On the Development of Activating Teaching Materials in Theoretical Physics

Von der Fakultät Mathematik und Physik der Universität Stuttgart
zur Erlangung der Würde eines Doktors der Naturwissenschaften
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Überblick

Ziel dieser Arbeit ist die Entwicklung und Testung von interaktiven und kognitiv aktivierenden Lehrmethoden und -materialien für die Theoretische Physik. Der Blick auf aktuelle Abbrecherquoten im Fach Physik in Deutschland und die fachdidaktische Forschung zur Effizienz von konventionellen Lehrveranstaltungen belegen die Notwendigkeit der Anpassung der Lehre an aktuell Studierende. Eine Möglichkeit, diese Probleme zu adressieren, sind Methoden, die Studierende kognitiv aktivieren und diesen zeitnah Feedback geben. Solche Methoden werden im schulischen Kontext und in Einführungsveranstaltungen an Hochschulen schon lange erfolgreich mit einem empirisch nachgewiesenen Mehrwert eingesetzt. Für fortgeschrittenere Vorlesungen, wie die Theoretische Physik, gibt es im deutschsprachigen Raum jedoch kaum Materialien und Lehrkonzepte, die solche Ansprüche erfüllen.

Diese Arbeit versucht diesen Missstand zu korrigieren, indem auf einem fachdidaktisch sinnvollem Fundament Materialien und Methoden entwickelt werden, die Studierende bestmöglich in Lehrveranstaltungen der Theoretischen Physik unterstützen. Es werden Materialien vorgestellt und beschrieben, die Vorlesungen in den Fächern Theoretische Mechanik, Elektrodynamik, Quantenmechanik und Thermodynamik/statistische Physik unterstützen können. Ein weiterer Anspruch an die entwickelten Materialien ist es, dass diese in Zukunft einfach und ohne viel Vorbereitung von Dozierenden und Tutor*innen übernommen und verwendet werden können. Darüber hinaus sollen die neuen Materialien nicht nur kognitiv aktivierend sein, sondern auch dort ansetzen, wo Studierende die meisten Probleme in aktuellen Lehrformaten haben.

Im Rahmen dieser Arbeit wurden Tutorien und Begleitseminare parallel zu Vorlesungen an der Universität Stuttgart und der Friedrich-Schiller-Universität Jena beobachtet und Probleme und Schwierigkeiten gesammelt, welche gehäuft auftreten. In Kontexten der Mathematischen Methoden der Physik (häufig unzureichende Ausbildung), bei Wechseln zwischen Repräsentationen (mathematisch, bildlich, sprachlich), bei der Anknüpfung an Vorwissen (Experimentalphysik und Schulphysik), bei der Identifikation und Trennung wichtiger und weniger wichtiger Inhalte und beim fachlich und zielgruppengerechten Argumentieren und Diskutieren treten immer wieder verschiedene Probleme auf.

Parallel zu den Beobachtungen wurden immer wieder neue Materialien, die diese Probleme adressieren, ausgearbeitet, getestet und verbessert. Grundregel für die Ausarbeitung war, dass die neuen Materialien auf Lehrmethoden beruhen, deren didaktischer Mehrwert bereits an anderer Stelle in der Physiklehre nachgewiesen wurde.

Die vorhin erwähnten Problemfelder werden im Rahmen dieser Arbeit mit verschiedenen didaktischen Methoden einzeln oder in Kombinationen adressiert. Dabei handelt es sich um das Lernen mit Lösungsbeispielen (worked examples), der Peer Instruction (Diskussion in Kleingruppen), der Versprachlichung und Verbildlichung von Formeln und der didaktischen Rekonstruktion und Elementarisierung physikalischer Inhalte und Methoden.

In dieser Arbeit wird beispielhaft beschrieben, wie Umsetzungen dieser Methoden in der Theoretischen Physik aussehen und wie diese in Lehrveranstaltungen integriert werden können. Darüber hinaus findet eine Einordnung und Kategorisierung der Ansätze statt und es werden Probleme und Herausforderungen bei der Umsetzung sowie daraus resultierende Weiterentwicklungen angesprochen. Diese Methoden eignen sich alle für andere Dozierende und Hochschulen. So können z.B. worked examples ohne größeren Aufwand in Übungsserien integriert oder Fragen mit anschließender Peer-Diskussion (Peer Instruction) in regulären Vorlesungen diskutiert werden.

Zur Entwicklung der Begleitmaterialien bietet sich das Modell der didaktischen Rekonstruktion an. Hier werden die relevanten Fachinhalte (Analyse der Sachstruktur oder fachliche Klärung) und die Perspektive bzw. der Stand der Lernenden (Studien zum Lernen sowie Erprobung und Evaluation von Lehreinheiten) gleichwertig in die didaktische Strukturierung der neuen Materialien einbezogen, was den Lernerfolg steigert. Alle Materialien und Methoden, die in dieser Arbeit vorgestellt werden, sind nach den Prinzipien dieses Modells entwickelt.

