. The preposition *under* is true, if LO is part of the S region relative to RO, and *beside* is true, if LO is a part of the regions W or E relative to some rectangle that is bigger than the bounding box of RO. Modifiers such as *just* and *far* place restrictions on the length of the distance from LO to RO. Summarising the relations defined by O’Keefe, most of them can be defined in terms of OP$_{P1}$, OP$_{O2}$, and OP$_{P2}$.

2.9.7 Fuhr, Socher, Scheering, and Sagerer 1995

Fuhr et al. (1995) describe a computational system for generating and understanding projective locative expressions in 3D space with respect to a task of constructing a toy-plane. They cover the projective locative expressions that are used in the relative frame of reference and that contain the terms *left*, *right*, *in front of*, *behind*, *above*, and *below*. Location of objects is represented by 3D bounding boxes which are aligned to the object's principal axes. The position of LO relative to RO is represented by an array of vectors, each vector representing a particular cell of an angular deviation grid. The direction of a vector represents the average direction of the cell and its length expresses the ratio of the object's volume that intersects with the cell. The grid is centered around the RO in a 45° pattern. It is a 3D extension of the partitioning that Hernandez (1994) uses for objects in *close proximity* in two-dimensional space, see Figure 2.38(b) in Section 2.9.10. Further note, that in (Hernandez, 1994) the bounding boxes are aligned externally to the horizontal and vertical axes of the coordinate system, and in (Fuhr et al., 1995) they are aligned intrinsically, so sides of the bounding boxes of two different objects are not necessarily aligned. In 3D the grid consists of 79 cells: the bounding box of the RO (1), one cell for each side (6), two cells for each edge (24), and six cells for each vertex (48). Relative position is represented by an array of 78 vectors. The bounding box of the RO is not associated with a direction. Let $\vec{e}_i$ be the unit vector that expresses the average direction of cell $cell_i$, then the array consists of the following vectors:

(76) $\vec{c}_i = \frac{|LO \cap cell_i|}{|LO|}\vec{e}_i$

The degree of applicability of a direction relation with respect to a pair of LO and RO is computed by a fuzzy applicability function that is applied to the vector representing each cell. Given that every direction relation is associated with a unit vector $\vec{e}_{dir}$, the term $|\angle(\vec{c}_i, \vec{e}_{dir})|$ denotes the angular deviation between that vector and the vector representing cell $cell_i$. The degree of applicability for cell $cell_i$ is defined as follows:

(77) $app(\vec{c}_i) = \begin{cases} 1 - \frac{|\angle(\vec{c}_i, \vec{e}_{dir})|}{90^\circ} & \text{if } |\angle(\vec{c}_i, \vec{e}_{dir})| \leq 90^\circ \\ 0 & \text{if } |\angle(\vec{c}_i, \vec{e}_{dir})| > 90^\circ \end{cases}$

The degree of applicability is maximal ($app(\vec{c}_i) = 1$) if the vector of the cell is aligned with the prototypical direction associated with the direction relation in question. It is minimal ($app(\vec{c}_i) = 0$) if the angle is greater than 90°. The overall degree of applicability is the weighted sum of the degree of applicability of each cell multiplied by the length of the vector representing it:

(78) $app = \sum_i |\vec{c}_i| \; app(\vec{c}_i)$

For generating projective locative expressions, Fuhr et al. (1995) pick the preposition that is associated with the direction relation with the highest degree of applicability. For a spatial interpretation of projective locative expressions, they activate those cells whose inherent direction is aligned with the prototypical direction specified by the projective preposition. The degree of alignment is determined by application of the applicability function to the corresponding acceptance area. All acceptance areas that receive a higher degree of applicability than a threshold of 0.3 count as spatial interpretation of the locative expression. Applied to the AD^{Grid} relations defined in Section 2.8 the cells with the following deviation are activated as possible acceptance areas: $0°$, $22.5°$, and $67.5°$. That means the relations of Fuhr et al. (1995) correspond to AD$_5^{Grid}$ relations.

2.9.8 Gapp 1994, 1996

Gapp (1994a, 1996) presents a system for generating locative expressions in 3D and 2D space. Gapp (1994a) provides direction relations corresponding to the projective prepositions *in front of, behind, to the right of, to the left of, above, below,* and *beside*. Gapp (1996) presents direction relations corresponding to cardinal directions. The system described in (Gapp, 1994a) handles locative expressions in the relative and the intrinsic frames of reference. It implements a default priority of the intrinsic frame of reference determined by *intrinsic fronts* over an intrinsic frame of reference evoked by *accidental fronts*, over the default relative frame of reference.

Gapp's relation schema employs angular representations and fuzzy applicability functions. It is intended to be applied to objects which are disconnected from each other. The location of objects is represented by geometric descriptions of their hull. Relative position of LO with respect to RO is represented by a vector $\vec{l}$ from the centroid of the RO ($loc_{cent}(ro)$) to the centroid of the LO ($loc_{cent}(lo)$) which is adjusted to a scale that is defined by the sides of RO's bounding box. Let $\vec{\rho}$ be the prototypical direction that is specified by the projective preposition with respect to a particular frame of reference. Let the function $proj(a, b)$ yield the orthogonal projection of a onto b, and let $\perp\vec{v}$ denote a vector that is orthogonal to $\vec{v}$. In 2D-space the sides of the bounding box of RO are defined by the terms $proj(ro, \vec{\rho})$ and $proj(ro, \perp\vec{\rho})$. Thus the vector $\vec{l}$ representing the position of LO relative to RO is defined by the following product of a vector and a matrix:

(79) $$\vec{l} = \overrightarrow{loc_{cent}(ro)loc_{cent}(lo)} * \begin{pmatrix} |proj(RO, \vec{\rho})| & 0 \\ 0 & |proj(RO, \perp\vec{\rho})| \end{pmatrix}$$

The angle α expresses the angular deviation $\vec{l}$ from the prototypical direction ρ:

(80) $$\alpha = |\angle(\vec{l}, \vec{\rho})|$$

The degree of applicability is the product of the degree of applicability with respect to direction and the degree of applicability with respect to distance:

(81) $$app(p) = app_{dir}(p) * app_{dist}(p)$$

The applicability functions app_{dir} and app_{dist} are monotone decreasing fuzzy applicability functions, the mathematical details are not specified in the papers. The function app_{dir} yields 1.0 for an angular deviation of $\alpha = 0°$. The value decreases with increasing angular deviation. It is zero for α greater than some maximum deviation (which is a free parameter). The function app_{dist} yields 1.0 for zero distance, and it decreases with increasing distance. It is zero when the distance exceeds some maximal distance (which is a free parameter).

Gapp (1994a, 1996) defines fuzzy geometric direction relations. Fuzzy values greater than zero indicate applicability (to some extent), a fuzzy value of zero indicates that the relation is not applicable. The relations are equivalent to AD_4^{cent} which is associated with a threshold of $90°$.

2.9.9 Olivier and Tsujii 1994

Olivier & Tsujii (1994) describe a system for automatic visualisation of spatial descriptions in 2D. They describe a treatment of the projective prepositions *in front of*, *behind*, *left of*, and *right of*. The system can interpret locative expressions with respect to all three frames of reference. The absolute frame of reference is implemented as a projection of the intrinsic orientation of a third object onto the RO. For example, in order to describe the position of an object relative to another object, we can make use of the intrinsic orientation of a room (if it has any orientation at all). In general, objects are associated with intrinsic orientations. The location of an object is represented by a single point, additionally there is information about the spatial extension of an object in arbitrary directions. Relative position is represented by means of a vector $\binom{x}{y}$. Direction relations are defined via cost functions from vectors to positive real numbers including zero. The cost P increases with increasing length and with increasing angular deviation of the vector from the prototypical direction ρ that is associated with the projective preposition.

(82) $$P = P_{prox} + P_{dir}$$

For the computation of P we assume a local coordinate system where the vertical axis, i.e. the y axis, is aligned with the prototypical direction of the relation, and the origin of the coordinate system is centered on RO. Relative coordinates of LO with respect to RO are given by x and y, the distance between RO and LO is given by d.

(83) $$\begin{aligned} x &= x_{LO} - x_{RO} \\ y &= y_{LO} - y_{RO} \\ d &= \sqrt{x^2 + y^2} \end{aligned}$$

The cost of the distance is dependent on the squared difference of distance d and some optimal distance L_{prox} and on a factor K_{prox}:

(84) $$P_{prox} = \frac{K_{prox}}{2}(d - L_{prox})^2$$

The cost of the direction is dependent on the distance of LO from the vertical axis of the local coordinate system and on some factor K_{dir}:

(85) $P_{dir} = \frac{K_{dir}}{2} x^2$

The actual values of K_{prox}, L_{prox}, and K_{dir} have to be adjusted to the task at hand. In Olivier & Tsujii (1994) K_{prox} and L_{prox} are linearly dependent on the sum of the spatial extension of LO and RO in the direction of the prototypical direction and K_{dir} is linearly dependent on the spatial extension of the RO perpendicular to the prototypical direction. Let $T_g(o)$ be the orthogonal projection of a spatially extended object o onto a directed axis g. Let the vector $\vec{\rho}$ indicate the prototypical direction of the relation and the vector $\vec{\rho_T}$ be perpendicular to $\vec{\rho}$; c_1, c_2, and c_3 are constants:

(86) a. $K_{prox,\vec{\rho}} = c_1 \cdot (|T_{\vec{\rho}}(ro)| + |T_{\vec{\rho}}(lo)|)$
b. $L_{prox,\vec{\rho}} = c_2 \cdot (|T_{\vec{\rho}}(ro)| + |T_{\vec{\rho}}(lo)|)$
c. $K_{dir,\vec{\rho}} = c_3 \cdot (|T_{\vec{\rho_T}}(ro)|)$

Olivier & Tsujii (1994) determine the global minimum of the cost function K. It describes the point which is the optimal candidate for the interpretation of locative expression in question. They do not provide a classification in the sense of Section 2.7.

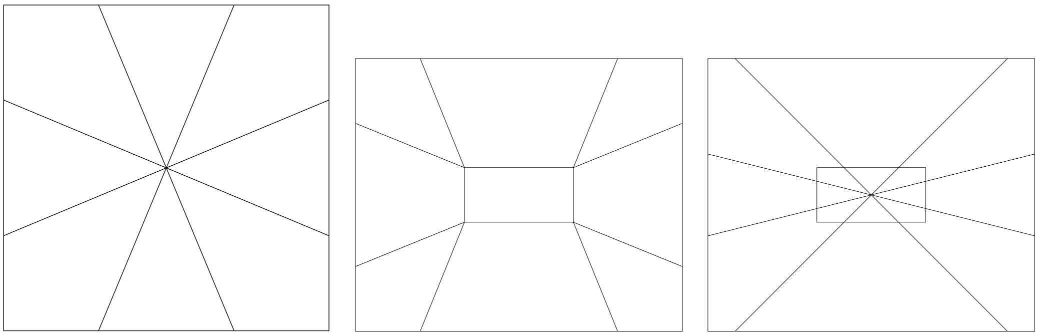
2.9.10 Hernandez 1994

Hernandez (1994) describes relation schemata for representing and reasoning about the location of 2D layout plans of offices. The relations defined comprise four base relations *right*, *left*, *front*, and *back* on various levels of granularity, and some fine-grained composite relations, e.g. *front-left*. Their application is considered in all frames of reference: intrinsic, relative, and absolute. The latter is used when the orientation is imposed by the space containing LO and RO, e.g. a room. The location of an object is represented by a centroid and a bounding box. Direction relations are defined via discrete applicability functions on the angular component of vectors.

Hernandez (1994) distinguishes between three distinct types of relation schemata whose application is controlled by domain restrictions. Either the objects *overlap*, they are in *close proximity*, or they are *far away*. The LO is in *close proximity* of the RO if LO and RO do not overlap and if the centroid of the LO falls within an area up to three times the maximum radius of the RO, where the maximum radius of RO is defined by the minimum bounding circle – the minimal circle that completely includes the RO. The LO is *far away* of RO if it is not in *close proximity* nor overlaps with RO.

Every relation schema defines sectors or regions which are associated with a particular direction relation. A direction relation applies to a particular pair of LO and RO if the centroid of the LO lies within the associated sector.

The first domain restriction only admits LO and RO that are *far away*. Both LO and RO are represented as centroids, and the half-lines defining the sectors start in



(a) For objects that are far away. (b) For objects in close proximity. (c) For overlapping objects.

Figure 2.38: Types of relation schemata used by

RO's centroid. As a default (Hernandez, 1994, p40,41) assumes 8 regular sectors with an angle of 45° as shown in Figure 2.38(a), but textual and situational context might indicate that only four sectors with 90° or even only two sectors with 180° should be used. If LO is on the borderline between two sectors, the procedure either recurs to a sector model with a smaller number of sectors, or to a complementary model with the same number of sectors (Hernandez, 1994, p51). The second domain restriction is that LO and RO are in *close proximity*. In that case the half-lines defining the sectors are shifted from the centroid of the RO to the corners of the bounding box, see Figure 2.38(b). Finally, if LO and RO overlap, space is partitioned into 8 sectors which start in the center of the RO and are aligned with the sides and the corners as shown in Figure 2.38(c). A precise description of the partitioning schema is missing, but the diagram suggests that the partitions ensure that every corner and each center of a side is on the middle axis of a sector.

For objects that are *far away* the relations defined by Hernandez are equivalent to AD_2^{cent} and AD_4^{cent}, for the relations of the other two distance conditions there are no equivalent relation schemata. But for objects that are in *close proximity* the relation schema AD^{Grid} might provide similar relations, and for *overlapping* objects the relations defined by AD_4^{cent} provide equivalent relations at least for the coarsest level of granularity with half-planes as acceptance areas.

2.9.11 Papadias and Sellis 1994

Papadias & Sellis (1994) and Papadias & Theodoridis (1997) present a relation schema for qualitative spatial reasoning in geographic information systems. They define relations for the cardinal directions *north*, *south*, *east*, *west*, *north-east*, *north-west*, *south-east*, and *south-west*, for the relation *same-level*. Additionally, they describe fine-grained versions of these relations indicated by one of the prefixes *restricted*, *strong*, *strong-bound*, *just*, *weak*, and *weak-bounded*. Objects are represented by bounding boxes aligned to the external coordinate system. An entire spatial scene is represented by a *spatial index* modelling the relative order of the corners of all bounding boxes. Locative direction relations are defined by discrete applicability functions in terms of interval relations or topological conditions on the 3×3 grid imposed by the sides of RO's bounding box, see Figure 2.39.

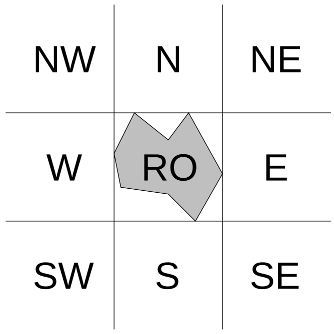


Figure 2.39: 3×3 grid defined by the bounding box of the RO.

Papadias & Sellis (1994) use the term *primitive relations* for relations that express the location of a point relative to the bounding box of the RO. The prefix *restricted* is used to mark *north*, *south*, *east*, and *west* as primitive relations, which are associated with the regions N, S, E, and W, respectively, relative to RO. The primitive relation *same-position* expresses that the point representing LO is a part of the bounding box around RO. The relation *same-level* is defined as disjunction of *restricted-east*, *restricted-west*, and *same-position*.

Direction relations between two bounding boxes are defined for the direction *north* and apply to the other principal cardinal directions in a similar fashion. Let $y_{\min}(a)$ and $y_{\max}(a)$ denote the minimum and maximum vertical coordinates of a, respectively, and $x_{\min}(a)$ and $x_{\max}(a)$ the minimum and maximum horizontal coordinates, and let $loc_b(lo)$ and $loc_b(ro)$ denote bounding boxes of LO and RO, respectively.

The relation *strong-north* is defined by the condition that $loc_b(lo)$ is a proper part of $NW \cup N \cup NE$, see (87) below for definitions in terms of axial representations and see Figure 2.40 for illustrations. The relation *strong-bounded-north* holds if $loc_b(lo)$ is a proper part of N (87-b), *strong-northeast* holds if $loc_b(lo)$ is a proper part of NE. LO is *just-north* of RO, if $loc_b(lo)$ is a tangential proper part of $NW \cup N \cup NE$ and externally connected to $E \cup loc_b(ro) \cup W$, see (87-d). The relation *weak-north* holds if $loc_b(lo)$ overlaps with $NW \cup N \cup NE$ but not with $SW \cup S \cup SE$, see (87-e), and *weak-bounded-north* holds if $loc_b(lo)$ is *weak-north* of $loc_b(ro)$ and a proper part of $N \cup loc_b(ro)$, see (87-f). The relation *weak-northeast* specifies that $loc_b(lo)$ overlaps with NE, and is a proper part of $loc_b(ro) \cup N \cup E \cup NE$, see (87-g), and finally, LO is *north-south* of RO if $loc_b(lo)$ overlaps with $NW \cup N \cup NE$, and with $SW \cup S \cup SE$, see (87-h). These are the relations defined in terms of axial representations:

(87) a. strong-north(p,q) : $y_{\min}(p) > y_{\max}(q)$

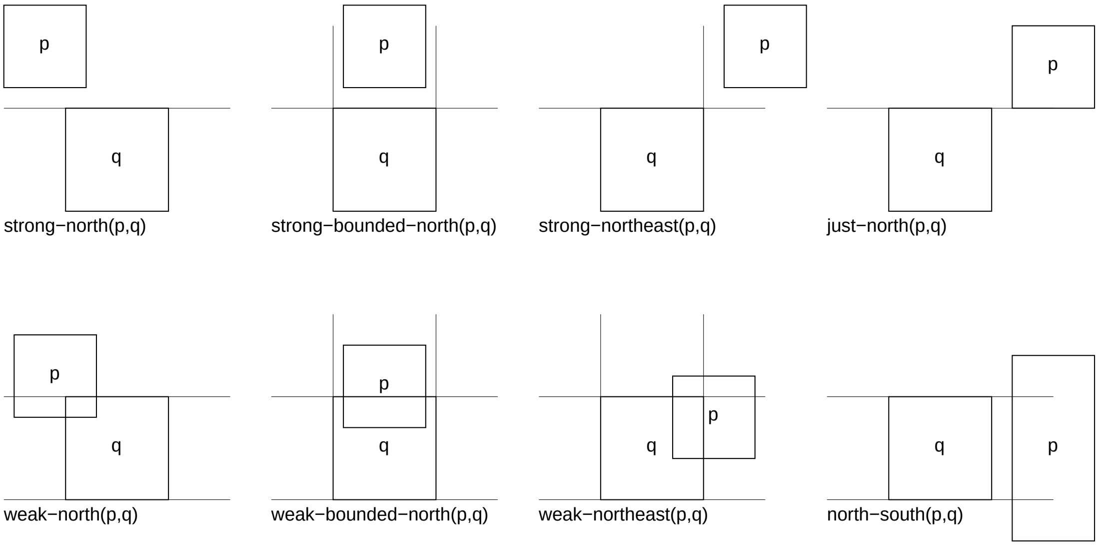


Figure 2.40: Refinements of the north relation as illustrated in Papadias & Sellis (1994)

b. strong-bounded-north(p,q) : $y_{\min}(p) > y_{\max}(q) \wedge x_{\min}(q) < x_{\min}(p) < x_{\max}(p) < x_{\max}(q)$

c. strong-northeast(p,q) : $y_{\min}(p) > y_{\max}(q) \wedge x_{\min}(p) > x_{\max}(q)$

d. just-north(p,q) : $y_{\min}(q) < y_{\max}(q) = y_{\min}(p) < y_{\max}(p)$

e. weak-north(p,q) : $y_{\min}(q) < y_{\min}(p) < y_{\max}(q) < y_{\max}(p)$

f. weak-bounded-north(p,q) : $y_{\min}(q) < y_{\min}(p) < y_{\max}(q) < y_{\max}(p) \wedge x_{\min}(q) < x_{\min}(p) < x_{\max}(p) < x_{\max}(q)$ [10]

g. weak-northeast(p,q) : $y_{\min}(q) < y_{\min}(p) < y_{\max}(q) < y_{\max}(p) \wedge x_{\min}(q) < x_{\min}(p) < x_{\max}(q) < x_{\max}(p)$

h. north-south(p,q) : $y_{\min}(p) < y_{\min}(q) \wedge y_{\max}(q) < y_{\max}(p)$

All relations are illustrated in Figure 2.40. The meaning of the prefixes can be described in the following way. The modifiers *strong*, *just*, and *weak* express conditions in terms of interval relations between the projections of LO and RO onto an axis aligned with the prototypical direction. The modifier *strong* holds if the projection of LO is discrete from RO's projection. The modifier *just* specifies external connection, and the modifier *weak* specifies overlap in the sense of the interval relation $overlaps^{-1}$ defined in (16) in Section 2.5 on page 36. The modifier *bounded* carries restrictions on projections onto an axis orthogonal to the prototypical direction, it holds if the projection of LO is a proper part of the projection of RO.

The relations defined by Papadias & Sellis (1994) are best matched by OP_{OP} relations – they are not equivalent but OP_{OP} relations imply the relations defined here.

[10]If you compare with the original definition, note, that I take the subformula $i < l$ in the last conjunct to be the subformula $j < l$.

strong-bounded-north and *weak-bounded-north* are implied by *north* from OP_{OP2}; *strong-north* is implied by *north* from OP_{OP4}; and *weak-north* is implied by OP_{OP5}. The combined relations such as *northeast* and *north-south* are not provided by the OP_{OP} relation schema. The relation *just-north* cannot be modelled by OP_{OP} relations, because OP_{OP} relations – as any of the relations we defined in Section 2.8 – cannot express the topological relation of a region being a *tangential part of* another region.

2.9.12 Topaloglou 1994

Topaloglou (1994) describes a logic for expressing "approximate" topological and directional relations in 2D space. He defines direction relations *straight-north*, *straight-east*, *straight-south*, *straight-west*, *northeast*, *northwest*, *southeast*, and *southwest*. The location of objects is represented by single points, and the position of LO relative to RO is represented by a vector. Direction relations are defined by discrete applicability functions. Topaloglou (1994) defines "approximate" direction relations by weakening the notion of vertical and horizontal collinearity. He replaces the notion of spatial coincidence by a vicinity predicate.

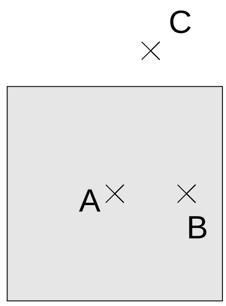


Figure 2.41: B is in the vicinity of A which is marked by the grey box, but C is not.

The vicinity of a point A is constructed as a square centered on A. In Figure 2.41, B is in the vicinity of A, but C is not. Vicinity makes it possible to define a point p as vertically collinear to another point q if it is in the vicinity of some point on the vertical axis through q. And similarly, p is horizontally collinear to q if it is in the vicinity of some point on the horizontal axis through q. All '*straight*' relations, e.g. *straight-north*, require this kind of horizontal or vertical collinearity. The relations corresponding to secondary cardinal directions, such as *northwest*, exclude both horizontal and vertical collinearity.

We can reformulate Topaloglou's relation schema in the terms we have been using in previous sections of this chapter by representing the location of RO as a rectangle which includes all points lying in the vicinity of points of RO. Axial representations of a point LO with respect to the rectangle defined by RO enable us to use standard definitions of spatial relations, e.g. p is *straight-north* of q if p is a part of the N region with respect to RO, etc. The relations defined in this way correspond to OP_{P1} and OP_{P2}.

2.9.13 Rajagopalan 1993

Rajagopalan (1993) proposes a relation schema for qualitative spatial reasoning that is capable of reasoning about relative positions in two-dimensional space and the effects of translational motion. It defines the direction relations *aligned-x, aligned-y, left-of, right-of, overlap-left-boundary*, and *overlap-right-boundary*. The location of objects is represented by bounding boxes which are aligned to an external coordinate system. Direction relations are defined by discrete applicability functions that are based on conditions on axial representations. Let $y_{\min}(a)$ and $y_{\max}(a)$ denote the minimum respective maximum vertical coordinate of a, and $x_{\min}(a)$ and $x_{\max}(a)$ the minimum and maximum horizontal coordinates.

(88)
a. left-of(A,B): $x_{\max}(A) < x_{\min}(B)$
b. aligned$_x$(A,B): $y_{\max}(A) > y_{\min}(B) \wedge y_{\min}(A) < y_{\max}(B)$
c. aligned$_y$(A,B): $x_{\max}(A) > x_{\min}(B) \wedge x_{\min}(A) < x_{\max}(B)$
d. overlapped(A,B): $aligned_x(A,B) \wedge aligned_y(A,B)$
e. overlap-left-boundary(A,B):
$overlapped(A,B) \wedge x_{\max}(A) \leq x_{\max}(B) \wedge x_{\min}(A) < x_{\min}(B)$
f. overlap-right-boundary(A,B):
$overlapped(A,B) \wedge x_{\min}(B) \leq x_{\min}(A) \wedge x_{\max}(A) > x_{\max}(B)$
g. right-of(A,B): $x_{\max}(B) < x_{\min}(A)$

The relations *aligned*$_x$ and *aligned*$_y$ denote overlap of two objects' vertical (*aligned*$_x$) and horizontal (*aligned*$_y$) projections, respectively. The relations *right-of* and *left-of* do not have any vertical restrictions. But the horizontal projections of their arguments have to be *disconnected* (see 'topological relations' in Section 2.3). The relations *overlap-right-boundary* and *overlap-left-boundary* require overlap of the LO's bounding box with the left and right boundary of the RO's bounding box, respectively. Vertical orientation relations like *above* and *below* are missing, but they can be defined in a similar manner to *left-of* and *right-of*. The relations *right-of* and *left-of* correspond to the relations *east* and *west* from OP$_{P2}$, respectively. The other two locative direction relations, *viz overlap-left-boundary* and *overlap-right-boundary*, require overlap of LO and RO which cannot be modelled by any of the relations defined in Section 2.8.

2.9.14 Abella and Kender 1993, 1994

Abella & Kender (1993), Abella & Kender (1994), and Abella (1995) present direction relations in 2D space that are used to generate descriptions of maps and x-ray images. They define direction relations corresponding to the projective terms *above*, *below*, *left*, and *right*. For each relation they define two variants, *strict* and *restricted* relations. The modifiers *somewhat* and *very* are discussed in combination with distance relations. The location of objects is represented by intrinsically aligned rectangles. They are aligned to the principal axes of inertia, and their sides are determined by the first and second momentum of inertia. Such rectangles are equal to or smaller than bounding boxes. Relative position is represented by two rectangles. For the purpose of post-fuzzification the parameters determining the rectangles are stored in a 12-dimensional vector conveying the center of each rectangle, the area, and defining the moments of inertia.

Direction relations are defined by fuzzy applicability functions which are based on discrete applicability functions on axial representations which are 'fuzzified' by post-processing with a fuzzification procedure.

Abella (1995) defines the relations *above*, *below*, *left*, and *right*, and additionally two variants of each relation indicated by the prefixes *strictly* and *restricted*. Relations are defined in terms of inequalities on the vertical and the horizontal axis. LO is *above* RO if LO is fully contained in the *north* half-plane consisting of the regions *NW*, *N*, and *NE* relative to RO. The relations *below*, *left*, and *right* are defined in the same manner. The relations *strictly-above, strictly-below, strictly-right*, and *strictly-left* are satisfied if LO does not overlap RO, but LO overlaps the *N*-region, *S*-region, *E*-region, and *W*-region, respectively. The *restricted* counterparts, namely *restricted-above*, *restricted-below*, *restricted-right*, and *restricted-left*, imply that the LO is completely contained in the *N*-region, *S*-region, *E*-region, and *W*-region, respectively. In Abella & Kender (1993) unmodified projective locative expressions with the terms *above*, *below*, *right*, and *left* are associated with *strict* relations, i.e. *strictly-above*, *strictly-below*, *strictly-right*, and *strictly-left*, when applied to particular domains.

In order to introduce tolerance to those discrete definitions, the relations are fuzzified such that they are assigned a fuzzy truth value indicating a degree of applicability. The idea is that spatial configurations that satisfy the discrete conditions of a relation receive the fuzzy truth value 1. Configurations that are close to satisfaction receive a value between 0 and 1 while the score decreases the bigger the distance of the LO from a position satisfying the discrete conditions. Abella & Kender (1993) describes a Monte-Carlo simulation to determine the fuzzy truth value. The parameters representing LO and RO are randomly varied a limited number of times and for each resulting configuration the algorithm checks the discrete truth conditions associated with the relation. Abella (1995) employs a fuzzy membership function that computes a fuzzy truth value from vertical and horizontal distance of the LO to the nearest position where it would fully satisfy the conditions, i.e. where the distances are 0. The following function is applied to vertical and horizontal distance, respectively:

(89)

$$f_\sigma(d) = e^{\frac{-d}{\sigma}}$$

The parameter d is either vertical or horizontal distance and σ is a positive value controlling the degree of fuzzification. A value close to 0 means almost no fuzzyfication, greater values for σ increase the degree of fuzzification. The truth values for vertical and horizontal inequalities are combined by standard fuzzy logic operations.

Locative expressions are generated depending on fuzzy truth values of the relation associated with the expression. Abella (1995) specifies the thresholds 0.33 and 0.66 that delineate relations whose negation is applicable [0,0.33[, from relations which cannot be applied [0.33, 0.66], and relations which are applicable]0.66,1]. Consequently, a locative expression contains a particular projective preposition if the corresponding relation has a truth value greater than 0.66. It contains negated projective prepositions, for example *not below*, if the corresponding relation has a truth value less than 0.33.

The upper interval [0.66,1] is split further into three sub intervals determining the generation of the modifiers *somewhat* and *very*. A truth value between 0.66 and 0.75 generates the modifier *somewhat,* and a truth value between 0.9 and 1 generates *very*. Additionally they define superlative expressions *topmost*, *leftmost*, *rightmost*, and *bottommost*. The modifiers *very* and *somewhat* are only applicable to distance prepositions such as *near* and *far*. The core relations defined by Abella & Kender correspond to OP_{P1} and OP_{P2}, but their interpretation is widened to some extent by the “fuzzification” procedure.

2.9.15 Schirra 1993

Schirra (1993) describes two systems that process locative expressions: one system generates locative expressions to describe parts of soccer games (SOCCER). It can describe static and dynamic two-dimensional scenes. The other system visualises locative expressions (ANTLIMA). The paper describes a treatment of projective locative expressions consisting of the prepositions *left, right, in front of*, and *behind* and of the modifiers *directly, more or less, approximately*, and *almost*. The system processes direction relations in the intrinsic and the relative frame of reference. The location of objects is represented by their hulls. The relative position of LO with respect to RO is represented by a vector connecting the proximal points. Additionally the vector is adjusted by some scaling vector derived from the spatial extension of RO. Direction relations are defined by fuzzy applicability functions on angular representations. The degree of applicability decreases with increasing deviation of the relative position vector from the prototypical direction and with increasing length. The mathematical details are not specified. An indication of the effect of modifiers is given by a Figure reproduced in Figure 2.42.

2.9.16 Wazinski 1992

Wazinski (1992) describes a system that generates locative expressions for describing the location of objects in pictures. The expressions consist of the following terms: *right, left, above, below, top, bottom*, and *center*. The system generates simple descriptions of the form “A is to the right of B.” and complex descriptions containing two relation terms like “A is above and to the right of B”.

Figure 2.42: Acceptance areas associated with modifiers in (Schirra, 1993)

Wazinski (1992) provides relation schemata for direction relations where the LO is a proper part of the RO, and relation schemata that are applicable to disconnected objects only. These schemata define direction relations for describing the position of an object in the picture relative to another object in the picture.

The location of objects is represented by their hull. In order to compute a representation of relative position the relation schema employs a combination of angular representations and an orthogonal grid. A degree of applicability is computed by means of fuzzy applicability functions. The orthogonal grid divides the plane into 9 regions with the bounding box of the RO defining the partition. Each cell is associated with an area weight which is the proportion of LO overlapping with that cell. The centroid of each intersection of LO and a cell is computed and the distance of that centroid from the bounding box of RO is associated with that cell. The degree of applicability for each cell i is the product of the area weight and a distance weight determined by the function $dist$.

(90) $app_i = \frac{|C_i|}{|LO|} * dist(relation, loc_{cent}(C_i), loc_b(RO))$

where $C_i = cell_i \cap LO$, and $dist$ is a fuzzy function dependent on the actual relation, the centroid (loc_{cent}) of the intersection of LO with respect to cell i, and the bounding box (loc_b) of RO. The details of the distance function are not given in the paper, but it says that for *above* the degree of applicability decreases with increasing horizontal distance and with increasing vertical distance.

