82 ANALYSIS OF REINFORCED CONCRETE BEAMS WITHOUT SHEAR REINFORCEMENT USING NON-LOCAL MICROPLANE MODEL

J. OŽBOLT and R. ELIGEHAUSEN
Stuttgart University, Germany

Abstract
The shear resistance of reinforced concrete beams without shear reinforcement is studied using the non-local microplane model and plane stress finite elements. The main objective of the present work is the study of the size effect. Calculated failure loads for geometrically similar specimens of four different sizes are compared with test data and the recently proposed size effect law. Results of the analysis as well as test results exhibit significant size effect. Observed failure is of the brittle type and is due to failure in tension-compression. Further studies with variations of the mesh size and load path demonstrated that results of the calculations using a non-local continuum are not influenced by the above parameters. However, the calculated failure loads depend on the characteristic length over which the strains are measured. In contrast to that, the calculated failure loads are inobjective when a local continuum is used and depend on mesh size, load path and convergence criteria. This is due to the stability of the numerical analysis, which may lead to an overriding of the critical failure mode and activating to a more stable one. In order to correctly predict failure load using local analysis and crack band approach it is necessary to check the stability of the numerical procedure.
Keywords: Size effect, Shear, Non-local Continuum, Microplane Model, Stability, Objectivity.

1 Introduction

Kani, (1967) was one of the first who demonstrated that the shear strength of identical concrete beams decreases with increasing beam depth and that the shear design provisions used at that time were unsafe for larger beams. Recently, several experimental studies confirm the size effect in a diagonal shear type of failure (Bazant and Sun, 1987; Walraven, 1990; Bazant and Kazemi, 1990; Walraven and Lehwalter, 1990). In the last two studies, the size effect law proposed by Bazant (1984) was confirmed. However, according to Walraven and Lehwalter, the aggregate size has no influence on the size effect law for a maximum aggregate size between 8 and 32 mm.

To improve the understanding of the mechanism of the diagonal shear failures and of the size effect on the failure load, four

Fracture Processes in Concrete, Rock and Ceramics Edited by J.G.M. van Mier, J.G. Rots and A. Bakker.
© 1991 RILEM. Published by E. & F.N. Spon, 2–6 Boundary Row, London SE1 8HN. ISBN 0 419 15870 7

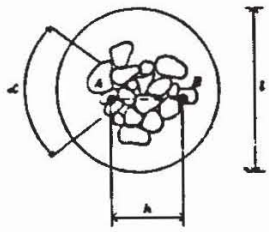

Fig. 1 Interaction among various orientations and at distance

geometrically similar specimen with different sizes are analyzed using the non-local microplane model and four node plane stress finite elements. Results of the numerical analysis are compared with above test results and size effect law.

In many applications of finite elements in failure simulations of concrete structures, a local continuum approach based on the crack band theory is used. Studies by Rots (1988) indicate that the results may be inobjective and may depend on the finite element size and orientation. To explore the reason for this inobjectivity, studies are performed using the microplane model and a local continuum. Varied are the mesh size and load path. Furthermore, it is checked weather objective results are reached when a non-local continuum is used.

In a numerical analysis based on a non-local continuum the key parameter is the characteristic length over which the strains are averaged. Therefore, the influence of this parameter on the diagonal shear failure load is studied.

2 Review of the non-local microplane model

The basic idea of the microplane models, that were initiated by G.I.Taylor (1938), is that the plastic slips in the modeling of the plasticity were calculated independently on various crystalographic planes, based on the resolved shear stress component, and were then superimposed to obtain the plastic microstrain. Recently (Bazant, 1984; Bazant & Gambarova, 1985; Bazant & Prat, 1988), this approach was extended to include strain softening of concrete, and was renamed more generally as the "microplane model", in recognition of the fact that the approach is not limited to plastic slip but can equally well describe cracking and strain softening damage. However, in order to prevent instability due to strain softening, the microplanes must be constrained kinematically rather than statically.

A basic requirement for a continuum model for a brittle heterogeneous material such as concrete is that it must correctly display the consequences of heterogeneity of the microstructure. A continuum constitutive model lumps the average response of a certain characteristic volume of the material (Fig. 1). In essence, one may distinguish two types of interactions among the particles or damage sites in the microstructure, which must be somehow manifested in the continuum model: (1) Interaction at **distance** among various sites (e.g., between A and B, Fig. 1); and (2) interaction **among various orientations** (see angle α in Fig. 1).

