# Sound Bridge Localization in Buildings by Structure-borne Sound Intensity Measurements* 

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#### Abstract

Sound Bridge Localization in Buildings by Structure-borne Sound Intensity Measurements Summary

A localization procedure is described which permits fast and accurate localization of point-like sound bridges in buildings by means of measurements of the vibrational intensity at a few points on the wall or floor. In a laboratory experiment a resolution of about 10 cm was achieved with seven points. Essential to the performance of the procedure are a favourable choice of the positions of the measurement points, a suitable weighting of the measured data, and a favourable choice of the band for the frequency average.


## Lokalisation von Schallbrücken in Gebäuden

 durch Messung der Körperschallintensität ZusammenfassungEs wird ein Meßverfahren vorgestellt, das eine schnelle und genaue Lokalisierung von punktförmigen Schallbrücken in Gebäuden erlaubt. Gemessen wird die Körperschallintensität an wenigen Punkten auf der Wand
oder dem Fußboden. In einem Laborversuch wurde mit sieben Punkten eine Genauigkeit von ungefähr 10 cm erreicht. Wesentlich für eine erfolgreiche Lokalisierung sind eine günstige Auswahl der Meßpunkte, eine geeignete Gewichtung der Meßergebnisse und eine günstige Wahl des Frequenzbereichs für die Mittelwertbildung.

La localisation des court-circuits acoustiques dans les immeubles au moyen de mesures de l'intensité vibratoire transmise par les structures en jeu

## Sommaire

On décrit un processus óperatoire qui permet une localisation rapide et précise des court-circuits acoustiques quasi-ponctuels qui peuvent exister dans les immeubles. Il suffit pour cela de mesurer l'intensité des ondes vibratoires en un nombre restreints de points dans les murs ou les planchers. Dans une expérience de laboratoire, on a obtenu une précision d'une dizaine de centimètres avec des mesures en sept points. Mais pour que le procédé atteigne des performances de cet ordre, il est indispensable d'avoir choisi favorablement les emplacements des points de mesurage, d'avoir opéré une pondération convenable des mesures effectuées et d'avoir bien sélectionné la bande de fréquences.

## 1. Introduction

Sound insulation in buildings is frequently impaired by sound bridges between the two parts of a double wall or between floating floor and structural floor. The removal of such deficiencies is usually quite expensive, unless the position of a sound bridge can be detected with reasonable accuracy. However, an effective, i.e. fast and accurate method of localization had been missing so far.

It is true that numerous measurements of the struc-ture-borne sound level at closely spaced points on the wall or the floor provide sufficient accuracy in many cases, but this procedure is very tedious. In search of a faster method we examined the practicability of lo-

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calization by means of structure-borne sound intensity measurements. The experimental results obtained from a double wall with sound bridges are promising with respect to both accuracy and speed.


## 2. Measurement of structure-borne sound intensity

Since the pioneering work of Noiseux [1] the problem of measuring the intensity of waves in beams and plates has been treated by various authors [2-6]. Sound bridges in walls and floors generate mainly bending waves, the intensity of which can be measured by two accelerometers:

$$
\begin{equation*}
I_{x} \approx-\frac{\sqrt{B \varrho h}}{\omega^{2} d} \operatorname{Im}\left\{\hat{a}_{2} \hat{a}_{1}^{*}\right\} \tag{1}
\end{equation*}
$$

The $x$-component $I_{x}$ of the bending wave intensity $I$ is proportional to the imaginary part of the cross spectrum of the accelerations $a_{1 / 2}=\hat{a}_{1 / 2} \exp (-\mathrm{i} \omega t)$, which are measured a distance $d$ apart from each other along the $x$-direction ( $B=$ bending stiffness, $\varrho=$ mass density, $h=$ thickness of the plate, positive $x$-direction from accelerometer 1 to 2). Eq. (1) is valid
in the limit of thin plates, where the bending wavelength $\lambda$,

$$
\begin{equation*}
\lambda=\sqrt[4]{\frac{E h^{2}}{3 \varrho\left(1-\sigma^{2}\right)}} \sqrt{\frac{\pi}{f}}, \tag{2}
\end{equation*}
$$

is longer than six times the thickness of the plate ( $E=$ Young's modulus, $\sigma=$ Poisson's ratio, $f=\omega / 2 \pi$ ). In addition, it is assumed that the measurement takes place outside the exponentially decaying nearfields of boundaries, inhomogeneities, and sound bridges. The latter restriction is discussed in [1]; the distance from the origins of nearfields should be at least half a wavelength. Furthermore, the distance $d$ between the accelerometers should be small compared to the wavelength $\lambda$. This distance may be optimized by minimizing the estimate of the relative error,

$$
\begin{equation*}
\frac{\varepsilon}{k d}+\frac{1}{6}(k d)^{2} \tag{3}
\end{equation*}
$$

where $\varepsilon$ is the phase mismatch between the two measuring channels ( $k=2 \pi / \lambda$ ).