Direction relations are associated with one or more cells. The overall degree of applicability of a particular direction relation is computed by summing up the degrees of the associated cells:

(91)
$$app = \sum_i app_i$$

Wazinski (1992) distinguishes between composite and elementary relations. Composite relations are associated with one single cell. For example "(right, top)" is associated with the region NE from the orthogonal projection model. Elementary relations are associated with three cells, e.g. "top" is associated with the region NW, N, and NE. The degree of applicability is used to find the direction relation with the highest degree of applicability. The relations are similar to the ones defined by OP$_{Grid2}$ and OP$_{Grid4}$.

2.9.17 Yamada et al. 1988

Yamada et al. (1988) describe a system that visualises projective locative expression conveying cardinal directions in two-dimensional space. The following directions are considered: *north*, *south*, *east*, *west*, *northeast*, *northwest*, *southeast*, and *southwest*. The location of an object is represented by a single point, and relative position of LO with respect to RO is represented by a vector. Direction relations are defined by cost functions on angles and distance. The overall cost is the sum of the distance cost and the direction cost.

(92) $P = P_{dist} + P_{dir}$

The global minimum of the cost function P determines a point which is the optimal interpretation of the locative expression that has to be visualised.

Yamada et al. (1988) assume that there is an optimal distance for a direction relation. Therefore, the distance cost is minimal if the distance between LO and RO is equal to that optimal distance. Let K, K_1, K_2, δ, and L be some constants of which L is the optimal distance. The distance cost between two points A and B is proportional to the squared difference of the distance from the optimal distance:

(93) $P_{dist}(A, B) = \frac{K}{2}(|\overline{AB}| - L)^2$

Let x and y be the relative coordinates of the LO with respect to the RO, and θ the angle between the horizontal axis and the prototypical direction counterclockwise.

(94) $x = x_{LO} - x_{RO}$
$y = y_{LO} - y_{RO}$

The direction cost is computed as follows:

(95) $P_{dir}(x, y) = \frac{K_1(y\cos\theta - x\sin\theta)^2 + K_2}{x\cos\theta + y\sin\theta + \frac{1}{\delta}}$

If we represent relative position by polar coordinates (α, d) the direction cost is computed as follows:

(96) $P_{dir}(\alpha, d) = \frac{K_1 d^2 \sin^2(\alpha-\theta)+K_2}{d\cos(\alpha-\theta)+\frac{1}{\delta}}$; only defined if $|\theta - \alpha| \leq 90°$

From this equation it is easy to see that the cost approximates infinity for 90 degrees deviation. The overall cost function is constrained by a discrete component which inhibits certain regions completely. P and P_{dir} are only defined for angular deviations equal or less than 90°. Under the assumption that objects with spatial extension are represented by their centroid, the relations are equivalent to the relations defined by AD_4^{cent}.

2.9.18 André et al. 1987

André et al. (1987) describes a system called *CITYTOUR* for describing and understanding German locative expressions that describe the location of static and dynamic objects on a 2D map representation. They define direction relations corresponding to pairs of a projective preposition and a modifier. The prepositions are '*vor*' (*in front of*), '*hinter*' (*behind*), '*rechts von*' (*to the right of*), and '*links von*' (*to the left of*). The modifiers are '*direkt*' (*directly*), '*recht gut*' (*pretty well*), and '*in etwa*' (*sort of*).

The location of an object is represented by its hull, and the virtual observer is represented as a point. The system can process relations in the intrinsic and the relative frame of reference. The intrinsic frame of reference can be employed if the RO has an intrinsic front, the relative frame of reference can be used for all objects. The coordinate system of the relative frame of reference is centred on the observer. In static cases the front-back-axis is constructed as the bisector between the two tangents from the observer to the RO as shown in Figure 2.43. The left-right-axis is perpendicular passing through the point of the observer. In case the orientation is induced by the

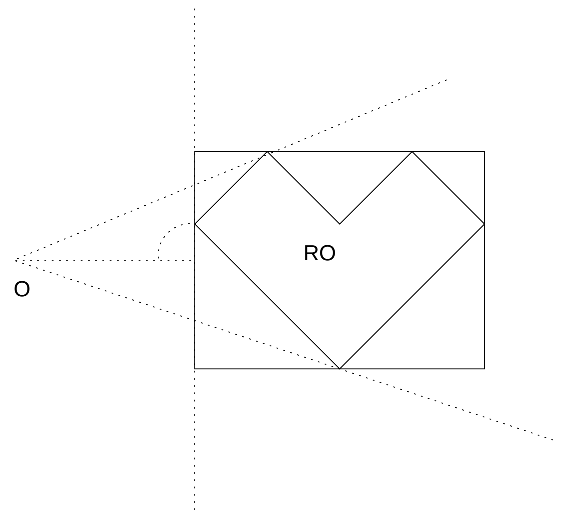


Figure 2.43: Aligning the relative frame of reference in André et al. (1987).

movement of the virtual observer, the front direction of the relative frame of reference is aligned with the direction of the movement. If the virtual observer is part of the bounding box of RO, the system does not determine a direction relation.

The relation schema is based on axial representations that are aligned with the coordinate system of the frame of reference. It is refined by 3 trapezoidal distance regions for each direction. Figure 2.44 shows an example of the direction relations associated with *directly-above*, *above*, *quite-well-above*, and *sort-of-above*. The degree of appli-

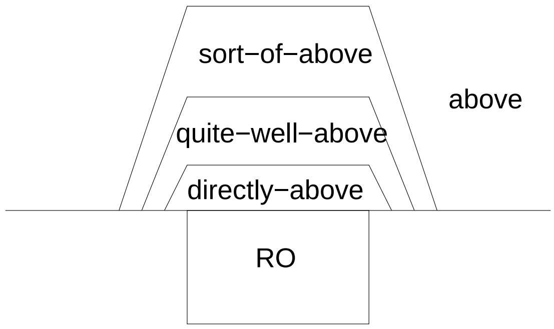


Figure 2.44: The acceptance region of the direction relation *above* and embedded acceptance regions for three different modifiers in (André et al., 1987).

cability is determined by a binary applicability function. A direction relation applies to a pair of LO and RO, if the centroid of the LO lies within the region that is determined by the direction relation and the bounding box of the RO. The mathematical details of the treatment of modifiers are not specified in the paper. The relations defined above correspond to OP_{P2}^{cent} defined in Section (41) on page 50.

Chapter 3

Semantics of Projective Locative Expressions

This chapter specifies the notion of *projective locative expressions* and defines alternative theories about their meaning. All semantic theories that are proposed at the end of this chapter are completely defined in terms the locative direction relations from Chapter 2. These semantic theories are evaluated by means of the procedure described in Chapter 4 against projective locative expressions from the HCRC Map Task corpus (see Chapter 5). The results are discussed in Chapter 6.

Section 3.1 defines the notion of projective locative expressions. Section 3.2 gives an overview of the factors that influence the semantics of these expressions. In Section 3.3 a formalism is defined that allows for representing the vagueness of projective locative expressions in terms of underspecified representations. Section 3.4 reviews formal semantic approaches to projective locative expressions. Section 3.5 defines a number of alternative geometrical semantic theories of projective locative expressions and points out the assumptions that have been made to fit those theories to the domain of the HCRC Map Task corpus.

3.1 Projective locative expressions

This section defines the range of linguistic expressions that are considered in this study. *Locative expressions* describe the location of an entity – I use the term *located object* (LO) to refer to that entity. Locative expressions consist of an expression denoting the LO and typically a locative prepositional phrase. The prepositional phrase can be connected to the expression denoting the LO by simple PP attachment or by a copula. The spatial reading of locative prepositional phrases establishes a spatial relation between the located object and the entity denoted by their argument – I use the term *reference object* (RO) to refer to that entity.

(1) a. The apple is to the left of the table.

b. The apple is on the table.

In both sentences the noun phrase *the apple* refers to the LO, and the noun phrase *the table* to the RO. Typically we distinguish between two kinds of spatial relations that are established by locative prepositional phrases: locative direction relations and topological relations. Topological relations specify the way in which the LO is spatially connected to the RO. Is the LO contained in the RO? Do they touch each other? A brief overview of topological relations has been given in Section 2.3. Topological prepositions that express topological relations are, for example, *in*, *on*, and *at*.

Projective prepositions (Herskovits, 1986) such as *above*, *below*, *to the right of*, and *to the left of* express locative direction relations which specify the location of the LO with respect to the RO by means of a prototypical direction. They are called *projective* because they "fundamentally involve the experience of viewing and the idea of a point of observation" (Herskovits, 1986, 156). An example of a projective locative expression is given in (1-a) (*The apple is to the left of the table*).

Locative expressions can additionally contain modifier phrases which modify the spatial relation established by the prepositional phrase. *Hedges* (Lakoff, 1973) such as *almost*, *sort of* and *just* are modifiers that strengthen or weaken the truth of a statement. Zwarts (1997) provides a more detailed analysis of modifiers of projective prepositional phrases and distinguishes between *distance modifiers* and *direction modifiers*. *Distance modifiers* constrain the distance between the LO and the RO. They can be realised as measure phrases or adverbs such as *10 centimetres, far, a bit*. *Direction modifiers* constrain the range of locative direction relations. Zwarts gives the examples *straight* and *diagonally*.

This work is concerned with projective locative expressions, in particular with expressions that contain the following projective terms:

(2) *right,left*
below, underneath, under, down
above, up, upwards, top, bottom
east, west, north, south

These terms can be embedded in a variety of expressions. In particular the terms *left* and *right* occur in composite expressions such as *left of, to/on the left of,* and *to/on the left-hand side of.* But also *top* and *bottom* typically occur in phrases like *on top of* and *at the bottom of.* Some terms are combined with the preposition *of* or *from* in order to render them fully functional as a projective prepositions, e.g., *south of* and *upwards from*.

There are two modifiers with which we will be particularly concerned :

(3) *directly*
slightly

Examples of projective locative expressions that are relevant for the present investigation are given in (4):

(4) *X is above Y.*
X is directly above Y.
X is to the right of Y.
X is slightly to the right of Y.

3.2 Aspects of meaning

This section provides an overview of the factors that have been reported in the literature to contribute to the meaning of projective locative expressions. These factors are direction, distance, functional relations between the LO and the RO, the frame of reference, the impact of occlusion of one object with respect to the viewer, and objects in the spatial context of LO and RO.

Locative direction relations Locative direction relations (see Chapter 2) are the principal components of the meanings of projective locative expressions. They classify relative position (meaning the location of a LO relative to the RO) with respect to prototypical directions. The previous chapter has shown how they integrate the influence of the objects' geometric properties such as spatial extension and shape.

Distance Distance is explicitly expressed in projective locative expressions by means of modifiers that constrain the distance between LO and RO, for example, "a bit", "one inch", and "far", e.g. (Zwarts & Winter, 2000), (Zwarts, 1997), (André et al., 1987). Some approaches to the meaning of projective locative expressions explicitly model an effect of distance on the meaning of unmodified projective locative expressions, see Kelleher (2003), Gapp (1994a), Olivier & Tsujii (1994), and Yamada et al. (1988). They define an optimal distance between LO and RO, and any deviation from this optimal distance decreases the degree of applicability. Matsakis et al. (2001) also uses a component which is sensitive to distance in this way.

The empirical study by Logan & Sadler (1996), however, suggests that distance has no such effect. Therefore, we assume that distance should not be modelled within the semantics of projective locative expressions but rather with the semantics of certain modifiers.

Frame of reference The selection of a single frame of reference is critical for applying locative direction relations to spatial configurations (see Section 2.4) and also for interpreting projective locative expressions (Carlson, 1999). The selection process is typically based on cues found in projective locative expressions and their context, suggesting which frame of reference should be used. Procedures that determine the selection of a particular frame of reference can be found in Kelleher (2003) (Olivier & Tsujii, 1994), (Gapp, 1994b), and (André et al., 1987).

The data we are concerned with in this study (see Section 5.1), however, always make use of the frame of reference that reflects the conventional use of printed texts and maps, see Section 2.4. Therefore, factors that influence the selection of a frame of reference do not play a part in this study.

Occlusion In real and virtual 3D environments occlusion of an object by another object is a factor that controls the use of the projective prepositions *in front of* and *behind* (Kelleher, 2003). Given a viewer and two objects x and y in a 3D scene, an object x occludes an object y with respect to a viewer v if and only if there is a straight line trough v, x and y such that the intersection of x is between the intersections of v and y. If x occludes y, x can be said to be *in front of* y, and y to be *behind* x.

Occlusion is not relevant for the domain of map task maps, since there are only very few objects on the maps which overlap, and we find only two occurrences of expressions in the data that contain the projective term *front*. The term *behind* is not present at all.

Functional relations Functional relations between objects influence the degree to which projective prepositions are applicable to spatial configurations. Carlson-Radvansky et al. (1999) describe experiments that show the dependency of the prepositions $above$ and $below$ on functional relations between objects such as a toothbrush and a tube of toothpaste or a coin and a piggy bank. Functional relations had the effect that the subjects tended to interpret and express direction relations between the objects' functional parts, instead of referring to the entire objects. In the first experiment subjects were asked to place two objects according to a locative expression. When they were asked to place a tube of toothpaste *above* a toothbrush, they showed the tendency to place the opening of the tube vertically above the head of the toothbrush. When they were asked to place a tube of oil paint above the toothbrush, this tendency was not as strong as in the previous case. The settings are identical from a geometric point of view, so the difference must be a conceptual one. Carlson-Radvansky et al. (1999) conclude that the effect is due to functional relations: there is a convention of putting toothpaste on a toothbrush, but there is no such convention of using paint on a toothbrush. In another experiment they showed subjects pictures containing a coin and a piggy bank and asked them to rate the degree of applicability (cf Section 2.6) of the statement *the coin is above the piggy bank*. The location of the slot of the piggy bank was systematically varied. The results showed that the highest degrees of applicability were assigned to those settings where the coin was vertically above the slot of the piggy bank regardless of the slot's location on the piggy bank. Again, the positions of the coin relative to the whole piggy bank were identical from a purely geometric point of view; the only difference was the place of the slot.

Coventry et al. (2001) report three experiments that systematically test for the influence of geometric and functional relations, respectively, on the prepositions *above*, *below*, *over*, and *under*. They presented subjects with pictures of a man holding an umbrella in different positions with rain coming down from different angles. On some pictures the umbrella fulfilled its function and protected the man from the rain, on some pictures it did not, so that the rain hit the man, in yet other pictures there was no rain, at all. Different configurations of rain were systematically combined with different positions of the umbrella. The subjects rated the applicability of prepositions to these combinations. Dependency of the ratings on alignment of rain and umbrella (so that the umbrella fulfilled its function) was interpreted as a positive effect of functional relations on the meaning of the corresponding preposition. The experiments showed that the prepositions *over* and *under* are very sensitive to functional relations, and although the prepositions *above* and *below* show some sensitivity, too, they are more strongly determined by geometric relations. The general view put forth in (Coventry & Garrod, 2004; Coventry, 1998; Coventry et al., 2005) is that the semantics of spatial relations is dependent on both functional and geometrical factors.

Concerning the data relevant for this study (Section 5.1) hardly any functional relations are found between any two landmarks of the map task maps. Most landmarks are isolated line drawings that are completely unrelated to any other landmark. The only exceptions are pairs involving bodies of water, as for example, (i) a bridge over a river, where the functional relation is established by the convention that bridges are used to cross that river; (ii) a beach at the sea, which is by definition at the border of some body of water; and (iii) a ship on the sea where the functional relation is established by ships being conventionally used as floating vehicles. There are a few more cases of landmarks which are functionally related to other landmarks in some way. However, the number of these cases is so small that it is quite harmless to assume that functional relations have no impact on the data.

Distractor objects Distractor objects are objects which appear in the same spatial context as the located object and the reference object of a locative expression. Herskovits (1986, p81) formulates the effect of distractor objects in the *shifting contrast principle*. Given a reference object and a preposition, if an object A receives a higher degree of applicability[1] than another object B, "then one can use that preposition to discriminate A from B – so that the locative phrase will be assumed true of A, but not of B." That means that a preposition that is appropriate to describe the location of LO relative to RO, might not longer be appropriate after a distractor object D has been introduced which fits the given combination of preposition and reference object better than the LO, see Figure 3.1.

[1]See Section 2.6.

(a) LO is above RO.

(b) LO is above RO – even in the presence of D.

Figure 3.1: Applicability of "LO is above RO" does not change when we add a distractor object D.

The system presented in (Kelleher, 2003, p279/280) interprets locative expressions in a similar way: it selects the object with the highest degree of applicability from a set of possible alternatives.

Carlson & Logan (2001) provide evidence against such a relativistic definition of the meaning of projective prepositions. Although they show that the presence of distractor objects decreases the degree of applicability, they show that this effect is unrelated to the relative placement of the distractor object. That means that the truth of projective locative expressions is independent of the presence of distractor objects.[2]

3.3 Vagueness

This section discusses vagueness of projective locative expressions. It has already been made clear in the Introduction of this thesis that we approach the question of the truth conditions of projective locative expressions under the presumption that projective locative expressions are vague. Thus we expect them to partition the domain into three sets of pairs of objects: one set for which the expression is *definitely true*; a second set for which it is *indefinite*, that means, it can be either *true* or *false*; and a third set for which the expression is *definitely false*.

Following the view on the vagueness of adjectives presented in Kamp (1975), I will assume that the vagueness of projective locative expressions has two general sources: (i) vagueness of the weight with which each of the different aspects of the meaning of a projective locative expression (see Section 3.2) contribute to the overall meaning of the expression; and (ii) vagueness with respect to the satisfaction of each of these aspects. The most important aspects that contribute to the meaning of projective locative expressions have been described in the previous section. And in general, all of these aspects, as well as the ways in which they combine, have to be taken as a sources of vagueness. I have argued in Section 3.2, however, that it is sufficient for this study to concentrate on the vagueness of one single aspect, namely the geometric properties

[2]This only holds of course under the restriction that we are not looking at comparative forms.

and relations which are captured by locative direction relations as defined in Section 2.8. The vagueness introduced by this single aspect is the problem of finding 'sharp' criteria, i.e. locative direction relations, which determine whether a certain projective locative expression is satisfied. I will aim at a semantics that is based on a pair of locative direction relations $\langle LDR_1, LDR_2 \rangle$ which – in accordance with our expectations *vis-á-vis* the meaning of projective locative expressions – partition the domain into three parts: one relation, LDR_1, demarcates the boundary between the pairs that make the expression *definitely true* and all pairs that can make it false; and the other relation, LDR_2, demarcates the boundary between all pairs that can make the expression true and all pairs that make it *definitely false*. In order to obtain a tripartite domain where the boundaries are in fact defined by LDR_1 and $LDR2$, there is the additional requirement that LDR_2 subsumes LDR_1, i.e., every pair of objects that satisfies LDR_1 also satisfies LDR_2. This requirement ensures that every pair of objects $\langle x, y \rangle$ that makes an expression *definitely true* ($LDR_1(x, y)$) can be true ($LDR_2(x, y)$), and that every pair of objects $\langle x, y \rangle$ that makes an expression *definitely false* ($\neg LDR_2(x, y)$) can be false ($\neg LDR_1(x, y)$).

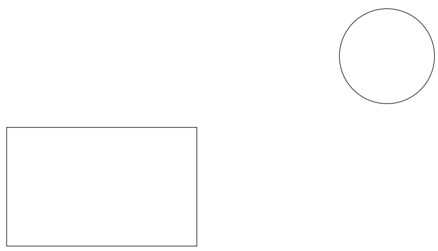


Figure 3.2: Is the circle above the rectangle?

Before we come to the formal details, let me illustrate the vagueness of projective locative expressions by the following example:

(5) The circle is above the rectangle.

It can be argued that the predication of the circle being *above* the rectangle with respect to Figure 3.2 is true in some respect, but also that it is false in some other respect. On the one hand, we can adopt the "vertical" perspective of *above* which specifies that LO is above RO if and only if there is at least one vertical axis which intersects RO and LO such that the (vertical) coordinates of the intersection with LO are greater than the (vertical) coordinates of the intersection with RO. The graphical interpretation of the "vertical" perspective is depicted in Figure 3.3(a); the circle should overlap with the grey area in order to count as "vertically" above the rectangle. However, the circle is not “vertically” above the rectangle, and so, from the “vertical” perspective sentence (5) is false.

(a) Acceptance area for “vertically” *above*.

(b) Acceptance area for “horizontally” *above*.

Figure 3.3: Example with acceptance areas for *above*.

On the other hand, we can adopt the “horizontal” perspective. According to the “horizontal” perspective LO is above RO if and only if there is a part of LO which is vertically higher than every part of RO. The graphical interpretation is depicted in Figure 3.3(b). The circle counts as “vertically” above the rectangle if it overlaps with the grey area. So, the circle is “horizontally” above the rectangle, and sentence (5) is true from the “horizontal” perspective.

Let us preliminarily accept these two perspectives as legitimate interpretations of the preposition *above*; “vertically above” defines the extension of the pairs that *definitely* count as *above*, and “horizontally above” defines the extension of all pairs that *can* count as *above*. The next question is one for a formalism that allows us to provide a semantics of *above* (i) that yields true for all pairs where the LO is “vertically above” the RO; (ii) that allows to classify pairs as either true or false when the LO is “horizontally above” but not “vertically above” the RO; and (iii) that yields false for all pairs where the LO is not “horizontally above” the RO.

Fine (1975) and Pinkal (1985) propose formal languages that admit vague terms allowing for indefinite truth values. Vague terms are evaluated with respect to models that provide access to all possible *precisifications* of those terms. By a *precisification* of a vague term we understand a way of making the term more precise. For example, earlier we adopted two different perspectives to interpret the term *above*. Each of these perspectives provides a precisification that is completely precise, i.e., each determines a definite truth value (true or false) for each pair of objects. On the one hand we interpreted above as “vertically above” and on the other hand as “horizontally above”.

Fine (1975) proposes truth conditions for vague sentences that are based on the *supervaluation* technique (van Fraassen, 1969): a vague sentence is true if it is true for all ways of making it completely precise. It is false if it is false for all ways of making it completely precise. In all other cases its truth-value is indefinite. The supervaluation technique is suitable to interpret *above* as true or false for all pairs of objects of the domain which make *above definitely true* or *definitely false*, respectively. But all pairs of objects from the domain which belong to the *indefinite* part remain indefinite.

Pinkal (1985, p180ff) introduces two modal operators which quantify over precisifications. They correspond to the natural language expressions *in all respects* and *in some respects*. Let A be a sentence of a vague language, then *in-all-respects(A)* is true if and only if it is true with respect to all precisifications of A. The formula *in-some-respects(A)* is true if and only if there exists at least one precisification of the given model in which A is true. Applying Pinkal's operators to sentence (5), we obtain *true* in one case in the given model and *false* in the other. With the operator *in-all-respects* the sentence comes out as false:

(6) *in-all-respects*(the circle is above the rectangle).

The circle is not above the rectangle *in all respects*; there is at least the interpretation of *above* as “vertically above” which is not satisfied. If the sentence is embedded under the operator *in-some-respects*, then it comes out as true:

(7) *in-some-respects*(the circle is above the rectangle).

The circle is above the rectangle in some respects, that is because it is “horizontally above” the rectangle.

These two example show that we can deal with vague sentences by embedding them under one of those two operators. Embedding it under the operator *in-all-respects* yields the truth value *false* and embedding it under *in-some-respects* yields *true*. Thus, we obtain the intended result; pairs of objects from the domain that are *definitely true* satisfy and those that are *definitely false* do not satisfy a vague expression under either operator. And pairs that count as *indefinite* relative to the meaning of a vague predicate can make a sentence containing that expression true or false depending on the operator embedding that expressions.

Underspecification and biased valuation I will now describe a way of dealing with vagueness that integrates the operators *in-some-respects* and *in-all-respects* in the valuation procedure. We first define a formalism that uses underspecified representations to represent vagueness. The idea has been described already in the beginning of this section. Vague terms partition the domain into three parts associated with *definitely true*, *indefinite*, and *definitely false*, respectively. Such a partition can be explicitly represented by means of at least two completely precise (*viz* non-vague) terms.

Let us define an extended predicate logic L with the additional feature of underspecified predicates. The language L is based on a predicate logic with the logical symbols $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\exists$, $\forall$, the variables v, v_1, v_2, v_3, $\ldots$, the constants c, c_1, c_2, c_3, $\ldots$, and the n-place predicate letters Q_i^n $(i, n \in \mathcal{N})$. The standard syntax of predicate logic is extended by the possibility of composing complex predicate symbols by means of the operator $\oplus$. The Q_i^n are the *atomic* predicate symbols of L. Two or more atomic predicates of the same arity n, $Q_{i_1} \ldots Q_{i_m} (m \geq 2)$ can be combined to make the complex n-place predicate symbol

(8) $Q_{i_1}^n \oplus \ldots \oplus Q_{i_m}^n$

Complex predicates are used to represent underspecification.

In the example above, we took "vertically above" and "horizontally above" as two completely precise interpretations of the preposition *above*. Now, let us assume that the semantics of *above* is expressed by a combination of the predicates A_{vert} and A_{hor} that correspond to these interpretations where the extension of A_{vert} determines all pairs of objects that definitely satisfy *above*, and the extension of A_{hor} determines all pairs of objects that *can* satisfy *above*. We combine A_{vert} and A_{hor} to a complex predicate and obtain the following underspecified representation:

(9) $(\lambda x.\lambda y)(A_{vert} \oplus A_{hor})(x, y)$

Before I define the semantics for L, let me describe the idea of the valuation procedure. The valuation procedure integrates the operators *in-some-respects* and *in-all-respects* by means of a two-valued flag which I call *bias*. A *positive bias* corresponds to the operator *in-some-respects*; vague predications evaluated with a positive bias are true if there is at least one way of interpreting the vague predication such that the predication is true. I call this bias *positive* because a valuation under positive bias shows "a positive predisposition" towards the formula – if there is any chance so the formula is evaluated as *true,* at all. A *negative bias* corresponds to the operator *in-all-respects*; vague predications evaluated with a negative bias are only true if they are true in all respects. That means, they are only true if all ways of interpreting the vague predication are true. Since the valuation procedure shows "a negative predisposition" towards the formula that is evaluated – if possible in some way the formula is evaluated as *false* – the bias is termed *negative* bias. I call the valuation method using a bias a *biased valuation*.

The semantics of L is defined with respect to a first order model M, an assignment function g, and a bias b. A model M for L is a pair $\langle U, I\rangle$ where U is a non-empty set and I is a function from Q_i^n onto n-place relations on U. An assignment function g maps variables onto elements of U. The *satisfaction value* of a formula ϕ of L with respect to a model $M = \langle U, I\rangle$ by the assignment g with respect to bias $b \in \{0,1\}$ is defined by the following recursion:

(10)
a. $[Q_i^n(v_{i_1} \ldots v_{i_n})]_{M,g,b} = 1$ iff $\langle g(v_{i_1}), \ldots, g(v_{i_n})\rangle \in I(Q_i^n)$
b. $[\neg\phi]_{M,g,1} = 1$ iff not $[\phi]_{M,g,0} = 1$ (iff $[\phi]_{M,g,0} = 0$)
c. $[\neg\phi]_{M,g,0} = 1$ iff not $[\phi]_{M,g,1} = 1$ (iff $[\phi]_{M,g,1} = 0$)
d. $[\phi \wedge \psi]_{M,g,b} = 1$ iff $[\phi]_{M,g,b} = 1$ and $[\psi]_{M,g,b} = 1$
e. $[\phi \vee \psi]_{M,g,b} = 1$ iff $[\phi]_{M,g,b} = 1$ or $[\psi]_{M,g,b} = 1$

f. $[\phi \rightarrow \psi]_{M,g,b} = 1$ iff $[\neg\phi]_{M,g,b} = 1$ or $[\psi]_{M,g,b} = 1$
g. $[(\exists v_i)\phi]_{M,g,b} = 1$ iff for some $u \in U$ $[\phi]_{M,g[u/v_i],b} = 1$
h. $[(\forall v_i)\phi]_{M,g,b} = 1$ iff for all $u \in U$ $[\phi]_{M,g[u/v_i],b} = 1$
i. $[Q^n_{i_1} \oplus \ldots \oplus Q^n_{i_l}(v_{i_1} \ldots v_{i_n})]_{M,g,1} = 1$ iff
there is a m with $1 \leq m \leq l$: $[Q^n_{i_m}(v_{i_1} \ldots v_{i_n})]_{M,g,1} = 1$
j. $[Q^n_{i_1} \oplus \ldots \oplus Q^n_{i_l}(v_{i_1} \ldots v_{i_n})]_{M,g,0} = 1$ iff
for all m with $1 \leq m \leq l$: $[Q^n_{i_m}(v_{i_1} \ldots v_{i_n})]_{M,g,0} = 1$

This extension of predicate logic is conservative. All formulae of L that do not contain the operator $\oplus$ are valuated in the standard way. The bias does neither affect the semantics of atomic predicates in L nor of the complex formulas that can be formed from these. The only effect of the bias is that it controls the valuation of complex predicates, see rules (10-i) and (10-j). A positive bias in the valuation makes a complex predication true if and only if there exists at least one atomic predicate Q_i that is part of the complex predicate and that is true. It is false if all atomic predicates are false. A negative bias in the valuation makes a complex predication true, if and only if all atomic predicates Q_i that are part of the complex predicate are true. It is false if there is at least on atomic predicate that is false. Tuples of individuals which make all predicates of an underspecified representation true or false, constitute cases for which $Q^n_{i_1} \oplus \ldots \oplus Q^n_{i_l}$ is ***definitely true*** or ***definitely false***, respectively. In other cases the predication is indefinite. The negation rules (10-b) and (10-c) invert the bias, all other rules pass the bias through the recursion without changing it.

$\neg$	
1	0
#	#
0	1

$\wedge$	1	#	0
1	1	#	0
#	#	#	0
0	0	0	0

$\vee$	1	#	0
1	1	1	1
#	1	#	#
0	1	#	0

$\rightarrow$	1	#	0
1	1	#	0
#	1	#	#
0	1	1	1

Table 3.1: Truth tables of strong Kleene logic $\mathbf{K}$

Relation to strong Kleene logic This calculus (let us refer to it by the letter $\mathbf{B}$ and more precisely, let us use $\mathbf{B}, \mathbf{1}$ for positive bias and $\mathbf{B}, \mathbf{0}$ for negative bias) can be mapped onto the three-valued strong Kleene logic ($\mathbf{K}$) which has the truth values 1 (true), $\#$ (indefinite) and 0 (false). The truth tables are shown in Table 3.1.

The complex predicates of $\mathbf{B}$ can evaluate to all three truth values. They evaluate to 1 (*viz* definitely true) if all of their atomic predicates evaluate to true (see (10-i)), they evaluate to 0 (*viz* definitely false) if none of their atomic predicates evaluate to true (see (10-j)), and otherwise, they evaluate to the indefinite truth value $\#$.

The relation between $\mathbf{B}$ and $\mathbf{K}$ is expressed by the following statements: (i) Only if a formula ϕ is true in B with negative bias, then ϕ is true in K. (ii) Only if a formula ϕ is false in B with positive bias, then ϕ is false in K. And (iii), only if a formula ϕ is true in B with positive bias and false with negative bias, then it is indefinite ($\#$) in K:

(11) (i) $[\phi]_{B,0} = 1 \Longleftrightarrow [\phi]_K = 1$
(ii) $[\phi]_{B,1} = 0 \Longleftrightarrow [\phi]_K = 0$
(iii) $[\phi]_{B,1} = 1 \wedge [\phi]_{B,0} = 0 \Longleftrightarrow [\phi]_K = \#$

Proof

1. First, we derive two auxiliary premises under the assumption that (11-i) and (11-ii) are true:

   - $[\phi]_{B,1} = 1 \Leftrightarrow$ not $([\phi]_{B,1} = 0) \Leftrightarrow$ not $([\phi]_K = 0) \Leftrightarrow [\phi]_K \neq 0$
   - $[\phi]_{B,0} = 0 \Leftrightarrow$ not $([\phi]_{B,0} = 1) \Leftrightarrow$ not $([\phi]_K = 1) \Leftrightarrow [\phi]_K \neq 1$

2. Then we can generally show that (11-3) is true if (11-i) and (11-ii) are true:

   - $[\phi]_{B,1} = 1 \wedge [\phi]_{B,0} = 0 \Leftrightarrow [\phi]_{B,1} = 1$ and $[\phi]_{B,0} = 0 \Leftrightarrow [\phi]_K \neq 0$ and $[\phi]_K \neq 1 \Leftrightarrow [\phi]_K = \#$

   So in order to prove the three statements in (11) we only need to show (11-i) and (11-ii)!