Eine ausführlichere didaktische Rekonstruktion ist in dieser Arbeit ebenfalls beispielhaft beschrieben. Hier wird das Thema der quantenmechanischen Verschränkung und der Ausschluss von versteckten lokalen Parametern behandelt. Am Ende der Rekonstruktion steht ein Kurs, der die interaktiven und aktivierenden Elemente bestmöglich einsetzt, um Studierende maximal zu unterstützen und zu aktivieren bzw. zu fordern.

Neben der Entwicklung von allgemeinen aktivierenden Materialien liegt ein Schwerpunkt dieser Arbeit auf der Ausbildung von Lehramtsstudierenden in der Theoretischen Physik. Hier ergeben sich weitere Spannungsfelder in der Ausbildung. Große Herausforderungen entstehen, wenn Studierende des Lehramts eine andere Fachkombination studieren als Physik und Mathematik. Bei allen anderen Kombinationen erhalten die Studierenden keine vergleichbare Mathematikausbildung, wie es im Fachstudium vorgesehen ist, die Inhalte des Physikstudiums bleiben aber teilweise die gleichen. Zur Unterstützung dieser Studierenden wird die Entwicklung und der Aufbau von Blockkursen beschrieben, die die Mathematischen Methoden der Physik vor der Theoretischen Mechanik und der Elektrodynamik wiederholen.

Ein weiteres Spannungsfeld in der Lehramtsausbildung ist die scheinbare Zusammenhangslosigkeit zwischen der Theoretischen Physik und der Schulphysik. Die Theoretische Physik mit ihrem stark vom Formalismus geprägten Aufbau erscheint zu weit vom Schulunterricht und der dort behandelten Physik entfernt zu sein. Dies führt Studierende häufig zu der Grundsatzfrage: *„Wofür benötige ich das in Zukunft?“* Wenn keine befriedigende Antwort gefunden wird, hat dies einen Mangel an Motivation zur Folge.

Die Entwicklung (nach den Prinzipien der didaktischen Rekonstruktion) neuer Seminare, die den Nutzen der Theoretischen Physik für angehende Lehrkräfte betonen sollen, stellt den Abschluss dieser Arbeit dar. Ziel dieser Seminare ist es, das Fachwissen, das Schulwissen und das fachdidaktische Wissen zu stärken, indem diese drei Aspekte an ausgewählten Themen explizit von Studierenden in Verbindung gebracht werden. Die neuen Seminare sind auf den vorher diskutierten Methoden aufgebaut.

Die Materialien, die im Rahmen dieser Dissertation entstanden sind, können online unter dem Link `https://doi.org/10.18419/darus-3972` aufgerufen und heruntergeladen werden.

Abstract

This work aims at the development and testing of interactive and cognitively activating teaching methods and materials for theoretical physics. The view on current dropout rates in physics and the physics education research on the efficiency of conventional courses prove the necessity to adapt teaching to current students. One way to address these problems is to use methods that cognitively activate students and provide them with short-term feedback. Such methods have long been used successfully in school contexts and university introductory courses with empirically proven added value. However, for more advanced lectures, such as theoretical physics, there are hardly any materials and teaching concepts in German language that meet such demands.

This thesis tries to correct this deficiency by developing materials and methods that support students in theoretical physics courses in the best possible way based on a reasonable foundation of physics education research. Materials that can support lectures in theoretical mechanics, electrodynamics, quantum mechanics, and thermodynamics/statistical physics are presented and described. Another requirement for the developed materials is that they can be quickly adopted and used by lecturers and tutors in the future with little preparation. Furthermore, the new materials should address the areas where students have the most problems.

In the context of this work, tutorials and accompanying seminars parallel to lectures at the University of Stuttgart and Friedrich Schiller University Jena were observed, and problems and difficulties that occur frequently were collected. In the contexts of mathematical methods of physics (often insufficient training), change between representations (mathematical, pictorial, linguistic), linking to previous knowledge (experimental physics and school physics), identification and separation of essential and less important contents, and professional and target group appropriate argumentation and discussion different problems occur again and again.

Parallel to the observations, new materials addressing these problems were repeatedly elaborated, tested, and improved. The basic rule for the elaboration is that the new materials are based on teaching methods of which an educational added value has already been demonstrated elsewhere in physics teaching.

The previously mentioned problem areas are addressed in the framework of this work with the educational methods of learning with worked examples, peer instruction (discussion in small groups), verbalization and visualization of formulas, and educational reduction and elementarization of physical contents and methods individually or in combinations.

This thesis describes examples of how these methods are implemented in theoretical physics and how they can be integrated into courses. Furthermore, classification and categorization of the approaches take place, and problems and challenges in the implementation and further developments are addressed. These methods are all suitable for other lecturers and universities. For example, worked examples can be integrated into exercise series without much effort, or questions with subsequent peer discussion (peer instruction) can be discussed in regular lectures.