The interactions at distance control the localization of damage. They are ignored in the classical, local continuum models but are

reflected in non-local models (Pijaudier-Cabot and Bazant, 1987). The non-local aspect is a requisite for a realistic description of the size effect, as well as for the modeling of fracture propagation in the form of a crack band.

According to the non-local concept, the stress at a point depends not only on the strain at the same point but also on the strain field in a certain neighborhood of the point (Kröner 1968; Krumhansl 1968; Levin 1971; Eringen & Edelen 1972). In the current study, an effective form of the non-local concept, in which all variables that are associated with strain softening are non-local and all other variables are local is used. The originally proposed non-local concept (Pijaudier-Cabot and Bazant, 1987) is here modified introducing additional weighting functions that control the averaging of the total (positive and negative) strain field into the directions of main principle stresses. An important advantage of this formulation, called non-local damage or non-local continuum with local strain, is that the differential equations of equilibrium as well as the boundary conditions are of the same form as in the local continuum theory, and that there exist no zero-energy periodic modes of instability.

The key parameter in the used non-local concept is the characteristic length l_c over which the strains are averaged and it has a significant influence on the results of the analysis. Bazant & Pijaudier-Cabot (1989), assumed that this length is a material parameter which can be taken as $3d_a$ (d_a = maximum aggregate size). However, in general 3D stress-strain situations the characteristic length is difficult to interpret as a material parameter depending on the concrete mix only, but may be influenced by other parameters as well such as stress-strain state and geometry of the particular problem. Principally, the characteristic length must be chosen so that together with the stress-strain law used in the smeared crack approach can correctly prescribe structural behavior.

The non-local microplane model as well as an effective numerical iterative algorithm for the loading steps, that is used in the finite element code is described in detail by Bazant and Ozbolt (1990).

3 Size effect study

3.1 Finite element meshes and material parameters

A size effect study of reinforced concrete beams, previously tested by Bazant and Kazemi (1990), is carried out. Four different sizes of geometrically similar specimen are analyzed. The geometry of the specimen used in the analysis are shown in Fig. 2. Test results for the specimens A - C where available.

Only one half of the specimen is analyzed i.e. a symmetric failure mode is forced. In the analysis four-node plane stress finite elements are used. The finite element meshes used in the analysis are shown in Fig. 3. Specimen A - C are analyzed using four integration points and specimen D is calculated by employing nine integration points.

The material parameters used in the analysis are chosen such that the uniaxial compression and tensile strength was about the same as in the experiment, (initial Young moduli E^0= 40000 MPa, Poisson's ratio ν= 0.18), and microplane model parameters taken such that the