3. For atomic predicates from L, see (10-a), (i) and (ii) are trivially true. Note, that atomic predicates from L cannot assume the indefinite truth value in K.

4. Suppose ϕ is a complex predicate $Q^n_{i_1} \oplus \ldots \oplus Q^n_{i_l}(v_{i_1} \ldots v_{i_n})$, see (10-i) and (10-j):

   - $[Q^n_{i_1} \oplus \ldots \oplus Q^n_{i_l}(v_{i_1} \ldots v_{i_n})]_{B,0} = 1$
     $\Leftrightarrow$ for all m with $1 \leq m \leq l : [Q^n_{i_m}(v_{i_1} \ldots v_{i_n})]_{B,0} = 1$
     $\Longleftrightarrow$ for all m with $1 \leq m \leq l : [Q^n_{i_m}(v_{i_1} \ldots v_{i_n})]_K = 1$
     $\Leftrightarrow [Q^n_{i_1} \oplus \ldots \oplus Q^n_{i_l}(v_{i_1} \ldots v_{i_n})]_K = 1$
   - $[Q^n_{i_1} \oplus \ldots \oplus Q^n_{i_l}(v_{i_1} \ldots v_{i_n})]_{B,1} = 0$
     $\Leftrightarrow$ there is no m with $1 \leq m \leq l$: $[Q^n_{i_m}(v_{i_1} \ldots v_{i_n})]_{B,1} = 1$
     $\Leftrightarrow$ for all m with $1 \leq m \leq l$: $[Q^n_{i_m}(v_{i_1} \ldots v_{i_n})]_{B,1} = 0$
     $\Longleftrightarrow$ for all m with $1 \leq m \leq l$: $[Q^n_{i_m}(v_{i_1} \ldots v_{i_n})]_K = 0$
     $\Leftrightarrow [Q^n_{i_1} \oplus \ldots \oplus Q^n_{i_l}(v_{i_1} \ldots v_{i_n})]_K = 0$

5. For the negation (10-b) and (10-c) we suppose that (11-i) and (11-ii) are true for ϕ, then we show that they hold for $\neg\phi$:

   - $([\neg\phi]_{B,0} = 1) \Leftrightarrow ([\phi]_{B,1} = 0)$
     $\Longleftrightarrow ([\phi]_K = 0) \Leftrightarrow ([\neg\phi]_K = 1)$
   - $([\neg\phi]_{B,1} = 0) \Leftrightarrow$ not $([\phi]_{B,1} = 1) \Leftrightarrow ([\phi]_{B,0} = 1)$
     $\Longleftrightarrow ([\phi]_K = 1) \Leftrightarrow ([\neg\phi]_K = 0)$

6. Similarly for the conjunction (10-d), we suppose that (11-i) and (11-iii) hold for both ϕ and ψ, then we show that they also hold for $\phi \wedge \psi$:

   - $([\phi \wedge \psi]_{B,0} = 1) \Leftrightarrow ([\phi]_{B,0} = 1$ and $[\psi]_{B,0} = 1)$
     $\Longleftrightarrow ([\phi]_K = 1$ and $[\psi]_K = 1) \Leftrightarrow ([\phi \wedge \psi]_K = 1)$
   - $([\phi \wedge \psi]_{B,1} = 0) \Leftrightarrow$ not $([\phi \wedge \psi]_{B,1} = 1) \Leftrightarrow$ not $([\phi]_{B,1} = 1$ and $[\psi]_{B,1} = 1)$
     $\Leftrightarrow ([\phi]_{B,1} = 0$ or $[\psi]_{B,1} = 0)$
     $\Longleftrightarrow ([\phi]_K = 0$ or $[\psi]_K = 0) \Leftrightarrow ([\phi \wedge \psi]_K = 0)$

7. The proofs for (10-e) to (10-h) are similar to the one for the conjunction.
   q.e.d.

The role of the bias might become clearer by the following observation. If a complex predicates contains at least two atomic predicates of which one is tautologically true and one tautologically false, then it is completely determined by the bias. Suppose, Q_t is a one-place predicate which is true for any argument, and Q_f a one-place predicate which is false for any argument.

(12) $\forall x : Q_t \oplus Q_f(x)$

An evaluation with a positive bias makes (12) true, because every x satisfies the property Q_t! An evaluation with a negative bias makes (12) false, because no x satisfies the property Q_f!

Example

Let us now see how the negation of sentence (5) comes out with *biased valuation*.

(13) The circle is not above the rectangle.

The model M represents the spatial configuration depicted in Figures 3.2 (a) and (b). Let us use the constant c_c to refer to the circle and the constant c_r to refer to the rectangle. Following the illustrations M satisfies the sentence that the circle is “horizontally above” (A_{hor}) the rectangle:

(14) $[A_{hor}(c_c, c_r)]_{M,g,b}$=1.

but M does not satisfy the sentence that the circle is “vertically above” (A_{vert}) the rectangle:

(15) $[A_{vert}(c_c, c_r)]_{M,g,b}$=0

We represent the semantics of *above* by the underspecified representation $A_{vert} \oplus A_{hor}$. The semantics of sentence (13) is represented by the following formula:

(16) $\neg(A_{vert} \oplus A_{hor})(c_c, c_r)$

First, we evaluate (17) with a positive bias:

(17) a. $[\neg(A_{vert} \oplus A_{hor})(c_c, c_r)]_{M,g,1}$ = 1 iff
b. $[(A_{vert} \oplus A_{hor})(c_c, c_r)]_{M,g,0}$ = 0 iff
c. not for all A $\in \{A_{vert}, A_{hor}\}$: $[A(c_c, c_r)]_{M,g,0}$ = 1

Since $A_{vert}(c_c, c_r)$ is false, (17-c) is satisfied. Therefore, (16) comes out true. Next, we evaluate the formula with a negative bias:

(18) a. $[\neg(A_{vert} \oplus A_{hor})(c_c, c_r)]_{M,g,0}$ = 1 iff
b. $[(A_{vert} \oplus A_{hor})(c_c, c_r)]_{M,g,1}$ = 0 iff
c. there is no A $\in \{A_{vert}, A_{hor}\}$ such that $[A(c_c, c_r)]_{M,g,1}$ = 1

Since $A_{hor}(c_c, c_r)$ is true, (18-c) is false. Therefore, (16) comes out false.

The choice of a positive or negative bias is motivated as follows: We choose a positive bias ($b = 1$), if we assume that a sentence/utterance has been produced by a speaker that was cooperative and had the intention to say something true. And we choose a negative bias ($b = 0$) if we are not cooperative ourselves and try to find an interpretation that makes the sentence/utterance false. A positive bias corresponds to Pinkal's operator *in-some-respects*, and a negative bias to the operator *in-all-respects*.

3.4 Formal semantics

This section describes different ways of modelling the semantic contribution of the components of projective locative expressions. First, I will present non-compositional approaches that treat projective prepositions and modifiers as units that are not decomposed further. Such a non-compositional approach is also pursued in this thesis (cf Section 3.5.) Second, I will present compositional approaches that provide semantic representations for modifiers and prepositions and define rules for combining them in order to compose the semantic representation of the whole expression. Finally, I will give an overview of the different semantics in the literature of how modifiers contribute to projective locative expressions.

As said earlier in this chapter, projective locative expressions consist of an expression denoting the LO, an expression denoting the RO, a phrase embedding the projective term, and optionally, one or more modifiers.

(19) The circle is just directly above the rectangle.

In this example *the circle* refers to the LO and *the rectangle* to the RO. The preposition *above* is the projective term and *just* and *directly* are modifiers.

Non-compositional approach Bierwisch (1988) treats locations as entities in their own right. The function $loc : D \mapsto L$ maps entities from a general domain D onto locations belonging to the same location domain L. A projective term is mapped onto a locative direction relation REL. The meaning of locative expressions is represented as a spatial relation between the locations of LO and RO.

(20) $REL(loc(lo), loc(ro))$

This predication can be directly interpreted in terms of any of the locative direction relations defined in Section 2.8.

Non-compositional approaches to projective locative expressions with modifiers are described in (André et al., 1987), (Schirra, 1993), and (Matsakis et al., 2001). They are non-compositional in the sense that combinations of modifiers and prepositions are treated as units, and semantic representations have to be specified for each possible combination of modifiers and prepositions. This is the general template for semantic mappings from modifiers and prepositions to spatial relations:

(21) $modifier^n \times preposition \mapsto REL$

There have to be rules for unmodified prepositions, (22-a), rules for prepositions with one modifier, (22-b), with two modifiers, (22-b), and so on.

(22) a. $preposition \mapsto REL$
b. $modifier \times preposition \mapsto REL$
c. $modifier \times modifier \times preposition \mapsto REL$

(André et al., 1987) and (Schirra, 1993) provide semantic mappings for unmodified prepositions and prepositions with one modifier. The acceptance area defined by the locative direction relation associated with an unmodified preposition $preposition$ is divided into subareas. Each of these areas is associated with a pair of a modifier and that preposition $\langle modifier, preposition\rangle$ The partitions are controlled by distance from the RO, see Section 2.9.18 Figure 2.44. Schirra (1993) defines subareas for the modifiers *directly*, *more or less*, *almost*, and *approximately*, see Section 2.9.15. André et al. (1987) defines subareas for the German modifiers *direkt* (*directly*), *recht gut* (*well*), and *in etwa* (*sort of*). Similarly, Matsakis et al. (2001) define fine-grained subrelations of locative direction relations and associate them with combinations of a modifier and a preposition, see Section 2.9.4. In particular, they provide mappings for combinations containing the following modifiers: *perfectly*, *nearly*, *mostly*, *loosely*, *somewhat*, *strongly*, *slightly*, and *a little*.

Compositional approach The following part provides background information about compositional approaches to modification in the literature. In this study, however, we adopt a non-compositional approach to modification.

Wunderlich & Herweg (1991) decomposes locative expressions into a binary localisation predicate LOC and a function $PROJ$. The localisation predicate $LOC(a, S)$ is true if and only if the object a is located in the *acceptance space* S. Two possible ways to define LOC are to use the topological relations *part-of* and *overlap*, respectively, between the location of the LO and the acceptance space S:

(23) $LOC(lo, S) \Leftrightarrow P(loc(lo), S)$
$LOC(lo, S) \Leftrightarrow O(loc(lo), S)$

The acceptance space S is a set of points which is obtained by applying the function $PROJ$ to RO:

(24) $LOC(LO, PROJ(RO))$

For example, the expression "*LO is above RO*" is represented by the formula (25) where $ABOVE(\cdot)$ is the function that determines the relevant acceptance space with respect to its argument:

(25) $LOC(lo, ABOVE(ro))$

Zwarts (1997) and Zwarts & Winter (2000) propose a thoroughly compositional semantics for modified locative expressions which builds upon the kind of semantic representation schematically derived in (24). They argue that the denotation of $PROJ(RO)$ cannot be a set of points, but that it is a set of arrows, i.e. finite, oriented straight lines in space. Arrows are specified by pairs of vectors where the first vector points to the origin of the arrow, and the second vector to the end point relative to the arrow's origin, see (Zwarts & Winter, 2000). This proposal is based on the assumption that modifiers modify prepositional phrases rather than prepositions, and that they denote functions MOD which work like filters on the extension of $PROJ(RO)$ – yielding a set of arrows, or, more precise, a subset of $PROJ(RO)$. Modified locative expressions can thus be represented by the following formula:

(26) $LOC(LO, MOD(PROJ(RO)))$

Zwarts (1997) and Zwarts & Winter (2000) define a semantics of locative expressions based on vectors, which is called *vector space semantics*. In vector space semantics the denotation of $PROJ(RO)$ is given by a set of arrows. The localisation predication $LOC(lo, S)$ is true, if and only if the set of points of the location of lo is a subset of the set of endpoints of the set S of arrows (Zwarts & Winter, 2000, Section 2.4). Modifiers are functions from sets of arrows to sets of arrows. They work like filters reducing the denotation of the unmodified term. For example, the set of arrows denoted by the expression *directly above X* is a subset of the denotation of *above X* with $DIRECTLY$ being the function determined by the adverb *directly*:

(27) $DIRECTLY(ABOVE(x)) \subset ABOVE(x)$

Modifiers Zwarts (1997, 2003) provides definitions for Dutch modifiers. The modifiers *vlak* (*right*) and *direct* (*directly*) are defined as distance modifiers. They let pass only those arrows whose length is close to zero. The modifiers *ver* (*far*) and *dicht* (*close*) constrain length in relative terms by comparing with a contextually given norm. The modifiers *hoog* (*high*), *laag* (*low*), and *diep* (*deep*) also constrain length in relative terms, but additionally specify constraints on the direction. *Recht* and *pal* (both mean *straight*) constrain the set of arrows to arrows that coincide with the prototypical direction[3] associated by the projective term. The opposite is the case with the modifier *schuin* (*diagonally*), it only applies to vectors that deviate from the prototypical direction between 0 and 90 degrees.

O'Keefe (2003) claims that the modifier *just* is a distance modifier which places restrictions on the distance between LO and RO.

In contrast to Zwarts, Rauh (1996) and (Herskovits, 1986) suggest that the modifiers *right* and *directly* constrain direction instead of distance. According to Rauh (1996, p211) the modifier *right* chooses those denotations which satisfy the relation particularly well. She stresses her opinion that *right* cannot exclusively be seen as a distance modifier. Herskovits (1986) adopts "ideal meanings" which can be shifted by principles. The attribution of the *ideal meaning* of a projective preposition without any shifting operations can be expressed by the modifier "directly" (Herskovits, 1986, p.185).

Zwarts & Winter (2000) introduce the distinction between two modes of modification: *non-projective modification* applies to the arrows as they are provided by the projective term, and *projective modification* operates on the projections of the arrows onto the axis defined by the prototypical direction. That means, that the directional component of those projected arrows is aligned with the prototypical direction after such a projection. The following projective locative expression describes Figure 3.4, it provides an example of projective modification which conveys that the vertical distance between the circle and the rectangle is 1 cm, but not that the actual distance between them is 1 cm.

(28) The circle is 1 cm above the rectangle.

According to Zwarts & Winter (2000), measure phrases can be used both as non-projective and as projective modifiers.

[3]Cf. Section 2.1.

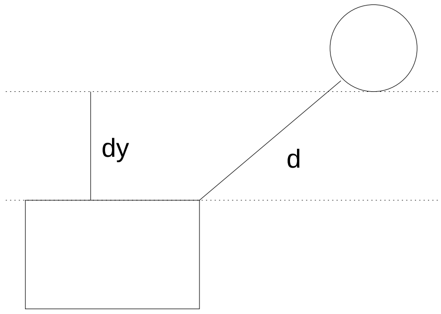


Figure 3.4: The Euclidean distance between the circle and the rectangle is d, the vertical distance is dy.

3.5 Semantic theories of projective locative expressions

This section connects projective locative expressions and direction relations defined in Section 2.8 by specifying lexical semantic theories of projective locative expressions. I will define a generic mapping from projective terms to locative direction relations. For each relation schema and range level this mapping will generate a lexical semantic theory that defines definite truth conditions which partition the domain into cases for which the corresponding expressions are true and cases for which they are false. Then, I will specify the format of lexical semantic theories of projective locative expressions that generate underspecified semantic representations. This format is going to be used for the analyses in Chapter 6.

NB: The semantic theories defined here are completely based on locative direction relations. They can only be applied to expressions that are interpreted with respect to a certain frame of reference and that there are no functional relations between the objects which the expressions refers to.

Let H^{RS_n} be a generic mapping from projective terms to locative direction relations defined by relation schema RS on range level n. Projective terms convey the prototypical direction that is associated with a projective locative expression. The following sets explicitly state this association for the directions $north$, $south$, $east$, and $west$ which correspond to the predicate names defined by the direction relation schemata in Section 2.8. The set $NORTH$ contains all projective terms that are associated with the prototypical direction $north$, the set $SOUTH$ contains the terms that are associated with $south$, and so on.

(29) a. $NORTH$ = {*above, top*[4] *, up, upwards, over, north*}
b. $SOUTH$ = {*below, underneath, beneath, bottom, down, under, south*}

[4]Note, that some of the projective terms used here are only parts of phrases, such as *top* as in *on top of*, *upwards* as in *upwards from*, *right* as in *to the right of*, and so on; compare with Section 3.1.

c. $WEST$ = {*left, west*}
d. $EAST$ = {*right, east*}

Let us define a lexical mapping H^{RS_n} for each relation schema RS and range level n. It maps projective terms onto one of the locative direction relations $north^{RS_n}$, $south^{RS_n}$, $east^{RS_n}$, and $west^{RS_n}$ depending on the prototypical direction that is associated with it.

(30)
$$H^{RS_n}(term) \mapsto \begin{cases} \lambda ro.\lambda lo.north^{RS_n}(lo, ro) & \text{if } term \in NORTH \\ \lambda ro.\lambda lo.south^{RS_n}(lo, ro) & \text{if } term \in SOUTH \\ \lambda ro.\lambda lo.west^{RS_n}(lo, ro) & \text{if } term \in WEST \\ \lambda ro.\lambda lo.east^{RS_n}(lo, ro) & \text{if } term \in EAST \end{cases}$$

This mapping is used for projective locative expressions by applying it to the unique projective term of the expressions. For example, the projective preposition *above* is a member of the set $NORTH$. Thus, it is mapped to $north^{RS_n}$ direction relations. The expressions *to the right of* and *on the right-hand side of* are mapped to $east^{RS_n}$ direction relations.

The mapping defines lexical semantic theories for all projective locative expressions that contain a projective term occurring in one of the sets in (29-d). Since this mapping is only controlled by the prototypical direction that is associated with a projective locative expression, it does not distinguish between distinct prepositions that are associated with the same direction, as for example, *below* and *underneath*, nor does it distinguish between unmodified and modified expressions, nor between distinct modifiers. For any two-dimensional spatial configuration consisting of a LO and a RO the corresponding locative direction relation determines a truth value, either *true* or *false*. As said in Section 2.3, locative direction relations are associated with domain restrictions. Spatial configurations that do not satisfy the domain restrictions, strictly speaking, make the relation *false*.

Kind of lexical mapping sought The kind of lexical mapping sought is a mapping from a projective preposition and a modifier to underspecified semantic representations each of which separates the domain into three sets: a set of pairs of objects which make the expression true, a set for which the truth value cannot be determined unequivocally, and a set of pairs of objects which make the expression false. For unmodified expressions and for some modified expressions the mapping H determines a relation schema RS, and two range levels k and l:

(31)
$$H_*(\langle mod, proj\rangle) \mapsto H^{RS_k}(proj) \oplus H^{RS_l}(proj)$$

The mapping will be such that all positive uses of $mod \times proj$ in the data will satisfy $H^{RS_l}(proj)$ and all negative uses will not satisfy $H^{RS_k}(proj)$. The following example illustrates a lexical mapping that maps the combination *directly above* onto relations of range level 1 and 2 from the relation schema OP$_P$:

(32) $H_*(\langle directly, above\rangle)$
$\mapsto H^{OP_{P1}}(north) \oplus H^{OP_{P2}}(north)$
$\mapsto \lambda x \lambda y.(north^{OP_{P1}} \oplus north^{OP_{P2}})(x, y)$

Recall that in Section 2.8 we defined groups of relations instantiating a given relation schema such that the relations of a lower range level were always subsumed by the corresponding relations from a higher range level. Therefore, the relations of the lowest range level are subsumed by any instantiation of the lexical mapping (32). For other modified expressions, such as those that are modified by *slightly* or *diagonally*, this implication is intuitively wrong. We can expect that these and maybe other modifiers exclude relations of lower range levels from their meaning.

Let us define one further lexical mapping H'_* which is used for such "diagonal" modifiers. It determines a relation schema RS and four range levels i, j, k and l with $i \leq j \leq k \leq l$:

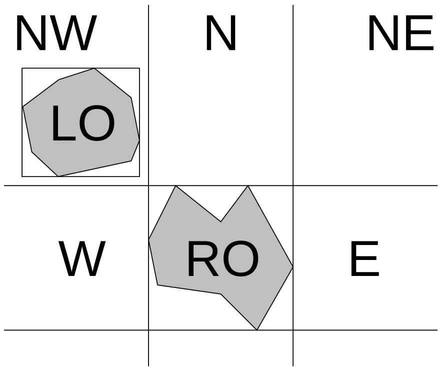
(33)

$$H'_*(mod \times proj) \mapsto H^{RS_k}(proj) \oplus H^{RS_l}(proj) \wedge \neg (H^{RS_i}(proj) \oplus H^{RS_j}(proj))$$

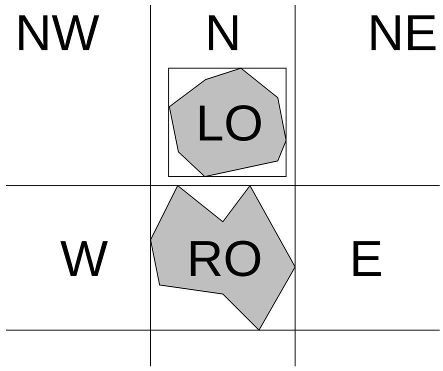
The mapping will be such that all positive uses of $mod \times proj$ in the data satisfy $H^{RS_l}(proj)$ but not $H^{RS_i}(proj)$ and all negative uses will either satisfy $H^{RS_j}(proj)$ or they will not satisfy $H^{RS_k}(proj)$. Let me give an example. Suppose the meaning of *diagonally above* is specified by means of OP$_{OP}$ relations such that it is *definitely true* if the LO is part of the NW or NE region, cf. Figure 3.5(a). It is *indefinite* if it overlaps with one of these regions and at most one of the regions N, E, and W, see for example Figures 3.5(c) and 3.5(d). It is *definitely false* for all other cases, e.g Figure 3.5(b):

(34) $north^{OP_{OP4}} \oplus north^{OP_{OP5}} \wedge \neg \, (north^{OP_{OP2}} \oplus north^{OP_{OP3}})$

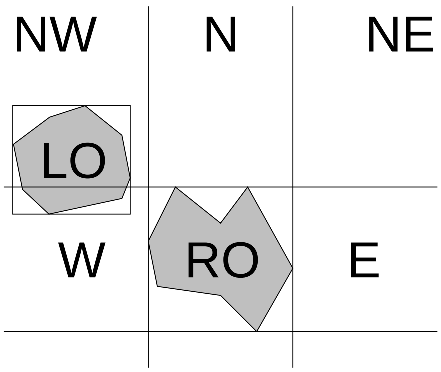
The first conjunct specifies a vague upper boundary; it is definitely true for spatial configurations that satisfy OP$_{OP4}$ and it can be true for cases satisfying OP$_{OP5}$. The second conjunct specifies a vague lower boundary. It is definitely true for spatial configurations not satisfying relations from OP$_{OP3}$ and it can be true for cases not satisfying relations from OP$_{OP2}$.



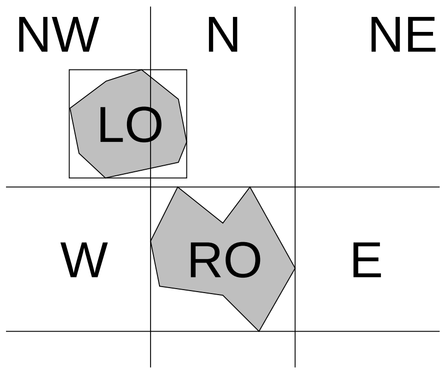
(a) $north^{OP_{OP4}} \wedge \neg north^{OP_{OP3}}$. Definitely diagonally north.



(b) $north^{OP_{OP2}}$. Definitely not diagonally north.



(c) $north^{OP_{OP5}} \wedge \neg north^{OP_{OP4}}$. Indefinite.



(d) $north^{OP_{OP3}} \wedge \neg north^{OP_{OP2}}$. Indefinite.

Figure 3.5: Examples illustrating the definition of “diagonally” in (34).

Chapter 4

A Method for Testing Semantic Theories

This chapter describes a method for testing semantic theories of projective locative expressions. This method will later be used to test the semantic theories specified in Chapter 3 with respect to data from the HCRC Map Task corpus described in Chapter 5. The results will be presented in Chapter 6.

The method is based on the model-theoretic notion of truth from formal semantics (Section 4.1). But instead of using models as formal devices we will use them here as a means to represent the spatial information of a specific spatial setting (a map of the Maptask experiments to be precise) in terms of a specific semantic theory (Section 4.2).

4.1 Evaluating semantic theories

In the previous chapter semantic theories of projective locative expressions were presented. These theories make precise predictions about the extension of certain projective prepositions. They clearly say whether or not a spatial configuration is correctly described by a preposition. This section describes a method for evaluating these theories against statements of projective locative expressions describing specific spatial configurations.

More specifically, a semantic theory T is evaluated with respect to a pair consisting of a locative expression e and a spatial configuration s. Let the term $[e]_T$ be the semantic representation of the expression e according to the semantic theory T. Now, given that a proficient user of the language in question testifies that e correctly describes the spatial configuration s, then $\langle e, s \rangle$ supports T if $[e]_T$ is compatible with s. If, however, $[e]_T$ is incompatible with s then $\langle e, s \rangle$ counts as evidence against T.

A simple and direct way of obtaining such pairs $\langle e, s \rangle$ for English is to collect natural language utterances or written sentences which native speakers of English have produced with the intention to describe the location of an object with respect to another object. As mentioned earlier, this study takes this kind of data from the HCRC Map Task corpus. This corpus and the preparation of the data for the evaluation will be described in Chapter 5.

Before we can apply such an evaluation method we have to be clear about the following two points: (i) How is a semantic representation $[e]_T$ constructed from an expression e? (See paragraph *Semantic representation* below). And (ii) what does it mean for a semantic representation $[e]_T$ to be compatible with some spatial configuration s? (This question is addressed in the paragraph *Deriving a hypothesis from the semantic representation*.)

Semantic representation In Section 3.5 we have already defined lexical mappings H^{RS} which map combinations of projective prepositions and modifiers onto locative direction relations from a relation schema RS. These mappings only provide part of the construction of a semantic representation. The complete mapping from projective locative expressions to semantic representations is specified as follows.

Projective locative expressions are only suitable for use in the evaluation procedure when they are conveyed by statements which speakers make in order to describe some specific spatial configuration. Let us formalise the occurrences of suitable projective locative expressions by feature structures of the following type:

(1)
$$\begin{bmatrix} \text{prep} & : & \text{a symbol denoting a projective preposition} \\ \text{mod} & : & \text{a list of symbols denoting modifiers} \\ \text{lo} & : & \text{a symbol uniquely referring to the located object} \\ \text{ro} & : & \text{a symbol uniquely referring to the reference object} \\ \text{use} & : & \text{a truth value} \end{bmatrix}$$

Feature structures of this type represent the preposition, $prep$, a list of modifiers, mod, and unique references to the located object, lo, and the reference object, ro, respectively. The values of the features $prep$ and mod are lexical items. The values of the features lo and ro are unique labels or unique names of the objects described by this expression.[1] The predication that is denoted by the expression is composed of the binary relation predicate that is obtained by applying a lexical mapping H^{RS} to the combination of the preposition and the modifiers. The arguments of the predication are the values of the features lo and ro. Let us use the notation $f.prep$ to refer to the value of the feature $prep$ from the feature structure f. And similarly, $f.mod$, $f.lo$, $f.ro$

[1]This way of coding LO and RO restricts the capacity of this representation format to locative expressions which uniquely refer to exactly one LO and one RO. Locative expressions containing quantified terms or plural terms cannot be formalised this way. However, the procedure can be extended to handle arguments which are specified by quantified terms under the condition that the domain specified by the restrictor can be determined exactly.

and $f.use$ to refer to the values of the respective features. The predication expressed the feature structure f is the following:

(2) $(H^{RS}(\langle f.prep, f.mod\rangle))(f.lo, f.ro)$

The last feature, use, specifies whether the expression is used positively ($f.use = true$) or negatively ($f.use = false$). With a positive use of a locative expression a speaker makes the statement that the predication conveyed by the expression is true, and with a negative use the speaker makes the statement that the corresponding predication is false.

Putting it all together, feature structures f_i of the type described above in (1) represent occurrences of locative expressions e_i. Semantic representations $[e_i]_{RS}$ of e_i are defined by the following mapping from feature structures f_i onto formulas of some formal language L_{RS} the vocabulary of which is defined by the relation schema RS.

(3) let $\pi_j := H^{RS}(\langle f_i.prep, f_i.mod\rangle)$

$$[e_i]_{RS} := [f_i]_{RS} := \begin{cases} \pi_j(f_i.lo, f_i.ro) & ;\text{ if } f_i.use = true, \\ \neg\pi_j(f_i.lo, f_i.ro) & ;\text{ if } f_i.use = false \end{cases}$$

Let me give an example to illustrate the construction of a semantic representation from a locative expression. The following two sentences contain locative expressions which are used positively and negatively, respectively. They describe the spatial configuration shown in Figure 4.1 on page 118.

(4) a. The triangle is above the rectangle.
b. The triangle is not to the left of the rectangle.

Given that t_1 is a symbol that uniquely refers to the triangle and r_1 a symbol that uniquely refers to the rectangle, the above statements are represented by the following feature structures:

(5) *a.*
$$\begin{bmatrix} \textit{prep} & = & \textit{above} \\ \textit{mod} & = & \textit{()} \\ \textit{lo} & = & t_1 \\ \textit{ro} & = & r_1 \\ \textit{use} & = & \text{true} \end{bmatrix}$$

b.
$$\begin{bmatrix} \textit{prep} & = & \textit{left} \\ \textit{mod} & = & \textit{()} \\ \textit{lo} & = & t_1 \\ \textit{ro} & = & r_1 \\ \textit{use} & = & \text{false} \end{bmatrix}$$

The mapping specified in (3) then produces the following two formulas from (5-a) and (5-b) for an arbitrary relation schema RS. Note, that $H^{RS}(above)$ yields the binary predicate $north^{RS}$, and $H^{RS}(left)$ yields the binary predicate $west^{RS}$, compare with page 109:

(6) a. $north^{RS}(t_1, r_1)$
b. $\neg west^{RS}(t_1, r_1)$

Deriving a hypothesis from the semantic representation At this point, we have a mapping that produces semantic representations $[e_i]_T$ of locative expressions e_i. Next, we want to set up hypotheses stating that $[e_i]_T$ is compatible with the spatial configuration s_i described by e_i:

(7) $[e_i]_T$ is compatible with s_i

Let us formalise the notion of *compatibility* by means of the model-theoretic notion of truth. In order to do so, we will assume that there is an algorithm – and in fact such an algorithm will be presented in the next section – which generates a model M_{T,s_i} for a theory T from a spatial configuration s_i such that M_{T,s_i} satisfies every (formal) sentence from the theory T which is about the objects of s_i.[2] Given such a model M_{T,s_i} the hypothesis derived from $\langle e_i, s_i \rangle$ for T can be formulated equivalently as

(8) $M_{T,s_i} \models [e_i]_T$

For any pair $\langle e_i, s_i \rangle$ and any theory T this hypothesis is either true or false. Based on such hypotheses we derive evidence for and against a theory T in the following way. If the hypothesis is true, then the pair $\langle e_i, s_i \rangle$ supports T. Otherwise, the pair $\langle e_i, s_i \rangle$ provides evidence against T.

4.2 Models of spatial configurations

This section specifies an algorithm that generates models M_{T,s_i} from specific two-dimensional spatial configurations s_i for a specific semantic theory T of projective locative expressions. The creation of such models is dependent on the semantic theory T which has been selected. Each such theory T generates from a given spatial configuration s_i its own model M_{T,s_i}, which reflects the interpretation T assigns to the primitive predicates, i.e. the relations. Thus, each model M_{T,s_i} is both a representation of the spatial configuration s_i from which it has been derived and an instantiation of the semantic theory T.

A model M for a formal language L is a structure $\langle U, I \rangle$ that consists of a set U of individuals, called the universe, and an interpretation function I. The interpretation function I assigns each constant c_i of L an individual $u_i \in U$

[2]NB: Formal sentences from the theory T are formulas of the corresponding formalism which a implied by T.

(9) $I(c_i) = u_i$

and I assigns each n-place predicate symbol P^n from L a set of n-tuples from the universe U such that

(10) $I(P^n) \subseteq U^n$.

The predication of the predicate P^n to the tuple of n constants $\langle c_1, \ldots, c_n \rangle$ is satisfied by M if and only if the application of the interpretation function I to each constant of that tuple yields an n-tuple that is a member of the extension of P_n:

(11) $M \models P^n(c_1, \ldots, c_n) \iff \langle I(c_1), \ldots, I(c_n) \rangle \in I(P^n)$

This notion of satisfaction provides us with a precise notion of truth for every predication $P^n(c_1, \ldots, c_n)$ with respect to a model M.