This estimate has been obtained for a plane wave ( $k d \gg 1$ ) propagating along the direction defined by the two accelerometers. The first term in (3) follows directly from eq. (1) while the second is due to a finite difference approximation in the derivation of eq. (1) [ 6,7$]$.

The deviation of the measured intensity direction $\left(\arctan \left(I_{y} / I_{x}\right)\right)$ from the true one can be shown to be of the order of $\varepsilon / k d$. For a selected pair of accelerometers with typically $\varepsilon \approx 0.01$ the relative error is minimized at $k d \approx 0.3(5 \%)$ resulting in a directional deviation of a few degrees.

For sound bridge localization we primarily need the direction of the intensity vectors in order to answer the question where the sound energy comes from. Therefore, the restriction to long wavelengths ( $\lambda>6 h$ ) need not be observed rigorously, because we may assume that the measured direction ( $k d<1$ understood) is still close to the direction of the actual intensity, even if its absolute value is no longer correctly described by (1).

The real part of the cross spectrum, which of course is measured simultaneously with the imaginary part, approaches the vibrational amplitude squared for $\boldsymbol{k d} \rightarrow 0$. In order to get an informative picture of the vibrational state of a plate the real part should be exploited, too, and displayed graphically together with the imaginary part. The vibrational level is represented by a circle, the radius of which is proportional to the real part of the cross spectrum, and the vibrational intensity by an arrow, the length of which is proportional to the (negative) imaginary part of the cross spectrum multiplied by $\cot (k d)$. With this choice the radius of the circle will be equal to the length of the
arrow, if only a single plane wave is present in the plate. It is shown in the appendix that (under certain conditions) the arrow can never cross the circle.

## 3. Localization procedure

The location of a point source in an infinite homogeneous isotropic plate is given by the intersection of the intensity directions at two arbitrary points. In the case of finite plates reflections of the vibrational intensity at boundaries or other inhomogeneities cause deviations from the radial propagation pattern around the point source in the infinite plate, especially far from the source. Hence, intensity measurements at only two points are no longer sufficient. One has to measure at several points instead, calculate all intersections, and estimate the location of the source by appropriate averaging. The effectiveness of such a localization procedure is mainly determined by
(i) the choice of measurement points,
(ii) the weighting of the intersections points,
(iii) the frequency average.

For the experiment described in the next section we adopted the following guidelines:
(i) Measure the intensity at four points regularly distributed over the wall or the floor. Usually these four vectors will lead to a preliminary estimate of the number and locations of sound bridges. Additional measurement points should then be selected around the supposed locations of the sound bridges at a distance of roughly half a wavelength.
(ii) The intersection points are classified according to Fig. 1. An intersection point is called a source, if both intensity arrows point away from it, and a sink, if both arrows point towards it. At an intersection point of the third type energy does not either enter or leave the wall: It just passes along a curved fluxline.


Fig. 1. Classification of intersection points.

Since these fluxlines are frequently closed curves indicating a circulation of energy (see Fig. 5), this type of intersection point is called a "circulation point". Points of this kind are discarded, because they are not very useful for localization purposes. Only source and sink points are to be dealt with as follows.

For each source (or sink) associated with the intensities $I_{i}$ and $I_{j}$ at the measurement points $i$ and $j$ the weight

$$
\begin{equation*}
\frac{\left|I_{i}\right|\left|I_{j}\right|}{d_{i} d_{j}}\left|\sin \alpha_{i j}\right| \tag{4}
\end{equation*}
$$

is used to calculate an average source (or sink) location ( $d_{i}=$ distance from point $i$ to intersection, $\alpha_{i j}=$ angle between $I_{i}$ and $I_{j}$ ). The weighting factor (4) may become plausible because of the following arguments: With a fixed uncertainty of the direction of $\boldsymbol{I}_{i}$ the spatial uncertainty of the intersection is proportional to the distance $d_{i}$; additionally it is dependent on the angle between $\boldsymbol{I}_{\boldsymbol{i}}$ and $\boldsymbol{I}_{\boldsymbol{j}}$. (For coinciding directions $I_{i}$ and $I_{j}$ intersection can be defined based on $\left|I_{i}\right|$ and $\left|I_{j}\right|$; the sine-factor in (4) should then be omitted.) Reflected intensity changes the direction of the intensity coming directly from the sound bridge. With decreasing distance from the location of the sound bridge the direct intensity becomes more and more dominant over the reflected intensity and the deflection of the total intensity from the direct one becomes smaller. Therefore the weight (4) is taken proportional to the magnitudes of the intensities. For the final location of a sound bridge, however, all measurements at points far from it should be discarded, because the reflected intensity may dominate there and worsen the accuracy of localization. The weighted average of the distances of all sources (or sinks) from the average source (or sink) characterizes the spatial uncertainty of the average location.
(iii) Intensity measurements at single frequencies are not very suitable for localization purposes, because reflections are virtually always present and disturbing. Fortunately, the effect of reflections can be suppressed, at least to some extent, by frequency averages. The frequency range most favourable is at medium frequencies with numerous moderately excited resonances. Strongly excited resonances should be avoided, because the antinodes act as strong sinks of vibrational energy and the intensity from the sound bridge is deflected towards them. Moreover, the frequency should be neither too low (nearfields) nor too high (higher modes, local resonances of bricks in masonry walls etc.). As mentioned in section 2, validity of simple bending-wave theory is not essential for localization purposes.