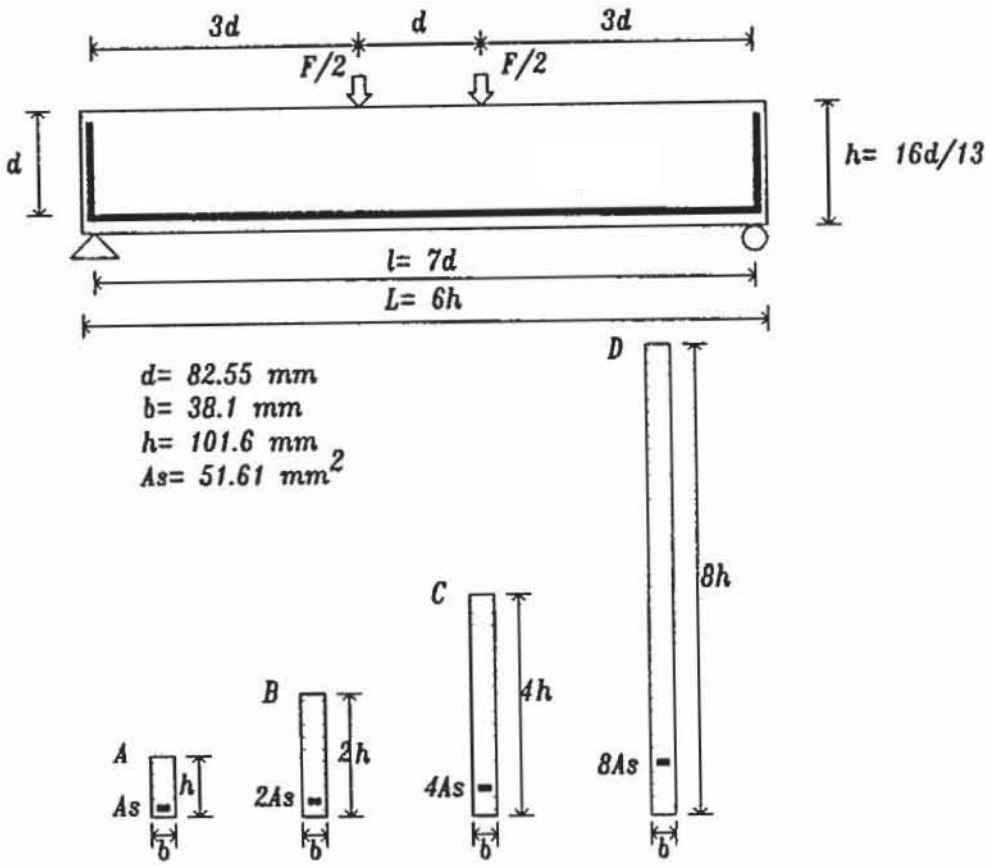

Fig. 2 Specimen geometry used in the analysis

calculated uniaxial concrete compression, and tensile strengths are f_c = 46 MPa and f_t = 2.9 MPa respectively. These values for the f_t and f_c are calculated using one finite element in a plane stress situation. The characteristic length of the non-local continuum was determined by fitting the average test results for the specimen B. It should be noted that this length, in the zone of interest, should not be smaller than at least two times the smallest finite element size since otherwise the analysis is equivalent to the classical local continuum analysis. In the present size effect study, the characteristic length was fixed as l_c = 40 mm. However, in the case of the smallest specimen (d= 82.55 mm) the characteristic length has been taken as l_c = 25 mm, because for l_c = 40 mm the strains over the half the beam depth would be averaged, what is not realistic.

Reinforced steel bars are modeled by introducing a corresponding smeared reinforcement area in the second row of the finite elements (see Fig. 3). The behavior of steel is taken as linear elastic, with an elasticity modulus E^s= 210000 MPa. It is assumed that there is no slip between concrete and steel. In the experiments this was assured by extending the steel bars at the beam ends over the beam depth (see Fig. 2).

The present numerical results are obtained by prescribing a force at the top of the specimen, i.e. by load control.

3.2 Results of the analysis

Load-displacement curves for four different specimen sizes are plotted

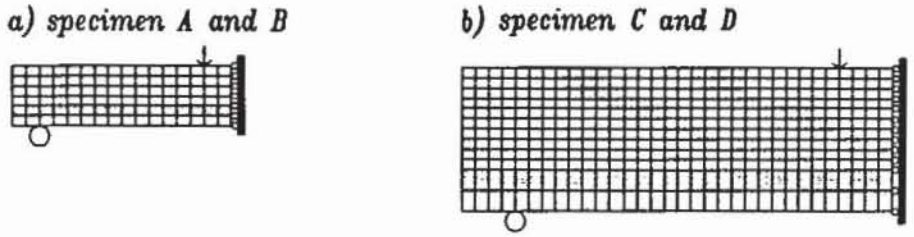

Fig. 3 Finite element meshes used in the analysis

in Fig. 4. Basically, the shape of the curves is bilinear. The first part is almost linear elastic up to the point where the first vertical bending crack is opening, following by the second, globally almost straight, but less steep line up to failure. The first part of the second line is consisting of a few local peaks that are due to the formation of the further bending cracks. The descending branch of the load-displacement curve could not be calculated probably because of the snap-back.

The maximum principle deformations of the specimen size C before failure are plotted in Fig. 5. Rectangulars plotted in this figure are proportional to the magnitude of the max. principle strains. The larger side of each rectangle is plotted in the direction perpendicular to the direction of the maximum principle strains. Fig. 5 indcates alarge tensile deformations at the compression part of the beam, in the direction perpendicular to the beam axis (horizontal cracking). Due to this, the concrete tensile strength is reduced and as a consequence, a critical diagonal crack, running from the bottom of the beam and approaching the compression zone at an angle (measured from the beam center line) of about $\alpha = 30^{\circ}$, causing brittle failure. Since before failure the critical crack is opening, there is almost no shear stress along the crack and failure is due to Mode I.

