Measurement of small movements and vibrations by laser photography

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Most of the holographic techniques applied so far for non-contacting measurements of surface displacements are highly sensitive. But the interpretation of the interference fringes obtained is usually rather difficult for a non-expert, especially for the study of tilts in the presence of lateral in-plane movements. Moiré techniques could be applied to overcome some of the difficulties encountered but they require a relatively flat surface, which needs to be specially prepared. The application of speckling, a well-known phenomenon in coherent optics, was found to be very convenient for the investigation of lateral in-plane displacements and vibrations in the presence of tilts. The speckle pattern can be regarded as related to the object and hence will be displaced if the object or part of it is moved. For the analysis of in-plane movements or tilts the speckle pattern is photographed in the image plane or the Fourier transform plane of the object, respectively, before and after the movement has taken place. From the diffraction effects of the recorded speckle patterns the movement of the object is determined. Different techniques to measure rigid-body translations, tilts and surface vibrations will be demonstrated and a comparison with other methods given together with the appropriate experimental results.

1 INTRODUCTION

In recent years, highly sensitive non-contacting methods have been developed for measuring surface displacements and mechanical oscillations by hologram interferometry. The holographic techniques are very often too sensitive to disturbances and interpretation of the interference fringes is rather difficult for superimposed movements. In addition, the determination of the exact plane of localization of the interference fringes is usually not very accurate [1].

However, a different technique using the speckle phenomenon has been found to overcome some of these problems. The application of speckle patterns has many potential advantages and it leads to a very simple engineering tool. It has many practical applications and the interpretation of the results is easy.

There are basically two complementary techniques to measure lateral in-plane movements. For measurement of very small displacements the interference effect between two speckle patterns is used, as first described by Lendeertz [5].

In this technique two illuminating waves are needed to form two speckle patterns of the optically rough object surface. The resultant interference pattern is recorded before and after deformation of the surface of interest. Point-by-point subtraction of these two images may be performed photographically. It yields bright and dark fringes on the image, representing half-wavelength contours of the displacement. Alternatively by matching the spatial frequency content of the image, determined by the aperture of the lens, to the resolving power of a vidicon camera it is possible to record the interference speckle pattern in electronic form.

Furthermore, surface vibration can be observed in real time by forming an image of the surface of interest together with the coherent uniform wavefront on a vidicon, for instance. Thus parts of the surface that are stationary retain a highly speckled image, but the speckle contrast is reduced in regions where the object vibrates. These regions give lower frequencies in the camera output and may be separated from the speckled regions by a high-pass filter placed between the camera and the monitor.

For the analysis of larger in-plane movements (larger than the smallest speckle dimension in the appropriate plane) two identical, but shifted speckle patterns can be recorded in the image plane. Young's fringes result when the developed plate (or film) is illuminated with coherent light. The spacing of the cosine fringes is inversely proportional to the in-plane translation. In-plane movements can be studied even in the presence of small tilts and movements parallel to the line of sight. Tilts, however, can be analysed very accurately by recording the speckle patterns in the Fourier transform plane of the object. To measure mechanical oscillations the speckle pattern can be recorded as a time-average exposure in the corresponding plane. The fringes obtained in this way are Bessel functions of order zero and first kind.

2 PHYSICAL DESCRIPTION OF THE SPECKLE PHENOMENON

The speckled appearance occurs everywhere in space when an optically rough surface is illuminated by highly coherent light such as from a laser. The main features of the speckle patterns in the Fourier transform plane as well as in the image have been investigated recently by a number of authors [2–4]

A physical model will now be used to explain briefly the main characteristics of speckling. Fig.1 shows schematically an optically rough object illuminated by a coherent wave converging to a focus in the neighbourhood of the point E. For simplicity in the diagram, transmitted illumination is shown. The roughness is assumed to be localized on the exit surface of the object. Let us consider the irregularities to comprise a random set of positive and negative lenses. The pencils of rays falling on two such lens elements are indicated in the diagram. The rays indicated with double arrows pass undeviated to the point E. The convex lens element at O_1 focuses rays at P_1, from which they will diverge as a relatively narrow pencil giving a wave W_1. The concave lens element at O_2 produces a pencil of rays coming from the virtual image P_2 which, because of the deeper curvature, diverges as a wider-angle pencil, giving the wave W_2. In the diffraction plane the total disturbance will be the resultant of the mutual interference of the coherent waves produced by all such elements of the object. These waves will have random mutual phases because of the random variation of the optical thickness of the object.

At a point such as Q_1 there will be light from all elements of the object whereas only those elements giving wide-angle scattering will send light to a point such as Q_2 which is further from the centre E of the pattern. Since the maximum intensity that can be produced increases with the number of interfering waves the envelope of the intensity in the diffraction pattern will be expected to have greater value at Q_1 than at Q_2. Fig.2(a) shows a typical example of the speckle pattern in the Fourier transform plane of a ground glass which has a superimposed amplitude grating. In addition a rectangular aperture is limiting the object field. In Fig.2(b) an immersion liquid (alcohol $n = 1.36$) has been used to reduce the optical roughness of the surface. As a result the width of the envelope of the speckle pattern becomes smaller without altering the size and shape of the individual speckle. Furthermore, the spectra of the grating become visible. In Fig.2(c) the spatial frequency of the particular ground glass is recorded.