## 4. Experiment

A double wall made of solid lime-sand bricks was built in our wall testing facility without flanking transmission. The wall was approximately 4 m wide and 3 m high, and contained two sound bridges (diameter 4 cm ), which could be switched on and off by means of screw spindles. The two leaves of the wall were 115 mm and 175 mm thick respectively, and separated by mineral fibre panels ( 5 cm thick). The outer faces were plastered ( 5 mm thick). The weighted sound insulation index $R_{\mathrm{w}}$ of 79 dB was reduced by a sound bridge to about 70 dB .

For the localization procedure the following equipment was used: The thinner leaf of the double wall was excited by a heavy shaker (RMS/SW 122) supplied with the power amplifier RMS/TGA 251. Vibration was measured on the thicker leaf by two accelerometers ( $B \& K 4370$, distance $\approx 5 \mathrm{~cm}$ ), two preamplifiers ( $\mathrm{B} \& \mathrm{~K} 2635$ ) and a dual channel analyzer (HP 3562 A ) in stepped-sine mode. The measured cross spectra were stored on disc and evaluated by a Pascal program on a desk top computer (HP 9000/216).

For the graphical representation of the results according to section 2 an estimate of the bending wavelength is needed. Assuming $E=8 \mathrm{GPa}, \sigma=0.3$, $h=0.175 \mathrm{~m}, \varrho=1800 \mathrm{~kg} \mathrm{~m}^{-3}$ yields $\lambda=0.84 \mathrm{~m}$ at 1000 Hz . Fig. 2 shows vibrational levels (circles) and intensities (arrows) at seven points on the wall, which is predominantly excited by the indicated sound bridge. The average source location as determined from the five intensities on the right hand side is only 8 cm away from the actual sound bridge location. The choice of the frequency average (from 500 Hz to 2000 Hz ) is essential to this result. If the vibrations are averaged over a wider band (from 90 Hz to 2500 Hz )


Fig. 2. Vibrational levels (circles) and intensities (arrows) on a leaf of the double wall excited by a sound bridge ( $\odot$ ) (frequency average from 500 Hz to 2000 Hz ).


Fig. 3. Vibrational levels (circles) and intensities (arrows) on a leaf of the double wall excited by a sound bridge ( $\bullet$ ) (frequency average from 90 Hz to 2500 Hz ).


Fig. 4. Vibrational levels (circles) and intensities (arrows) on a leaf of the double wall excited by a sound bridges ( $\odot$ ) (frequency average from 900 Hz to 2000 Hz ).


Fig. 5. Vibrational levels (circles) and intensities (arrows) on a leaf of the double wall without sound bridges (frequency average from 900 Hz to 2000 Hz ).
localization is clearly worse (Fig. 3): Strongly excited resonances dominate and lead to high vibrational levels and to relatively small intensities. (If one were to localize the sound bridge exclusively on the basis of these vibrational levels, one would guess that it should be near the maximum in the quadrant at the lower left!)

Under favourable circumstances it is possible to infer the existence of more than one sound bridge from only a few measurement points. Fig. 4 clearly indicates a sound bridge somewhere near the left edge of the wall, but only the assumption of an additional sound bridge can explain the almost vanishing intensity at the centre pointing to the left and the relatively high intensity at the upper right. The sound bridge near the left edge (without the sound bridge near the centre) could be localized within 12 cm with seven measurement points, although one would have had to suspect a result less accurate because of nearfields caused by the left edge of the wall.

Finally, Fig. 5 shows the vibrational behaviour of the wall without sound bridges. There is no convincing indication of a sound bridge; the vibrational energy is more or less circulating. The vibrational levels close to the edges of the wall permit a crude characterization of the boundary conditions: The wall is not rigidly clamped. The edges, in particular the upper one, may be regarded as sound sources because of a residual flanking transmission at high frequencies.