In Fig. 6 the nominal stresses at failure $\sigma_N = F_U/2bd$ (F_U = peak load), obtained in the numerical analysis and in the experiments (average values) are compared with the size effect law proposed by Bazant (1985):

$$\sigma_N = B\, f_t (1 + \beta)^{-1/2}, \qquad \beta = d/d_0 \tag{1}$$

The optimum values for the parameters B and d_0 are obtained by a linear regression analysis of the experimental results. It can be seen that the agreement between the results of the numerical analysis and the test data is good and that the failure loads must be described by the size effect law, i.e. the nominal stresses at peak load decrease with increasing specimen size. As expected, due to the fact that the failure type is brittle, the size effect is close to linear elastic fracture mechanics.

Bazant size effect law and size effect law proposed by Walraven and Lehwalter, (1990) are compared in Fig. 7. Values of the shear strengths are plotted relative to the beam depth d= 200 mm. Differences between these two predictions are large if the smaller beam depth is considered. This is probably due to the fact that the Walraven and Lehwalter (1990) proposed size effect law for the concrete mix with the maximum aggregate size between 8 and 32 mm.

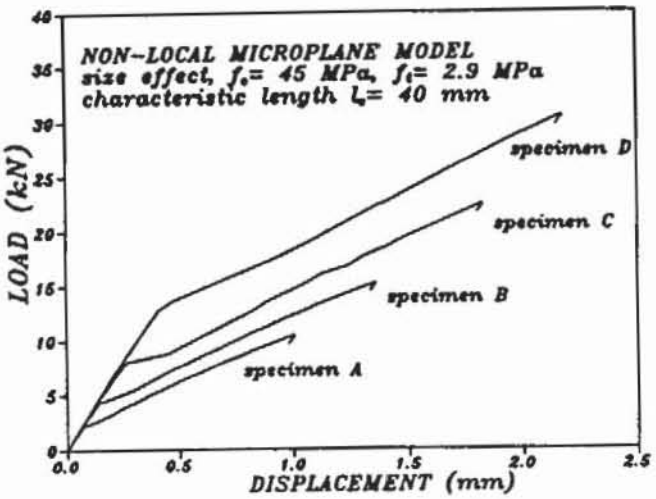

Fig. 4 Load-displacement curves for different specimen sizes

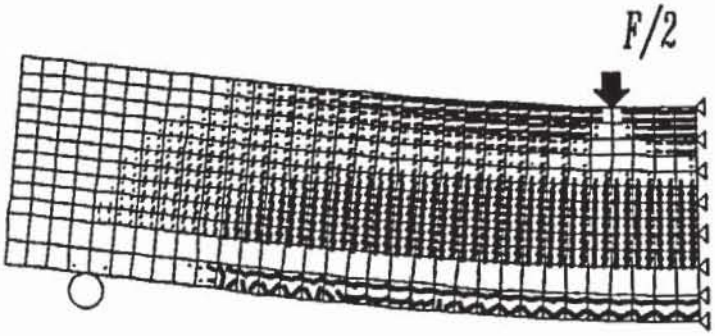

Fig. 5 Maximal principle deformations at peak load

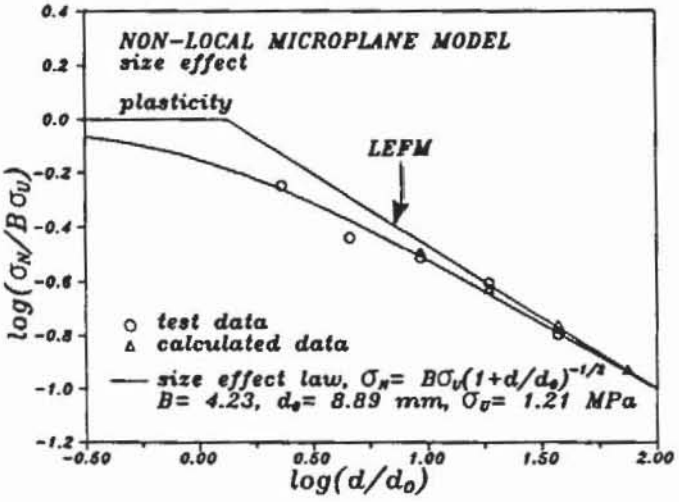

Fig. 6 Comparison between calculated and test data and size effect law

However, Bazant size effect law for the present example is calculated using the specimen with maximum aggregate size of 1.4 mm. Due to this,

prediction employing Bazant size effect law is more close to linear elastic fracture mechanics.