For every relation schema RS defined in Section 2.8 we specified a semantic Theory T in Section 3.5 defining four locative direction relations $north^{RS}$, $south^{RS}$, $east^{RS}$, and $west^{RS}$. These locative direction relations provide precise notions of truth with respect to two-dimensional spatial configurations s. In this way it is possible to determine the truth value of any locative direction relation for every possible pair of objects from s. Let us assume that each object of s is associated with a unique constant c_i, so that these constants can be used to uniquely refer to the objects of s.

The following steps generate a model $M_{T,s} = \langle U, I \rangle$ for T from a spatial configuration s. First, the universe U is created. For each spatial object of s there is a unique constant c_i, and for each constant c_i a new individual u_j is inserted into U. The interpretation function I is to map c_i onto u_j. Thus every model $M_{T,s}$ constructed in this way satisfies the following condition:

(12) $U = \{u \mid \exists c_i : u = I(c_i)\}$

Second, the extension of all locative direction relations defined by RS is constructed. Every relation rel^{RS} defined by RS is applied to all possible pairs of objects in s referred to by the corresponding pair of constants $\langle c_k, c_l \rangle$. Only if the pair $\langle c_k, c_l \rangle$ satisfies the truth conditions $rel^{RS}(c_k, c_l)$ the pair of the corresponding individuals from U is added to the extension of rel^{RS}. Every model $M_{T,s}$ constructed in this way satisfies the following condition:

(13) $M_{T,s} \models rel^{RS}(c_k, c_l) \iff \langle I(c_k), I(c_l) \rangle \in I(rel^{RS})$

Semantic representations of projective locative expressions as they are defined in the previous section are always of one of the following two forms (see page 115) where r in M is a binary relation and c_1 and c_2 are constants:

(14) a. $r(c_1, c_2)$

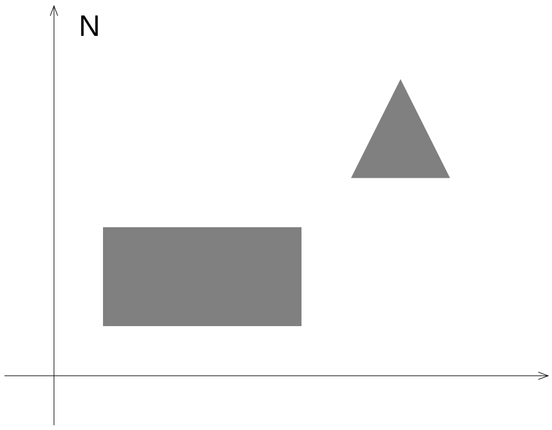


Figure 4.1: A two-dimensional spatial configuration consisting a rectangle r_2 and a triangle t_2. The direction of the vertical axis is associated with the direction *north*.

b. $\neg r(c_1, c_2)$

It is easy to see that models $M_{T,s}$ generated above satisfy $r(c_1, c_2)$ if and only if the relation denoted by r applies to the objects referred to by c_1 and c_2 in the spatial configuration s. And, similarly, models $M_{T,s}$ satisfy $\neg r(c_1, c_2)$ if and only if the relation denoted by r in M does not apply to the objects referred to by c_1 and c_2 in the spatial configuration s.

Example Let me illustrate the generation of a model by an example. We want to generate a model $M_1 = \langle U_1, I_1 \rangle$ of the spatial configuration shown in Figure 4.1 for the theory defined by the relation schema OP$_{P2}$. There are two constants r_1 and t_1, and the relation schema OP$_{P2}$ defines four locative direction relations corresponding to the main cardinal directions. A complete application of all relations of OP$_{P2}$ to all combinations of constants yields the following model:

(15) $M_1 = \langle U_1, I_1 \rangle$ where $U_1 = \{u_1, u_2\}$ and I_1 is defined as follows:

a. $I_1(r_2) = u_1$
b. $I_1(t_2) = u_2$
c. $I_1(north^{OP_{P2}}) = \{\langle u_2, u_1 \rangle\}$
d. $I_1(south^{OP_{P2}}) = \{\langle u_1, u_2 \rangle\}$
e. $I_1(east^{OP_{P2}}) = \{\langle u_2, u_1 \rangle\}$
f. $I_1(west^{OP_{P2}}) = \{\langle u_1, u_2 \rangle\}$

The model maps the constants r_1 and t_1 onto the individuals u_1 and u_2, respectively. The individual u_1 identifies the rectangle and u_2 the triangle. Following the definitions of the associated locative direction relations the triangle, u_2, is *north* and *east* of the rectangle, u_1, and u_1 is *south* and *west* of u_2.

Now, we can check the hypotheses in (16) derived for the sentences in (4) (‘*the triangle is above the rectangle*’ and ‘*the triangle is not to the left of the rectangle*’):

(16) a. $M_1 \models north^{OP_{P2}}(t_1, r_1)$
b. $M_1 \models \neg west^{OP_{P2}}(t_1, r_1)$

M_1 satisfies both formulas. Therefore, the combination of the corresponding locative expressions and the spatial configuration in Figure 4.1 support the semantic theory T defined by OP$_{P2}$.

Chapter 5

Data

The data that will be discussed in this chapter is based on the HCRC Map Task corpus (Anderson et al., 1991; Isard, 2001). The Map Task corpus contains uses of locative projective expressions and representations of the corresponding spatial situations. The aim of this chapter is to document all steps that have been carried out to extract projective locative expressions from the corpus and to prepare them for the application of the procedure described in Chapter 4. Details of the data and the results of the application will be presented in Chapter 6.

The first section describes the HCRC Map Task corpus which is a collection of route description dialogues. The routes which had to be described in the dialogues were specified by means of maps consisting of schematic landmarks. As part of the route descriptions the speakers used projective locative expressions to describe the location of landmarks with respect to other landmarks. For each schematic map a polygon model is created (Section 5.2). All locative projective expressions which meet certain requirements are marked and annotated with reference links and semantic information (Section 5.3).

5.1 HCRC Map Task corpus

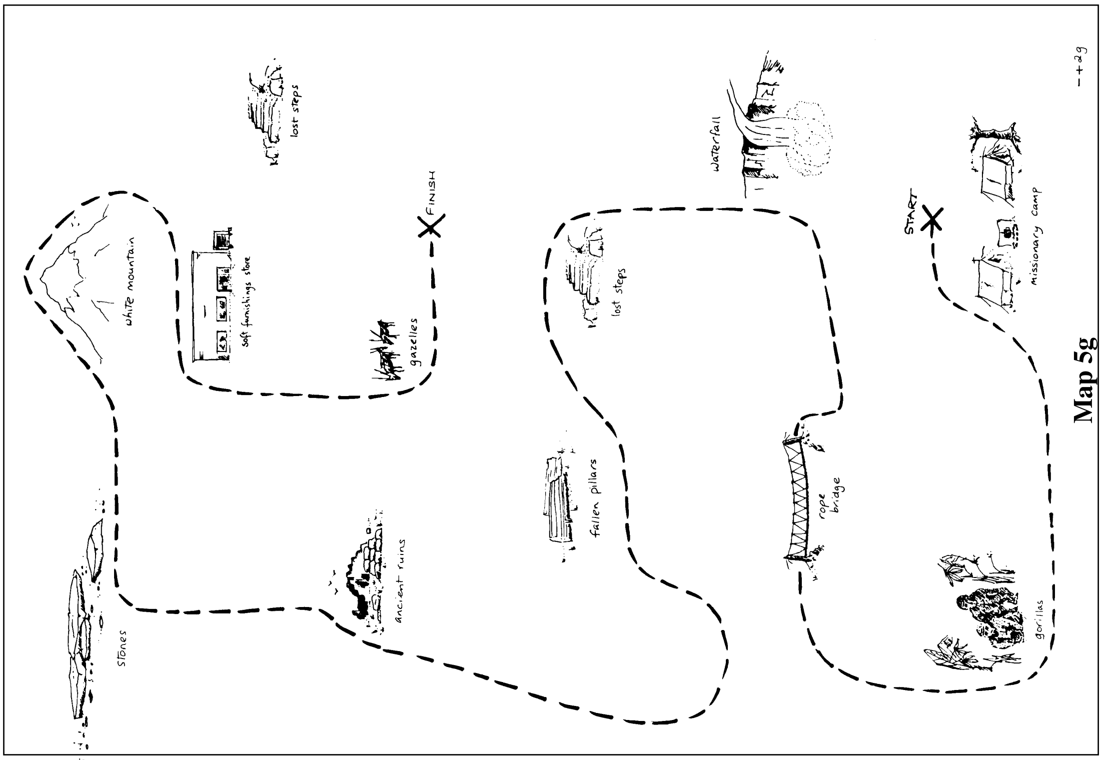
The *HCRC Map Task Corpus* (Anderson et al., 1991) is a collection of dialogues of people trying to accomplish a *Map Task*. Map Tasks are route description tasks where one subject tries to explain a route printed on a map to another subject. The characteristics of the particular version applied in the HCRC Map Task experiments are explained in Section 5.1.1. The collection comprises 128 dialogues which have been recorded with 32 subjects, each of whom took part in 4 experiments. The complete data collection consists of (i) recordings of the dialogues, (ii) their transcriptions, (iii) various annotation layers, and (iv) electronic copies of all maps. Section 5.1.2 introduces the maps and Section 5.1.3 the transcriptions and some relevant annotation layers. I will draw the reader's attention to a subtask of the Map Task in Section 5.1.4 – besides describing the route the participants also talk about the location of landmarks appearing on this map. The solution of such *localisation tasks* is a are frequently a key to the success of the actual route description task. The parts of the corpus that a part of attempts to solve localisation tasks provide the empirical data used in this thesis.

5.1.1 Task

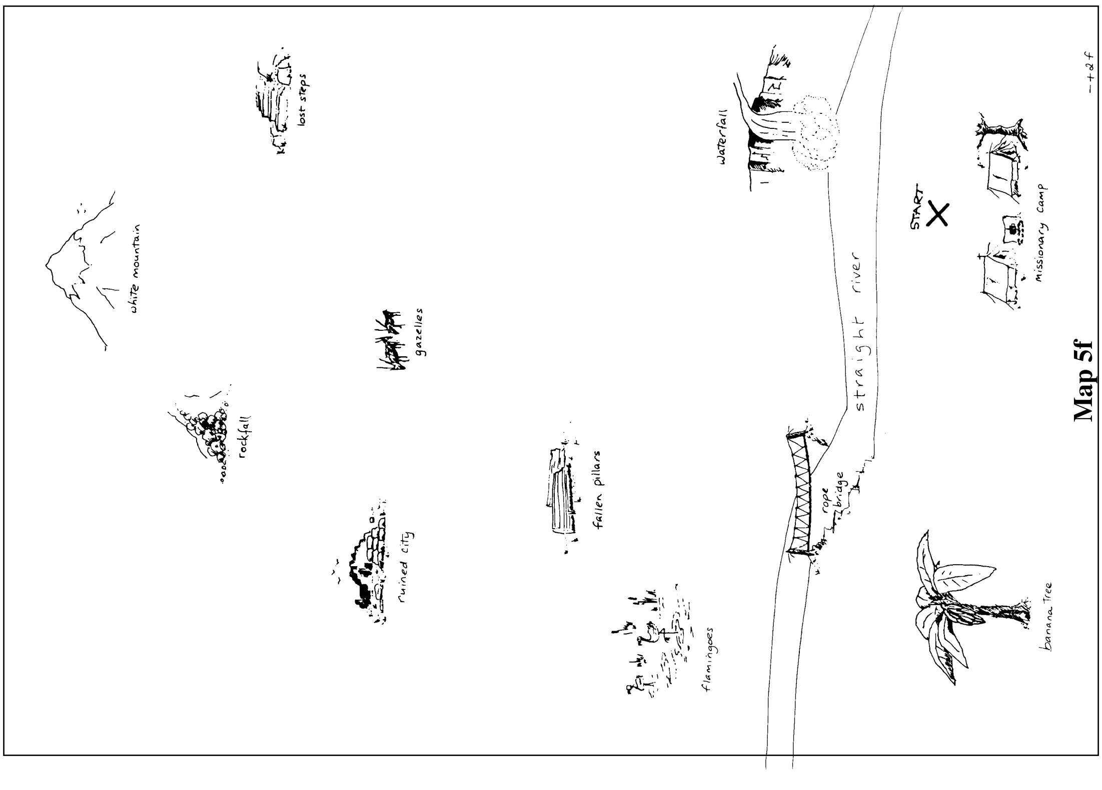
In the Map Task described in (Anderson et al., 1991) two participants engage in a conversation about a route that is printed on a map. Each of the participants in this task has a schematic map containing line drawings of objects, so called *landmarks*. The map of one participant – the *instruction giver* – has a path drawn in it which does not appear on the map of the other participant – the *route follower*. Their joint task is to replicate that path on the map of the route follower. Communication between the participants is restricted to natural language communication and, in some experiments, the participants additionally have eye contact. In all experiments, the participants cannot see the other participant's map and they are not supposed to use gestures or to show their maps to each other. The task can be accomplished only by means of what the participants say to one another. Additional difficulty is introduced by giving maps to the instruction giver and the route follower which slightly deviate from each other in that the landmarks on the two maps do not match exactly. The participants are warned in advance, but they are not told what kinds of mismatches to expect. These mismatches add an additional task to the actual route description task: the participants need to align their information about the landmarks before they can use them in the route description task (see Section 5.1.4).

5.1.2 Maps

The maps used in the HCRC Map Task experiments are schematic maps containing line drawings that serve as landmarks. Each landmark is associated with a textual label. The maps are grouped in pairs: there is a map for the instruction giver and one for the route follower. I will also use the terms *giver map* and *follower map*. Figure 5.1 shows an example of a pair of maps. 16 different pairs of maps have been used to collect the entire corpus, so that for each map there are 8 dialogues. None of the pairs consists of identical maps. Nevertheless, any two maps belonging to a single pair are supposed to represent the same situation. There are three kinds of differences between the maps in a pair. Some landmarks simply do not appear on the other map. For example in Figure 5.1 starting from the top left of the giver map, there are *stones*, a *soft furnishing store, lost steps* in the middle, and a *straight river*, all of which do not appear on the follower map. The follower map, however, contains the landmarks *rockfall* and *flamingos* which do not appear on the giver map. The second kind of difference between the maps are distinct textual labels – identical line drawings appear at the same location but they are associated with distinct textual labels. In Figure 5.1 for example, there is a landmark on the left-hand side in the upper middle which appears as *ancient ruins* on the giver map and as *ruined city* on the follower map. In



(a) giver map *m5g*



(b) follower map *m5f*

Figure 5.1: Pair of maps number 5

the third kind of difference both line drawing *and* label are different – two distinct line drawings with distinct labels appear at the same location of the corresponding maps. For example, look at the bottom left corner of the maps in Figure 5.1. On the giver map there are *gorillas* and on the follower map there is a *banana tree*. Every giver map in the data collection contains one pair of duplicated landmarks. The giver map in Figure 5.1, for example, contains two copies of the landmark *lost steps*: one is at the top right, the other one is in the center. There are no follower maps in the data collection which contain duplicated landmarks.

The data collection provides electronic copies of the maps. They are supplemented by a table containing all textual labels defining unique identifiers (*ID*) for each label – a prefix indicating the map and a symbol derived from the label itself. For example, the label *ancient ruin* in map 5 has the ID `m5_ancient_ruin`. The table containing these identifiers additionally describes on which maps the labels and the corresponding landmarks appear. For example it specifies that the label *ancient ruin* appears on the giver map once, and that it doesn't appear on the follower map (compare the maps on page 123).

5.1.3 Annotation layers

Transcriptions and annotations are coded in a multi-layered structure (Isard, 2001). We use the *Conversational Games* layer and the *Conversational Moves* layer (Carletta et al., 1997) to construct a coherent sequential transcription of the dialogue that has been freed of any overlaps of utterances, i.e., when the participants talk simultaneously. *Conversational moves* define utterance units. They are annotated with information about who speaks and with information about the type of the conversational move. Carletta et al. (1997) distinguish between the following types: *commands*, *statements*, *preparation* moves, different kinds of *questions*, and different kinds of *responses*. The structure of the entire dialogue is determined by *conversational games* which partition the dialogue into segments and define an ordering between them. The *Conversational Games* layer enables us to derive a sensible sequential ordering of conversational moves even if they overlap or have been uttered simultaneously.

The *Landmark References* layer (Bard et al., 2000) marks nominal expressions that refer to landmarks and codes co-reference by means of identifiers that belong to the corresponding textual labels (see above). The expressiveness of this coding layer is limited to refer to textual labels and it is sufficient for marking co-reference. However, it should be clear that we cannot use the same coding scheme if we want to encode the specific landmark a nominal expression refers to. Certainly, textual labels or their IDs can be uniquely resolved to landmarks in most cases, in particular, when there is only one landmark with that particular textual label on the map *and* when we know which map we are talking about. However, if there are two landmarks with the same textual label on the same map then this coding scheme cannot distinguish between the two. Similarly, it cannot distinguish a landmark on the giver map from a landmark with the same textual label on the follower map. A different ID scheme which extends the one of the *Landmark References* layer is described in Section 5.3.2.

5.1.4 Localisation subtask

The participants of a Map Task experiment try to align their information about the landmarks of the maps, when they become aware that there are mismatches between them. In contrast to the route description task, the *localisation subtask* is symmetric: the instruction giver needs information about the follower map in order to describe the route in a way that the route follower can understand easily. And the route follower needs information about the giver map in order to understand the route directions from the instruction giver.

The alignment process comprises two aspects. One aspect is that the participants identify labels from the other map with landmarks on their own map. This is trivial if a landmark does not have a duplicated twin and the textual labels are identical on both maps. For example, the landmark *white mountain* appears exactly once on both the giver map and the follower map in Figure 5.1. A more interesting case of identification is at hand when the labels of two corresponding landmarks are distinct, as for example, the *ancient ruins* on the giver map of Figure 5.1 and *ruined city* on the corresponding follower map. The following utterance is taken from dialogue *q2ec3*:

(1) FOLLOWER: i'm at the *ancient ruins*.

This shows that the route follower adopts the label *ancient ruins* from the giver map. Identification can also go wrong. The dialogue section shown in (2) – this one too is taken from dialogue *q2ec3* – presents a case where the participants identify the *rockfall* from the follower map with the *stones* from the giver map.

(2)
a. GIVER: now, have you got some stones at the top?
b. FOLLOWER: i've got rock fall.
c. GIVER: okay, right, if you head it up (*sic!*) ... towards the *rock fall*.
. . .
d. GIVER: okay, then, so you're at you're at the ... stones now?
e. FOLLOWER: yeah.
. . .
f. FOLLOWER: under the *stones*?
g. GIVER: under the stones.

In (2-c) the instruction giver adopts the label *rock fall*, but changes back to *stones* in (2-d), which makes the route follower adopting the label *stones* in (2-e). The lack of any other talk about correlating the landmarks in a different way and the confirmation in (2-g) suggest that the participants have identified those landmarks even though they do not match (compare with Figure 5.1).

Figure 5.2: Current position in a route description task with map 5.

The other aspect of the alignment process is the reconstruction of the location of a missing landmark. In such a case, the participant who has that landmark on his or her map describes its location or responds to questions about its location. We find three kinds of localisations in the corpus: a landmark's location can be specified (i) relative to other landmarks, (ii) relative to the whole map, and (iii) relative to the current position of the path. All following examples relate to map 5 shown in Figure 5.2. The examples in (3) show localisations of landmarks relative to other landmarks:

(3) a. Is the rope bridge *below the fallen pillar*?
b. The rope bridge is *to the left of the waterfall*.

This kind of localisation frequently contains projective locative expressions. The next two examples in (4) specify the location of a landmark with respect to the map as a whole:

(4) a. Do you have gorillas *at the bottom*?
b. The fallen pillars are *in the center of the map*.

Here, landmarks are localised by specifying parts of the map with the phrases *the bottom* and *the center of the map*. The located objects are specified to be located in the regions specified by those phrases. The fundamental difference to projective locative expressions is that the reference object (here it is the entire map) *contains* the located object. We also find localisations relative to the current position of the route description task. For example, Figure 5.2 shows a section of the giver map with an arrow pointing to the current position of the route. The arrow also indicates the direction of the route.

(5) a. The white mountain is *in front of me*.
b. The white mountain is *to my right*.

These sentences locate the landmark *white mountain* with respect to the current position. It is typical for such localisations that they occur with distinct instantiations of the relative frame of reference, compare with Section 2.4. The description in (5-a) uses the relative frame of reference from the perspective of a virtual observer that inhabitates the two-dimensional plane and who has just come the way shown by the arrow. This virtual observer is directly facing the *white mountain*, therefore, the speaker might use the prepositional phrase *in front of* to describe the relation between that mountain and that current position of the route. The description (5-b) is in the relative frame of reference from the perspective of the real observer who looks directly at the whole map. Here, the speaker uses the prepositional phrase *to my right* to describe the relation between the mountain and the current position.

It should be noted that localisation of a landmark relative to other landmarks could in principle make use of the perspective from the virtual observer, too. But we do not find any such cases in the corpus.

5.2 Polygon models of maps

This section describes the representation format of *polygon models* which explicitly represents the identity of objects and their spatial properties. It is used to represent the maps of the Map Task. Polygon models define unique identifiers for all landmarks and explicitly represent contour and location of each landmark. Unique identifiers of landmarks are used in the annotation process described in Section 5.3. The procedure described in Chapter 4 is used to apply spatial relations from Section 2.8 to spatial objects of the polygon models defined in this section.

The representation format of the maps as they are available from the original data collection of the corpus is a bitmap format which does not contain any explicit information about landmarks – neither about the identity of landmarks nor their geometric properties. Therefore, I manually created polygon models of the maps. For all landmarks they provide us with their contour and their location so that we can compute their geometric properties and arbitrary geometric relations that hold between them. Landmarks are represented by closed polygons approximating their contour. The location of the polygon correlates with the location of the corresponding landmark on the original map. Polygons enable us to derive geometric properties of landmarks such as their area, their centroid, bounding boxes, etc. An example is given in Figure 5.3 which shows a copy of giver map 5 and the corresponding polygon model.

We have drawn a polygon around each landmark such that it tightly surrounds the whole drawing including marginal parts of the background. The polygons approximate the contours of the landmarks in more or less rough detail, so that fine features are lost, but significant protrusions are preserved. The strategy of drawing polygons around the entire drawings is based on the observation that the path on the map task maps leads in almost all cases around the entire drawings of landmarks. More specifically, the paths on the giver maps do not touch parts of any drawings nor do they lead through the space between separated parts of any drawing. There are only a few exceptions to this observation: *footbridge* on *m1g*, *rift valley* on *m2g*, *rope bridge* on *m5g*, *m10g*, and *m13g*, and finally, *iron bridge*, *mountain stream*, and *green bay* on *m15g*. Although in these cases the path overlaps with the drawings of the landmarks they do not provide evidence against this strategy. The paths cross the drawings of the bridges in a conventional way, that means that they go *over* bridges. The same holds for the valley, here the path goes through the deepest point of the valley. So, although the paths overlap with these landmarks, they do it in a way that is precisely determined by the drawing of the landmark. The landmarks *mountain stream* and *green bay* differ from other

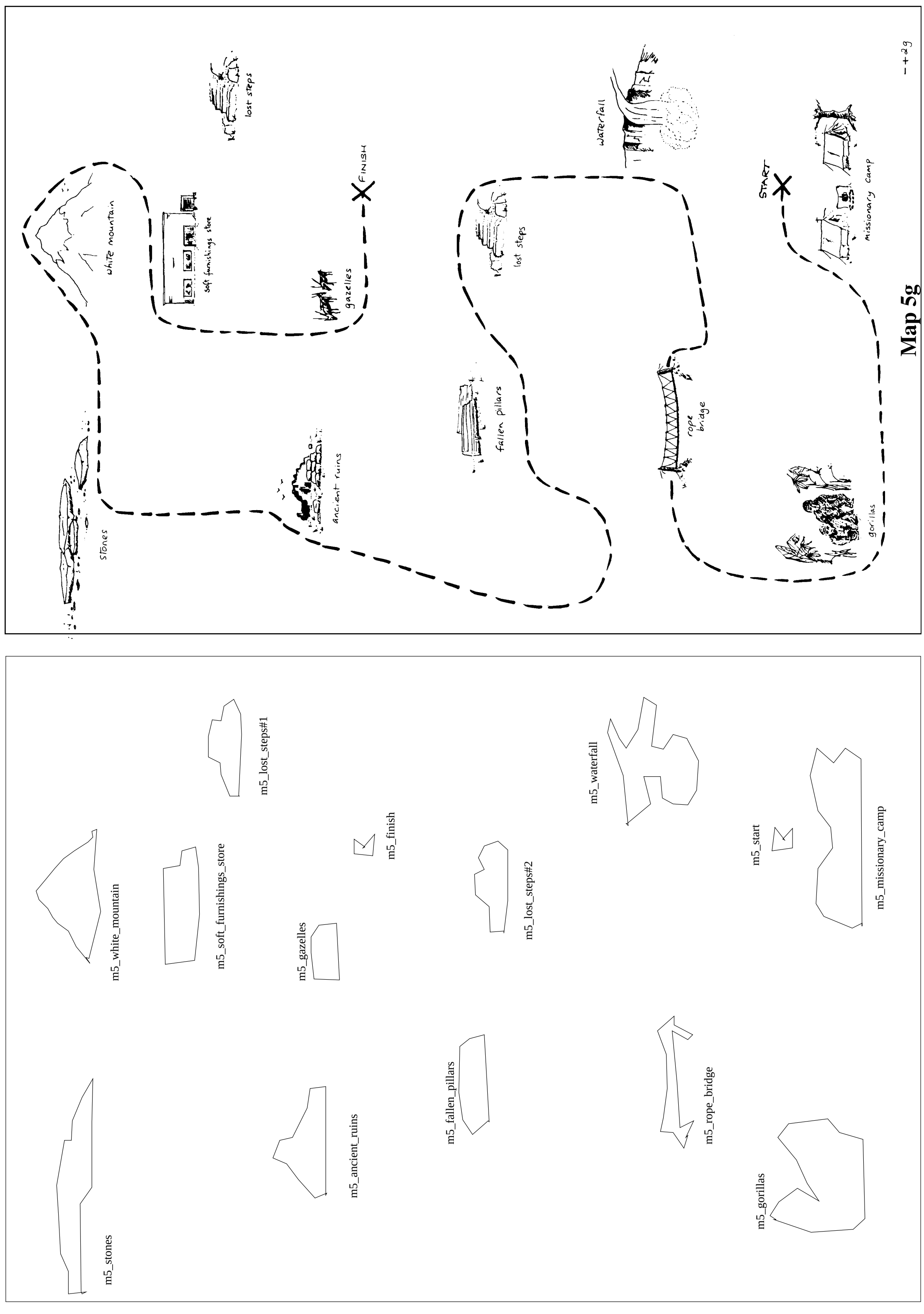
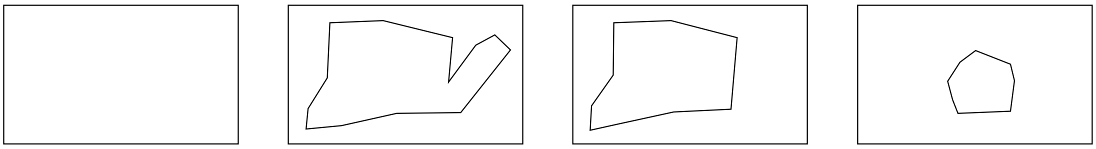


Figure 5.3: Polygon model of giver map 5 (*m5g*).

landmarks on the maps in that they are structural features of the landscape represented by the map. Besides that, they have a clear contour line, so that there is no distinction between figure and background anyway. I conclude that the map task participants use the entire drawings as landmarks and that there is no need to split landmarks into figure and background.



(a) Landmark "*old mill*" or "*mill wheel*" (b) Contour of mill and background (c) Contour of mill (d) Contour of mill wheel

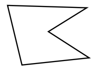
Figure 5.4: Alternative contours representing a landmark

Let me illustrate the strategy with a few examples. The first drawing in Figure 5.4 is a landmark taken from map 12. On the giver map it is associated with the label *old mill* and on the follower map with the label *mill wheel*. The drawing contains a mill consisting of a shed and a water wheel in the foreground and a brook in the background. The second picture shows a polygon representing the landmark consisting of the whole drawing including the mill and the brook. Alternatively, the label *old mill* could also be used to only refer to the mill. In that case the landmark would be represented by a polygon like the one displayed in (c). And the label *mill wheel* could be used to only refer to the wheel part of the mill the contour of which is represented by the polygon in (d). Although these two alternatives are plausible, only the polygon displayed in (b) complies with our strategy which surrounds the whole drawings.

Another example is shown in Figure 5.5(a). The landmark with the label *ancient ruins* is a drawing of a ruin and some background including three birds above the ruin. The polygon that represents this landmark also includes the birds. We slightly deviated from our strategy for drawing polygons around the crosses that mark *start* and *finish* of the paths. In order to explicitly represent the intersection of the cross and also its spatial extension, the polygons start in the center of the cross, go through all four endpoints, and end again in the center. An example of such a polygon is displayed in Figure 5.5(b).

Each polygon is associated with a *landmark identifier*. *Landmark identifiers* are symbols that uniquely identify landmarks on maps. They are composed of two or if necessary three components: (i) the number of the map and (ii) the label of the landmark. For example, the polygon that represents the landmark *ancient ruins* on giver map 5 is associated with the landmark identifier `m5_ancient_ruins`. If two landmarks appearing on one map share the same label, then a third component is appended to the other two components. It is (iii) a component that is a number distinguishing between double occurrences of a label on a single map. The landmark that is closest

(a) Ancient ruins and polygon



(b) Polygon of a start cross (without the cross)

Figure 5.5: Example contours of landmarks.

to the top of the map is marked with '*#1*' and the other one with '*#2*'. For example, on map 5 there are two landmarks with the label *lost steps*. The corresponding polygons are associated with the symbols `m5_lost_steps#1` and `m5_lost_steps#2`, respectively. Landmark identifiers are printed in Figure 5.3 in arbitrary locations around the corresponding polygon. We use these symbols to refer to the polygons of the polygon model and also to refer to landmarks on a map. The first two components of landmark identifiers are identical to the symbols used in the reference coding layer of the map task corpus (see Section 5.1.3). The third component is introduced in order to distinguish between double occurrences of landmarks on one map.

Polygon models are finite, approximate representations of two-dimensional spatial configurations. They model spatial properties such as contour and location by means of closed polygons. Polygons are defined by lists of two dimensional points. These points define the boundary of an area in the Euclidean plane of which it is possible to determine for any arbitrary point whether it lies inside or outside that area. The boundary of that area is defined by a set of finite straight lines. One line connects the first point and the second point. Another line connects the second and the third point, etc. Finally, the last point of the list and the first point are connected. Let POL be the set of all possible polygon specifications:

(6) $POL := \langle \mathcal{R} \times \mathcal{R} \rangle^n$ for all $n \in \mathcal{N}$ with $n > 0$

Formally, a polygon model defines a set of polygon specifications each of which is tied to an identifier. A polygon model $G = \langle ID, p \rangle$ is a structure that consists of a non-empty set of symbols ID and a total function $p : ID \mapsto POL$ which maps each symbol of ID onto a polygon specification from POL.

For each map m in the Map Task corpus we create a polygon model $G_m = \langle ID_m, p_m \rangle$ where ID_m is a set of landmark identifiers and p_m a function that maps each landmark identifier onto a polygon specification from POL. Polygon models establish the connections between names of landmarks and their spatial properties. For example, the landmark identifier `m2_start` from giver map 2 is associated with the following polygon specification:

(7) p_{m2g}(`m2_start`) = ($\langle$2640, 345$\rangle$, $\langle$2880, 195$\rangle$, $\langle$2385, 180$\rangle$, $\langle$2475, 555$\rangle$, $\langle$2895, 510$\rangle$, $\langle$2640, 345$\rangle$)

The polygon that is defined by this list of points is depicted in Figure 5.5(b). The index $m2g$ in the example is a symbol that uniquely determines giver map 2. We call such symbols *map identifiers*. They are composed according to the following convention: the letter '*m*' followed by the number of the map, followed by the letter '*g*' or '*f*' indicating the version where the letter '*g*' is used for maps of the instruction givers and '*f*' for maps of the route followers.

Counterpart Relation The participants of the map task experiments assume that both maps represent the same spatial situation even though the maps do not match exactly, see Section 5.1. Because they assume this, the participants try to identify the labels used by the other participant with landmarks on their own map. This is trivial for landmarks which occur exactly once on both maps with the same label. In other cases it takes a little more conversational effort. But as soon as such an identification has been established, the participants can use either label to refer to a landmark on their own map. For example in dialogue `q1nc1` a landmark appears on the map of the instruction giver as *old mill* whereas it has the label *mill wheel* on the map of instruction follower. Nevertheless, the instruction follower uses the label of the giver map *m12g* to refer to the *mill wheel* on his or her map:

(8) FOLLOWER: i'd if i need to go ... ... beneath the old mill right, okay?