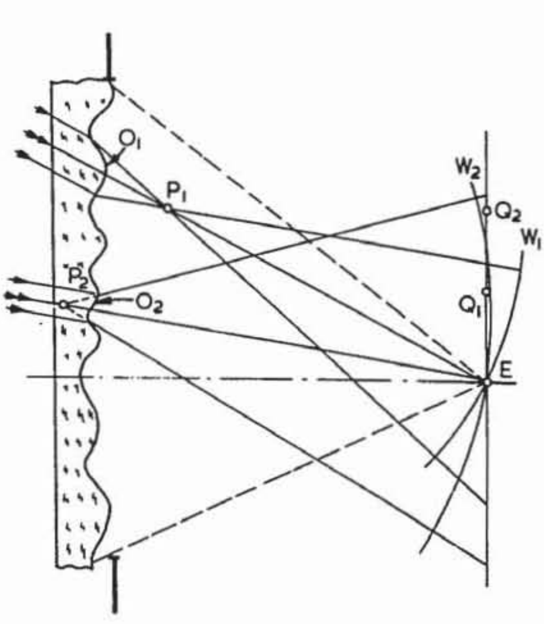

Fig.1 Waves scattered by an optically rough surface.

The smallest size of detail which can occur in the diffraction pattern is determined by the angular size of the diffracting object as seen from the observation plane. It corresponds to the two interfering waves with the greatest mutual inclination. It can be shown that the contrast of the speckles can reach unity [3].

By the image formation of a rough object illuminated with coherent light a speckled diffraction pattern occurs in the entrance pupil. This random illumination appears in the exit pupil, so that the speckling in the image is determined by considering the exit pupil to act as a rough object. Hence the smallest speckle dimension is determined by the total converging angle of the image-forming pencil.

3 MEASUREMENTS OF IN-PLANE MOVEMENTS

The application of speckle patterns to study in-plane translations and rotations has been investigated recently [5–9]. To simplify the argument let us consider the surface itself to be characterized by the coherent speckle pattern generated by illuminating it with laser light. Photographing the speckle pattern before and after the movement of the surface has taken place will lead to two practically identical, but shifted, speckle patterns from which the in-plane translation and rotation can be derived by analysing its optical diffraction effect. Young's interference fringes are obtained when the developed photographic plate is illuminated with coherent light. The fringe spacing is inversely proportional to the amplitude of the translation. A point-by-point analysis of the speckle pattern is useful when the movements are different locally. In this way the stress of a material can also be derived.

For a harmonic in-plane oscillation of the object the amplitude and orientation of the movement can be obtained by stroboscopic illumination; for example, by illuminating the object at its extreme positions. In this way, the problem is reduced to a translation measurement giving cosine fringes in the Fraunhofer diffraction pattern as previously. Alternatively, vibratory movement can be measured by a single time-average exposure of the speckle pattern (exposure over a number of oscillations). It was shown in [8] that the position probability density function of the speckles is given for $-\xi_0 < \xi < \xi_0$ as

$$P(\xi) = \frac{\mathrm{rect}(\xi/\xi_0)}{\pi\sqrt{(\xi_0^2 - \xi^2)}} \tag{1}$$

where ξ_0 = amplitude of oscillation and $P(\xi) = 0$ outside this range. This predicts the greatest likelihood of an individual speckle being at the ends of its travel. Illuminating the speckle pattern recorded as a time-average exposure leads to Bessel functions of order zero and first kind in the diffraction pattern (Fourier transformation).

The fringes obtained by illuminating the speckle patterns recorded in the image plane with coherent light are multiplied by the autocorrelation of the exit pupil of the lens system used to form an image of the object. Hence, they are seen at an angle twice as large as the exit pupil is seen at from the image plane.

Small out-of-plane movements and tilts have practically no consequence on the accuracy of the results, but could reduce the contrast of the fringes as a result of a decorrelation of the speckle patterns.

The translation $\Delta\xi$ can be found from the fringe spacing p in the Fraunhofer diffraction plane, and the amplitude of oscillation ξ_0 from the width $\frac{1}{2}p_1$ from the origin to the first minimum of the zero-order Bessel function, for instance:

$$\Delta\xi = \frac{\lambda f_1}{Mp}, \quad \xi_0 = 0.76\,\frac{\lambda f_1}{Mp_1} \qquad (2)$$

where f_1 is the focal length of the lens or the distance between the developed photographic plate and the focal plane depending on the particular arrangement for obtaining the Fraunhofer diffraction pattern. M is the lateral magnification and λ the wavelength of light.

An interesting result can be obtained when the speckle pattern of an oscillating object is recorded as a time-average exposure and has superimposed on it the speckle pattern of the stationary object. This leads to a modified Fraunhofer diffraction pattern. We obtain no longer an intensity distribution of $J_0^2(\,)$ but $|K + J_0(\,)|^2$. K is a constant depending partly on the illumination of the stationary and time-averaged recording of the speckle patterns. If $K > |K_0|$ (K_0 = amplitude of the first minimum of J_0) the minima occur for $J_0'(\,) = 0$ and $J'' > 0$. This result may sometimes be useful when larger movements are analysed with the same experimental configuration.

For the experiments the object needs to be illuminated by a plane or spherical wave from a laser. The quality of the wavefront illuminating the object is not critical. An image of the object is then formed with a lens by double exposure or as a time-averaged exposure. The developed plate is then illuminated by a spherical wave, for instance, and the fringe pattern is studied in the focal plane. For the experiments we used a HeNe-laser (λ = 632.8 nm) and mostly Agfa Scientia 8E 75 plates (and sometimes films) to record the speckle patterns.