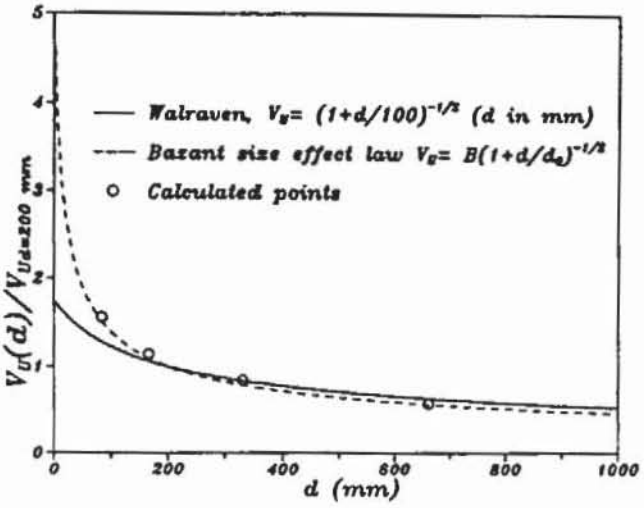

Fig. 7 Comparison between different size effect predictions

4 Objectivity and stability of the numerical analysis

Currently, failure simulations of reinforced concrete structures is often performed using the smeared crack approach and the crack band theory. However, studies by Rots (1988), indicate that in many cases results can be inobjective and depend on the finite element choice. In the analysis of the reinforced concrete beams without shear reinforcement using microplane model and local continuum the same problem arised.

In order to demonstrate the inobjectivity of the local analysis and make it more clear, specimen C has been analyzed using two different meshes. The same type of the mesh as shown in Fig. 3 have been used. The first mesh was relatively coarse, with 112 finite elements, and in a second mesh the number of elements was doubled, i.e. the size of the finite elements has been decreased by the factor of two. The resulting load-displacement curves are plotted in Fig. 8. This figure clearly demonstrates large mesh sensitivity. Namely, using a finer mesh, the failure load is much higher than in the case of a coarse mesh. Furthermore, if a coarse mesh is used, failure is due to diagonal shear, while the finer mesh specimen exhibits arc action and the calculated failure is due to failure of concrete in compression. It is interesting that refining the mesh, the real failure mode is somehow overridden by the arch action. For load up to approximately the failure load that is observed for the coarse mesh specimen, the convergency is relatively bad for both meshes. After that, for the fine mesh specimen suddenly the convergency becomes much better since a more stable mode is activated. It seems that one point exists (stability point), after which the diagonal shear failure mode is switched into a more stable failure mode - arch action. The same problem arises when in the coarse mesh instead of four integration points nine integration points is used.

To check this assumption, a stability analysis of the finer mesh

specimen is carried out. Namely, the determinant of the symmetric tangent stiffness matrix, K_t^*, is calculated. The matrix K_t^* is calculated by symmetrization of the nonsymmetric tangent stiffness matrix K_t, obtained employing the microplane model, as follows:

