# A numerical and experimental study of wavy ice-structure in an asymmetrically cooled parallel-plate channel

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## INTRODUCTION.

Ice formation of flowing water in a pipe or a channel, whose wall is kept at a uniform temperature below the freezing temperature of the water, is a basic engineering problem. It introduces many practical problems, such as pressure drop, diminution of flow rate and sometime, breakage of the pipe as a result of flow blockage by ice. The phenomenon of freezing of flowing water involves interactions between the turbulent flow, the shape of the ice layer and the heat transfer at the ice-water interface. Under certain conditions these interactions result in an instability of the ice layer. This instability is caused by the strong laminarization of the turbulent flow due to converging ice layers in the entrance region of the cooled channel and results in a wavy ice structure. Wavy ice layers with one wave, occuring in a parallel-plate channel subjected to symmetrically cooled walls were investigated experimentally by Seki et al. [1] and by Weigand and Beer [2]. More recently Weigand and Beer [3] were able to predict numerically the shape of wavy ice layers with one wave occuring in a symmetrically cooled channel. Wavy ice layers in a parallel-plate channel with one wave in the case of asymmetrically cooled walls were investigated experimentally by Tago et al. [4] and by Weigand and Beer [5] No numerical calculation of asymmetric wavy freezing fronts was done in the past. Therefore, the subject of this paper is the presentation of a numerical model for calculating steady state ice layers with one wave in the entrance region of an asymmetric cooled channel. The method is based on a work performed by Weigand and Beer [3]. The given numerical study is supported by a detailed experimental investigation.

## ANALYSIS.

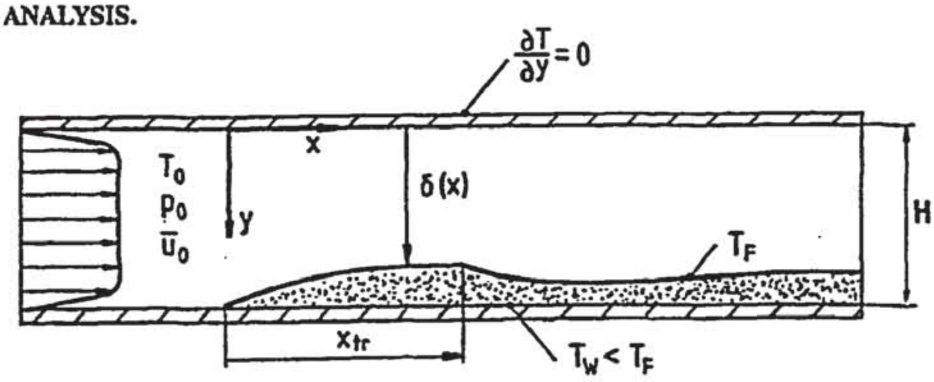


Fig. 1. Physical model and coordinate system.

Consideration is given to a turbulent flow entering an asymmetrically cooled parallel-plate channel

(Fig. 1) with a fully developed turbulent velocity profile and with the uniform temperature $T_0$. In the chill region, the wall temperature of the lower wall is maintained at a constant value $T_w$, which is below the freezing temperature $T_F$ of the fluid and therefore, a frozen layer is generated at the cooled lower wall. The upper wall of the two dimensional channel was perfectely isolated. Therefore, the heat flux through this wall is zero.

Assuming an incompressible, Newtonian fluid with constant fluid properties, the steady-state boundary layer equations for the fluid can be written as [6]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_L}\frac{\partial p}{\partial x} + \nu_L\frac{\partial}{\partial y}\left[\left(1 + \varepsilon_m^*\right)\frac{\partial u}{\partial y}\right] \tag{2}$$

$$0 = -\frac{\partial p}{\partial y} \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = a_L\frac{\partial}{\partial y}\left[\left(1 + \frac{Pr}{Pr_t}\varepsilon_m^*\right)\frac{\partial T}{\partial y}\right] \tag{4}$$

where $\varepsilon_m^* = \varepsilon_m/\nu_L$ is the eddy viscosity and $Pr_t$ is the turbulent Prandtl number according to Cebeci [6]. The boundary conditions belonging to eqs. (1) - (4) are given by

$$\begin{aligned} x &= 0 &&: p = p_0 \;,\; u = \text{given} \;,\; T = T_0 \\ y &= 0 &&: u = v = 0 \;,\; \partial T/\partial y = 0 \\ y &= \delta(x) &&: u = v = 0 \;,\; T = T_F \end{aligned} \tag{5}$$