Fig.3 illustrates the different fringe patterns obtained from the same region of a tuning fork used in an electronic watch (ν = 1050 Hz). Fig.3(a) is the result of the stroboscopic illumination (illumination at the extreme positions, ΔT = 30 μs). Cosine fringes are obtained in this way. By contrast, Fig.3(b) shows a photograph of the fringes obtained by a time-average recording of the speckle pattern. The fringes are Bessel function of order zero and first kind. Fig.3(c) shows the fringes obtained from the superimposed speckle patterns, namely from the time-average recording superimposed on the speckled image of the stationary object. A wider fringe spacing results, as expected. An F/2.8 lens working at $M = -3$ was used for these experiments. The amplitude of oscillation was $2\,\xi_0$ = 19 μm (peak/peak).

Fig.4 shows the interference fringes obtained by holographic interferometry with stroboscopic illumination of the tuning fork. A more detailed description of the tuning fork used in a new design of an electronic watch and the stroboscopic illumination is found in reference [9]. As a consequence of additional tilts occurring for different regions of the tuning fork the fringes obtained were not localized in one plane. The Fraunhofer diffraction patterns of the speckle pattern recorded as a time-average exposure in the image plane are shown for different regions of the tuning fork in Fig.5. The amplitude of oscillation changed from 19 μm to 9.6 μm (peak/peak).

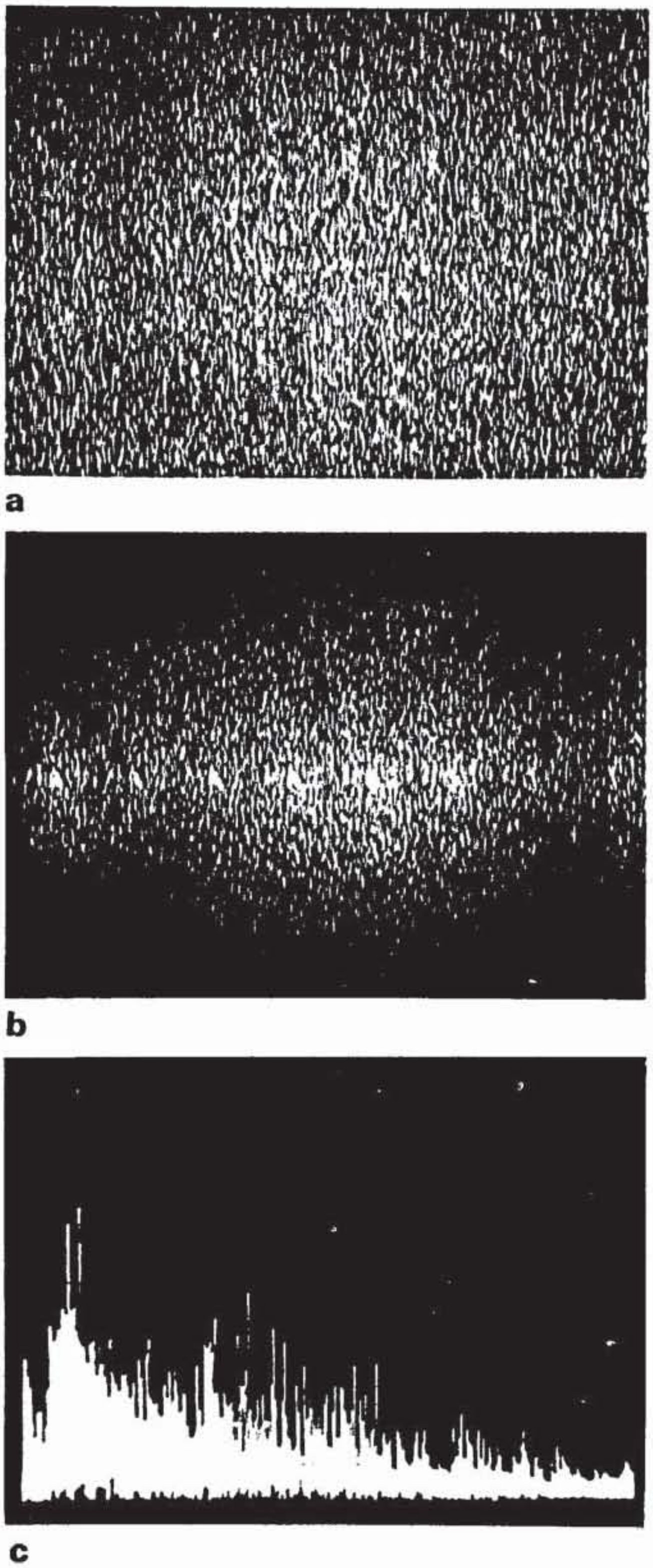

Fig.2 (a) Diffraction patterns of a ground glass with an amplitude grating superimposed on it.
(b) Diffraction pattern as in (a) but with an immersion-liquid on the rough surface (n = 1.36).
(c) Frequency spectrum of the ground glass.