$$K_t^* = \frac{1}{2} (K_t + K_t^T) \tag{2}$$

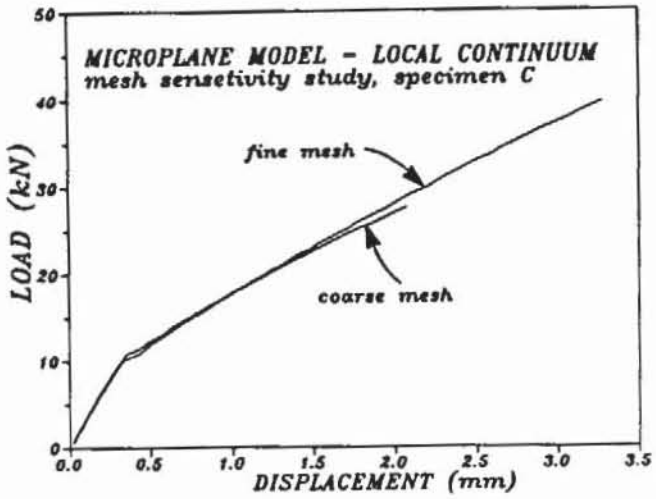

Fig. 8 Finite element mesh sensitivity in local analysis

The matrix K_t^T is transpose of the matrix K_t. Up to the load approximately equal to the failure load that is due to the coarse mesh specimen, $\det|K_t^*| > 0$, and $\det|K_t^*| < 0$ after that point. Since the determinant of the tangent stiffens matrix K_t^* changed the sign, it is clear that a bifurcation point is passed somewhere between these two calculations (de Borst, 1987). As shown by Bazant (1988), the path that occurs after the bifurcation point must minimize the second order work $\Delta W = \delta f \, \delta u/2$ where δf is the increment of the prescribed force and δu the calculated displacement increment. It is evident that in the present example the path taken does not minimize the second order work since the analysis indicates that an arch action is activated which is not the most critical path. This clearly shows that in order to detect the bifurcation (failure) point in the local continuum analysis, a stability analysis should be carried out.

A similar problem is present in the local analysis if different loading paths or different convergence criteria (Kazic, 1990) are prescribed. To demonstrate the influence of a loading path, in Fig. 9 load-displacement curves for the specimen size B, using the finite element mesh shown in Fig. 3a, are plotted. In the employed local continuum analysis, the calculated failure load is much higher if displacement control is used rather than load control. In contrast, using a non-local continuum, calculated failure loads are almost the same for both cases.

In many practical applications, where fracture of the structure is due to fracture in tension, local continuum analysis based on the crack band approach can correctly predict failure load. However, in the case of diagonal shear failure, where failure is due to failure in tension-compression, crack band concept can not objectively predict failure load. Reason is probably due to the fact that using this

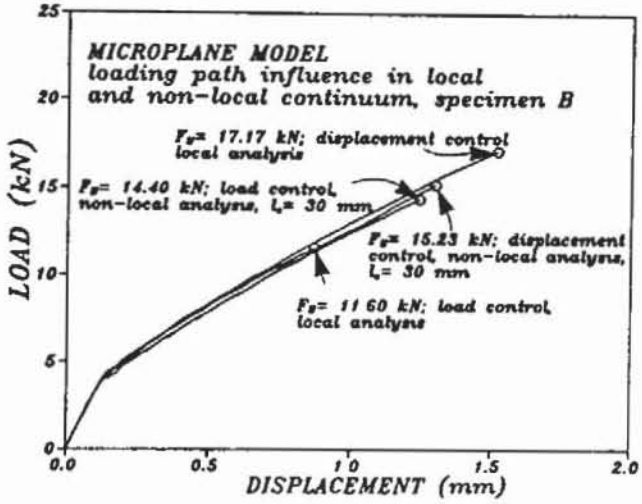

Fig. 9 Load path sensitivity

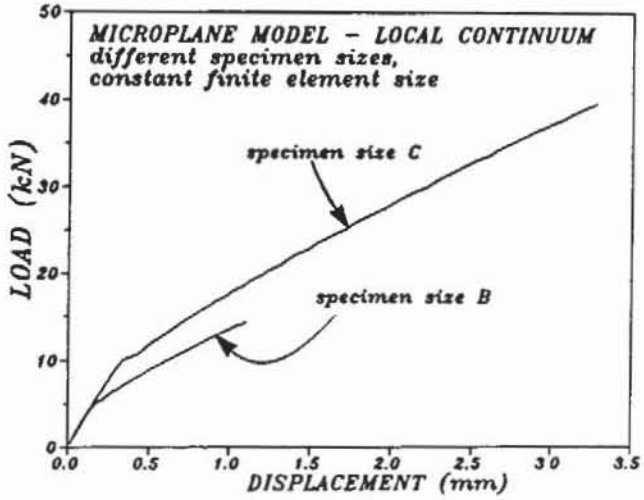

Fig. 10 Inobjectivity with respect of the specimen size

concept there is no interaction between a tension and a compressive part of the structure, i.e. there is no interaction over a distance. To demonstrate this, specimen of two different sizes (Specimen B and C, Fig. 2) are analyzed. In both cases the size of the finite elements were constant and the same, so that according to the crack band approach σ-ε curves are constant. Since the increase of the size is by a factor of two it would be expected that the failure load for the larger specimen is maximum two times larger (no size effect) or approximately 1.4 times larger with the size effect, than the failure load of the smaller specimen. However, the results show (see Fig. 10) that the failure load for the larger specimen is more than two times larger, due to the fact that in the case of the larger specimen instead of a diagonal shear failure, an arc action is formed. This indicates that a local analysis, even if it is used with the crack band approach, can not objectively predict diagonal shear failure.

