HYSTERETIC BEHAVIOR OF DEFORMED REINFORCING BARS UNDER SEISMIC EXCITATIONS

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SUMMARY

A mathematical model of a deformed bar anchored in a concrete block, recently developed by the authors, has been generalized to cover the bond conditions found in exterior and interior joints of reinforced concrete frames subjected to severe earthquake loadings. The analytically predicted response compares well with the results of a series of tests conducted in Berkeley for monotonic and cyclic loadings. An extensive numerical investigation has been carried out to show the influence on the anchored bar behavior of different parameters such as: 1) Severity of hysteretic requirements, 2) characteristics of steel (namely yield stress and strain hardening), 3) anchorage length. The results of this investigation are used to offer some practical recommendations and conclusions.

1. INTRODUCTION

Efficient earthquake-resistant design of structures requires accurate prediction of their inelastic response (hysteretic behavior). In cases of R/C moment/resistant space frames this hysteretic behavior is very sensitive to the observed deterioration in stiffness that occurs with the increase in severity of the hysteretic requirements. One of the main sources of stiffness degradation is the increase in bond slippage during cyclic loading of the main reinforcing bars along their anchorage lengths in their supports and/or joints. This increase in slippage is a consequence of bond deterioration. Thus it is of utmost importance to be able to predict such bond-slipage behavior.

For that purpose in this paper the mathematical model of a deformed bar embedded in a well confined concrete block and subjected to generalized excitations presented in /2/ is extended to cover the bond conditions found in the unconfined end regions of joints of reinforced concrete frames. This extension is based on the elaboration of test results /3,5/. Furthermore the analytically predicted response of anchored bars with long embedment length is compared with available experimental results /1/. Finally the model is used to investigate the influence of various parameters on the hysteretic response of anchored bars.

2. MATHEMATICAL MODEL OF ANCHORED BARS

Only a short description is given here. Full details can be found in /4/.

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The actual behavior of a bar of finite length embedded in a concrete block is idealized as a one-dimensional problem and modeled using the ordinary nonlinear differential equation \( dN(x)/dx + q(x) = 0 \), where \( q(x) = \gamma \cdot d \cdot \sigma(x) \) and \( N(x) = A_s \cdot \varepsilon(x) \), with \( d \) diameter of the bar and \( A_s \) area of the cross section. The equation connects the axial force in the bar, \( N \), to the resultant per unit length of the bond stresses on the perimeter of the bar, \( q \). It has to be coupled with the constitutive laws for steel and bond, \( \sigma(x) = \varepsilon(\varepsilon(x)) \) and \( \sigma = \varepsilon(s(x)) \), where \( s(x) \) is the slip along the bar and \( \varepsilon(x) \) the steel strain which is set equal to \( ds/dx \), thus neglecting, for simplicity, the deformation of concrete. Boundary values are specified at the two end points of the bar. Together with the differential equation they define a non-linear two point boundary value problem, which is solved here numerically by using a "shooting technique". A similar technique has also been used in /6/.

The used local bond stress-slip relationship \( \gamma = \hat{\nu}(s(x)) \), as formulated in /2/, is illustrated in Fig. 1. It takes into account all significant parameters that appear to control the behavior observed in experiments and consists of a monotonic envelope, unloading -, friction -, reloading - branch and reduced envelope. Details about the different branches are given in /2-5/. However, it is worth mentioning here, that the degradation of bond resistance (reduced envelope and reduced frictional resistance) is connected to some cumulative damage parameters, formulated as a function of the total dissipated energy. This assumption, which is rationally acceptable in the range of low cycle fatigue, is the basis for an easy generalization of the bond model to the case of random excitations and results in a satisfactory agreement between calculated and observed cyclic bond behavior /3/.

The constitutive bond model presented in /2/ is valid only for well confined concrete regions. But the bond conditions in a joint vary along the embedment length.

For an interior joint three different regions have been identified in /1/ (see Fig. 2). They show differences both in the shape of the monotonic envelopes, different for positive and negative slip, and in the rate of bond degradation. Of course, there is a gradual variation in the bond behavior when proceeding from an unconfined region 1 or 2 to the confined one in the middle part of the joint. To cover this behavior, the analytical local bond model has been modified as follows:

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**Fig. 1:** Analytical model for local bond stress-slip relationship

**Fig. 2:** Bond stress-slip relationships under monotonic loading for different regions in a joint
- instead of only one, two different monotonic envelopes were specified for positive and negative slip values (compare Fig. 2);

- the way of computing damage parameters was generalized to cover the different bond behavior for loading in tension or compression, but it yields the same results as before when positive and negative envelopes are equal, as it is the case in the confined region.