The useful range for the application of the described speckle method to measure in-plane translations may be 5 μm $< \Delta\xi <$ 0.5 mm. Furthermore, rigid body displacements parallel to the line of sight ($\Delta z \leqslant$ 1 mm) have practically no

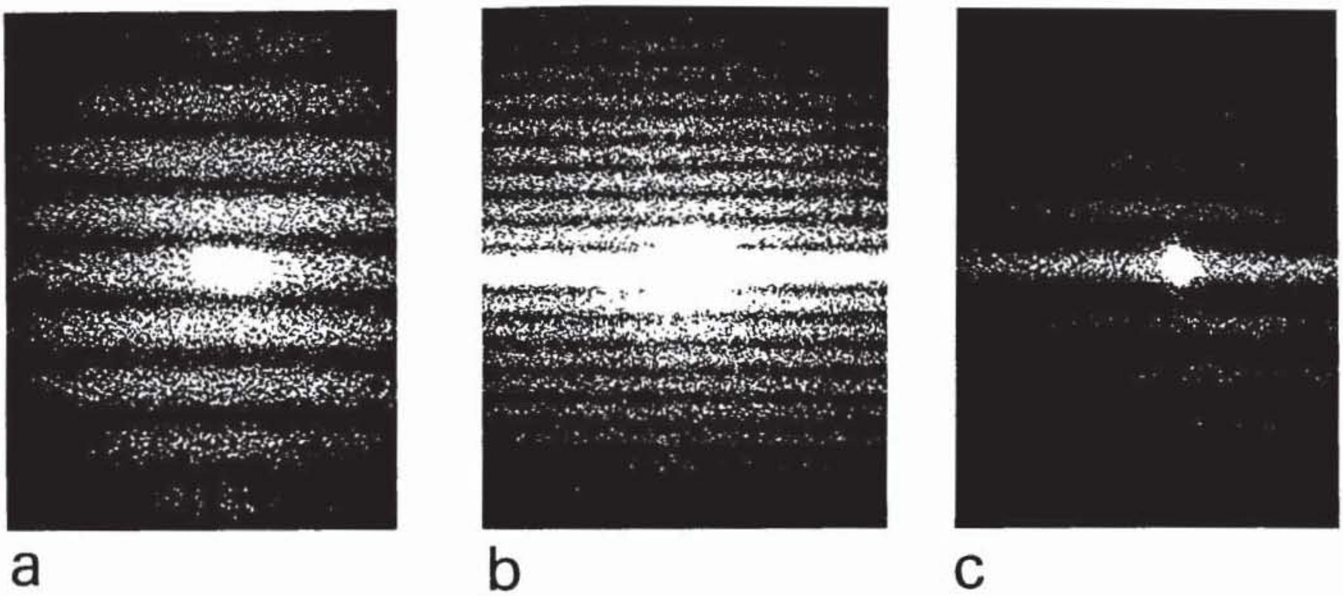

Fig.3 Comparison of the fringes obtained in the Fraunhofer diffraction patterns obtained from a mechanical in-plane oscillation ($2\xi_0 = 19\ \mu m$, peak/peak). The speckle patterns were recorded in the image plane.
(a) for a stroboscopic illumination of the object.
(b) as a time-average exposure.
(c) as a superposition of the stationary speckle pattern on that of the time-average exposure.

consequences on the in-plane measurements. A slight decrease in contrast was observed.

Small additional tilts γ ($\gamma < 1^\circ$ for an F/2.8 lens) did not disturb appreciably the contrast of the fringes obtained for in-plane translation measurements. However, a low-aperture image-forming lens led to a reduction of the contrast even for smaller tilts, especially when the in-plane translations were small. Therefore, a reasonably high-aperture lens should be used to measure in-plane translations in the presence of small tilts. In addition, the lens should be well corrected to measure in-plane translations in the presence of in-plane rotations.

Fig.4 Interference fringes of the tuning fork obtained by holographic interferometry (stroboscopic illumination).

4 MEASUREMENTS OF TILTS

The novel feature of the analysis of tilts* is that the lateral shift of the speckle pattern in the Fourier transform plane is due to a tilt in another plane, the object plane [10]. An additional small in-plane translation in the object plane introduces a linear phase change proportional to it in the Fourier transform plane of the object and has therefore no consequences since we record the intensity only.

It was shown in [10] that the original speckle pattern in the Fourier transform plane of the object described by $a(x)$ is shifted laterally as a result of a small tilt γ to $a[x - \gamma(1 + \cos\beta)]$ (Fig.6). β is the angle of incidence of the light. For $\beta = 0$ (normal incidence) the identical speckle pattern is rotated by 2γ. In this special case the optically rough surface acts like a mirror. The original and shifted speckle pattern are recorded in the Fourier transform plane by a double exposure technique (before and after a tilt) or as a time-average exposure for an oscillating object. Illuminating the developed plate (or film) yields Young's interference fringes in the Fraunhofer diffraction pattern of the plate. They are multiplied by the autocorrelation function of the illuminated object field. Cosine interference fringes are obtained for a single tilt introduced between the two exposures. Bessel functions of order zero and first kind occur by a time-average recording of an

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* It should be noted that rotations with the axis of rotation orthogonal to the line of sight are referred to as tilts.