We model this kind of relation between the landmarks of a pair of maps by a counterpart relation cp that relates landmark identifiers to each other. The motivation for this counterpart relation is that it simplifies the annotation process of the reference coding layer (Section 5.3) because the annotators can derive the landmark identifier from the labels used by the participants and do not need to distinguish between different landmark identifiers of giver map and follower map. The counterpart relation $cp : ID \times ID$ is an equivalence relation. It is defined between landmark identifiers referring to landmarks that occupy the same location relative to the origin of the corresponding map. Each landmark identifier always has at least one counterpart, i.e. the landmark identifier itself, and at most two, namely the landmark identifier itself and the one from the map of the other participant. An example for a landmark identifier having two counterparts is `m12_old_mill` from map *m12g*. It has the trivial counterpart `m12_old_mill` and the counterpart `m12_mill_wheel` from the map *m12f*.

Merged Polygon Models The polygon models of a pair of maps belonging to the same dialogue are merged to a *merged polygon model*. Merged polygon models explicitly represent congruence and difference with respect to the landmarks of the single maps.

A merged polygon model $G' = \langle G_g, G_f, cp\rangle$ consists of a polygon model $G_g = \langle ID_g, p_g\rangle$ of the giver map, a polygon model $G_f = \langle ID_f, p_f\rangle$ of the follower map, and a binary counterpart relation $cp : ID_g \cup ID_f \times ID_g \cup ID_f$ between all landmark identifiers of both polygon models.

Let's define a convenience function $pol(l, m)$ that combines p_g and p_f such that we can use the landmark identifiers of one map to refer to their counterparts on the other map. The function $pol(l, m)$ returns a polygon specification from the polygon model G_m if $p_m(l)$ is defined or if $p_m(l')$ is defined, where l' is a counterpart of l:

(9) $$pol(l, m) = \begin{cases} p_g(l'); & \text{if there is a } l' \in ID_g \text{ such that } cp(l, l') \\ & \text{ and } m \text{ refers to a giver map} \\ p_f(l'); & \text{if there is a } l' \in ID_f \text{ such that } cp(l, l') \\ & \text{ and } m \text{ refers to a follower map} \\ undefined; & \text{otherwise} \end{cases}$$

Going back to the above example (8) where the route follower uses the label *old mill* from the giver map to refer to the landmark with the label *mill wheel* on his or her map. The polygon specification from *m12f* can be obtained by using one of the landmark identifiers, either the one associated with *old mill* or the one associated with *mill wheel*. Applying the function pol to either alternative yields the same polygon specification:

(10) pol(`m12_old_mill`, `m12f`) = pol(`m12_mill_wheel`, `m12f`)

Summary Polygon models of the Map Task maps represent landmarks as closed polygons with a specific location in a coordinate system. The function $pol(l, m)$ yields polygon specifications from pairs of a landmark identifier l and a map identifier m. In order to get the polygon specification of a particular landmark on a particular map it is possible to use either the actual landmark identifier of the landmark or its counterpart.

5.3 Manual annotation of locative expressions

This section describes the procedure of manually identifying and annotating locative expressions in the Map Task corpus. We collect locative expressions and provide a syntactic analysis, a reference analysis, and information about the commitment the speakers exhibit towards each locative expression. The result of the annotation procedure is an exhaustive list of occurrences of locative expressions in the Map Task corpus which describe the location of a landmark relative to another landmark.

The annotations provide information that is required by the method described in Chapter 4: (i) they provide the input to the lexical semantic theories defined in Section 3.5; (ii) they establish reference links between the arguments of locative expressions and spatial objects; and (iii) they indicate whether a locative expression is used positively or negatively. The annotations are used in Chapter 6 to evaluate semantic theories of location descriptions.

5.3.1 Annotation target

The *annotation target* is the part of a locative expression that is marked and annotated with information. The aim of the manual annotation procedure is to identify and mark locative expressions which are used to describe spatial relations between landmarks. They are uttered by the participants of the map task experiment trying to accomplish *localisation subtasks* (cf. Section 5.1.4). Instead of marking entire locative expressions we only mark the term that denotes a spatial relation. Most frequently they are prepositions or prepositional phrases behaving like prepositions, as for example in (11) and (12):

(11) It's just [*above*] the picket fence.

(12) The forest is [*to the right of*] the village.

We also find spatial relations denoted by verbal descriptions, e.g. (13), and descriptions of the configuration of a group of landmarks, e.g. (14):

(13) Is the start actually [*touching*] the lake?

(14) The ravine, the forest, and the village, they are kind of in a [*triangle*].

As said above, we only study locative expressions which describe spatial relations between landmarks. In most cases the expressions specifying the located object and the reference objects refer to specific landmarks on specific maps. Far fewer cases contain quantified expressions, such as *nothing* and *anything*. Quantified expressions refer to landmarks, if their domain is implicitly or explicitly restricted to landmarks. (15-a) presents an example where the restrictor is given implicitly; in (15-b) it is given explicitly:

(15) a. I've got *nothing* above the caravan park.
b. I don't have *any obstacles* above the caravan park.

In both utterances the *located object* is specified to be the empty set of landmarks.

For identifying the annotation target we need to collect the single parts of each locative expression first. Note that these can be distributed over multiple utterances. Then we analyse what types of objects are related to each other. Only if the description expresses a relation between landmarks does the corresponding term specifying the relation qualify as an annotation target. These occurrences have to be distinguished from descriptions of the location of a landmark relative to the current position of the path, relative to the map as a whole, and relative to parts of landmarks (compare with Section 5.1.4). We annotate each annotation target with one or more feature structures which are composed of four layers: *reference coding layer*, *commitment coding layer*, *syntax coding layer*, and *logical structure coding layer*. Each annotated feature structure specifies a particular interpretation of the corresponding locative expression in the dialogue. In the example in (16) we find two interpretations of one locative expression – one for each participant:

(16) A: There is a level crossing [*below*] the lake.
B: That's right.

In one interpretation A describes a spatial relation between the landmarks with the labels *level crossing* and *lake* on his or her map. The affirmative response of B gives rise to another interpretation. B claims that the description given by A also refers to his or her map. The difference between both interpretations is that the participants each refer to their own map, and thus they describe relations between different pairs of objects. If the two maps always matched exactly, then both participants would describe the same spatial configuration with the same locative expression. And distinguishing between two interpretations would just be redundant multiplication. But since the maps are distinct from each other, the participants can describe distinct spatial configurations with the same locative expression. Therefore in cases like (16), we must distinguish between two interpretations.

The reason for this need of distinction should become clearer with the following example. Suppose speaker A has two landmarks with the label *field* on his or her map. Let us refer to them with the identifiers `field#1` and `field#2`. The landmark referred to by `field#1` is is in the center of the map and the landmark referred to by `field#2` is at the bottom. Speaker B has only `field#2` on his or her map. Now, suppose that speaker A uses the label *field* to refer to `field#1`, while speaker B interprets *field* as referring to `field#2`:

(17) A: Do you also have a house [*to the right of*] the field?
B: Yes, but it is quite far away.

Again, A and B use the same locative expression to describe a spatial relation between distinct pairs of landmarks. A describes a relation between the *house* and `field#1`, and B interprets the expression as describing a relation between the *house* and `field#2`.

5.3.2 Reference coding layer

A locative expression refers to a map, or more precisely, to a tuple of landmarks of a map, if a speaker uses it to describe the location of these landmarks relative to each other. The *reference coding layer* links the preposition (or the prepositional phrase) with the landmarks to which the corresponding locative expression refers. We annotate three different kinds of reference links. There are links to landmarks that play the role of *located objects*, then there are links to indicate the role of *reference objects*, and links to the maps themselves. Links to maps are necessary to resolve the links to the

Figure 5.6: A section of map *m2f*

landmarks, and moreover, they define the domain of quantified expressions such as "nothing". Each annotation contains one link to a *located object*, a set of links to one or more *reference objects* – multiple *reference object* links are, for example, necessary for the ternary relation *between* – and a link to the map which the locative expression refers to. This last link is specified by a map identifier (see Section 5.2):

(18) *Features of the reference coding layer*

$$\begin{bmatrix} \text{LO} & \text{: Landmark identifier} \\ \text{RO} & \text{: Set of Landmark identifiers} \\ \text{MAP} & \text{: Map identifier} \end{bmatrix}$$

Landmarks are specified in terms of unique identifiers defined by the *polygon model*. Every annotation needs exactly one identifier for the *located object* (`LO`) slot. If the *located object* is given by negatively quantified terms, e.g. "nothing" or "anything", then we use the special identifier `#unspec`. The `RO` slot is filled with the identifiers of all landmarks that are used as *reference objects*. The `MAP` slot is filled with a map identifier. For example, (19) presents an utterance of a speaker who describes a situation from *giver map* 2 which is depicted in Figure 5.6.

(19) The hideout is [*above*] the gold mine.

The preposition *above* receives the following annotation on the reference coding layer:

(20)

$$\begin{bmatrix} \text{LO} & = \texttt{m2\_outlaws\_hideout} \\ \text{RO} & = \left\{ \texttt{m2\_goldmine} \right\} \\ \text{MAP} & = \texttt{m2f} \end{bmatrix}$$

These features specify reference to *follower map* number 2. The located object is given by the landmark with the label *outlaws hideout*, the reference object by the landmark with the label *gold mine*.

5.3.3 Commitment coding layer

The commitment coding layer specifies the way in which a participant is committed to the truth of a particular locative expression with respect to his or her map. Coding of commitment is dependent on the value of the `MAP` feature described above. If it refers to a giver map, the commitment of the instruction giver is coded. Otherwise, if it refers to a follower map, the commitment of the route follower is coded.

We distinguish between five different modes of commitment: (i) `positive` commitment means that the participant attributes the corresponding locative expression to his or her own map. (ii) The contrary is indicated by `negative` commitment. Negative commitment means that the participant explicitly rejects such an attribution. We keep explicit rejections separate from *implicit rejections* (`impl-negative`) which are based on pragmatic reasoning. If the participant is offered a choice of options of which he or she selects one, then the other options are implicitly rejected. For example, in (21) A offers B the choice between two options:

(21) A: Is the level crossing to the right or to the left of the east lake?
B: It's to the right.

Since B selects the option *to the right*, he or she implicitly rejects the other option *to the left of*. Lack of acknowledgement is annotated with the value `non-committing`. If a participant leaves a question unanswered then we interpret this reaction such that he or she does not show any commitment. For example, (22) shows the same question, but a different response:

(22) A: Is the level crossing to the right or to the left of the east lake?
B: Just go past the lake.

B does not select any of the options offered by A. Therefore, we annotate both occurrences of projective prepositions (i.e. *to the right* and *to the left of*) with the commitment value `non-committing`. This value is also used to mark that a locative expression does not make any sense because it has not been completed. (v) Positive and negative commitment can be overridden by the value `cancelled` if the participant overtly corrects his or her commitment at a later point in the dialogue. For example, look at the utterances shown in (23). They are all utterances from the instruction giver in dialogue *q4ec3*. The instruction giver first indicates that he or she is committed to the truth of a particular locative expression, but he or she cancels this claim in the course of the dialogue. The asterisk (*) indicates reference to the *boat house* in the center of giver map 7.

(23) q4ec3.124$_{query-yn}$ G: oh do you not have another one* ... to your right ... of the concealed hideout
q4ec3.133$_{clarify}$ G: no i want you to go to the boat house* on your right ... on your ... on your left
q4ec3.137$_{explain}$ G: well i've got one ... ... i've got one* ... ... to the left of the concealed hideout

The first utterance in (23) suggests[1] that the instruction giver presumes that the *boat house* is to the right of the *concealed hideout* on giver map 7. The second utterance already indicates that he or she has discovered his or her mistake. The third utterance finally corrects it. The following structure defines all possible values for the commitment feature:

(24) *Feature of the commitment coding layer*

$$\begin{bmatrix}\text{COMM}: & (\texttt{positive}\,|\,\texttt{negative}\,|\,\texttt{impl-negative}\,| \\ & \texttt{cancelled}\,|\,\texttt{non-committing}) \end{bmatrix}$$

How do speakers show commitment to a statement? There are in principle two ways: either they make an assertion of a statement themselves or if another speaker says something containing a statement, then they can show their commitment to that statement by an appropriate response. For the annotations, we consider four patterns of conversational interaction to analyse in which way the participants commit to locative expressions: (i) assertions, (ii) questions/answer pairs, (iii) other responses, and (iv) implications from questions:

Assertion If a locative expression occurs in an assertion from a participant about his or her own map, then it refers to the speaker's map. We annotate it with `positive` commitment if the statement itself is positive. Otherwise, if the statement is negative we annotate `negative` commitment. The next example shows a positive statement and its negation:

(25) a. The gold mine is directly below the hideout.
b. The gold mine is not directly below the hideout.

Utterance (25-a) exhibits `positive` commitment, and utterance (25-b) exhibits `negative` commitment.

Question/answer pairs If a participant asks a question containing a locative expression referring to the other participant's map, then we have to consider both the question and the corresponding answer to determine the commitment of the other participant. One kind of question simply requires positive or negative responses (*aka* yes-no-questions). Positive responses like "*yes*" indicate `positive` commitment of

[1]Questions can reveal something about the speaker's attitude. Compare with paragraph *Implications from questions*).

the addressee, negative responses like "*no*" indicate `negative` commitment. The following example shows a question of *A* and two alternative answers by *B*:

(26) A : Is the gold mine directly [*below*] the hideout?
B : (i) yes / (ii) no.

If the answer is (26-i) then we fill the `COMM` slot with the value `positive`, if it is (26-ii) we fill in the value `negative`. The following feature structure shows the features of the reference coding layer and the commitment coding layer of a possible annotation of the annotation target *below* in (26). Suppose that the corresponding locative expression refers to the map `m2f`. The numbers of the alternative commitment values correspond to the numbers of the answers in (26):

(27) $$\begin{bmatrix} \text{LO} & = \texttt{m2\_goldmine} \\ \text{RO} & = \left\{\texttt{m2\_outlaws\_hideout}\right\} \\ \text{MAP} & = \texttt{m2f} \\ \text{COMM} & = \text{(i) } \texttt{positive} \text{ / (ii) } \texttt{negative} \end{bmatrix}$$

Another kind of question (*aka* choice-question) provides sets of alternative locative expressions and requests the addressee to choose one of them. We annotate the locative expression which is selected by the addressee with `positive` commitment. The other locative expressions are marked with the value `impl-negative` since they are implicitly rejected. The value `impl-negative` indicates that another locative expression was preferred over the one marked with it. It can only be interpreted as negation under the assumption that choice-questions imply exclusiveness of their parts. For example look at the following question/answer pair:

(28) A: Is the level crossing [*to the right*], [*to the left of*], or [*in line with*] the east lake?
B: It's to the right.

The answer in (28) picks the option *to the right*, so the corresponding locative expression refers to B's map and B is positively committed to it. The other options, i.e. *to the left of* and *in line with*, also refer to B's map but here we annotate `impl-negative` in the respective `COMM` slots. In the example, the answer exactly repeats the option, but it is only required that the option is implied by the answer. We assume that modifiers such as *just*, *directly*, and *slightly* imply the use of the corresponding unmodified locative expressions. In the example above, the utterance *it's slightly to the right* would have the same consequences for the annotations as the actual utterance *it's to the right*.

For yes-no-questions and choice-questions we assume that *above*, *below*, *right* and *left* can exclude each other, so that the following locative expressions occurring in the questions are annotated *impl-negative*:

(29) A: Is the level crossing [*to the right*] or [*to the left of*] the east lake?
B: It's above.

(30) A : Is the gold mine directly [*below*] the hideout?
B : It's to the left.

In both examples participant B does respond with answers that imply the negation of the options provided by the questions.

Other responses We have just seen that answers to questions display the commitment of the addressee. Responses to assertions can have the same effect. If one participant makes an assertion containing a locative expression, then the other participant can respond and agree or disagree with the purpose of communicating that the same locative expression also refers to his or her map. In the following example participant A asserts a locative expression and B assents to it:

(31) A: There is a telephone box [*below*] the east lake.
B: That's right.

The effect is that the annotation target *below* is annotated with two feature structures. We annotate the first structure, because participant A asserts the corresponding locative expression. Reference and commitment annotations are displayed in (32-a) given that participant A's map is *m0g*. The second structure is annotated because of the response of participant B. He or she says that the description "the telephone box is below the east lake" also applies to his or her map. The corresponding annotation structure is given in (32-b) assuming that participant B's map is *m0f*:

(32) a.
$$\begin{bmatrix} \text{LO} & = \texttt{m0\_telephone\_box} \\ \text{RO} & = \left\{\texttt{m2\_east\_lake}\right\} \\ \text{MAP} & = \texttt{m0g} \\ \text{COMM} & = \texttt{positive} \end{bmatrix}$$
b.
$$\begin{bmatrix} \text{LO} & = \texttt{m0\_telephone\_box} \\ \text{RO} & = \left\{\texttt{m2\_east\_lake}\right\} \\ \text{MAP} & = \texttt{m0f} \\ \text{COMM} & = \texttt{positive} \end{bmatrix}$$

Implications from questions In general, questions do not display in which way the speaker is committed to the content of the question. When a participant addresses a question to the other participant whether a particular locative expression applies, then we cannot infer anything directly from the question whether or not that is the case. Nevertheless, we adopt an assumption that enables us to derive information about the

Figure 5.7: A section of map *m2f* with the landmarks *rapids* and *manned fort*

way the speaker is committed to such a locative expression. The participants in the Map Task experiments have good reason to believe that most of the landmarks on the other participant's map match the landmarks on their own map. Among other possibilities (compare with Section 5.1.4) they check this by asking for the location of landmarks which appear on their own map with respect to other landmarks which appear on their own map, too. When the participants use such questions they actually describe their own map. Therefore, we assume that such questions show that the speaker is committed to the corresponding assertion, and thus to the claim that the locative expression he or she uses is true of the mentioned landmarks on his or her own map. Of course, this presupposes that the located object and all reference objects of a locative expression appear on the speaker's map. The example (33) shows an utterance taken from dialogue *q2ec4* which relates to the section of map *m2f* displayed in Figure 5.7. The structure in (34) shows the features of the reference coding layer and the commitment coding layer which we attribute to the annotation target *below*:

(33) have you got a manned fort [*below*] the rapids?

(34)
$$\begin{bmatrix} \text{LO} & : \texttt{m0\_manned\_fort} \\ \text{RO} & : \left\{\texttt{m2\_rapids}\right\} \\ \text{MAP} & : \texttt{m2f} \\ \text{COMM} & : \texttt{positive} \end{bmatrix}$$

In (33) the route follower checks the existence of the landmark *manned fort* on the giver map giving specific information about its location on his or her own map, namely that it is *below the rapids*. Independent of an answer of the instruction giver we assess positive commitment of the route follower in such cases.

5.3.4 Syntax coding layer

Syntactic annotations enumerate the constituents of a locative expression and specify syntactic relations which hold between them. Here we restrict ourselves to a partial syntactic analysis. We specify (i) the relation term of the locative expression (`REL`), (iii) a distance modifier (`DIST`), (iv) a list of other modifiers (`MOD`), and (v) a symbol expressing syntactic coordination with other locative expressions (`SCONJ`). The type of the features for the syntax annotation layer is shown here:

(35) *Features of the syntax coding layer*

$$\begin{bmatrix} \text{REL :} & \text{Relation term} \\ \text{MOD :} & \text{List of modifiers} \\ \text{DIST :} & \text{String containing a distance modifier} \\ \text{SCONJ :} & \text{Symbol indicating conjunction class.} \end{bmatrix}$$

The relation term (`REL`) is the principal lexical item that determines the spatial relation. In cases where the annotation target is a simple preposition the feature `REL` takes the preposition itself, in cases where the annotation target is a complex phrase the name of the relation is given by the noun or adverb, e.g. *left* in “to the left of”. Distance modifiers (`DIST`) unambiguously relate to distances between located object and reference objects. They express a measure of length either in quantitative terms (“two inches”), in relative terms (“halfway”), or in vague terms (“a bit”, “far”). All other modifiers like hedges (vagueness modifiers) and direction modifiers are listed in the modifiers slot (`MOD`). The example below, see (36), contains a locative expression with the modifier *slightly*. The feature `SCONJ` encodes syntactic coordination of locative expressions where the locative expressions share the same syntactic realisation of the terms specifying the reference objects, as for example:

(36) The mill is above and slightly to the left of the mountain.

Here we have two locative expressions: one can be paraphrased with “the mill is above the mountain” and the other with “the mill is slightly to the left of the mountain”. The annotations are attributed to the corresponding annotation targets *above* and *to the left of*, respectively. Since the locative expressions are coordinated by a conjunction and they share the same argument, namely the expression *the mountain*, we mark them as being syntactically coordinated. Technically, the value taken by the feature `SCONJ` is a symbol that specifies a conjunction class. All feature structures that are annotated with the same conjunction class are coordinated to each other. In the example, we would annotate the following two feature structures, (37-a) to *above* and (37-b) to *to the left of*. The syntactic coordination of these feature structures is specified by having the same value assigned to the `SCONJ` feature. This symbol is arbitrarily chosen, in the example it is *c0*, but it is unique in the corpus, so that it uniquely specifies a set of coordinated feature structures. Here, it is the set of the following two feature structures:

(37) a. $\begin{bmatrix} \text{REL} = & \text{above} \\ \text{MOD} = & () \\ \text{DIST} = & \text{'' ''} \\ \text{SCONJ} = & \text{c0} \end{bmatrix}$

b. $\begin{bmatrix} \text{REL} = & \text{right} \\ \text{MOD} = & \text{(slightly)} \\ \text{DIST} = & \text{'' ''} \\ \text{SCONJ} = & \text{c0} \end{bmatrix}$

5.3.5 Logical structure

The capability of expressing logical structure with the annotation scheme presented here is very limited. In general, all locative expressions of one dialogue are interpreted in conjunction. And the commitment feature determines whether a locative expression is interpreted as a positive statement X or as a negated statement $\neg X$. For example, suppose that there is a dialogue which contains three relevant locative expressions X, Y, and $\neg Z$. We assume that all three locative expressions are true, that means that the following formula is true:

(38) $X \wedge Y \wedge \neg Z$

For most cases this expressiveness is sufficient. Only for a small number of occurrences of locative expressions we have to represent a negation taking scope over two or more locative expressions like the example shown in (39) which is taken from dialogue *q2ec6*:

(39)	q2ec6.96$_{query-yn}$	G:	so is it ... to your right of the stone creek ... and just up a bit
	q2ec6.97$_{reply-n}$	F:	no

We annotate the annotation targets *to your right of* and *up*. The conjunction of the corresponding locative expressions is rejected by the route follower. Note that the route follower does not reject each locative expression separately. That means that we do not want to represent a conjunction of two negated locative expressions, but a negation taking scope over a conjunction of positive locative expressions such as the following formula:

(40) $\neg(V \wedge W)$

In order to code negated conjunctions we introduce the additional feature `LCONJ`. Like the syntactic coordination feature `SCONJ` it takes a symbol indicating a conjunction class. In contrast to the `SCONJ` feature which expresses syntactic coordination of two locative expressions sharing the same realisation of the reference object, the `LCONJ` feature is used to mark locative expressions which are logically coordinated under negation.

(41) *Features of the logical structure coding layer*

$\begin{bmatrix} \text{LCONJ :} & \text{Symbol indicating logical conjunction class.} \end{bmatrix}$

We link the conjuncts with each other by means of the `LCONJ` feature and mark them as negated by setting `COMM = negative`. The annotation targets in example (39) receive the values for the features `LCONJ` and `COMM` as shown in (42) and (43). The symbol *class1* is an arbitrary symbol that specifies a conjunction class. The conjunction class associated with *class1* in this example only consists of these two feature structures:

(42) *to your right of*

$\begin{bmatrix} \text{LCONJ =} & \text{class1} \\ \text{COMM =} & \texttt{negative} \end{bmatrix}$

(43) *up*

$\begin{bmatrix} \text{LCONJ =} & \text{class1} \\ \text{COMM =} & \texttt{negative} \end{bmatrix}$

This combination of the features `LCONJ` and `COMM` enforces an interpretation of (39) which can be paraphrased as follows: The instruction follower says that it is not the case that the manned fort is to the right of the stone creek and up a bit.

5.3.6 Annotation procedure

The annotations were carried out in two passes. Each dialogue was annotated by two annotators, a third annotator resolved differences between the annotations.

In the first pass, the annotators read an entire dialogue and mark the utterances which contain an annotation target. After both annotators have finished we compute the differences and return them to the annotators, so that each annotator goes over these parts of the corpus again. Another computation of the differences on the re-annotated data yields cases which both annotators have seen twice and disagree about. These differences are resolved by the annotator who has not yet seen the data.

In the second pass, the annotators look at the utterances which have been selected in the previous pass. They mark the annotation target and add the annotations of the layers described above, i.e. reference coding layer, commitment coding layer, syntax coding layer, and logical structure. We compute the differences again, but instead of giving them back to the annotators they are resolved by a third annotator directly.

Chapter 6

Analyses

This chapter describes the results of the automatic application (cf. Chapter 4) of the lexical semantic theories of projective locative expressions from Chapter 3 to the empirical data described in Chapter 5.

The first section gives an overview of the locative expressions that have been found in the data. Some specific semantic theories for projective locative expressions proposed in the literature are evaluated in Section 6.2. The evaluation of the whole set of semantic theories defined in Chapter 3 is evaluated in Section 6.3. Based on the results of the evaluation an automatic algorithm determines semantic theories of projective locative expressions in terms of the underspecification formalism from Chapter 3. Sections 6.4 and 6.5 analyse projective locative expressions that are modified by the modifiers *directly* and *slightly*, respectively. Specific semantic theories of modified projective locative expressions are determined based on the analysis in the same way as for unmodified expressions. The relation between semantic theories of unmodified and modified projective locative expressions is discussed in Section 6.6.

6.1 Description and preparation of the data

This section gives an overview of the data that was obtained by annotating locative expressions in the HCRC Map Task corpus as described in the previous chapter. Furthermore, this section describes the steps that are carried out to prepare this data in order to evaluate semantic theories of locative expressions with it.

Overall we identified 1367 occurrences of locative expressions which describe the location of a landmark relative to another landmark.[1] These occurrences were annotated with 1643 feature structures (compare with Section 5.3). We ignore all repetitions of locative expressions in the same dialogues: If a locative expression involving the same locative expression, the same modifiers, if any, the same landmarks and the

[1]These expressions do not comprise descriptions of the location of a landmark with respect to the entire map nor relative to the current position from the route description task, compare with Section 5.1.4.

same truth value (see ‘commitment’ on page 5.3.3) occurs more than once in a given dialogue, then all occurrences except the first one are discarded.[2] An example of a repetition within the same utterance is the following:

(1) There is a circle **above** the rectangle. It is **above** the rectangle and to the left of the triangle.

After all repetitions have been removed there are 1228 locative expressions remaining. Table 6.1 shows the most frequent prepositions and projective terms.[3]

above	196	down	51	along	15
below	187	on level	28	under	14
left	138	at	28	beneath	14
right	107	up	26	next to	13
between	82	beside	25	near	11
underneath	80	in line	18	bottom	11

Table 6.1: Most frequent prepositions and projective terms after removing all repetitions of locative expressions in the same dialogue.

There are 1121 positive uses of locative expressions, i.e. the commitment feature is `COMM=positive`, and 86 negative uses (63 annotated with `COMM=negative`, and 23 with `COMM=impl-negative`). 12 locative expressions were annotated as `non-committing`. Among the remaining 9 feature structures 3 were annotated with `COMM=cancelled`, and 6 could not be assigned a proper annotation. The latter 9 cases are described in detail in Appendix A.

Preparation In order to be able to apply the evaluation method from Chapter 4 the data set is further reduced in the following way: (i) all those expressions that are marked as `non-committing` are removed. (ii) All those expressions are removed that are logically embedded in the discourse other than conjunction or conjunction of the negation (see Section 5.3.5). (iii) All those expressions are removed that do not contain one of the following projective terms: *above, below, left, right, underneath, down, up, beneath, under, bottom, top, west, east, south, north, over, upwards*.

There are 751 expressions in the data satisfying these three conditions. The frequency distribution of the projective terms is given in Table 6.2. 410 of these projective locative expressions occur without modifiers, 341 are modified in some way. Table 6.3 shows the 8 most frequent modifiers combining with projective prepositions.

[2] Technically, a feature structure b is a repetition of a feature structure a if the following condition is satisfied:
$a.LO = b.LO \wedge a.RO = b.RO \wedge a.DIAL = b.DIAL \wedge a.REL = b.REL \wedge a.COMM = b.COMM$

[3] Note, that some terms listed in the table such as *left, right* and *bottom* have to be complemented to play the role of a preposition. Compare with page 92.

above	171	up	22	east	5
below	157	beneath	14	south	4
left	121	bottom	10	north	4
right	96	under	10	over	3
underneath	74	top	8	upwards	1
down	46	west	5		

Table 6.2: Frequencies of the projective terms of all relevant projective locative expressions. The total size is 751.

The second column contains the frequency of the modifier given that there is only one single modifier (e.g., *directly above*) and the third column contains the total count of the modifier counting all its occurrences within projective locative expressions. The symbol **distance** stands for distance modifiers such as *one inch*.

modifier	single	total
just	96	117
distance	92	111
directly	47	57
slightly	23	31
right	16	20
sort of	9	14
straight	8	9
like	6	10

Table 6.3: Most frequent modifiers including distance modification. The column '*single*' contains the frequencies of expression containing exactly one modifier and *total* the total frequency of the corresponding modifier modifying a projective term.

Altogether, the data contains 410 locative expressions for evaluating the semantics of unmodified expressions, 47 locative expressions for evaluating the semantics of locative expressions that are (exclusively) modified by *directly* and 23 locative expressions for evaluating the semantics of locative expressions that are (exclusively) modified by *slightly*.

6.2 Evaluation of the semantics of unmodified projective locative expressions

This section evaluates some specific semantic theories (see Section 3.5) for unmodified projective locative expressions which correspond to theories proposed in the literature, see Section 2.9. More specifically, semantic theories from the following studies will be evaluated: Kelleher (2003), O'Keefe (1996), Fuhr et al. (1995), Gapp (1994a), Abella

term	freq	term	freq
above	86	under	5
left	85	down	5
below	77	up	5
right	63	west	3
underneath	52	south	2
beneath	7	north	2
bottom	7	upwards	1
top	7	east	1
		over	1

Table 6.4: Frequency of the projective terms in unmodified locative expressions.

& Kender (1993), and André et al. (1987). These theories will be evaluated according to the test procedure specified in Chapter 4.

NB: It should be noted that most of these theories are intended to be used in three-dimensional environments. Here, however, we evaluate these theories only with locative expressions describing two-dimensional data. It is therefore important to keep in mind that the evaluation only tests these theories under the assumption that they are independent of the spatial domain. But it doesn't say anything about their performance in the spatial domain intended by their authors.

Data The evaluation is carried out on unmodified projective locative expressions from the data set prepared in Section 6.1. It consists of 410 locative expressions and the corresponding spatial configurations. In the data set there are 387 positive uses of projective locative expressions and 23 negative uses. The frequency distribution of the projective terms is described in Table 6.4.

Evaluations André et al. (1987) specify a semantics of the German counterparts of the prepositions *to the right of, to the left of, in front of* and *behind* in terms of OP^{cent}_{P2} relations. For the evaluation all locative expressions were extracted from the data that contained the projective terms *right* and *left*, respectively, implying variants of the corresponding prepositions such as *to the left of, on the left of, on the left-hand side of* and so on. The results of the evaluation of these theories and the results of all following evaluations are shown in Table 6.5.

Abella & Kender (1993) and Abella (1995) specify a semantics for *above*, *below*, *left*, and *right* which are based on OP_{P1} and OP_{P2} relations but they are not exactly equivalent to these relations, since Abella & Kender 'fuzzify' the relations yielding relations which have an extended range compared to the corresponding OP_{P1} and OP_{P2} relations. The theories based on OP_{P1} and OP_{P2} are evaluated with locative expressions from the data containing the prepositions *above*, *below* and all variants of *left* and *right* such as *to the left of, on the left of, on the left-hand side of* and so on.

Gapp (1994a) models the semantics of *above*, *below*, *to the right of*, *to the left of*, *behind* and *in front of* with AD_4^{cent} relations. The semantics is evaluated with locative expressions containing the projective terms *above*, *below*, *right* and *left*.