## 5. Conclusions

Sound bridges in buildings can be effectively localized by means of structure-borne sound intensity measurements. In a laboratory experiment an accuracy of about 10 cm was achieved by intensity measurements at only seven points on a double wall. Disturbing effects of reflections at boundaries and of inhomogeneities can be suppressed to a considerable extent by a favourable choice of the positions of the measurement points, a suitable weighting of the measured data, and a favourable choice of the frequency average. The simultaneous graphical representation of vibrational level and intensity greatly helps to obtain a correct interpretation of the vibrational state. The localization procedure described in this paper is superior - with respect to both accuracy and speed - to numerous measurements of vibrational levels only. Even though the sound bridges tested here were essentially point-like, we expect the procedure to perform effectively in the case of more extended sound bridges as well. A detailed account of our investigations is available on request [7].

## Appendix

A general, single-frequency field of propagating bending waves may be described by the amplitude

$$
\begin{equation*}
a=\int_{-\pi}^{\pi} A(\vartheta) \exp (\mathrm{i} \boldsymbol{k} \cdot \boldsymbol{r}) \mathrm{d} \vartheta \tag{A1}
\end{equation*}
$$

where $k$ is the (real) wave-vector of a plane wave travelling in $\vartheta$-direction with amplitude $A(\vartheta)$. If the accelerometers measure $a_{1}$ and $a_{2}$ along the $x$-direction, the real part of the cross spectrum is given by

$$
\begin{aligned}
\operatorname{Re}\left\{\hat{a}_{2} \hat{a}_{1}^{*}\right\}_{x}= & \operatorname{Re} \iint_{-\pi}^{\pi} A(\vartheta) A^{*}\left(\vartheta^{\prime}\right) \\
& \cdot \exp \left[\mathrm{i} k d\left(\cos \vartheta+\cos \vartheta^{\prime}\right) / 2\right] \mathrm{d} \vartheta \mathrm{~d} \vartheta^{\prime} \\
\approx & \operatorname{Re} \iint_{-\pi}^{\pi} A(\vartheta) A^{*}\left(\vartheta^{\prime}\right) \mathrm{d} \vartheta \mathrm{~d} \vartheta^{\prime}=\left|\int_{-\pi}^{\pi} A(\vartheta) \mathrm{d} \vartheta\right|^{2}
\end{aligned}
$$

where $k d \ll 1$ has been assumed in the last step ( $k=|\boldsymbol{k}|$ ). Accordingly,

$$
\begin{align*}
& -\cot (k d) \operatorname{Im}\left\{\hat{a}_{2} \hat{a}_{1}^{*}\right\}_{x}  \tag{A3}\\
& \quad \approx \frac{1}{2} \iint_{-\pi}^{\pi} A(\vartheta) A^{*}\left(\vartheta^{\prime}\right)\left(\cos \vartheta+\cos \vartheta^{\prime}\right)
\end{align*}
$$

where also $k d \ll 1$ has been used. For the orientation of the accelerometers along the $y$-direction the cosines in (A 3) have to be replaced by sines. The squared length of the intensity arrow is therefore proportional to

$$
\begin{align*}
& \frac{1}{4} \iiint_{-\pi}^{\pi} \iint A(\vartheta) A\left(\vartheta^{\prime}\right) A^{*}\left(\vartheta^{\prime \prime}\right) A^{*}\left(\vartheta^{\prime \prime \prime}\right)\left\{\cos \left(\vartheta-\vartheta^{\prime \prime}\right)\right. \\
& \quad+\cos \left(\vartheta-\vartheta^{\prime \prime \prime}\right)+\cos \left(\vartheta^{\prime}-\vartheta^{\prime \prime}\right) \\
& \left.\quad+\cos \left(\vartheta^{\prime}-\vartheta^{\prime \prime \prime}\right)\right\} \mathrm{d} \vartheta \mathrm{~d} \vartheta^{\prime} \mathrm{d} \vartheta^{\prime \prime} \mathrm{d} \vartheta^{\prime \prime \prime} . \tag{A4}
\end{align*}
$$

This integral (which is positive, because it is a sum of squares) is certainly not greater than a reference integral in which all cosines have been replaced by 1. However, this reference integral is equal to the square
of (A 2), which is proportional to the radius of the circle representing vibrational level. Hence, if the same constant of proportionality is used, the arrow can never cross the circle. The limiting case of the arrow touching the circle is reached, if and only if the wave field reduces to a single plane wave, $A(\vartheta)=A \delta(\vartheta-\theta)$, propagating along direction $\theta$.

It is stressed that the above statements are true under the following conditions: (i) There is only one type of wave present in the homogeneous plate, namely propagating bending waves of a certain frequency; nearfields are not allowed. (ii) The accelerometer distance is small compared to the bending wavelength. (iii) Damping is negligible over the distance $d$. In practice, one is often not sure how accurately these conditions are met, because the bending wavelength may be difficult to estimate.

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