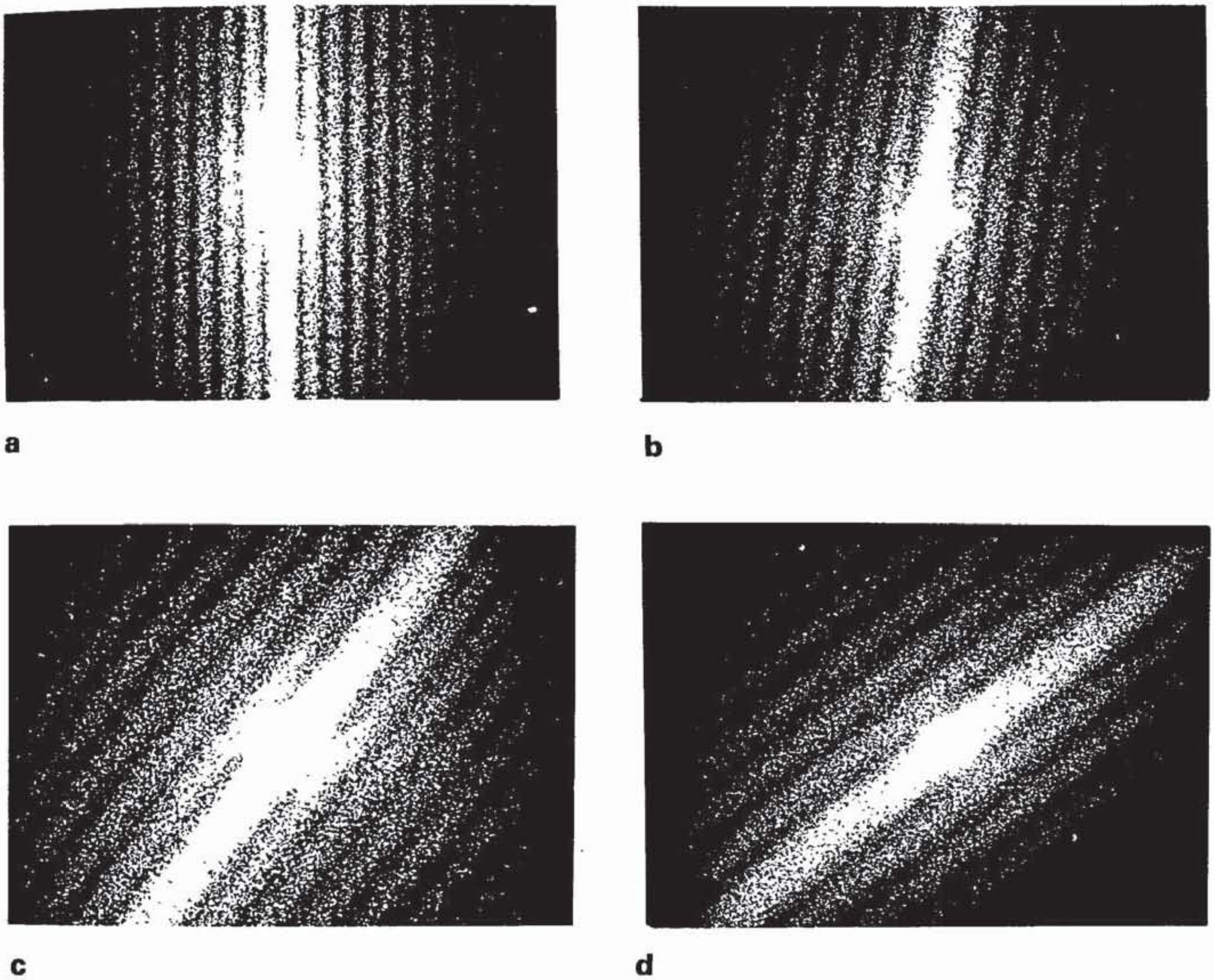

Fig.5 Fraunhofer diffraction patterns obtained for different regions of the tuning fork recorded as a time-average exposure in the image plane.

oscillating object. From the width of the cosine fringes *a* the tilt angle γ in the object is found to be

$$\gamma = \frac{\lambda f_1}{f(1 + \cos\beta)a} \tag{3}$$

and the maximum amplitude γ_0 of the oscillating angular tilt is, by denoting the separation of the first minimum from origin of the Bessel function by $a_1/2$,

$$\gamma_0 = 0.76\,\frac{\lambda f_1}{f(1 + \cos\beta)a_1} \tag{4}$$

where f is the focal length of the lens used to perform the optical transform of the object. Typical results are shown in the following photographs. In Fig.7 the speckle pattern was recorded in the Fourier transform plane of the object before and after tilts were introduced in turn, namely $\gamma_\xi = 60''$ in the ξ direction and $\gamma_\zeta = 60''$ perpendicular to it. Equidistant fringes perpendicular to each other are

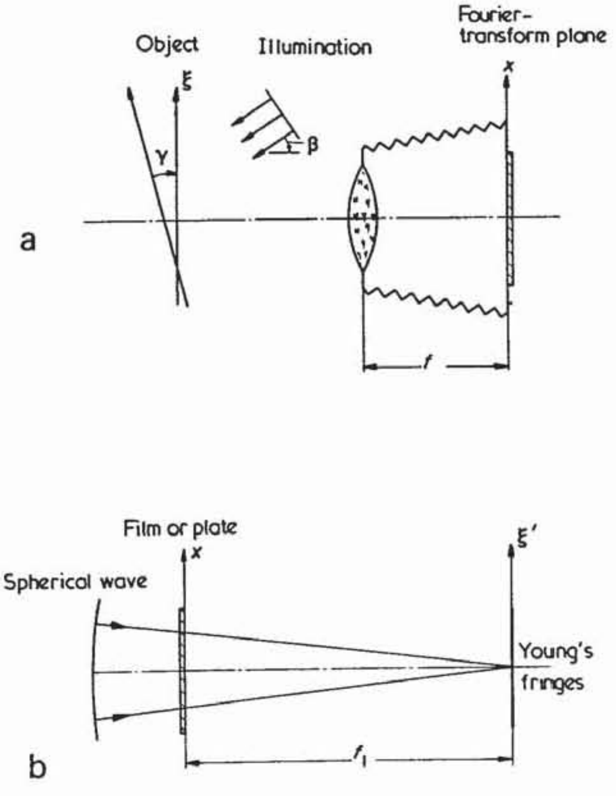

Fig.6 Experimental arrangement for tilt measurements by speckling.
(a) recording stage, (b) display of the fringes.

obtained. A familiar picture is shown in Fig.8 where 7 multiple exposures with 6 equal tilts of 30" between them were introduced. Multiple beam fringes as from a set of 7 identical equidistant pinholes are observed. The fringes become sharper when the number of equal tilts is increased.