Fuhr et al. (1995) specify the semantics of *in front of*, *behind*, *to the right of*, *to the left of*, *above* and *below* with AD_5^{Grid} relations. These theories are evaluated with locative expressions containing the projective terms *above*, *below*, *left*, and *right*, respectively.

O'Keefe (1996) and O'Keefe (2003) suggest that the semantics of *above* and *below* is modelled by OP_{P2} relations, and the semantics of *on top of*, *beneath* and *underneath* by OP_{P1} relations.[4] These theories are evaluated with locative expressions containing the corresponding preposition, i.e. OP_{P2} relations are evaluated with locative expressions containing *above* and *below*, and OP_{P1} relations with the prepositions *beneath*, *underneath*, *at the top of* and *on top of*.

Kelleher (2003) specifies a semantics for locative expressions containing the prepositions *in front of*, *behind*, *to the right of*, *to the left of*, *above*, and *below* with $\mathrm{AD}_{O5}^{int/cent}$ relations. They are evaluated with locative expressions containing the projective terms *above*, *below*, *left* and *right*.

Results and Discussion The results of the evaluations are described in Table 6.5. The baseline is computed by assuming that the semantics is modelled by a relation which is true for arbitrary pairs of objects, so that it is correct for all positive uses but incorrect for all negative uses. Let us say that a particular theory is acceptable according to this evaluation if it scores better results than the baseline.

Theory		projective terms	total	baseline	correct
OP_{P2}^{cent}	(André et al., 1987)	left, right	149	92.6%	91.3%
OP_{P1}	(Abella & Kender, 1993)	above, below, left, right	312	93.9%	24.4%
OP_{P2}	(Abella & Kender, 1993)	above, below, left, right	312	93.9%	88.5%
AD_4^{cent}	(Gapp, 1994a)	above, below, left, right	312	93.9%	97.1%
AD_5^{Grid}	(Fuhr et al., 1995)	above, below, left, right	312	93.9%	96.5%
OP_{P1}	(O'Keefe, 1996)	beneath, underneath, top	66	95.5%	27.3%
OP_{P2}	(O'Keefe, 1996)	above, below	163	95.1%	95.7%
$\mathrm{AD}_{O5}^{int/cent}$	(Kelleher, 2003)	above, below, left, right	312	93.9%	95.5%

Table 6.5: Evaluation of semantic theories with unmodified projective locative expressions which contain the projective terms required by the theories. The number of expressions found in the data is given in the '*total*' column. The term *top* stands for the projective prepositions *on top of* and *at the top of*, *left* stands for projective prepositions such as *to the left of*, *on the left of*, *on the left-hand side of*, and so on. The baseline is computed by evaluating a relation that is true for any pair of objects. The final column shows the percentage of expressions from the data that were predicted correctly by the corresponding semantic theory.

[4] O'Keefe also specifies a semantics for *over* and *under* but there are not enough data for an evaluation. For the same reasons the semantics of *north of*, *south of*, *west of*, and *east of* proposed by Yamada et al. (1988) is not evaluated here.

The following semantic theories do not achieve better results than the baseline: (André et al., 1987) for *left* and *right*, (Abella & Kender, 1993) for *above, below, left* and *right*, and (O'Keefe, 1996) for *beneath, underneath* and *on top of*. But note that the evaluation results for (Abella & Kender, 1993) only show a lower bound of an evaluation of the original semantic theories. Any fuzzification of the corresponding relations as described in (Abella & Kender, 1993) improves the results. The other semantic theories (Gapp, 1994a), (Fuhr et al., 1995), (O'Keefe, 1996, OP_{P2}), and (Kelleher, 2003) obtain better results than the baseline; they are acceptable according to this evaluation.

The next section shows that an explicit treatment of vagueness systematically yields better evaluation results. For example, Gapp's theory is based on AD_4^{cent} relations and correctly predicts 97.1% of the evaluation data. In the next section a theory is presented that is based on a combination of relations from AD_4^{cent} and AD_2^{cent}; it correctly predicts 99.5% of the evaluation data.

6.3 Analysis of the semantics of unmodified projective locative expressions

This section evaluates all semantic theories of unmodified projective locative expressions as they are described in Section 3.5.

A heuristic determines underspecified semantic theories of projective locative expressions based on the results of the evaluations. These theories specify semantic representations that consist of pairs of locative direction relations. Relations from the lower range level (cf. Section 2.1) hold for all spatial configurations which make the corresponding expression true in every respect. And relations from the upper range level hold for all spatial configurations which *can* make the corresponding expression true.

Data The evaluations are carried out on the same data used in the previous Section. It is the set of unmodified projective locative expressions prepared in Section 6.1 which consists of 410 locative expressions and the corresponding spatial configurations. In the data set there are 387 positive uses of projective locative expressions and 23 negative uses.

Method The aim of this analysis is to determine lexical mappings from projective prepositions onto pairs of locative direction relations for a given relation schema RS:

(2) $$H_*^{RS}(prep) \mapsto \langle H^{RS_k}(prep), H^{RS_l}(prep)\rangle$$

where k and l denote range levels of RS with $k \leq l$ so that for an arbitrary $prep$ $H^{RS_l}(prep)$ subsumes $H^{RS_k}(prep)$. The heuristic which determines these mappings from the data is such that every mapping satisfies the following conditions:

1. For every positive use of a locative expression with the projective preposition $prep$ and the located object lo and the reference object ro the complex predicate $H^{RS_k}(prep) \oplus H^{RS_l}(prep)(lo, ro)$ is true when valuated with a positive bias and false for every negative use also valuated with a positive bias.

2. There are no other range levels i and j with $i < j$ such that $H^{RS_i}(prep) \oplus H^{RS_j}(prep)(lo, ro)$ satisfies condition (1), and $n < i$ or $j < m$.

Such mappings produce underspecified semantic representations (see Section 3.3) which are true for the whole set of data – this is of course only possible if the relation schema RS provides appropriate relations that are on range levels which are low enough to reject all spatial configurations described by negative uses and high enough to accept all spatial configurations described by positive uses, respectively – and there are no other such underspecified semantic representations which fit the set of data more tightly.

The range levels k and l for a relation schema RS are determined in the following way: The upper range level l is determined according to the evaluation results of the positive uses. l is set to the lowest range level that correctly predicts the maximum number of positive uses. The lower range level k is set to the highest range level from the evaluation results that provides the maximum number of correctly predicted negative uses. If k is greater than l then k is reset to the same value as l. In Tables 6.6 and 6.7 the range levels determined by this heuristic are marked in bold face.

Re-rating We have to be clear about the consequences of determining lexical mappings in this way. The data determines the setting of the range levels k and l such that the semantics of every positive use of a locative expression in the data is correctly modelled by the relation determined by H^{RS_l} and that the semantics of every negative use is correctly modelled by H^{RS_k}. The implication of this approach is that there is at least one negative use in the data the semantics of which is correctly modelled by H^{RS_k} but not by any other mapping H^{RS_i} with $i > k$. Similarly, there is at least on positive use in the data the semantics of which is correctly modelled by H^{RS_l} but not by any other mapping H^{RS_j} with $j < l$.

This method is problematic when we assume that the data contains errors – either the speakers might have made an error such as confusing left and right or there might be errors in the annotations. We want to avoid that the range levels of the mappings are determined on the basis of erroneous occurrences of locative expressions. In order to handle this problem, the truth values of all data that led to setting a range level were re-rated by "*informants*" which were native speakers of English and didn't have any connection to the original HCRC Map Task experiments. More specifically, each such occurrence was presented to all informants with enough dialogue context so that the located object and the reference object could be identified. The projective preposition in question (and the modifiers, if any sections) were marked in bold face. For each locative expression the corresponding spatial configuration containing the LO and the

RO was presented, too. The informants had to decide whether the speakers used the locative expression intentionally or whether it was a false use, e.g. confusion of left and right or a slip of the tongue. Altogether 3 informants rated 26 pieces of data. The final rating was determined by the majority of the votes. The data and the ratings are presented in Appendix B.

Two pairs of locative expressions and spatial configurations were rated as false. They were temporarily removed from the data and not considered for determining upper and lower range levels of the semantic theories. All semantic theories corresponding to all range levels provided by the relation schemata OP_P, OP_O, OP_{OP}, OP_{Grid}, AD^{cent}, AD^{prox}, AD_P^{int}, AD_O^{int}, and AD^{Grid} from Section 2.8 are applied to the data in the way described in Chapter 4.

Results and Discussion The results are shown in Tables 6.6 and 6.7. The number of correct and incorrect predictions are listed for positive uses, negative uses and for the total set (i.e. positive and negative uses). Horizontal lines separate range levels which provide pairwise disjoint relations from range levels providing relations with the property of complement disjointness and from range levels which do not provide any of these inferential properties. For example, in Table 6.2 OP_{P1} provides pairwise disjointness and OP_{P2} and OP_{P3} complement disjointness. Note, that the relation schemata based on the topological relation *overlap* (*viz* OP_O and AD_O^{int}) do not provide any of these disjointness properties.

For each mapping H_*^{RS} specifying an underspecified semantic theory two range levels are determined according to the heuristic described above:

OP_P: $\langle \mathrm{OP}_{P1}, \mathrm{OP}_{P4} \rangle$
OP_O: $\langle \mathrm{OP}_{O2}, \mathrm{OP}_{O4} \rangle$
OP_{OP}: $\langle \mathrm{OP}_{OP3}, \mathrm{OP}_{OP9} \rangle$
OP_{Grid}: $\langle \mathrm{OP}_{Grid2}, \mathrm{OP}_{Grid6} \rangle$
AD^{cent}: $\langle \mathrm{AD}_2^{cent}, \mathrm{AD}_4^{cent} \rangle$
AD^{prox}: $\langle \mathrm{AD}_2^{prox}, \mathrm{AD}_5^{prox} \rangle$
AD_P^{int}: $\langle \mathrm{AD}_{P3}^{int}, \mathrm{AD}_{P5}^{int} \rangle$
AD_O^{int}: $\langle \mathrm{AD}_{O1}^{int}, \mathrm{AD}_{O5}^{int} \rangle$
AD^{Grid}: $\langle \mathrm{AD}_3^{Grid}, \mathrm{AD}_6^{Grid} \rangle$

These pairs of range levels define semantic theories which are evaluated with the same data set as in the previous section (cf Section 6.2, page 149). The baseline is computed by assuming the semantics is modelled by a relation which is true for arbitrary pairs of objects. It correctly accepts all positive uses but it incorrectly accepts all negative uses, too. It is simply computed by dividing the number of positive uses by the total size of the data. For each relation schema the column labelled '*simple*' describes the range level which yields the best evaluation results among the simple theories that map a preposition onto a single relation.

OPP (total:405, non-applicable:3)									
	positive uses			negative uses			total		
level	correct		incor.	correct		incor.	correct		incor.
1	80	(0,209)	302	23	(1,000)	0	103	(0,254)	302
2	345	(0,903)	37	21	(0,913)	2	366	(0,904)	39
3	379	(0,992)	3	18	(0,783)	5	397	(0,980)	8
4	381	(0,997)	1	12	(0,522)	11	393	(0,970)	12
OPO (total:405, non-applicable:3)									
	positive uses			negative uses			total		
level	correct		incor.	correct		incor.	correct		incor.
1	143	(0,374)	239	23	(1,000)	0	166	(0,410)	239
2	256	(0,670)	126	23	(1,000)	0	279	(0,689)	126
3	377	(0,987)	5	17	(0,739)	6	394	(0,973)	11
4	382	(1,000)	0	12	(0,522)	11	394	(0,973)	11
5	382	(1,000)	0	7	(0,304)	16	389	(0,960)	16
OPOP (total:405, non-applicable:3)									
	positive uses			negative uses			total		
level	correct		incor.	correct		incor.	correct		incor.
1	56	(0,147)	326	23	(1,000)	0	79	(0,195)	326
2	80	(0,209)	302	23	(1,000)	0	103	(0,254)	302
3	249	(0,652)	133	23	(1,000)	0	272	(0,672)	133
4	345	(0,903)	37	21	(0,913)	2	366	(0,904)	39
5	374	(0,979)	8	18	(0,783)	5	392	(0,968)	13
6	379	(0,992)	3	18	(0,783)	5	397	(0,980)	8
7	381	(0,997)	1	14	(0,609)	9	395	(0,975)	10
8	381	(0,997)	1	12	(0,522)	11	393	(0,970)	12
9	382	(1,000)	0	7	(0,304)	16	389	(0,960)	16
OPGrid (total:408, non-applicable:0)									
	positive uses			negative uses			total		
level	correct		incor.	correct		incor.	correct		incor.
1	64	(0,166)	321	23	(1,000)	0	87	(0,213)	321
2	169	(0,439)	216	23	(1,000)	0	192	(0,471)	216
3	348	(0,904)	37	21	(0,913)	2	369	(0,904)	39
4	369	(0,958)	16	19	(0,826)	4	388	(0,951)	20
5	384	(0,997)	1	17	(0,739)	6	401	(0,983)	7
6	385	(1,000)	0	15	(0,652)	8	400	(0,980)	8
7	385	(1,000)	0	12	(0,522)	11	397	(0,973)	11

Table 6.6: Application of semantic theories based on orthogonal projection relation schemata to unmodified projective locative expressions. For each range level the table provides the frequencies of correct and incorrect application of the corresponding semantic theory divided into positive uses, negative uses and total number. The numbers in brackets indicate the proportion of correct cases with respect to the sum of positive, negative and all uses, respectively.

ADcent (total:408, non-applicable:0)									
	positive uses			negative uses			total		
level	correct		incor.	correct		incor.	correct		incor.
1	206	(0,535)	179	23	(1,000)	0	229	(0,561)	179
2	305	(0,792)	80	23	(1,000)	0	328	(0,804)	80
3	364	(0,945)	21	19	(0,826)	4	383	(0,939)	25
4	385	(1,000)	0	15	(0,652)	8	400	(0,980)	8
5	385	(1,000)	0	10	(0,435)	13	395	(0,968)	13

ADprox (total:404, non-applicable:4)									
	positive uses			negative uses			total		
level	correct		incor.	correct		incor.	correct		incor.
1	240	(0,630)	141	23	(1,000)	0	263	(0,651)	141
2	300	(0,787)	81	23	(1,000)	0	323	(0,800)	81
3	343	(0,900)	38	21	(0,913)	2	364	(0,901)	40
4	377	(0,990)	4	16	(0,696)	7	393	(0,973)	11
5	381	(1,000)	0	11	(0,478)	12	392	(0,970)	12

ADIntP (total:404, non-applicable:4)									
	positive uses			negative uses			total		
level	correct		incor.	correct		incor.	correct		incor.
1	15	(0,039)	366	23	(1,000)	0	38	(0,094)	366
2	108	(0,283)	273	23	(1,000)	0	131	(0,324)	273
3	229	(0,601)	152	23	(1,000)	0	252	(0,624)	152
4	344	(0,903)	37	21	(0,913)	2	365	(0,903)	39
5	368	(0,966)	13	17	(0,739)	6	385	(0,953)	19

ADIntO (total:404, non-applicable:4)									
	positive uses			negative uses			total		
level	correct		incor.	correct		incor.	correct		incor.
1	259	(0,680)	122	23	(1,000)	0	282	(0,698)	122
2	336	(0,882)	45	22	(0,957)	1	358	(0,886)	46
3	365	(0,958)	16	15	(0,652)	8	380	(0,941)	24
4	380	(0,997)	1	10	(0,435)	13	390	(0,965)	14
5	381	(1,000)	0	7	(0,304)	16	388	(0,960)	16
6	381	(1,000)	0	4	(0,174)	19	385	(0,953)	19

ADGrid (total:405, non-applicable:3)									
	positive uses			negative uses			total		
level	correct		incor.	correct		incor.	correct		incor.
1	81	(0,212)	301	23	(1,000)	0	104	(0,257)	301
2	263	(0,688)	119	23	(1,000)	0	286	(0,706)	119
3	300	(0,785)	82	23	(1,000)	0	323	(0,798)	82
4	356	(0,932)	26	21	(0,913)	2	377	(0,931)	28
5	377	(0,987)	5	17	(0,739)	6	394	(0,973)	11
6	382	(1,000)	0	12	(0,522)	11	394	(0,973)	11
7	382	(1,000)	0	6	(0,261)	17	388	(0,958)	17

Table 6.7: Application of semantic theories based on angular deviation to unmodified projective locative expressions.

schema	baseline	simple	correct	complex	correct %
OP_P	94.4%	OP_{P3}	96.8%	$\mathrm{OP}_{P1}\oplus\mathrm{OP}_{P4}$	98.5%
OP_O	94.4%	OP_{O3}	96.1%	$\mathrm{OP}_{O2}\oplus\mathrm{OP}_{O4}$	98.8%
OP_{OP}	94.4%	OP_{OP5}	96.8%	$\mathrm{OP}_{OP3}\oplus\mathrm{OP}_{OP9}$	98.8%
OP_{Grid}	94.4%	OP_{Grid5}	97.6%	$\mathrm{OP}_{Grid2}\oplus\mathrm{OP}_{Grid6}$	99.5%
AD^{cent}	94.4%	AD_4^{cent}	97.6%	$\mathrm{AD}_2^{cent}\oplus\mathrm{AD}_4^{cent}$	99.5%
AD^{prox}	94.4%	AD_4^{prox}	95.9%	$\mathrm{AD}_2^{prox}\oplus\mathrm{AD}_5^{prox}$	98.5%
AD_P^{int}	94.4%	AD_{P5}^{int}	93.9%	$\mathrm{AD}_{P3}^{int}\oplus\mathrm{AD}_{P5}^{int}$	95.4%
AD_O^{int}	94.4%	AD_{O4}^{int}	95.1%	$\mathrm{AD}_{O1}^{int}\oplus\mathrm{AD}_{O5}^{int}$	98.8%
AD^{Grid}	94.4%	AD_5^{Grid}	96.1%	$\mathrm{AD}_3^{Grid}\oplus\mathrm{AD}_6^{Grid}$	98.8%

Table 6.8: Evaluation with the whole set of unmodified locative expressions. Simple theories are based on single relations, complex theories are based on underspecified semantic representations consisting of pairs of relations.

All simple theories except AD_P^{int} obtain better results than the baseline. All underspecified semantic representation (‘*complex*’) obtain better results than the baseline. All of these theories except for AD_P^{int} almost achieve 100% coverage of the data. The errors are due to those cases in the data which are not applicable to the corresponding relation schema and to those cases which have been removed in the re-rating process described above.

Only a semantics based on AD^{cent} provides disjointness of complements. All other theories allow for spatial configurations which might be described by a certain preposition as well as by its complement preposition.

The lower range levels of the theories OP_P, OP_{OP}, OP_{Grid}, AD^{cent}, AD^{prox}, and AD^{Grid} provide pairwise disjointness.

For theories based on OP_P and AD_P^{int} there are not enough range levels defined to cover the whole set of data.

6.4 Projective locative expressions modified by ‘*directly*’

This section evaluates all semantic theories specified by the mappings H^{RS} in Section 3.5 with projective locative expressions that are modified by the adverb *directly*. Based on the evaluation results semantic theories for projective locative expressions modified by *directly* are proposed. The method applied in this section is the same as the one from the previous section.

Data The evaluations are carried out on the set of data prepared in Section 6.1. More specifically, those projective locative expressions are selected which contain exactly one modifier, namely the adverb *directly*. The data set consists of 41 positive uses and

prepositions	freq
above	20
below	19
underneath	3
beneath	2
west	1
under	1
down	1

Table 6.9: Projective terms of locative expressions that are exclusively modified by the adverb *directly*.

4 negative uses. The frequency distribution of the projective prepositions is shown in Table 6.9.

Method As in the previous section it is the aim of this analysis to determine lexical mapping from pairs of the modifier *directly* and a projective preposition onto pairs of locative direction relations for a given relation schema RS.

(3) $H_*^{RS}(\langle\text{‘}\textit{directly}\text{’}, prep\rangle) \mapsto \langle H^{RS_k}(prep), H^{RS_l}(prep)\rangle$

The relations are determined according to the heuristic described in the previous section. Again, all critical locative expressions which were decisive for the selection of a particular relation were re-rated as before (cf. Appendix B). But this time no expressions were removed from the data.

Results and Discussion The results are shown in Tables 6.10 and 6.11. The number of correct and incorrect predictions are listed for positive uses, negative uses and for the total set (i.e. positive and negative uses). Horizontal lines separate range levels which provide pairwise disjoint relations from range levels providing relations with the property of complement disjointness and from range levels which do not provide any of these disjointness properties.

The range levels provided by AD^{cent}, AD^{prox} and AD$_O^{int}$ are not low enough to correctly reject all negative uses.

All semantic theories of projective locative expressions modified by *directly* are at least complement disjoint if the underlying relation schema provides any disjointness properties (OP$_P$, OP$_{OP}$, OP$_{Grid}$, AD^{cent}, AD^{prox}, AD$_P^{int}$, and AD^{Grid}). That means, if a spatial configuration is correctly described by the combination of *directly* and some projective preposition $prep$, then this spatial configuration cannot be correctly described by the complement preposition of $prep$. For example, ‘*directly above*’ excludes ‘*below*’.

The following theories also provide pairwise disjointness: OP$_{OP}$, OP$_{Grid}$, AD^{cent}, AD^{prox}, and AD^{Grid}. That means that *directly* $prep$ excludes the use of *directly* $prep'$ where $prep'$ is a preposition that is associated with a direction that is not aligned with

OPP (total:45, non-applicable:0)							
	positive uses		negative uses		total		
level	correct	incor.	correct	incor.	correct		incor.
1	10	31	4	0	14	(0,311)	31
2	41	0	0	4	41	(0,911)	4
3	41	0	0	4	41	(0,911)	4
4	41	0	0	4	41	(0,911)	4

OPO (total:45, non-applicable:0)							
	positive uses		negative uses		total		
level	correct	incor.	correct	incor.	correct		incor.
1	36	5	4	0	40	(0,889)	5
2	41	0	1	3	42	(0,933)	3
3	41	0	0	4	41	(0,911)	4
4	41	0	0	4	41	(0,911)	4
5	41	0	0	4	41	(0,911)	4

OPOP (total:45, non-applicable:0)							
	positive uses		negative uses		total		
level	correct	incor.	correct	incor.	correct		incor.
1	9	32	4	0	13	(0,289)	32
2	10	31	4	0	14	(0,311)	31
3	41	0	1	3	42	(0,933)	3
4	41	0	0	4	41	(0,911)	4
5	41	0	0	4	41	(0,911)	4
6	41	0	0	4	41	(0,911)	4
7	41	0	0	4	41	(0,911)	4
8	41	0	0	4	41	(0,911)	4
9	41	0	0	4	41	(0,911)	4

OPGrid (total:45, non-applicable:0)							
	positive uses		negative uses		total		
level	correct	incor.	correct	incor.	correct		incor.
1	7	34	4	0	11	(0,244)	34
2	30	11	2	2	32	(0,711)	13
3	41	0	0	4	41	(0,911)	4
4	41	0	0	4	41	(0,911)	4
5	41	0	0	4	41	(0,911)	4
6	41	0	0	4	41	(0,911)	4
7	41	0	0	4	41	(0,911)	4

Table 6.10: Evaluation frequencies of projective locative expressions that are modified by *directly*. Application of orthogonal projection relation schemata.

ADcent (total:45, non-applicable:0)							
	positive uses		negative uses		total		
level	correct	incor.	correct	incor.	correct		incor.
1	38	3	3	1	41	(0,911)	4
2	41	0	1	3	42	(0,933)	3
3	41	0	0	4	41	(0,911)	4
4	41	0	0	4	41	(0,911)	4
5	41	0	0	4	41	(0,911)	4

ADprox (total:45, non-applicable:0)							
	positive uses		negative uses		total		
level	correct	incor.	correct	incor.	correct		incor.
1	40	1	2	2	42	(0,933)	3
2	41	0	1	3	42	(0,933)	3
3	41	0	1	3	42	(0,933)	3
4	41	0	0	4	41	(0,911)	4
5	41	0	0	4	41	(0,911)	4

ADIntP (total:45, non-applicable:0)							
	positive uses		negative uses		total		
level	correct	incor.	correct	incor.	correct		incor.
1	8	33	4	0	12	(0,267)	33
2	25	16	4	0	29	(0,644)	16
3	39	2	1	3	40	(0,889)	5
4	41	0	0	4	41	(0,911)	4
5	41	0	0	4	41	(0,911)	4

ADIntO (total:45, non-applicable:0)							
	positive uses		negative uses		total		
level	correct	incor.	correct	incor.	correct		incor.
1	41	0	1	3	42	(0,933)	3
2	41	0	1	3	42	(0,933)	3
3	41	0	0	4	41	(0,911)	4
4	41	0	0	4	41	(0,911)	4
5	41	0	0	4	41	(0,911)	4
6	41	0	0	4	41	(0,911)	4

ADGrid (total:45, non-applicable:0)							
	positive uses		negative uses		total		
level	correct	incor.	correct	incor.	correct		incor.
1	11	30	4	0	15	(0,333)	30
2	41	0	1	3	42	(0,933)	3
3	41	0	1	3	42	(0,933)	3
4	41	0	0	4	41	(0,911)	4
5	41	0	0	4	41	(0,911)	4
6	41	0	0	4	41	(0,911)	4
7	41	0	0	4	41	(0,911)	4

Table 6.11: Evaluation frequencies of projective locative expressions that are modified by *directly*. Application of angular deviation schemata.

the direction associated with $prep$. For example, '*A is directly above B*' excludes '*A is directly to the left of B*', '*A is directly to the right of B*' and '*A is directly below B*'.

Similarly, the lower range levels of all of the former theories (OP$_P$, OP$_{OP}$, OP$_{Grid}$, AD^{cent}, AD^{prox}, AD$_P^{int}$, and AD^{Grid}) provide pairwise disjointness.

The underspecified theories determined for each relation schema are evaluated to the whole set. The baseline for each theory is 91.1%; it is the proportion of positive uses with respect to the sum of positive and negative uses. All semantic theories are

relation schema	theory	correct
OP$_P$	OP$_{P1}$$\oplus$OP$_{P2}$	100.0%
OP$_O$	OP$_{O1}$$\oplus$OP$_{O2}$	100.0%
OP$_{OP}$	OP$_{OP2}$$\oplus$OP$_{OP3}$	100.0%
OP$_{Grid}$	OP$_{Grid1}$$\oplus$OP$_{Grid3}$	100.0%
AD^{cent}	AD$_1^{cent}$$\oplus$AD$_2^{cent}$	97.8%
AD^{prox}	AD$_1^{prox}$$\oplus$AD$_2^{prox}$	95.6%
AD$_P^{int}$	AD$_{P2}^{int}$$\oplus$AD$_{P4}^{int}$	100.0%
AD$_O^{int}$	AD$_{O1}^{int}$$\oplus$AD$_{O1}^{int}$	93.3%
AD^{Grid}	AD$_1^{Grid}$$\oplus$AD$_2^{Grid}$	100.0%

Table 6.12: Evaluation of the underspecified semantic theories of projective locative expressions modified by *directly*.

better than the baseline.

6.5 Projective locative expressions modified by '*slightly*'

This section evaluates all semantic theories specified by the mappings H^{RS} in Section 3.5 with projective locative expressions that are modified by the adverb *slightly*. Based on the evaluation results semantic theories for these modified expressions will be proposed.

Data The evaluations are carried out on the set of data prepared in Section 6.1. More specifically, those projective locative expressions are selected which contain exactly one modifier, namely the adverb *slightly*.

Method After a first evaluation according to the method described in Chapter 4 critical expressions have been re-rated (cf Section 6.3). No expressions had to be removed from the data set because no expression was rated as false. There are 22 positive uses and one negative use. All types of prepositions are listed with frequencies in Table 6.15. We analyse *slightly* as a '*diagonal*' modifier, cf. Section 3.5, page 110. The corresponding lexical mappings map pairs consisting of the modifier *slightly* and a

OPP (total:23, non-applicable:0)							
	positive uses		negative uses		total		
level	correct	incor.	correct	incor.	correct		incor.
¬1	0	22	1	0	1	(0,043)	22
2	8	14	1	0	9	(0,391)	14
3	20	2	1	0	21	(0,913)	2
4	21	1	1	0	22	(0,957)	1

OPO (total:23, non-applicable:0)							
	positive uses		negative uses		total		
level	correct	incor.	correct	incor.	correct		incor.
1	1	21	1	0	2	(0,087)	21
2	1	21	1	0	2	(0,087)	21
3	22	0	1	0	23	(1,000)	0
4	22	0	1	0	23	(1,000)	0
5	22	0	0	1	22	(0,957)	1

OPOP (total:23, non-applicable:0)							
	positive uses		negative uses		total		
level	correct	incor.	correct	incor.	correct		incor.
1	0	22	1	0	1	(0,043)	22
2	0	22	1	0	1	(0,043)	22
¬3	0	22	1	0	1	(0,043)	22
4	8	14	1	0	9	(0,391)	14
5	20	2	1	0	21	(0,913)	2
6	20	2	1	0	21	(0,913)	2
7	21	1	1	0	22	(0,957)	1
8	21	1	1	0	22	(0,957)	1
9	22	0	0	1	22	(0,957)	1

OPGrid (total:23, non-applicable:0)							
	positive uses		negative uses		total		
level	correct	incor.	correct	incor.	correct		incor.
1	0	22	1	0	1	(0,043)	22
¬2	0	22	1	0	1	(0,043)	22
3	8	14	1	0	9	(0,391)	14
4	19	3	1	0	20	(0,870)	3
5	22	0	1	0	23	(1,000)	0
6	22	0	1	0	23	(1,000)	0
7	22	0	1	0	23	(1,000)	0

Table 6.13: Frequencies of the evaluation of projective locative expressions that are modified by *slightly*. Application of orthogonal projection relation schemata. Horizontal lines separate range levels which provide pairwise disjoint relations from range levels providing relations with the property of complement disjointness and from range levels which do not provide any of these inferential properties. Bold face range levels are included in the semantic theory, the symbol ¬ marks range levels which are explicitly excluded from the theory.

ADcent (total:23, non-applicable:0)							
	positive uses		negative uses		total		
level	correct	incor.	correct	incor.	correct		incor.
1	1	21	1	0	2	(0,087)	21
2	5	17	1	0	6	(0,261)	17
3	11	11	1	0	12	(0,522)	11
4	22	0	1	0	23	(1,000)	0
5	22	0	0	1	22	(0,957)	1

ADprox (total:22, non-applicable:1)							
	positive uses		negative uses		total		
level	correct	incor.	correct	incor.	correct		incor.
1	0	21	1	0	1	(0,045)	21
¬2	0	21	1	0	1	(0,045)	21
3	5	16	1	0	6	(0,273)	16
4	19	2	1	0	20	(0,909)	2
5	21	0	0	1	21	(0,955)	1

ADIntP (total:22, non-applicable:1)							
	positive uses		negative uses		total		
level	correct	incor.	correct	incor.	correct		incor.
1	0	21	1	0	1	(0,045)	21
2	0	21	1	0	1	(0,045)	21
¬3	0	21	1	0	1	(0,045)	21
4	8	13	1	0	9	(0,409)	13
5	17	4	0	1	17	(0,773)	5

ADIntO (total:22, non-applicable:1)							
	positive uses		negative uses		total		
level	correct	incor.	correct	incor.	correct		incor.
¬1	0	21	1	0	1	(0,045)	21
2	4	17	1	0	5	(0,227)	17
3	12	9	1	0	13	(0,591)	9
4	17	4	1	0	18	(0,818)	4
5	21	0	0	1	21	(0,955)	1
6	21	0	0	1	21	(0,955)	1

ADGrid (total:23, non-applicable:0)							
	positive uses		negative uses		total		
level	correct	incor.	correct	incor.	correct		incor.
1	0	22	1	0	1	(0,043)	22
¬2	0	22	1	0	1	(0,043)	22
3	2	20	1	0	3	(0,130)	20
4	11	11	1	0	12	(0,522)	11
5	22	0	1	0	23	(1,000)	0
6	22	0	1	0	23	(1,000)	0
7	22	0	0	1	22	(0,957)	1

Table 6.14: Frequencies of the evaluation of projective locative expressions that are modified by *slightly*. Application of angular deviation relation schemata.

prepositions	f
right	7
above	3
left	3
below	3
up	2
east	2
underneath	1
down	1
beneath	1

Table 6.15: Projective terms of locative expressions with *slightly* as a single modifying phrase

projective preposition onto quadruples of locative direction relations for a given relation schema RS. The quadruples determine a semantic representation which is defined by four range levels i, j, k and l with $i \leq j \leq k \leq l$:

(4) $H_*^{RS}(\langle\text{'slightly'},\ prep\rangle) \ \mapsto\ H^{RS_k}(prep) \oplus H^{RS_l}(prep) \wedge \neg H^{RS_i}(prep) \oplus H^{RS_j}(prep)$

The term $H^{RS_k}(prep) \oplus H^{RS_l}(prep)$ corresponds to the semantic representations of unmodified expressions (cf Section 6.3). The term $\neg H^{RS_i}(prep) \oplus H^{RS_j}(prep)$ excludes lower ranges from the extension specified by the previous term. The mapping will be such that all positive uses of $\langle mod, prep\rangle$ in the data satisfy $H^{RS_l}(prep)$ but not $H^{RS_i}(prep)$ and all negative uses will either satisfy $H^{RS_j}(prep)$ or they will not satisfy $H^{RS_k}(prep)$.