It is interesting that in a local analysis, with increasing mesh refinement the failure loads increase. Further more, the failure mode is changed from a diagonal shear failure, to failure of the concrete in compression, caused by an arc action. This is in contradiction with

experience from the local continuum analysis of the concrete structure where mesh refinement usually lead to strain localization and decrease of a failure loads. However, in present case mesh refinement increase the possibility that the most critical failure mode is override by another one, that is more stable. To detect a correct failure mode in the case of a diagonal shear using a local continuum approach, stability analysis should be carried out.

The demonstrated drawbacks of the local continuum analysis can be overcome if a non-local continuum approach is employed. Namely, in all analyzed examples where the non-local microplane model is used the calculated failure mode was of diagonal shear type. However, it is important that the finite element mesh is fine enough so that the characteristic length is at least two times the smallest size of the finite element. Otherwise, the continuum is too much local and the same problems, as demonstrated in the local analysis, may arise.

5 Influence of the characteristic length

In the non-local type of the finite element analysis the key parameter is the characteristic length, that define the volume of the material over which the strains are averaged. It is still not clear, whether this length is a material parameter or depends also on the geometry of the structure and the stress-strain state.

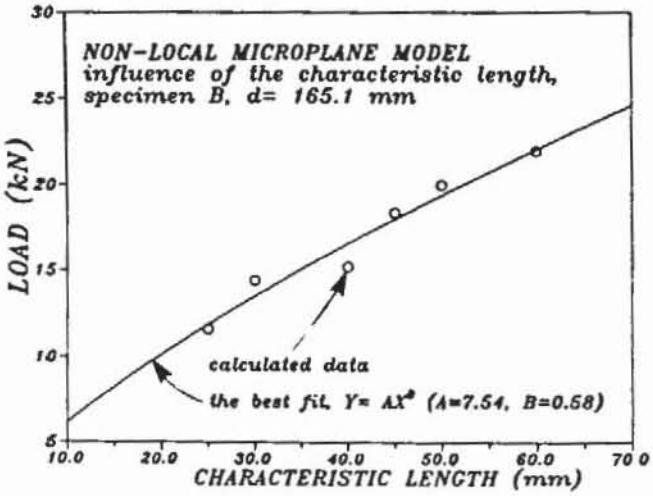

Fig. 11 Influence of the characteristic length

Here, the influence of the characteristic length on the diagonal shear failure load, of reinforced concrete beams, without shear reinforcement, is demonstrated. The study is performed using the specimen size B and the finite element mesh shown in Fig. 3a (four integration points). Geometry, finite element mesh and microplane material parameters are fixed. Six analysis are carried out with l_c = 20, 30, 40, 45, 50 and 60 mm. The smallest l_c value is about the finite element size. The largest characteristic length is approximately 1/3 of the beam depth. The load-displacement curves of all analyzed beams are similar and all fail in diagonal tension. However, the failure load increased with increasing characteristic

length (Fig. 11). Comparison between the calculated data and the average test data indicate that the optimum value of l_c, for the present case, is between 40 and 45 mm.

Since Fig. 11 clearly indicates a significant influence of the characteristic length on the failure load, further studies are needed to investigate the correlation between basic concrete properties i.e. tensile strength, fracture energy, specimen geometry and strain state and this length. However, presently the characteristic length should be chosen such that together with the assumed stress-strain relations the fracture of concrete structures is correctly prescribed. As already mentioned, the size of the l_c should be at least two times the smallest finite element size. On the other side, l_c should not be to large relative to the specimen geometry, since in such a case the strain field from the bottom to the top of the beam is averaged, what is not realistic.

6 Conclusions

A analysis of reinforced concrete beams without shear reinforcement, using the non-local microplane model, indicates significant size effect. Comparison of calculated failure loads with test data and size effect law demonstrate good agreement.

For all analyzed specimen sizes, observed failure was of diagonal shear type, unexpected and brittle. Failure is due to failure of concrete in tension-compression.

Results of the finite element analysis are inobjective if local continuum approach is used. Large mesh sensitivity and load path dependency is observed. As a consequence of a local continuum, the most critical failure mode can be easily overridden by a more stable one. It is observed that local analysis, even if the crack band approach is employed, can not objectively predict the failure. In order to detect the real failure mode in local continuum finite element analysis, stability analysis should be carried out.