For measurements of rotational oscillations (tilts) even in the presence of lateral in-plane oscillations, the speckle pattern was recorded as a time-average exposure in the Fourier transform plane of the object. The corresponding fringe patterns were photographed in the Fraunhofer diffraction pattern of the developed photographic plate.

In Fig.9(a), (b), (c) and (d) the fringe patterns are shown for different voltages applied to the coils of the electromagnetic head of the tuning fork. The fringes obtained from the superposition of the speckle pattern of the stationary object on that of the time-averaged recording are shown in Fig.9(c) and (d) for higher voltages (larger tilts); the minima of intensity occur for $J_0'(\ ) = 0$ and $J_0'' > 0$. The exposure time for the stationary object was chosen to be half that of the oscillating one, namely 2 s ($f = 50$ mm, F/4 and $f_1 = 120$ mm). The tilt fringes of an oscillating quartz ($\nu = 8000$ Hz) are shown in Fig.10.

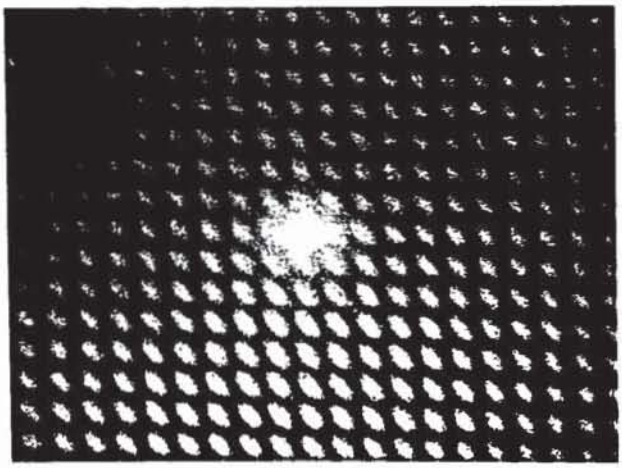

Fig.7 Photographs of the fringes obtained from two doubly exposed speckle patterns recorded in the Fourier transform plane of the object (an Al-plate). The tilts were $\gamma_\xi = 60''$ for the second and $\gamma_\zeta \perp \gamma_\xi$ ($\gamma_\zeta = 60''$) for the third exposure ($\beta = 45°$).

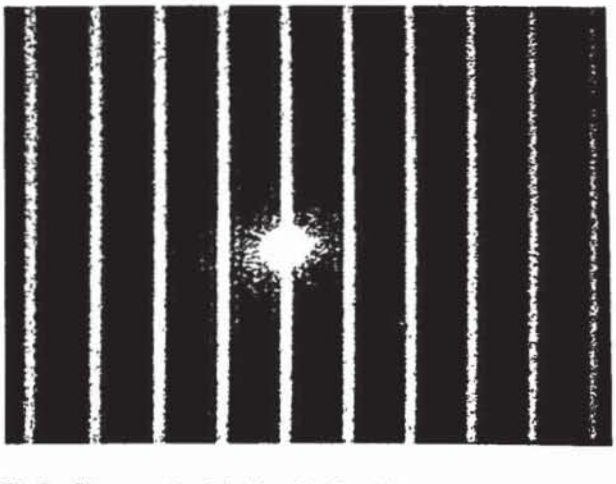

Fig.8 Photograph of the Frauhhofer diffraction pattern obtained from a multiple-exposed speckle pattern recorded in the Fourier transform plane of a Cu-plate with 6 equal tilts of $\gamma = 30''$ between the 7 exposures ($\beta = 45°$).

It should be noted that in the case where the light is multiple-scattered, for example by translucent surfaces, the speckle pattern becomes non-correlated and no fringes are visible.

5 CONCLUSION

The method described for analysing superimposed movements may become a very useful engineering tool and has many potential applications. No reference wave is needed and the practical implementations for analysis are not very demanding. No special processing and no sophisticated equipment is necessary to perform the investigations. A conventional photographic camera can be used to photograph the speckle patterns in the image or Fourier transform plane, as the case may be.

In-plane translations can be studied in the presence of small tilts γ. Tilts can be separated from in-plane movements by recording the speckle patterns in the Fourier transform plane of the object. The results obtained were very accurate (better than 2%). For a more quantitative study of superimposed movements it may be useful to record a doubly-exposed speckle pattern (before and after a movement) at an out-of-focus position [12]

The described speckle method can also be applied to measure phase variations. A phase variation may either lead to a in-plane lateral shift of the speckle pattern or a tilt depending on the particular arrangement.

In the described speckle method the requirements on the spatial and temporal coherence are rather relaxed compared with those of holographic techniques. A pulsed laser source could be used to perform the experiments.

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7 REFERENCES

1 Froehly, C., Monneret, J., Pasteur, J. and Viénot, J. Ch. 'Etude des faibles déplacements d'objets opaques et de la distortion optique dans les lasers à solide par interférométrie holographique'. *Optica Acta*, vol.16 (1969) pp.343–362.