In the solid region, one dimensional heat conduction is assumed. Therefore, the temperature distribution in the solid crust can be shown to be

$$(T_s - T_F)\,/\,(T_w - T_F) = (y - \delta)\,/\,(H - \delta)\;; \quad \delta \le y \le H \tag{6}$$

Eqs. (1) - (5) are coupled with eq. (6) by the interface energy equation

$$k_s\frac{\partial T_s}{\partial y} = k_L\frac{\partial T}{\partial y} \quad ; \quad y = \delta(x) \tag{7}$$

Deissler [7] found out that the turbulent Reynolds shear stress can be taken approximately as constant along a streamline in a highly accelerated flow, which yields a "laminarization"

$$\overline{u'v'}(\psi) = (\overline{u'v'}(\psi))_{x=0} \qquad \psi = \text{const.} \tag{8}$$

where $\psi$ is the streamfunction defined in the common way [6]. Equation (8) was used for calculating $\varepsilon_m^*$ in the region $0 \le x \le x_{tr}$. The initial distribution of the turbulent shear stress for $x = 0$ can easily be obtained with the help of a mixing length model [6].

For $x > x_{tr}$ the acceleration due to the sharply increasing thickness of the ice layer ceases and the flow recedes to its originally turbulent state. In this region, where the heat transfer at the solid-liquid interface is strongly enhanced, the eddy viscosity was calculated with a modified mixing length model according to Moffat and Kays [8]. The transition point $x_{tr}$ was correlated from measurements. For more detailed informations concerning the turbulence modelling, the reader is refered to [3]. The calculation of the frozen layer involves an iteration procedure. Initially a distribution of the ice layer is assumed. With this variation of $\delta(x)$ the conservation equations (1) - (4) are solved with the help of an implicit finite-difference method, which is known in literature as the Keller box-method [6]. After solving the eqs. (1) - (4) in conjunction with the boundary conditions eq. (5), a new distribution of $\delta(x)$ is calculated by inserting the yet known temperature gradient at the solid-liquid interface into eq. (7). This iteration process is repeated until $\Delta\delta = |\delta^{(i)} - \delta^{(i-1)}| < 0.01$ H at every axial position for two succesive iterations.

RESULTS AND DISCUSSION.

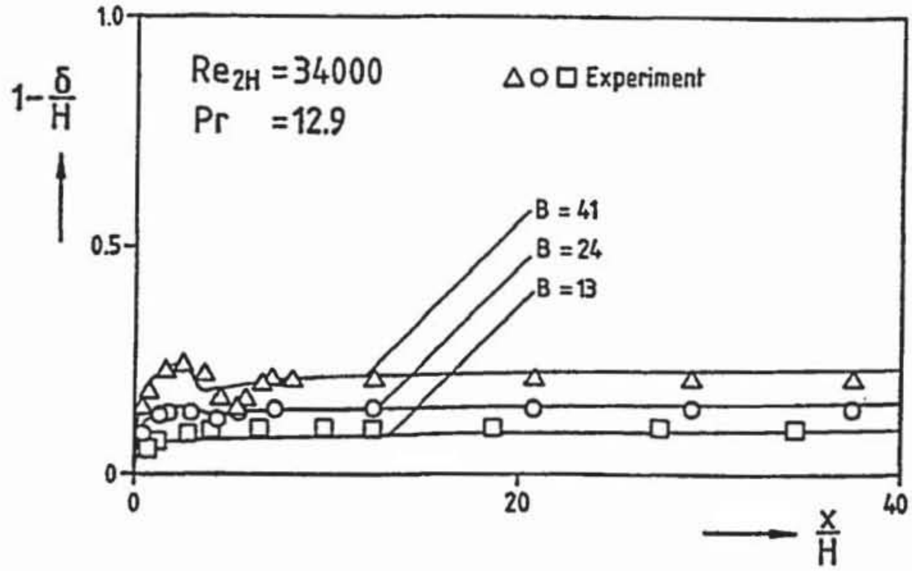


Fig. 2. Influence of the cooling parameter B on the shape of the ice layers.