Using the non-local finite element analysis with characteristic length that is at least two times larger than the smallest finite element size, the critical failure mode was always obtained. However, as expected, significant influence of the characteristic length on the calculated results is demonstrated. Further studies are needed in order to clarify the relation between characteristic length and concrete fracture parameters, specimen geometry and stress-strain state.

7 References

Bazant, Z.P., Size Effect in Blunt Fracture: Concrete, Rock,Metal. J. of Eng. Mechanics, ASCE, 110, No. 4, Apr. 1984, pp.518-535.

Bazant, Z.P., and Sun H.-H., Size effect in Diagonal Shear Failure: Influence of Aggregate Size and Stirrups, ACI Material Journal, V. 84, No. 4, 1987, pp. 259-272.

Bazant, Z.P, and Kazemi, M.T., Size effect on diagonal shear failure of beams without stirrups. Internal Report, Center for Advanced

Cement-Based Materials, Northwestern University.
Bazant, Z.P., and Gambarova, P.G., Crack shear in concrete: crack band microplane model. J. Struc. Eng., ASCE,110(9), pp. 2015-2035, 1984.
Bazant, Z.P., Microplane model for strain-controlled inelastic behavior, Mechanics of Engineering Materials, C.S. Desai and R.H. Gallager, eds., John Wiley & Sons, Chichester and New York, chap. 4, 1984, pp. 45-59.
Bazant, Z.P., and Prat, P.C., Microplane model for brittle-plastic material - parts I and II. J. of Eng. Mech., ASCE, 114(10), pp. 1672-1702, 1988.
Bazant, Z.P., and Pijaudier-Cabot, G., Measurement of Characteristic Length of Nonlocal Continuum. J. of Eng. Mech, ASCE, 115 (4), 1989, pp. 755-767.
Bazant, Z.P., and Cedolin, L., Stability of structures: elastic, inelastic, fracture and damage theories and codes, Oxford University Press, New York, 1990 (in press).
Bazant, Z.P., and Ozbolt, J., Nonlocal microplane model for fracture, damage and size effect in structures. J. of Eng. Mech., ASCE, 116, (11), 1990, pp. 2485-2505.
R. de Borst, Stability and Uniqueness in Numerical Modeling of Concrete Structures, Colloquium on Computational Mechanics of Concrete Structures, IABSE, August 1987, pp. 161-176.
Eringen, A.C., and Edelen, D.G.D., On non-local elasticity. Int. J. of Eng. Sci. 10, pp 233-248, 1972.
Kani, G.N., How Safe are Our Large Concrete Beams ?. ACI Journal, No. 64-12, March 1967, pp. 128-141.
Kazic, M. and Eligehausen, R., Influence of Anchoring Technique on Shear Strength of Reinforced Concrete Beams. Report in Preparation, Stuttgart University, Institut für Werkstoffe in Bauwesen, December, 1990.
Kröner, E., Interrelations between various branches of continuum mechanics. Mech. of Generalized Continuua, E. Kröner, ed., Springer, W.Berlin, pp. 330-340, 1968.
Krumhansl, J.A., Some considerations of the relations between solid state physics and generalysed continuum mechanics. Mech. of Generalized Continuum, E. Kröner, Ed., Springer, W.Berlin, pp. 298-331, 1968.
Levin, V.M., The relation between mathematical expectation of stress and strain tensor in elastic micro-heterogeneous media. Prikl. Mat. Mehk. 35, pp. 694-701, 1971 (in Russian).
Pijaudier-Cabot, G., and Bazant, Z.P., Nonlocal damage theory. J. of Eng. Mech., ASCE, 113, pp.1512-1533, 1987.
Rots, J.G., Computational modeling of concrete fracture. Thesis presented to Delft University of Technology, at Delft, The Netherlands, September 1988.
Taylor, G.I., Plastic strain in metals. J. of Inst. of Metals, Vol. 62, pp. 307-324, 1938.
Walraven, J., Scale Effect in Beams with Unreinforced Webs, Loaded in Shear. Annual Report, Delft University of Technology, Delft, The Netherlands, V.1, 1990, pp. 103-112.
Walraven, J. and Lehwalter, N., Einfluss des Masstabs in schubbeanspruchten Bauteilen ohne Schubbewerung. Beton und Stahlbetonbau 85, 1990, H. 9, pp. 228-232.