2 Goldfischer, L. I. 'Autocorrelation function and power spectral density of laser-produced speckle patterns'. *J. Opt. Soc. Am.*, vol.55 (1965) pp.247–253.

3 Hopkins, H. H. and Tiziani, H. J. 'Speckling in diffraction patterns and optical images formed with the laser'. *Proc. Intern. Symp. Holography*, Besançcon (1970) pp.viii–xvi.

4 Dainty, J. C. 'Some statistical properties of random speckle patterns in coherent and partially coherent

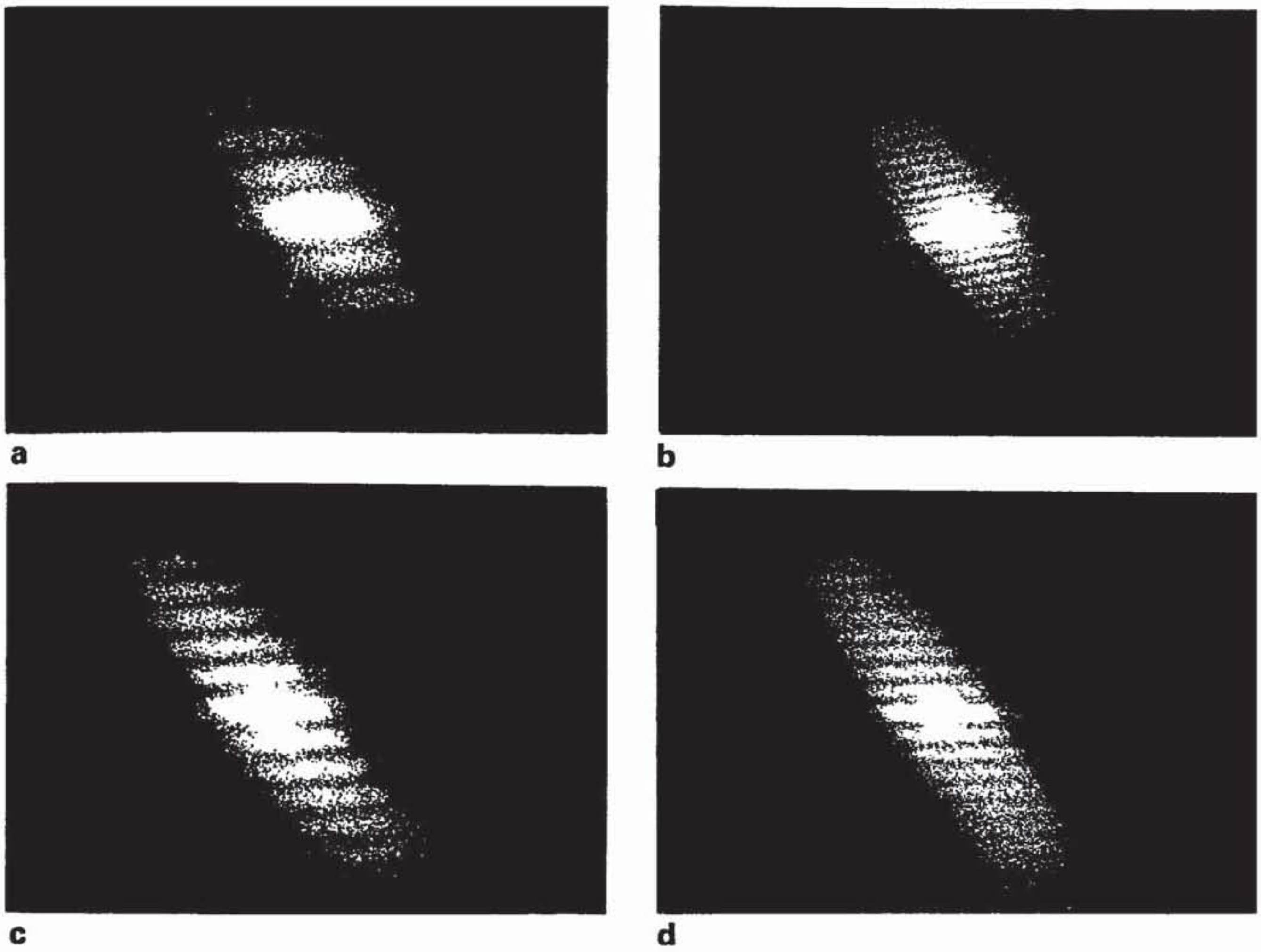

Fig.9 Photographs of the Fraunhofer diffraction pattern obtained from time-average exposures recorded in the Fourier transform plane of the tuning fork. The voltages applied to the coils of the electromagnetic heads were 1.0, 2.5 and 2.5 and 3.5 V for (a), (b), (c) and (d). For (c) and (d), however, the time-average exposure speckle pattern was superimposed on that of the stationary tuning fork (the tilts γ_0 were 3.9×10^{-4}, 10×10^{-4} and 14×10^{-4}).

illumination'. *Optica Acta,* vol.17 (1970) pp.761–772.

5 Leendertz, J. A. 'Interferometric displacement measurement on scattering surfaces utilising speckle effect'. *J. Phys. E., Sci. Instrum.*, vol.3 (1970) pp.214–218.

6 Archbold, E., Burch, J. M. and Ennos, A. E. 'Recording of in-plane surface displacement by double-exposure speckle photography', *Optica Acta,* vol.17 (1970) pp.883–898.