By specifying different bond laws along the embedment length a gradual transition of the bond behavior from the unconfined to the well confined region can be modelled.

A non-linear and a bilinear model were used for the stress-strain relationship of the bar in alternative. Because preliminary results showed the anchored bar behavior to be not substantially different for the two choices, the bilinear model, which is computationally more economical, was adopted.

3. COMPARISON OF ANALYTICAL PREDICTIONS OF THE RESPONSE OF ANCHORED BARS WITH TEST RESULTS

The numerical model of an anchored bar was used to compare the predicted response with experimental results obtained in some of the Berkeley tests /1/. In those tests the diameter of the bar was \(d_b = 25\, \text{mm}\), the anchorage length \(l_d = 25\, d_b\), the compression strength of concrete \(f' c \approx 30\, \text{N/mm}^2\) and the yield stress \(f_{y} \approx 500\, \text{N/mm}^2\). Other parameters characterizing the model, in particular the characteristic values for the local bond stress-slip relationships along the embedment length were chosen on the basis of available information as documented in /3,4/.

The force-slip relationships for the pulled bar end, obtained numerically, agree satisfactorily with corresponding experimental results, as can be seen from Figs. 3a, 4a and 5a,b. In Figs. 3b, 4b and 5c,d the distribution of steel strain, normal force, slip and bond force along the embedment length are plotted for characteristic points of the pertinent load history. As can be observed, numerical and experimental distributions agree less well than the corresponding normal force-slip relationships.

One reason for that, which explains especially well differences in strain distribution, is that after yielding of the bar a small difference between calculated and measured bar stress results in a large difference between corresponding steel strains, due to the small slope of the strain-hardening branch. Another reason is that the model approximates complicated end effects, such as the formation of a fractured concrete cone and the consequent loss of bond at the pulled bar end, only through a particular choice of different bond laws at different points of the bar. This choice, which should be representative of the average behavior, will in general fit some tests better than others, due to the random nature of actual bond phenomena.

Despite the mentioned problems the obtained accuracy of reproduction of experimental results seems sufficient for practical applications.
Fig. 4: Comparison of experimental and analytical results, test 13 of /1/

Fig. 3: Comparison of experimental and analytical results, test 3 of /1/
Fig. 5: Comparison of experimental and analytical results, test 14 of 1/
4. NUMERICAL STUDIES

The influence of some important parameters on the behavior of anchored beam bars was investigated by determining the model response to imposed histories of displacements (slip), just as it would be done in an experiment. In all numerical studies the bar (d_b = 25 mm) was pulled-pushed with forces of equal magnitude at both bar ends to simulate conditions at an interior joint. The assumed bond behavior along the embedment length was the same as in the previous comparison with test results.

4.1 Severity of hysteretic requirements

In this series of numerical tests the anchorage length (l_d = 25 d_b) and the steel characteristics (yield stress f_y = 450 N/mm² and strain-hardening ratio E/H = 0.017) were held fixed in order to study the influence of different loading histories. They consisted of cycles between constant slip values, which were chosen to give specified steel strains under monotonic loading. In this way hysteretic requirements are often expressed in literature. Note, that as bond is damaged by cycling, the steel strains reached after the first cycle may be smaller than the specified values.

In Fig. 6 the normal force-slip relationships are plotted for the following three cases: a) 6 cycles with peak strain value \( \varepsilon = + \varepsilon_y \), where \( \varepsilon_y \) is the yield strain (2.2 %), b) 3 cycles with \( \varepsilon = + 5 \% \) followed by 3 cycles with \( \varepsilon = + 10 \% \) and c) 3 cycles with \( \varepsilon = + 15 \% \) followed by 3 cycles with \( \varepsilon = + 30 \% \). For comparison the response of the anchored bar under monotonic loading is shown as well. As expected stiffness and strength of the anchorage were increasingly reduced with increasing hysteretic requirements. Even cycling between slip values corresponding to peak steel strains \( \varepsilon = \varepsilon_y \) gave a considerable reduction of the maximum resistance and the deformability at maximum resistance compared to monotonic loading, Fig. 6a. This is caused by the loss of bond at the end regions which simulates the formation of a concrete cone during tension loading. In the studied case (l_d = 25 d_b) cycling between slip values corresponding to peak steel strains \( \varepsilon \geq 10 \% \) caused a severe damage of bond (Fig. 6b,c).