Tables 6.13 and 6.14 show the frequencies of correct and incorrect application of semantic theories. It separately lists the frequencies for positive and negative uses. Horizontal lines separate range levels which provide pairwise disjoint relations from range levels providing relations with the property of complement disjointness from range levels which do not provide any of these disjointness properties. The range levels k and l, which correspond to the range levels of an unmodified expression, are determined according to the heuristic specified in Section 6.3. They are marked in bold face in the result tables. The range levels i and j (for excluding lower range levels) are marked with the symbol $\neg$. The upper range level j is set to the highest range level in the evaluation results which provides the no correct positive uses or, if there is no such range level, the exclusive part is omitted completely. The lower range level i is set to the highest range level which provides the maximum number of correct negative uses. If i is greater j, then i is reset to j.

Results and Discussion The underspecified theories determined for each relation schema are shown in Table 6.16. If the same range levels are combined by the underspecification operator $\oplus$ we just write this range level once without underspecification.

relation schema	theory	correct
OP_P	$\mathrm{OP}_{P4} \wedge\neg\ \mathrm{OP}_{P1}$	95.7%
OP_O	OP_{O3}	100.0%
OP_{OP}	$\mathrm{OP}_{OP8} \oplus \mathrm{OP}_{OP9} \wedge\neg\ \mathrm{OP}_{OP3}$	100.0%
OP_{Grid}	$\mathrm{OP}_{Grid5} \wedge\neg\ \mathrm{OP}_{Grid2}$	100.0%
AD^{cent}	AD_4^{cent}	100.0%
AD^{prox}	$\mathrm{AD}_4^{prox} \oplus \mathrm{AD}_5^{prox} \wedge\neg\ \mathrm{AD}_2^{prox}$	95.7%
AD_P^{int}	$\mathrm{AD}_{P4}^{int} \oplus \mathrm{AD}_{P5}^{int} \wedge\neg\ \mathrm{AD}_{P3}^{int}$	95.7%
AD_O^{int}	$\mathrm{AD}_{O4}^{int} \oplus \mathrm{AD}_{O5}^{int} \wedge\neg\ \mathrm{AD}_{O1}^{int}$	95.7%
AD^{Grid}	$\mathrm{AD}_5^{Grid} \wedge\neg\ \mathrm{AD}_2^{Grid}$	100.0%

Table 6.16: Evaluation of the underspecified semantic theories of projective locative expressions modified by *slightly*.

The theories were evaluated with respect to the same data set. All of them are equal to or greater than the baseline of 95.7% (22 positive uses divided by a total number of 23 expressions).

Theories based on the relation schemata AD^{cent} and AD^{Grid} provide complement disjointness. All other theories do not provide any disjointness properties. The theories based on OP_O and AD^{cent} cannot be analysed as '*diagonal*' modifiers since they do not exclude any lower range levels. All other theories can be analysed as '*diagonal*' modifiers. The part that excludes the lower ranges is pairwise disjoint for the following theories: OP_P, OP_{OP}, OP_{Grid}, AD^{prox}, and AD^{Grid}. That means a description with the combination of *slightly* and $prep$ rules out a description with combination of *slightly* and $prep'$ where $prep'$ is a preposition that is associated with a direction that is not aligned with the direction associated with $prep$. For example, '*A is slightly above B*' blocks '*A is slightly to the left of B*', '*A is slightly to the right of B*' and '*A is slightly below B*'.

6.6 Conclusions

This section summarises the semantic theories for projective locative expressions that have been determined in the preceding sections and discusses the relations between the semantics of unmodified expressions and expressions modified by *directly* and *slightly*.

Table 6.17 summarises all theories for projective locative expressions that were determined in the previous sections. The first part of the table summarises the semantic theories for unmodified projective locative expressions. They are specified in terms of underspecified combinations of a lower range level Lo and an upper range level Up. The column "CD of Up" describes whether the relations of the upper range level provide complement disjointness (CD); and the column "PD of Lo" describes whether the relations of the lower range level provide pairwise disjointness (PD). The second part of the table summarises the semantic theories for projective locative expressions modified by *directly* and describes disjointness properties of the relations of the corresponding range levels. The third part summarises the semantic theories for projective locative expressions modified by *slightly* and it describes disjointness properties of the corresponding relations. *IncUp* specifies the range level of the relations which determine the part of the domain that *can* make the expression true and the relations from range level *ExUp* determine the part of the domain which is explicitly excluded from the part determined by *IncUp*.

	unmodified (Lo⊕Up)	CD of Up	PD of Lo	directly (Lo⊕Up)	CD of Up	PD of Up	PD of Lo	slightly (IncLo⊕IncUp∧¬ExUp)	CD of IncUp	PD of ExUp
OP$_P$	1⊕4	-	yes	1⊕2	yes	-	yes	4 ∧¬ 1	-	yes
OP$_O$	2⊕4	-	-	1⊕2	-	-	-	3	-	-
OP$_{OP}$	3⊕9	-	yes	2⊕3	yes	yes	yes	8⊕9 ∧¬ 3	-	yes
OP$_{Grid}$	2⊕6	-	yes	1⊕3	yes	yes	yes	5 ∧¬ 2	-	yes
AD^{cent}	2⊕4	yes	yes	1⊕2	yes	yes	yes	4	yes	-
AD^{prox}	2⊕5	-	yes	1⊕2	yes	yes	yes	4 ⊕ 5 ∧¬ 2	-	yes
AD$^{int}_P$	3⊕5	-	-	2⊕4	yes	yes	yes	4⊕5 ∧¬ 3	-	-
AD$^{int}_O$	1⊕5	-	-	1⊕1	-	-	-	4 ⊕ 5 ∧¬ 1	-	-
AD^{Grid}	3⊕6	-	yes	1⊕2	yes	yes	yes	5∧¬ 2	yes	yes

Table 6.17: Summary of semantic theories and their disjointness properties. CD means complement disjointness, PD pairwise disjointness.

For each relation schema three semantic theories were determined from the analysis of the data: one for unmodified expressions, one for expressions modified by *directly* and one for expressions modified by *slightly*.

The theories for *directly* and *slightly*, more specifically the upper range levels of these theories are subsumed by (the upper range level of) the theory of unmodified expressions for all relation schemata. This is not very surprising since this inference was assumed in process of annotating the corpus, see page 138. However, this assumption does not rule out in principle that there could have been cases in the data leading to theories for *directly* and *slightly* that were not subsumed by the corresponding theory for unmodified expressions.

The theories based on AD^{cent} are the only theories that provide complement disjointness for unmodified projective locative expressions and expressions modified by *slightly*. However, they do not exclude any low range levels from the range determined by the semantics of expressions modified by *slightly*.

The semantic theories based on OP$_{OP}$, OP$_{Grid}$, AD^{prox} and AD^{Grid} satisfy all disjointness properties listed in the table except for complement disjointness of unmodified expressions and expressions modified by *slightly*,

Table 6.18 shows specific relations between the theories that are based on the same relation schema. The first column shows for which relation schemata the upper range level of the theory for *directly* is subsumed by the lower range level of the theory for unmodified expressions. If this subsumption relation is satisfied then the modification

	*directly*Up $\subseteq$ *unmod*Lo	*directly*Up $\subseteq$ *slightly*ExUp
OP_P	-	-
OP_O	yes	-
OP_{OP}	yes	yes
OP_{Grid}	-	-
AD^{cent}	yes	-
AD^{prox}	yes	yes
AD_P^{int}	-	-
AD_O^{int}	yes	yes
AD^{Grid}	yes	yes

Table 6.18: Comparison of semantics of unmodified expressions and expressions modified by slightly and directly, respectively.

with *directly* implies that the corresponding unmodified expression is definitely true and cannot be false, e.g.:

(5) *A is directly above B* $\Longrightarrow$ *A is above B in every respect.*

In contrast to the indirect way of determining the lower range level *unmodLo* of a theory for unmodified expressions in Section 6.3, lower range levels can be directly inferred from occurrences of projective locative expressions modified by *directly* if the inference rule of (5) holds.

The second column shows for which relation schemata the upper range level of the theory for *directly* is subsumed by the range level that determines which lower range levels are excluded by expressions modified by *slightly*. If this subsumption relation is satisfied then the modification with *directly* implies that the corresponding expression modified by *slightly* is false, e.g.:

(6) *A is directly above B* $\Longrightarrow$ *A is not slightly above B*

The following relation schemata provide theories which satisfy all of these properties: OP_{OP}, AD^{prox}, AD_O^{int}, and AD^{Grid}.

Pulling together the information of disjointness properties and the relation between the theories for modified and unmodified expressions, the relations of which relation schema should be selected as the basis for a semantics of projective locative expressions? Here we can only come to a conditional conclusion: if there are cues from other resources that prepositions and their complement prepositions are mutually exclusive then AD^{cent} should be selected. On the other hand, if there are cues that expressions modified by *directly* exclude (i) negative uses of the corresponding unmodified expressions and also (ii) modification by *slightly*, then AD^{Grid}, AD^{prox} or OP_{OP} are good candidates.

It should be clear that these theories have been determined on the basis of a small collection of locative expressions and a very limited spatial domain, and thus it is an open question how far these results can be transferred to other domains. In particular it should be noted that these theories do not account for cases where the frame of reference needs to be computed and where functional relations have an impact on the description of the spatial relation.

Chapter 7

Summary and Closing Remarks

7.1 Summary

This thesis has investigated semantic theories of projective locative expressions on the basis of independently collected conversational data. The investigation has been based on a novel formal approach to testing extensional semantic theories of projective locative expressions with pairs of spatial configurations and statements about them. This approach has been implemented as a computational procedure and applied to data from the HCRC Map Task corpus (Anderson et al., 1991).

In this study the semantic of projective locative expressions is based on locative direction relations. Locative direction relations are spatial relations that describe the location of an object lo relative to a reference object ro by means of a prototypical direction ρ.

Two kinds of semantic theories have been defined. Firstly, 'classical' semantic theories which divide the domain into two parts – either the meaning of an expressions is true with respect to a particular spatial configuration or it is false. A semantic theory T_{RS} is defined by a mapping H^{RS} from projective prepositions ($prep$) or pairs of a modifier and a preposition (mod, $prep$) onto locative direction relations rel_ρ:

(1) $H^{RS}(prep) \mapsto rel_\rho$
$H^{RS}(mod, prep) \mapsto rel_\rho$

Every such rel_ρ defines the extension of the corresponding projective locative expression. A locative predication involving mod and $prep$ and terms referring to a located object lo and a reference object ro is true if and only if rel_ρ holds between lo and ro. A set of the 'classical' theories was specified by systematically combining different techniques for defining locative direction relations found in the literature. However, projective locative expressions are vague to some extent. That means, that there are spatial configurations for some expressions that are true in some respect but false in another respect.

Therefore, secondly, 'composite' semantic theories T'_{RS} have been defined that map projective locative expressions onto pairs of locative direction relations $\langle rel^i_\rho, rel^j_\rho \rangle$. Each of these pairs divides the extension of an expression in definitely true (rel^i_ρ) and possibly true (rel^j_ρ). More precisely, rel^i_ρ is true for all spatial relations that can be expressed in every respect by expressions consisting of mod and $prep$. And rel^j_ρ is true for all spatial relations that *can* be described by such an expression. Consequently, rel^j_ρ is false for spatial relations that cannot be described by an expression consisting of mod and $prep$ in any respect. A semantics is defined for this formalism based on models of first-order logic. It determines a classical truth value for sentences of 'composite' theories.

The semantic theories T_{RS} (both the 'classical' and and the 'composite' theories) defined by each mapping H^{RS} have been tested against projective locative expressions from the HCRC Map Task corpus (Anderson et al., 1991) which is a collection of route description dialogues. The routes which had to be described in the dialogues were specified by means of maps consisting of schematic landmarks. In order to collect a set for testing all those locative expressions have been extracted from the corpus which the speakers used to make statements about the location of landmarks relative to other landmarks.

The testing procedure has been based on a model-theoretic analysis of the data. The collected utterances have been translated into a semantic representation formalism and the maps about which the utterances make claims have been transformed into structures that turn into models for this representation formalism (in the canonical sense of model-theoretic semantics) when combined with each of the theories T_{RS} that are to be tested. That is, each map is turned into as many models as there are testable theories; and the model-theoretic truth definition for the representation formalism then assigns for each utterance about that map a truth value to its translation in each of these models. If the truth value in a model agrees with the truth judgement evinced by the speaker, that provides positive evidence for the theory corresponding to that model. If truth value and judgement disagree, then that constitutes negative evidence for the corresponding theory.

An important subtask was the design of a formal method for generating canonical models for projective locative expressions from the maps as they are used in the map task. Each of the testable theories specifies semantic definitions for the different projective locative expressions in terms of geometric properties which are directly identifiable in the abstract geometrical maps into which the original maps were transformed (see below). Therefore, each such definition assigns to the corresponding projective locative expression a model-theoretic interpretation (i.e. an extension) for each map. In this way each theory turns each abstract map into a canonical model for the projective locative expressions of the representation formalism.

Computational implementation The testing algorithm has been implemented as a computational procedure. The data has been prepared in the following way.

First, the maps that were used in the map task experiments have been transformed into more abstract geometrical structures – rectangular parts of the Euclidean planes within which all landmarks are represented as polygons. Since each of the testable theories specifies semantic definitions for the different projective locative expressions in terms of geometric properties that are explicitly represented in the abstract maps and can be automatically recognised, each such definition assigns to the corresponding projective locative expression an extension for each abstract map, and these extensions can be determined by automatic means.

Second, all locative expressions were marked in the corpus if they described the location of a landmark with respect to another landmark. Then they were annotated with feature structures containing the information required for the evaluation procedure. In particular, they were annotated with the preposition ($prep$) a list of modifiers (mod) links to the objects (lo and ro) which the expressions refer to, and information about whether the speakers indicated that an expression is true or false. For each theory T_{RS} mappings from these feature structures were defined that turned them into formulas of the semantic representation formalism for T_{RS}. For each of these semantic representations the truth value was determined with respect to the corresponding model generated in the first step.

Results The evaluation shows good results for the ‘classical’ semantic theories. But none of them correctly separates all positive uses of projective locative expressions from all negative uses. A analysis of the errors suggests that these theories cannot correctly model both the semantics of negative and positive uses.

A data-driven approach was taken to determine ‘composite’ semantic theories for unmodified projective locative expressions and for expressions modified by *directly* and *slightly* The evaluation results systematically improved with respect to the corresponding ‘classical’ semantic theories.

Further analysis of the ‘composite’ theories showed that the meaning of complement prepositions did not come out as mutually exclusive for most those theories. But there are indications that a (correct) positive use of the modifier *directly* is mutually exclusive with negative uses of the corresponding unmodified expression, as well as with the positive use of the modifier *slightly*.

Conclusions This thesis presents a novel approach for testing semantic theories of projective locative expressions with conversational data. The testing procedure is based on a formal-semantic analysis of the natural language data and on a novel technique to translate the corresponding spatial data into specific models of formal semantics. These procedures were implemented and applied to data from the HCRC Maptask corpus. They proved to be a powerful tool for the analysis of both semantic theories and data.

7.2 Closing remarks

In order to proceed in the direction of this piece of research it would be desirable to evaluate the semantic theories of projective locative expressions with data derived from other domains. There are two questions. First, do the theories proposed in this work generalize to other domains or are they overfitting the data from the HCRC Maptask corpus? Second, how do these theories perform in domains where there are functional relations between the objects of the domain and where the frame of reference is not constant? It should be clear that for such domains the semantic theories from this work would have to be extended such that frame of reference computation, direction relations, and functional relations are integrated into single theories.

Another direction of continuing this work would be to reduce manual work and increase the number of automatic procedures. In principle, the testing procedure is fully automatical and thus it provides a good basis for machine learning approaches and inferential statistical analyses. However, it is only fully automatical as long as the data are prepared in the ways described, and most of the actual preparation of the data was done manually and turned out to be very labour intensive. Therefore, it would be highly desirable to have as much as possible of the preparations done automatically, too. The challenge is to find procedures that automatically produce unique spatial representations and unique feature structures as specified in Chapter 4, yet without them being dependent on the semantic theories that are to be tested. As for the geometrical representations, we can take such an independence for granted, since the problem of producing a spatial representation from a spatial configuration is one of recognising the identity of objects and assigning a location to them, but not one of classifying relative position. As for the feature structures that are some kind of pragmatic and semantic representations of uses of projective locative expressions, it is not clear to what extent the construction procedure is independent from the relevant semantic theories. The type of the feature structures is repeated here from Chapter 4:

$$\begin{bmatrix} \text{prep} & : & \text{a symbol denoting a projective preposition} \\ \text{mod} & : & \text{a list of symbols denoting modifiers} \\ \text{lo} & : & \text{a symbol uniquely referring to the located object} \\ \text{ro} & : & \text{a symbol uniquely referring to the reference object} \\ \text{use} & : & \text{a truth value} \end{bmatrix}$$

A computational procedure that provides analyses of projective locative expressions in such a way has to cope with a whole bunch of problems inherited from natural language processing, in particular (i) parsing, (ii) co-reference and reference analysis, and (iii) discourse and dialogue analysis. On all levels the question is whether we can resolve ambiguous analyses in order to obtain unique feature structures, and if so, if we can do this without relying on the semantic theories to be tested. The preposition ($prep$) of locative expressions and the modifiers (mod) are analysed by syntactic parsing of the relevant utterances. The syntactic analysis also provides the phrases that refer to lo and ro. The reference of these phrases has to be determined by a component that resolves anaphoric expressions and determines the reference of an expression with respect to given spatial representations. An analysis of the discourse or dialogue, respectively, determines logical and pragmatical embedding of the expression in the context. From such an analysis the value for the feature use should be derivable, i.e. a truth value that indicates whether the expression is used positively or negatively.

Kapitel 8

Zusammenfassung

Die Semantik Projektiver Lokativer Ausdrücke

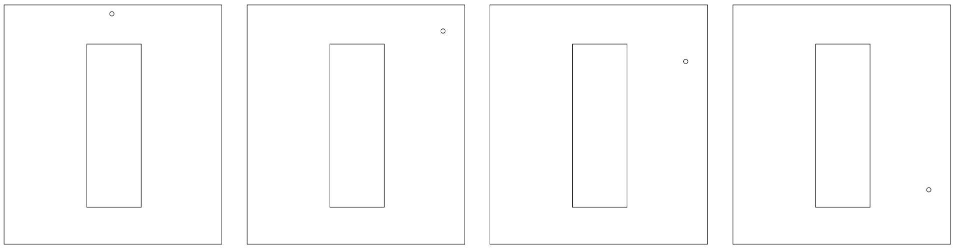
Einführung – Kapitel 1

In dieser Arbeit werden geometrische Wahrheitsbedingungen von *projektiven, lokativen Ausdrücken* anhand von Wegbeschreibungsdialogen untersucht. Projektive, lokative Ausdrücke beschreiben die Position eines Objektes, des lokalisierten Objekts oder auch LO, relativ zu einem Referenzobjekt, RO, durch Angabe einer Richtung. Die Richtung wird typischerweise durch projektive Präpositionen (Herskovits, 1986) wie zum Beispiel *über*, *unter*, *rechts von* und *links von* ausgedrückt. Der folgende Satz beinhaltet einen projektiven, lokativen Ausdruck:

(1) Der Kreis ist über dem Rechteck.

Die Präposition *über* bestimmt die Richtung *oben*, der Ausdruck *der Kreis* bezeichnet das LO und *dem Rechteck* das RO. Die Bedeutung solcher Ausdrücke soll hier in einer Weise untersucht werden, dass jedem dieser Ausdrücke Wahrheitsbedingungen zugeordnet werden, die dann für beliebige räumliche Konstellationen bestimmen, ob dieser Ausdruck auf diese Konstellation zutrifft, also wahr ist, oder eben nicht, also falsch ist. Dabei beschränken wir uns hier auf Fälle, in denen die Bedeutung von projektiven, lokativen Ausdrücken ausschließlich durch geometrische Wahrheitsbedingungen bestimmt wird, so dass andere Faktoren[1] vernachlässigt werden können. Um die Fragestellung zu verdeutlichen seien in Abbildung 8.1 vier Beispiele gegeben, die jeweils ein Rechteck und einen sehr kleinen Kreis zeigen. Für alle vier Konstellationen von (a) bis (d) können wir uns die Frage stellen, ob Satz (1) auf sie zutrifft. Entsprechende Erklärungsansätze, die in dieser Arbeit untersucht werden, basieren allesamt auf lokative Richtungsrelationen.

[1] Siehe zum Beispiel Coventry & Garrod (2004).



(a) (b) (c) (d)

Abbildung 8.1: Verschiedene Konstellationen eines Kreises und eines Rechtecks.

Lokative Richtungsrelationen – Kapitel 2

Lokative Richtungsrelationen rel_ρ klassifizieren relative Positionen, d.h. die Position eines Objektes relativ zu einem anderen Objekt, nach prototypischen Richtungen ρ, wie zum Beispiel *oben*, *unten*, *rechts links*, *vorne* und *hinten*. Das Bestimmen, ob eine Richtungsrelationen rel_ρ auf ein Paar von Objekten lo und ro zutrifft, erfolgt in drei Schritten. Zuerst wird eine Repräsentation der relativen Position für lo relativ zu ro erzeugt. Als Zweites wird der Grad der Anwendbarkeit von rel_ρ auf die Repräsentation ermittelt und damit auch auf das Paar der Objekte $\langle lo, ro\rangle$. Als Drittes wird entschieden, ob der Grad der Anwendbarkeit hinreichend hoch ist, so dass das Paar $\langle lo, ro\rangle$ die Relation rel_ρ erfüllt.

Bei Repräsentationen von relativer Position unterscheiden wir vornehmlich zwischen *Winkelrepräsentationen* und *achsenbasierten Repräsentationen*. Winkelrepräsentationen stellen relative Position mithilfe eines oder mehrerer Winkel bzw. Vektoren dar. Der Grad der Anwendbarkeit wird dann über die Abweichung der entsprechenden Winkel von der prototypischen Richtung ermittelt. Um zu entscheiden, ob die entsprechende Relation erfüllt ist, gibt es unterschiedlich Bedingungen, die insbesondere dann verschieden sind, wenn sich die Repräsentationen in der Anzahl der Winkel bzw. Vektoren unterscheiden.

Achsenbasierte Repräsentationen bestehen aus orthogonalen Projektionen der relevanten Objekte auf ein oder mehreren Achsen. Pro Dimension wird ein Paar räumlich ausgedehnter Objekte durch ein Paar von Intervallen dargestellt. Äquivalent können die Objekte auch als ein Paar von Rechtecken repräsentiert werden, bei denen die Eckpunkte von den entsprechende Intervallgrenzen der achsenbasierten Repräsentationen bestimmt werden. Mithilfe des Rechtecks des Referenzobjekts kann die Ebene in Regionen aufgeteilt werden, wie in Abbildung 8.2 dargestellt. Der Grad der Anwendbarkeit wird dann über den Grad der Überlappung des lokalisierten Objekts mit verschiedenen Kombinationen dieser Regionen errechnet.

Es wurden mehrere Relationsschemata RS spezifiziert, die lokative Richtungsrelationen für die Haupthimmelsrichtungen (*nord*, *west*, *süd*, *ost*) definieren. Jedes Relationsschema definiert pro Richtung eine Reihe von Relationen mit unterschiedlicher

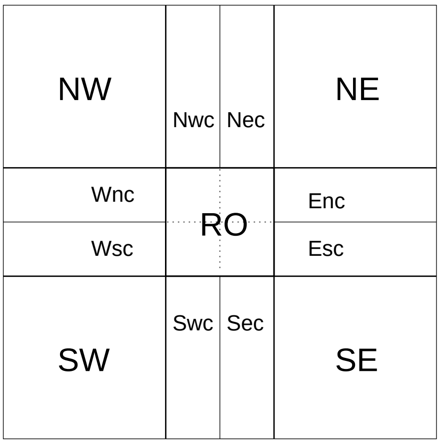


Abbildung 8.2: Aufteilung der Ebene um das Referenzobjekt in 12 Regionen .

Abdeckung, so dass Relationen mit höherer Abdeckung die entsprechenden Relationen mit niedrigerer Abdeckung komplett Umfassen. Der Abdeckungsgrad wird durch die Indizes i und j angegeben:

(2) $\forall lo, ro : rel^{RS_i}(lo, ro) \Rightarrow rel^{RS_j}(lo, ro)$ with $i \leq j$

In diesem Kapitel werden die Relationsschemata AD^{cent}, AD^{prox}, AD_P^{int}, AD_O^{int} und AD^{Grid} definiert, die auf Winkelrepräsentationen basieren, und die Relationsschemata OP_P, OP_O, OP_{OP} und OP_{Grid}, die auf achsenbasierte Repräsentationen basieren.

Semantik Projektiver Lokativer Ausdrücke – Kapitel 3

In diesem Kapitel werden zwei Arten semantischer Theorien definiert, die die Bedeutung von projektiven, lokativen Ausdrücken modellieren. Als Erstes werden projektive Präpositionen direkt auf die lokativen Richtungsrelationen abgebildet, die im vorherigen Kapitel definiert worden sind.

(3) $H^{RS}(mod) \mapsto rel_\rho$
$H^{RS}(mod, prep) \mapsto rel_\rho$

Diese Bedeutungsdefinitionen bestimmen eindeutig einen Wahrheitswert eines projektiven, lokativen Ausdrucks – ein Ausdruck ist entweder wahr in Bezug auf eine räumliche Konstellation oder er ist falsch.

Allerdings ist allgemein bekannt – und es wird auch an den Daten gezeigt –, dass projektive, lokative Ausdrücke eine vage Bedeutungskomponente haben. Das heisst, es gibt für alle diese Ausdrücke entsprechende räumliche Konstellationen, für die der Wahrheitswert nicht eindeutig bestimmt werden kann, und in gewisser Hinsicht ist der Ausdruck wahr, aber in anderer Hinsicht ist er falsch.

Um diese Vagheit zu modellieren, werden als Zweites Theorien vorgeschlagen, die projektive, lokative Ausdrücke auf Paare von Richtungsrelationen $\langle \pi_i, \pi_j \rangle$ abbilden. Wobei jedes dieser Paare die Extension des Ausdrucks in definitiv wahr (π_i) und möglicherweise wahr (π_j) aufteilt. Im speziellen ist π_i wahr für alle Paare von Objekten, die in jeglicher Hinsicht durch den Ausdruck bestehend aus mod und $prep$ korrekt beschrieben werden. Und π_j ist wahr für alle Paare von Objekten, die durch den Ausruck bestehend aus mod und $prep$ korrekt beschrieben werden *können*. Folglich ist π_j nur für die Paare falsch, die in keinerlei Hinsicht durch diesen Ausdruck beschrieben werden können.

Für den vorgeschlagenen Formalismus wird ein Valuationsverfahren vorgestellt, das anhand von Modellen der Prädikatenlogik erster Stufe den Wahrheitswert von Sätzen des Formalismus bestimmt.

Methode – Kapitel 4

Die semantischen Theorien, die im vorherigen Kapitel definiert wurden, werden anhand des *HCRC Maptask* Korpus (Anderson et al., 1991) getestet. Das Korpus besteht aus Wegbeschreibungsdialogen, in denen Wege beschrieben werden, die durch Karten mit schematischen Landmarken vorgegeben sind.

Das Testverfahren basiert auf einer modelltheoretischen Analyse der Daten. Die gesammelten Äußerungen werden in einen semantischen Repräsentationsformalismus übersetzt, und die Karten, über die diese Äußerungen Aussagen treffen, werden in geometrische Strukturen überführt, die dann in Modelle (im kanonischen Sinn der modelltheoretischen Semantik) für diesen Repräsentationsformalismus umgeformt werden, indem sie mit jeder der zu testenden Theorien T_{RS} kombiniert werden. Jede Karte wird also in genau so viele Modelle umgeformt, wie es zu testende Theorien gibt. Die modelltheoretischen Wahrheitsbedingungen weisen dann der semantischen Repräsentation jeder Äußerung über jede solche Karte einen Wahrheitswert zu entsprechend des jeweiligen Modells. Wenn der Wahrheitswert im Modell mit der Wahrheitswertbeurteilung des Sprechers übereinstimmt, stellt das positive Evidenz für die Theorie, die dem Modell entspricht, dar. Und wenn sich Wahrheitswert und Wahrheitswertbeurteilung unterscheiden, stellt das negative Evidenz für diese Theorie dar.

Als wichtiger Teil der Aufgabe wurde eine formal Methode entwickelt, die kanonische Modelle für die semantischen Theorien aus den Karten des *Maptask* generiert. Jede der zu testenden Theorien spezifiziert semantische Definitionen für die verschiedenen projektiven, lokativen Ausdrücke anhand von geometrischen Eigenschaften, die direkt aus den abstrakten geometrischen Karten ermittelt werden können, in die die originalen Karten des *Maptasks* umgewandelt wurden (siehe unten). Daher weist jede solche Definition den entsprechenden projektiven, lokativen Ausdrücken eine modelltheoretische Interpretation – und zwar eine Extension – für jede Karte zu. So wandelt jede Theorie eine abstrakte Karte in ein kanonisches Modell für projektive, lokative Ausdrücke des Repräsentationsformalismus um.

Das Testvervahren wurde implementiert. Dafür wurden die Daten folgendermaßen aufbereitet: Zum einen wurden die Karten, die in den *Maptask* Experimenten benutzt wurden, in abstrakte geometrische Repräsentationen umgewandelt, und zwar in rechteckige Ausschnitte der euklidischen Ebene in der die Landmarken als Polygone repräsentiert wurden. Da alle zu testenden Theorien semantische Definitionen für projektive, lokative Ausdrücke bestimmen und diese Definitionen sich auf geometrische Eigentschaften beziehen, die direkt in den abstrakten geometrischen Karten repräsentiert sind, kann die Extension der Ausdrücke automatisch ermittelt werden.

Zum anderen wurden im Korpus alle projektiven, lokativen Ausdrücke markiert, die die Position einer Landmarke relativ zu einer anderen Landmarke beschreiben. Desweiteren wurden sie mit Merkmalstrukturen annotiert, die die benötigten Informationen für das Testverfahren enthalten. Insbesondere wurde die Präposition des lokativen Ausdrucks annotiert ($prep$), eine Liste der Modifikatoren (mod), Verweise auf die Objekte (lo und ro), auf die sich der Ausdruck bezieht, und Information darüber, ob die Sprecher den Ausdruck für wahr oder falsch halten. Für jede Theorie T_{RS} wurden Abbildungen von diesen Merkmalstrukturen auf semantische Repräsentationen ensprechend T_{RS} definiert. Diese semantischen Repräsentationen wurden dann automatisch am entsprechenden Modell, das im ersten Schritt generiert worden ist, evaluiert.

Daten – Kapitel 5

Als empirische Grundlage der Studie diente das *HCRC Maptask* Korpus (Anderson et al., 1991), das sowohl projektive, lokative Ausdrücke enthält als auch die von ihnen beschriebenen räumlichen Konstellationen.

Wir haben geometrische Repräsentationen der Karten erstellt, in denen die Landmarken der Karten als Polygone dargestellt werden. Diese Repräsentationen erlauben den direkten computationellen Zugriff auf Position und räumliche Ausdehnung von Landmarken auf den entsprechenden Karten.

Zum Testen der Theorien wurden alle lokativen, projektiven Ausdrücke im Korpus markiert, die die Sprecher benutzen, um die Position einer Landmarke relativ zu einer anderen Landmarke zu beschreiben. Diese Vorkommen wurden dann mit Merkmalstrukturen annotiert, die semantische und pragmatische Informationen für den jeweiligen Ausdruck enthalten. Insbesondere wurden die Merkmale wie in (4) dargestellt annotiert. Die Merkmale `LO` und `RO` nehmen als Werte Symbole an, die eindeutig auf das lokalisierte Objekt bzw. auf das Referenzobjekt verweisen. `REL` und `MOD` geben die projektive Präposition bzw. die Modifikatoren des Ausdrucks an. Das Merkal `COMM` bestimmt, ob ein Sprecher deutlich macht, dass er den Ausdruck, der durch die übrigen Merkmale bestimmt wird, für wahr (`positive`) oder für falsch hält (`negative` und `impl-negative`). Das Merkmal `COMM` erhält den Wert `non-committing`, wenn keine dieser Interpretation ausreichend belegt werden kann, und den Wert `cancelled`, wenn der Sprecher seine Aussage revidiert.