7 Köpf, U. 'Ein kohärent-optisches Verfahren zur Messung mechanischer Schwingungen', *Optik,* vol.33 (1971) pp.517–521.

8 Tiziani, H. J. 'Application of speckling for in-plane vibration analysis'. *Optica Acta,* vol.18 (1971) pp.891–902.

9 Tiziani, H. J. 'Optische Methoden zur Schwingungsanalyse der Stimmgabel einer elektronischen Uhr'. *Optik,* vol.34 (1972) pp.442–455.

10 Tiziani, H. J. 'A study of the use of laser speckle to accurately measure small tilts of optically rough surfaces'. *Opt. Commun.*, vol.5 (1972) pp.271–276.

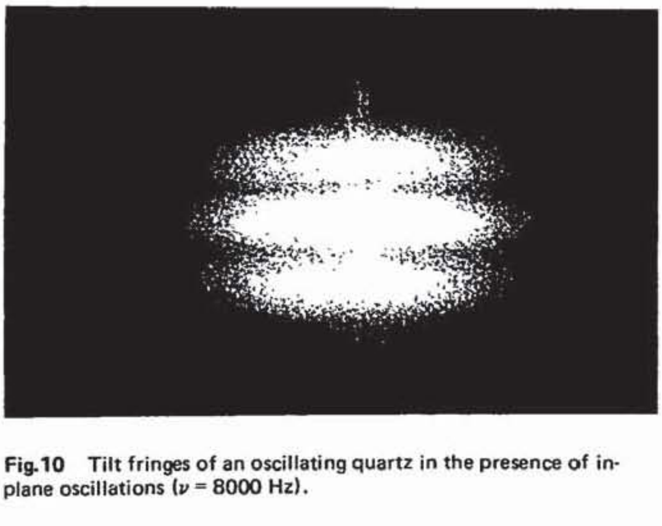

Fig.10 Tilt fringes of an oscillating quartz in the presence of in-plane oscillations ($\nu = 8000$ Hz).

11 Tiziani, H. J. 'Analysis of mechanical oscillations by speckling'. *Appl. Opt.*, vol.11 (1972) pp.2911–2917.

12 Archbold, E. and Ennos, A. E. 'Displacement measurement from double-exposure laser photographs'. *Optica Acta,* vol.19 (1972) pp.253–271.

Résumé

MESURE DE PETITS MOUVEMENTS ET DE VIBRATIONS PAR PHOTOGRAPHIE LASER

La plupart des techniques holographiques appliquées jusqu'à maintenant aux mesures sans contact des déplacements de surface sont hautement sensibles. Mais l'interprétation des franges d'interférence obtenues est généralement assez difficile pour un non expert, spécialement pour l'étude d'inclinaisons en présence des mouvements latéraux dans le plan de l'objet.

Les techniques de moiré pourraient être utilisées pour surmonter certaines des difficultés rencontrées mais elles nécessitent une surface relativement plane, qui doit être préparée spécialement.

L'application du mouchetage, un phénomène bien connu en optique cohérente, a été trouvée très adéquate pour l'étude des déplacements latéraux dans le plan et des vibrations en présence d'inclinaisons. Le mouchetage peut être considéré comme solidaire a l'objet et, de ce fait, sera déplacé si l'objet ou partie de celui-ci est déplacé. Pour l'analyse des mouvements dans le plan de l'objet et pour des inclinaisons, le mouchetage est photographié dans le plan image ou le plan de la transformée de Fourier de l'objet, respectivement, avant et après le mouvement. A partir des effets de diffraction du mouchetage enregistré, le mouvement de l'objet est déterminé.

Diverses techniques de mesure de translations de corps rigides, inclinaisons et vibrations de surface seront exposées et une comparaison avec d'autres méthodes sera discutée conjointement avec les résultats expérimentaux correspondants.

Zusammenfassung

MESSUNG VON KLEINEN BEWEGUNGEN UND SCHWINGUNGEN DURCH LASER PHOTOGRAPHIE

Die meisten holographischen Techniken, die bis jetzt für die berührungslose Messung von Oberflächenveränderungen angewendet werden, sind hochempfindlich. Aber die Deutung der erhaltenen Interferenzstreifen ist normalerweise ziemlich schwierig für Laien, besonders bei der Untersuchung von Verkippungen in der Gegenwart von seitlichen Bewegungen in der Ebene.

Die Moiré-Technik konnte angewendet werden, um einige der auftretenden Schwierigkeiten zu uberwinden, aber sie verlangt eine relativ ebene Oberflache, die speziell vorbereitet werden muss.

Die Granulation, ein in der koherenten Optik bekanntes Phänomen, erwies sich als geeignet, um bei seitlichen Verschiebungen und Schwingungen in der Ebene in der Gegenwart von Verkippungen angewendet zu werden. Die Granulation kann als mit dem Objekt zusammenhängend betrachtet werden, und sie wird infolgedessen bewegt, wenn das Objekt oder ein Teil von ihm bewegt wird. Für die Analyse von Bewegungen oder Verdrehungen in der Ebene wird die Granulation in der Bildebene oder Fourier-transformierten Ebene des Objektes photographiert, u.z. vor und nach der Bewegung. Aus den Beugungseffekten der festgehaltenen Granulation wird die Bewegung des Objektes bestimmt.

Verschiedene Techniken zur messung der Verschiebung von Festkörpern, die Verdrehungen und Amplituden von Uberflachen Schwingungen werden demonstriert, und ein Vergleich mit anderen Methoden wird zusammen mit geeigneten experimentallen Ergebnissen angestellt.