4.2 Characteristics of steel

In this set of numerical tests the anchorage length was again l_d = 25 d_b and the loading history was chosen as 3 cycles with \( \varepsilon = + 15 \% \) plus 3 cycles with \( \varepsilon = + 30 \% \) (previous case c). Yield stresses and strain hardening ratios E/H were varied in their respective practical range.

Fig. 7 shows the influence of the yield stress, which was varied between 300 N/mm² and 600 N/mm². Under monotonic loading the strength of an anchorage was almost independent on the yield stress. However, the deformability at maximum resistance increased considerably with decreasing yield stress (Fig. 7a). The deterioration of the resistance of anchorages caused by cyclic loading increased significantly with increasing yield stress (compare Fig. 7b with Fig. 6b and 7c). In particular, for a yield stress of 600 N/mm² (Fig. 7c) the strains aimed at could not even be reached and the bond was almost completely damaged by some cycles.
Fig. 6: Influence of the hysteretic requirements on the response of anchored beam bars

Fig. 7: Influence of yield stress on the response of anchored beam bars
Increasing the strain hardening ratio $E_1/E_0$ had qualitatively the same effect as increasing the yield stress, however, the influence was less pronounced.

The reason for the superior behavior of anchorages with lower yield stresses and/or lower strain hardening ratios is that in both cases when reaching the given peak strains lower steel stresses are developed. Therefore smaller bond stresses are necessary for the force transfer and less damage is observed.

4.3 Anchorage length

The influence of the anchorage length on the bar response was studied using the same hysteretic requirements as given in section 4.2. A bar with a yield stress $f_y = 450 \text{ N/mm}^2$ and a strain hardening ratio $E_1/E_0 = 0.017$ was employed. The main results are plotted in Fig. 8.

Under monotonic loading the strength of an anchorage increased, as expected, with increasing embedment length. While for $l_e = 15 \phi_b$ the bar was pulled out before reaching yield, bars with $l_e = 25 \phi_b$ reached a peak strain value $\varepsilon \approx 40 \%$. The response of anchorages with a length $1_e \geq 35 \phi_b$ was almost identical in the plotted slip range (Fig. 8a). Cycling loading caused a severe pinching of the hysteretic loops of anchorages with $l_e = 25 \phi_b$ (Fig. 6c). This was due to an almost complete damage of bond along the entire embedment length. On the contrary anchorages with a length of $l_e = 45 \phi_b$ showed stable hysteretic loops (Fig. 8b) and very limited bond damage.

Fig. 8: Influence of anchorage length on the response of anchored beam bars

5. CONCLUSIONS AND RECOMMENDATIONS

From the results obtained in this study the following main observations can be made:

(1) The proposed mathematical model allows to predict with accuracy sufficient for practical purposes the response of deformed reinforcing bars anchored in joints of ductile moment resisting reinforced concrete frames under generalized excitations.
(2) Under otherwise constant conditions the performance of anchorages during cycling loading is strongly influenced by the hysteretic requirements. Because damage of bond along the embedment length and the corresponding end slip significantly increase with increasing hysteretic requirements, it is important to know more precisely the cyclic steel strain histories likely to occur during strong ground motions.

(3) For constant hysteretic requirements the performance of anchorages improves with increasing anchorage length and with decreasing yield stress and decreasing strain hardening ratio $\frac{E_f}{E_0}$.

(4) The necessary anchor length is significantly influenced by hysteretic requirements and required performance of the anchorage during cycling loading. If hysteretic requirements as given in section 4.2 are assumed and almost stable hysteretic loops are required, an anchor length of approximately 25 $d_a$ or 35 $d_b$ is necessary for Grade 40 ($f_y \approx 275$ N/mm$^2$) or Grade 60 ($f_y \approx 415$ N/mm$^2$) deformed bars respectively. These values are valid for interior joints with a specified concrete strength $f'_c = 30$ N/mm$^2$. The recommended anchor lengths agree well with the values proposed in /7/.

(5) If the width of columns at interior joints is smaller than the proposed anchor length, the formation of plastic hinges in girders near column faces should be avoided by detailing the reinforcement in an appropriate way to avoid excessive slip and damage of bond. This is in accordance with earlier recommendations /8/. Otherwise the influence of slip of main beam bars on the dynamic response of ductile moment resisting R/C frames under strong ground motions may become too important to be neglected in the analysis. The analytical model presented herein provides a basis to formulate simplified joint models, which realistically take into account the influence of slip.

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