(4)
$$\begin{bmatrix} \text{LO :} & \text{eindeutiger Schlüssel für ein räumliches Objekt} \\ \text{RO :} & \text{eindeutiger Schlüssel für ein räumliches Objekt} \\ \text{REL :} & \text{projektiver Term} \\ \text{MOD :} & \text{Liste von Modifikatoren} \\ \text{COMM :} & (\texttt{positive} \mid \texttt{negative} \mid \texttt{impl-negative} \mid \\ & \texttt{cancelled} \mid \texttt{non-committing}) \end{bmatrix}$$

Ergebnisse – Kapitel 6

Es wurden verschiedene semantische Theorien für projektive, lokative Ausdrücke mit dem Testverfahren und den Daten aus den vorherigen Kapiteln evaluiert. Die Evaluierung der Theorien aus den Arbeiten von (Kelleher, 2003), (O'Keefe, 1996), (Fuhr et al., 1995), (Gapp, 1994a), (Abella & Kender, 1993) und (André et al., 1987) zeigt, dass all diese Theorien Probleme haben, die Semantik von negativen Verwendungen von projektiven, lokativen Ausdrücken korrekt zu modellieren.

Ein ähnliches Ergebnis ergab die Evaluierung von ‚klassischen' semantischen Theorien, die eine einfache Extension definieren. Diese ‚klassische' Theorien wurden repräsentativ für verschiedener Techniken aus der Literatur spezifiziert. Die Evaluierung zeigte, dass keine dieser Theorien alle positive Verwendungen in den Daten korrekt von den negativen Verwendungen trennt.

Alternativ wurden Theorien aus Paaren von klassischen Theorien zusammengesetzt. Diese ‚kombinierten' Theorien teilen den Gegenstandsbereich in drei Teile. Wie bereits oben erwähnt, sind diese Teile mit den bestimmten Wahrheitswerten, Wahr und Falsch, und einem unbestimmten Wahrheitswert verknüpft. Gemäß der Evaluierung verbessern die ‚kombinierten' Theorien die Semantik im Vergleich zu den ‚klassischen' Theorien systematisch.

Unter den ‚kombinierten' Theorien gab es nur eine Theorie über unmodifizierte, projektive, lokative Ausdrücke, bei der projektive Präpositionen ihre Komplemente ausschließen. Dieses Ergebnis wirft die Frage auf, ob sich Komplementpräpositionen wie zum Beispiel *above* (über) und *below* (unter) überhaupt generell ausschließen.

Drei der ‚kombinierten' Theorien zeigten die folgenden Eigenschaften: eine Verwendung des Modifikators *directly* schließt sowohl eine negative Verwendung des entsprechenden unmodifizierten Ausdrucks aus als auch den Gebrauch des Modifikators *slightly* mit dem entsprechenden unmodifizierten Ausdruck.

Schluss – Kapitel 7

Diese Arbeit beschreibt eine neuartige Methode zum Testen von semantischen Theorien über lokative Ausdrücke mit Gesprächsdaten. Das Testverfahren basiert auf einer formalsemantischen Analyse der Gesprächsdaten und auf einem Algorithmus, der Modelle im Sinne der formalen Semantik aus rein räumlichen Repräsentationen generiert. Dieses Verfahren wurde implementiert und konkret eingesetzt, um semantische Theorien mit Daten des *HCRC Maptask* Korpus zu testen. Es erwies sich als ein effektives Verfahren zur Analyse der semantischen Theorien und ebenso zur Analyse der Daten.

Appendix A

Expressions removed from the data

Some locative expressions have been removed from the data, either because they were marked as *cancelled*, i.e. the speaker indicates at a later point in the dialogue that the locative expression is incorrect (cf Section 5.3.3.), or because they could not be annotated appropriately according to the guidelines of the annotation scheme.

Cancelled occurrences The following two utterances contain locative expressions which are corrected instantly:

(1) q4nc4.266$_{explain}$ G: the youth hostel is about it's a couple of centimetres **to the left** eh **to the right** and a couple of centimetres above the alpine garden right so it's very c–

(2) q5ec6.153$_{check}$ F: directly **above below** the s– ... start

In the first utterance *to the left* is revised to *to the right*, and in the second utterance *above* is corrected to *below*.

Another locative expression annotated with *cancelled* is shown in move 124 of dialogue dialogue *q4ec3*. The relation expressed by *to your right of* is corrected in move 133 to *on you left*.

(3)

q4ec3.122$_{query-yn}$	G:	do you have a boat house to your ... left
q4ec3.123$_{reply-w}$	F:	right ... at the bottom
q4ec3.124$_{query-yn}$	G:	oh do you not have another one ... **to your right ... of** the concealed hideout
q4ec3.125$_{reply-y}$	F:	yeah right of the concealed hideout uh-huh
...		
q4ec3.133$_{clarify}$	G:	no i want you to go to the boat house **on your right** ... on your ... **on your left**
q4ec3.134$_{explain}$	F:	i don't have one on my left
q4ec3.135$_{explain}$	G:	thought you did

Problematic occurrences The following locative expressions were problematic in some respect and were removed from the data. In the following utterance the preposition *on the level* is overridden by *between*:

(4) q1ec5.142$_{query-yn}$ F: that **on the ... ... the level** ... between the fort ... mm

In the following case the anaphoric relation of *that* is not clear. Therefore, we cannot determine the RO unequivocally.

(5) q4ec2.218$_{reply-w}$ G: well the finish is just between rock fall and great lake it's n– ... it's not ... ... directly between them but it's **on the same level as** that

In the next part of dialogue *q1nc3*, the route follower states that the *finish* of the route is *right above the fort*. This expression has been removed from the data because we cannot determine the kind of commitment that the speakers exhibit towards this expression. Since the *finish* is not marked on the follower's map, the response of the instruction giver is decisive. He or she responds by the clarification that the finish is *about an inch above the fort*. It is not clear whether *about an inch above* implies *right above* and thus confirms it, or whether it rejects it.

(6)	q1nc3.432$_{clarify}$	G:	it's above
	q1nc3.433$_{acknowledge}$	F:	right **above** the fort och i've just got a general gist there
	q1nc3.434$_{clarify}$	G:	'bout an inch above the fort

In (7) the instruction giver describes the location of the *poisoned stream* which only occurs on his or her map, but it is described relative to the landmark *slate mountain* which does not occur on the giver map. Therefore, this piece of information is not reliable, and was removed from the data.

(7)	q7ec6.216$_{query-w}$	F:	where's the poison stream come from slate mountain
	q7ec6.217$_{explain}$	G:	there isn't a slate mountain on this one
	q7ec6.218$_{acknowledge}$	F:	right
	q7ec6.220$_{reply-w}$	G:	but it's ... below ... slate mountain

The next utterance contains the phrase "*nothing else but a saloon bar*" to describe the located object. This description is too complex to be captured by the annotation scheme.

(8) q6nc4.188$_{explain}$ F: there's just a noose and then like ... to the left there's stone creek and below that there's a saloon bar but nothing else

Similarly, the following part of dialogue *q7nc4* contains a locative expression *a poisoned stream near there*. Where *near there* refers to some location underneath the pyramid. This description of the reference object is too complex to be captured by the annotation scheme.

(9)	q7nc4.194$_{explain}$	G:	but underneath that we've got a cobbled street
	q7nc4.195$_{query-yn}$	G:	have you got that
	q7nc4.196$_{reply-n}$	G:	no
	q7nc4.197$_{explain}$	G:	i've got a poisoned stream near there

The following utterance contains a description which is underspecified with respect to which object is the located object and which the reference object:

(10)	q4nc7.278$_{align}$	G:	right if you think ... see between this ... thi– see how the saxon barn and the rope bridge are above each other

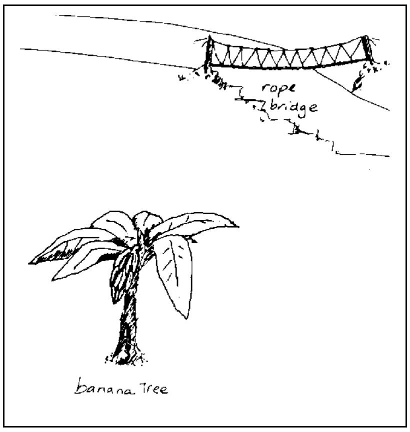
Either the *saxon barn* is above the *rope bridge* or the *rope bridge* is above the *saxon barn*. The spatial configuration is depicted in Figure A.1. The expression is removed from the data because no unique feature structure could be determined.

Figure A.1: Map 13: saxon barn and rope bridge.

Appendix B

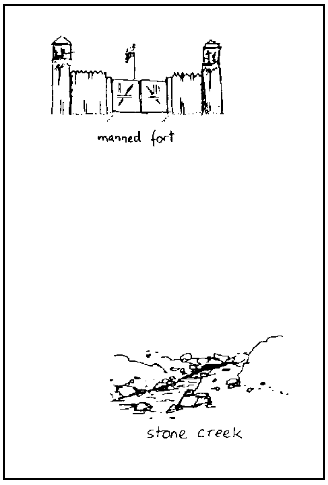
Re-rating of locative expressions

The truth values of the following locative expressions with respect to the corresponding spatial configurations were re-rated by three native speakers of english. More specifically, each locative expression was presented with enough dialogue context so that the located object and the reference object could be identified. The projective preposition in question and the modifiers, if any, were marked in bold face. Particles negating the relevant locative expressions and the phrase denoting the LO, if there were more than one LO, were also marked in bold face. For each locative expression the corresponding spatial configuration containing the LO and the RO was presented, too. The raters had to decide whether the speakers used the locative expression intentionally or whether it was a false use, e.g. confusion of left and right or a slip of the tongue. Altogether 3 raters rated 26 pieces of data. The rating was determined by the majority of the votes.



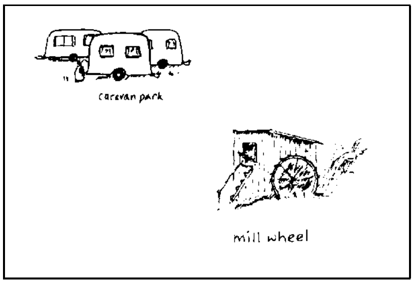
(1)	q6nc3.45$_{explain}$	G:	banana tree i've not got that at all
	q6nc3.46$_{explain}$	F:	it's a tree with big leaves and ... it's got a bunch of bananas on it
	q6nc3.47$_{ready}$	G:	well
	q6nc3.48$_{instruct}$	G:	just go left
	q6nc3.49$_{align}$	G:	okay
	q6nc3.50$_{reply-y}$	F:	right
	q6nc3.51$_{query-yn}$	F:	so should i stop there
	q6nc3.52$_{explain}$	F:	it's just underneath the rope bridge
	q6nc3.53$_{acknowledge}$	G:	underneath the rope bridge
	q6nc3.54$_{explain}$	F:	you know it's ... it's about five centimetres ... ... down and **slightly to the left of** it

number of ratings for		re-rated
`true`	`false`	truth value
3	0	`true`



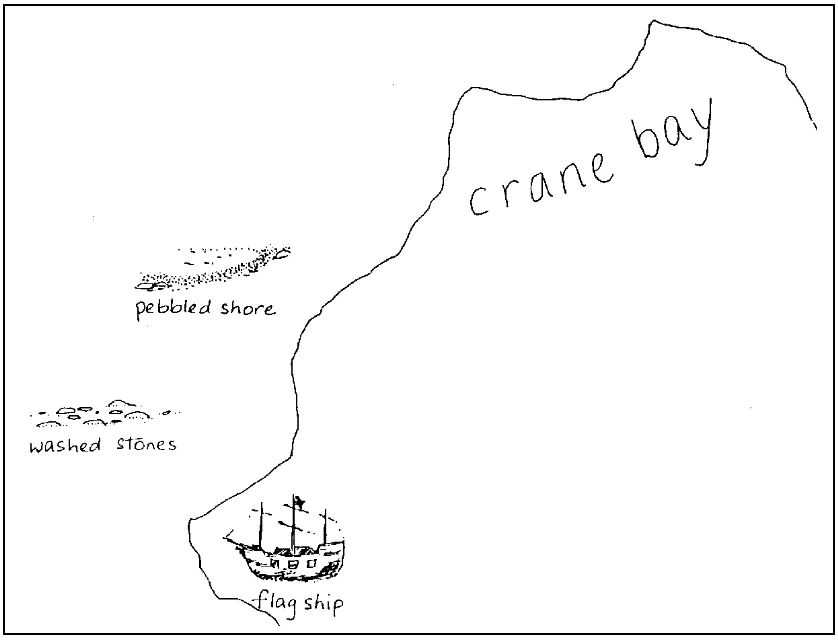
(2) q2ec6.89$_{query-w}$ G: where's the fort
...
q2ec6.96$_{query-yn}$ G: so is it ... to your right of the stone creek ... and just up a bit
q2ec6.97$_{reply-n}$ F: no
q2ec6.98$_{explain}$ F: **to my left** and up a wee bit ...

number of ratings for		re-rated
`true`	`false`	truth value
3	0	`true`



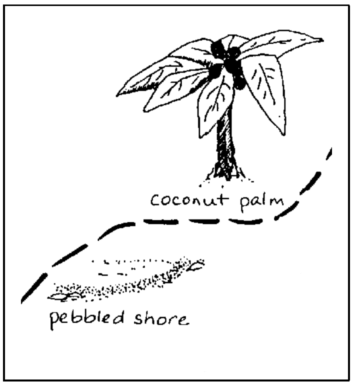
(3) q5nc7.25$_{explain}$ F: i have a mill wheel ...
q5nc7.26$_{acknowledge}$ G: mm well
q5nc7.27$_{query-yn}$ G: is it sort of ... ... down and **to the left of** the caravan park
q5nc7.28$_{reply-y}$ F: uh-huh

number of ratings for		re-rated
`true`	`false`	truth value
0	3	`false`



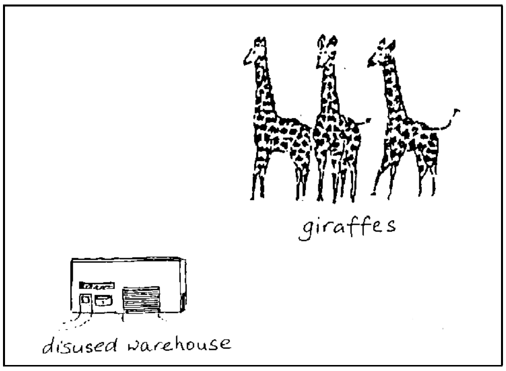
(4) q4ec3.174$_{query-w}$ G: have you got anything **below** pebbled shore
q4ec3.175$_{reply-w}$ F: washed stones
q4ec3.176$_{ready}$ G: well
q4ec3.177$_{reply-w}$ F: and flagship ... and **bay**

number of ratings for		re-rated
`true`	`false`	truth value
2	1	`true`



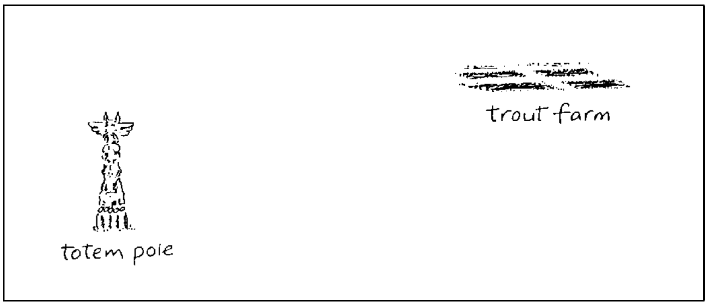
(5) q4ec5.183$_{query-w}$ F: where where is the object eh you're trying to avoid
q4ec5.184$_{reply-w}$ G: it's a– above pebbled shore
q4ec5.185$_{uncodable}$ F: it's
q4ec5.186$_{reply-w}$ G: **to the right of** pebbled shore just above
q4ec5.187$_{acknowledge}$ F: above and to the right right i understand now i–

number of ratings for		re-rated
`true`	`false`	truth value
3	0	`true`



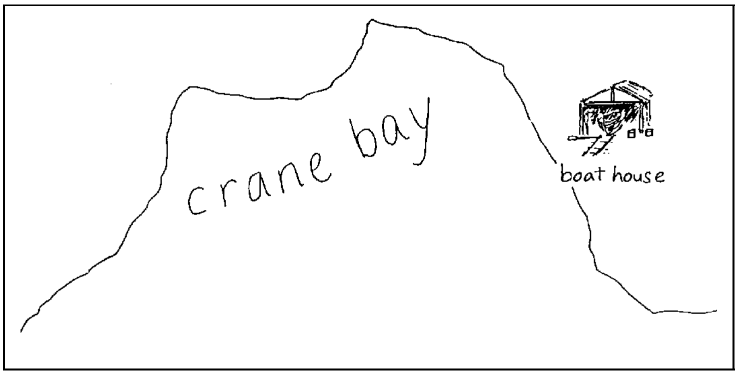
(6) q4nc2.264$_{explain}$ G: i don't have a a disused warehouse on mine
q4nc2.265$_{acknowledge}$ F: oh right
q4nc2.267$_{explain}$ F: well i– ... it's just parallel to it ... like ... just ehm ... ... well **not underneath** the giraffes ... you know over ... ... to the left

number of ratings for		re-rated
`true`	`false`	truth value
3	0	`true`



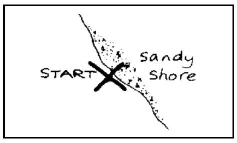
(7) q3nc7.66$_{query-yn}$ G: is totem pole **below** the trout farm
q3nc7.67$_{reply-n}$ F: **no** i–
q3nc7.68$_{reply-w}$ F: well it's kind of opposite it

number of ratings for		re-rated
`true`	`false`	truth value
3	0	`true`



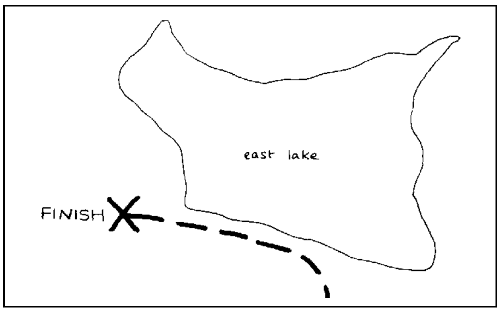
(8) q4nc3.329$_{explain}$ F: okay i'm directly above the boat house now
q4nc3.330$_{check}$ G: you're above it
q4nc3.331$_{reply-y}$ F: mmhmm
q4nc3.332$_{explain}$ F: my my boat house is ... down **below** crane bay
q4nc3.333$_{explain}$ G: i haven't got that

number of ratings for		re-rated
`true`	`false`	truth value
1	2	`false`



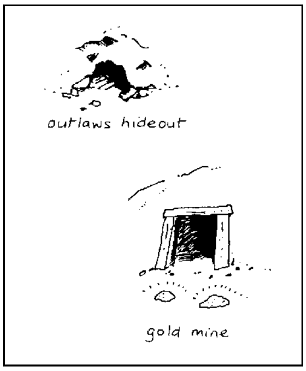
(9) : q6nc1.2$_{instruct}$ G: you start **to the left of** sandy shore ...

number of ratings for `true`	`false`	re-rated truth value
3	0	`true`



(10) q1nc1.611$_{instruct}$ G: but that's where you're meant to end up ... **underneath** east lake

number of ratings for `true`	`false`	re-rated truth value
2	1	`true`



(11)

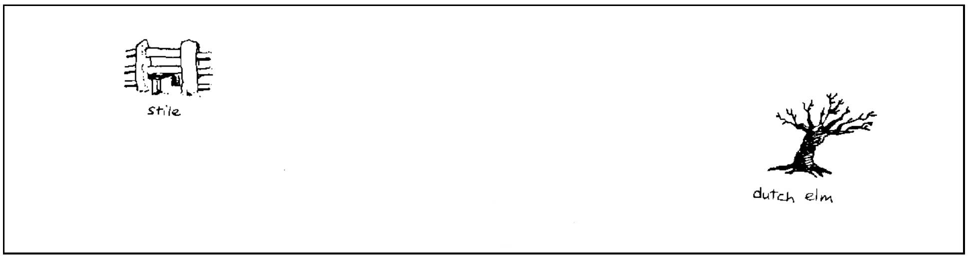
q2nc6.75$_{explain}$	F:	right i've got a gold mine here ...
q2nc6.76$_{acknowledge}$	G:	a gold mine
q2nc6.77$_{query-w}$	G:	where about
q2nc6.78$_{reply-w}$	F:	er just ehm just ... to the right and above it
q2nc6.79$_{query-yn}$	G:	... is it bet– ... bet– ... between you and the outlaws' hideout
q2nc6.80$_{reply-n}$	F:	ye– ... ... no
q2nc6.81$_{clarify}$	F:	it's er ... it's on the other side of the outlaws' ... ... outlaws' hideout
q2nc6.82$_{query-w}$	G:	on the right or left
q2nc6.86$_{clarify}$	F:	well ... ... the outlaws' hideout hideout's **at the top** ... ... and then **below** ... is the gold mine
q2nc6.84$_{acknowledge}$	G:	okay

number of ratings for		re-rated
`true`	`false`	truth value
3	0	`true`

(12)

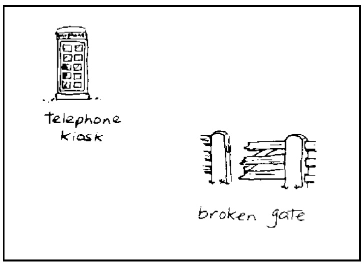
q2ec4.204$_{check}$	G:	you've got a gold mine a gold mine you got a gold mine ... below that
q2ec4.207$_{reply-y}$	F:	yeah
q2ec4.208$_{query-yn}$	G:	far below it
q2ec4.209$_{uncodable}$	F:	erm
q2ec4.210$_{query-yn}$	G:	directly below it
q2ec4.211$_{reply-n}$	F:	**not directly below** it
q2ec4.212$_{explain}$	F:	just ... ... just ... below the "u" ... ... if you take it down from the hideout ... ... er that ... that's just ... that's just at the left-hand side

number of ratings for		re-rated
`true`	`false`	truth value
3	0	`true`



(13)	q2nc2.224$_{align}$	G:	this dutch elm you have ... if you look across the page to the stile again
	q2nc2.225$_{acknowledge}$	F:	uh-huh
	q2nc2.226$_{query-w}$	G:	... is it beneath the stile ... or
	q2nc2.227$_{reply-w}$	F:	**slightly beneath** it ... ... the base of the dutch elm is about ... ... maybe half an inch ... below the bottom of the stile
	q2nc2.228$_{acknowledge}$	G:	uh-huh

number of ratings for		re-rated
`true`	`false`	truth value
3	0	`true`

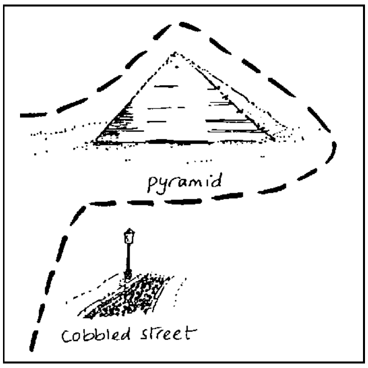


(14)	q6ec2.16$_{acknowledge}$	G:	a broken gate
	q6ec2.17$_{query-yn}$	G:	directly below the telephone kiosk
	q6ec2.18$_{reply-n}$	F:	oh **not directly below** no

number of ratings for		re-rated
`true`	`false`	truth value
3	0	`true`

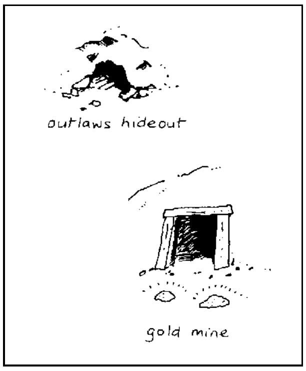
(15)	q4ec8.37$_{query-w}$	G:	now where's the overgrown gully ...
	...		
	q4ec8.41$_{query-w}$	G:	and eh to the ... left or right of highest viewpoint
	q4ec8.42$_{reply-w}$	F:	it's beneath it
	q4ec8.46$_{check}$	G:	it's directly beneath it
	q4ec8.45$_{reply-w}$	F:	it's ... **to the right-hand side**

number of ratings for `true`	`false`	re-rated truth value
3	0	`true`



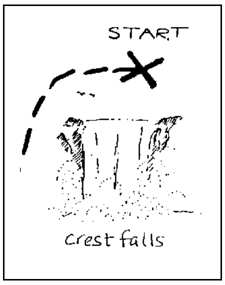
(16)	q7ec4.133$_{explain}$	G:	below the pyramid there's a cobbled street ... ... and
	q7ec4.134$_{query-yn}$	F:	directly below it
	q7ec4.135$_{reply-w}$	G:	ehm ... it's ... **below the ... ... western side** of the pyramid ...

number of ratings for `true`	`false`	re-rated truth value
3	0	`true`



(17)	q2nc4.130$_{instruct}$	G:	okay well ... you go up towards it ... but draw just beneath the words outlaws' hideout
	q2nc4.131$_{acknowledge}$	F:	okay
	q2nc4.132$_{check}$	G:	is that right over in the right-hand side
	q2nc4.133$_{reply-y}$	F:	yeah
	q2nc4.134$_{acknowledge}$	G:	okay
	q2nc4.135$_{explain}$	F:	and there's a gold mine **directly beneath** it

number of ratings for		re-rated
`true`	`false`	truth value
3	0	`true`

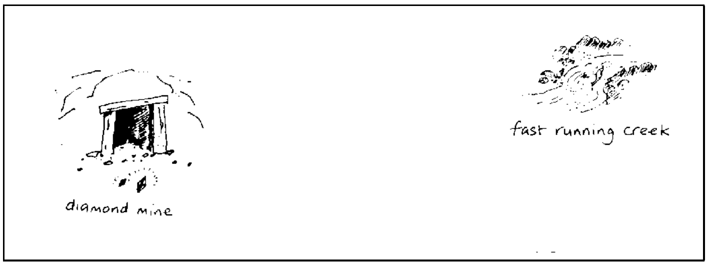


(18)	q7ec4.4$_{query-yn}$	G:	ha– ... have you got crest falls ...
	q7ec4.5$_{reply-y}$	F:	mmhmm
	q7ec4.6$_{explain}$	G:	well ... ... **directly above** them ... ... maybe about ... ... three quarters of an inch ... above there is the start point
	q7ec4.7$_{acknowledge}$	F:	yeah

number of ratings for		re-rated
`true`	`false`	truth value
3	0	`true`

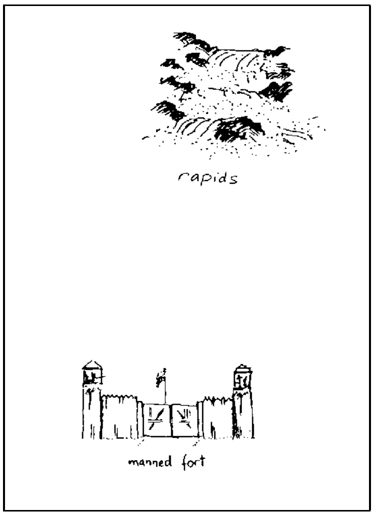
(19)	q3ec6.1$_{instruct}$	G:	right it starts **directly above** the crest falls if you go ... ... to the left of your page just to the edge of the crest falls
	q3ec6.2$_{acknowledge}$	F:	mmhmm

number of ratings for		re-rated
`true`	`false`	truth value
3	0	`true`



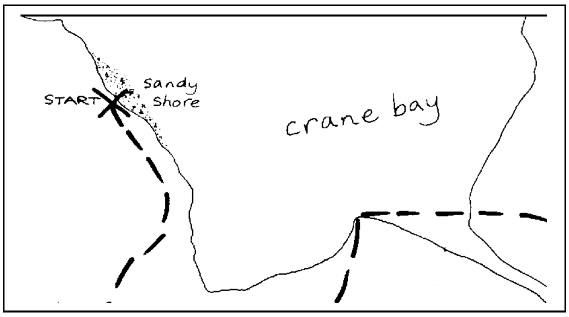
(20) q1ec5.42$_{acknowledge}$ G: fast running creek
q1ec5.43$_{query-w}$ G: where ... is that ... on a level with the diamond mine or is it ... slightly ... below ...
q1ec5.44$_{reply-w}$ F: it's **just slightly above**

number of ratings for `true`	`false`	re-rated truth value
3	0	`true`



(21) q2ec4.155$_{query-yn}$ F: have you got a manned fort ... ... below the rapids ...
q2ec4.156$_{reply-n}$ G: no
q2ec4.157$_{explain}$ F: 'cause i've got that interrupting ... ... er ... like dir– ... almost directly below it ... ... **slightly to the left** i've got a manned fort

number of ratings for `true`	`false`	re-rated truth value
3	0	`true`



(22) q2nc7.12$_{explain}$ F: um ... ... i've got a sort of river thing ... ... crane bay crane bay that's it

...

q2nc7.21$_{check}$ F: and it's ... **slightly to the right of** the start

q2nc7.22$_{reply-y}$ G: yeah ...

number of ratings for		re-rated
`true`	`false`	truth value
3	0	`true`

(23) q4ec8.37$_{query-w}$ G: now where's the overgrown gully ...

...

q4ec8.46$_{query-w}$ G: and eh to the ... left or right of highest viewpoint

q4ec8.45$_{reply-w}$ F: it's ... to the right-hand side ...

q4ec8.57$_{acknowledge}$ G: to the right-hand side of safari truck

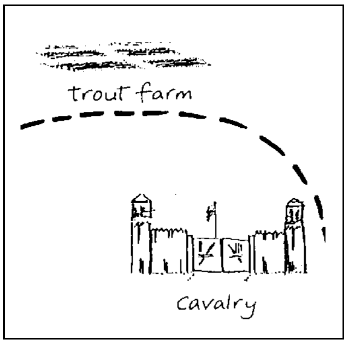
q4ec8.48$_{clarify}$ F: **slightly to the right-hand side**

q4ec8.49$_{acknowledge}$ G: right of this one right okay

q4ec8.50$_{clarify}$ F: right to the middle of highest viewpoint anyway

q4ec8.51$_{acknowledge}$ G: okay right

number of ratings for		re-rated
`true`	`false`	truth value
3	0	`true`



(24) q7ec1.165$_{explain}$ G: there's a ... fort ... ... cavalry fort

...

q7ec1.168$_{check}$ F: underneath the trout farm

q7ec1.169$_{clarify}$ G: underneath the trout farm ... ... slightly **slightly east** ... ... and underneath it ...

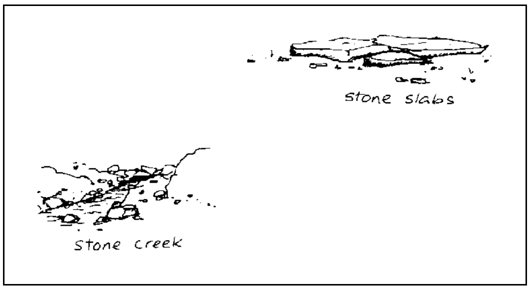
number of ratings for		re-rated
`true`	`false`	truth value
3	0	`true`

(25) q3ec1.137$_{clarify}$ G: cavalry it's a ... ... fort ... thing

...

q3ec1.143$_{clarify}$ G: it's almost south of the sou– ... the trout farm ... it's south ... and **slightly to the east**

number of ratings for		re-rated
`true`	`false`	truth value
3	0	`true`



(26)	q2ec4.158$_{query-yn}$	G:	you got another stone creek
	q2ec4.159$_{acknowledge}$	F:	eh oh right ...
	q2ec4.160$_{query-yn}$	G:	and stone slabs
	q2ec4.161$_{reply-y}$	F:	yeah it's right the way down below the manned fort ...
	q2ec4.162$_{query-yn}$	F:	have you not got anything in between the rapids and the stone creek ... down at the bottom ...
	q2ec4.163$_{reply-n}$	G:	no
	q2ec4.164$_{reply-y}$	G:	well yeah
	q2ec4.165$_{explain}$	G:	**slightly to its right** there's some stone slabs

number of ratings for		re-rated
`true`	`false`	truth value
2	1	`true`

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