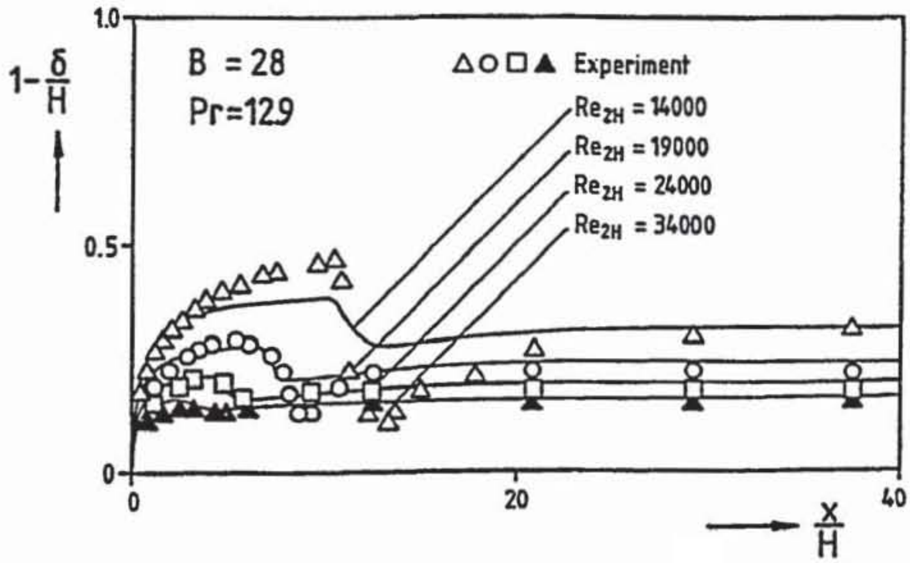


Fig. 3. Influence of the Reynolds number on the shape of the ice layers.

Fig. 2 and Fig. 3 show comparisons between calculated and measured ice layers ploted as a function of the axial coordinate. In both figures $(1 - \delta/H) = 0$ denotes the cooled lower wall and $(1 - \delta/H) = 1$ the isolated upper wall of the parallel-plate channel. Fig. 2 elucidates that an increasing wall cooling parameter $B = k_s/k_L(T_F - T_w)/(T_0 - T_F)$ results in a thicker ice layer for a given value of the Reynolds number $Re_{2H} = \bar{u}_0 2H/\nu_L$ (where $\bar{u}_0$ is the mean velocity at the entrance of the test section). For $B = 13$ a smooth ice layer is formed at the cooled wall of the channel. Increasing the cooling parameter B, results in a more pronounced laminarization of the flow in the entrance region of the cooled channel and a wavy ice layer with one small wave can be observed. A further increase in B leads to the development of a wavy ice layer with a steeper gradient of $\delta$ in the diffusor region of the wave. Fig. 3 visualizes the effect of a variation of the Reynolds number on the ice layer thickness for a fixed value of B. It is obvious that an increasing Reynolds number results in a decreasing ice layer thickness. This is due to the intensified heat transfer from the flowing liquid to the solid crust with growing values of $Re_{2H}$. The figure shows that the agreement between theory and experiment is quite good for $Re_{2H} = 24000$ and $Re_{2H} = 34000$. For lower values of the Reynolds number it can be observed that the theoretical model is not able to predict the minimum value of the ice layer thickness in the diffusor region. This can be attributed to three dimensional effects appearing in the diffusor region and also to the simple turbulence model which was used for the calculations for $x > x_{tr}$ [3]. Additionally, it can be recognized that for $Re_{2H} = 14000$ the theoretical results deviate from measurements for $x < x_{tr}$. This signifies that a full laminarization of the flow has taken place in the entrance region of the cooled channel [9]. In such a case the shape of the ice layer for $x < x_{tr}$ can be calculated with good accuracy by neglecting the eddy viscosity in the conservation equations ($\varepsilon_m^* = 0$). The turbulence model according to Deissler, eq. (8), overpredicts the heat transfer from the fluid to the solid crust in such an extreme case.

## CONCLUSIONS.

According to the experimental and theoretical results of the present investigation concering the freezing of water flow between asymmetrically cooled parallel plates with the occurence of laminarization effects, the following major conclusions may be drawn:

- A strong coupling exists between the nature of the flow and the shape of the solid-liquid interface. The development of wavy ice-layers can be attributed to the flow laminarization in the entrance region of the parallel-plate channel.
- Increasing the Reynolds number for a given value of B and Pr tends to stabilize the ice layer (Fig. 3).
- Increasing the cooling parameter B for a fixed value of the Reynolds number tends to destabilize the ice layer (Fig. 2).
- A simple method is presented for calculating the axial distribution of wavy ice layers with one wave for steady-state conditions.

## REFERENCES.

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