The model of educational reconstruction can be used to develop the accompanying materials. Here, the relevant subject content (analysis of subject structure or subject clarification) and the perspective, or state, of the learners (studies on teaching and learning, as well as testing and evaluation of teaching units) are equally incorporated into the educational structuring of the new materials, which increases learning success. All materials and methods presented in this thesis are developed according to the principles of this model.

A more detailed educational reconstruction is also described in this work as an example. Here, the topic of quantum mechanical entanglement and excluding hidden local parameters is addressed. The result of the reconstruction is a course that best uses interactive and activating elements to maximally support and activate students.

In addition to the development of general activating materials, this work focuses on training student teachers in theoretical physics. Here, other areas of tension in education arise. Significant challenges arise when student teachers study a subject combination other than physics and mathematics. In all other combinations, students do not receive comparable mathematics education to that provided in the physics-only curriculum, but some of the physics course content remains the same. To support these students, the development and design of block courses that review the mathematical methods of physics prior to theoretical mechanics and electrodynamics is part of this work.

Another tension in teacher education is the apparent disconnection between theoretical and school physics. With its structure strongly influenced by formalism, theoretical physics seems to be located too far from school teaching and the physics covered there. This impression can lead students to the fundamental question: *"What do I need this for in the future?"* If no satisfying answer is found, this results in a lack of motivation.

The closure of this thesis is the development (according to the principles of educational reconstruction) of new seminars designed to emphasize the benefits of theoretical physics for prospective teachers. These seminars aim at strengthening the university content knowledge, the school knowledge, and the pedagogical content knowledge by explicitly linking these three aspects in the context of selected topics by students. The new seminars are based on the methods previously discussed.

The materials produced as part of this dissertation can be accessed and downloaded online at https://doi.org/10.18419/darus-3972. The materials are available in German.

Contents

Abbreviations

cf.	confer, compare
CK	content knowledge
CHSH	Clauser, Horne, Shimony, and Holt
DSK	in-depth school knowledge
e.g.	for example
EPR	Einstein, Podolsky, and Rosen
i.e	that is
MER	model of educational reconstruction
PCK	pedagogical content knowledge
SK	school knowledge
UK	university knowledge
viz.	videlicet, namely
WE	worked examples
z.B.	zum Beispiel

1 | Introduction

1.1 Motivation

In university teaching lecturers may get the impression that physics learning is generally difficult and that responding to the student's initial situation and needs is essential. A closer look at the situation of current physics students in Germany reveals a particular need for action. For several years, the dropout rates in German physics studies have been increasing from $26\,\%$ for the graduating class of 1999 [1] to over $50\,\%$ for the graduating class of 2020 [2].In addition, students often explain their dropout with subject-specific requirements in the physics curriculum [3]. These facts raise the question of whether parts of the current student generations are no longer suitable for studies or whether this is a false impression.

Study results by Buschhüter et al. [4] show that the subject knowledge of first-year students in 2013 decreased compared to that of first-year students in 1978. Here, the knowledge of over 2000 students was measured using an older nationwide entrance test [5, 6]. First-year students' math skills have improved or worsened in some cases over the decades depending on the subject area [7]. However, the 2013 students consistently performed worse in the physics questions than their comparison group of 1978 [4]. In addition, it has been shown that some students do not reach the required target level after reasonable progress in their studies [8]. Even worse, after five semesters, some students do not reach the level their fellow students in the year average group already had in the first semester [9].

However, these results do not prove that current students are worse than previous generations. The higher performances in some of the subject areas of mathematics prove that the students have not become worse in general. If anything, these data (looking at the ever-increasing dropout rates) show that something is amiss in the relationship between studies and students. In many cases, studies and students do no longer seem to be a good match. This has three possible consequences for universities: First, they can complain politically about the conditions and declining level. Second, they can lower the requirements of their study programs to raise the number of students back to an appropriate level. Furthermore, third, they can adapt their teaching to current students' prior knowledge and needs.

The first consequence could be a supporting factor in the long run but without guarantee. The second point should never be an option. Consequently, only the adaptation of teaching can help acutely. This work would like to contribute to this because there are different possibilities to adapt the teaching to the needs of the students.

The first point of reference for improvements in university teaching is the study by Richard Hake [10]. In 1998, the learning gains of entire cohorts (with over 6000 students) in mechanics lectures were measured at several colleges and universities. Learning gains g are defined via the knowledge x_{after} after a course and prior knowledge

x_{before} of the corresponding topic,

$$g = \frac{x_{\text{after}} - x_{\text{before}}}{1 - x_{\text{before}}}.$$

The background of this definition is the assumption that learning gain becomes more complicated when prior knowledge is high. The average learning gain in conventional courses is less than $25\,\%$. Conventional courses are lecture in front-of-class teaching, exercise series with simple problem-solving tasks as homework, and laboratory practicals with a focus on data generation and analysis.

If, however, the course format is changed to interactive elements, in which the learners are specifically asked to do something themselves and frequently receive feedback, the learning gain increases to $48\,\%$ on average even though these new formats still need to be optimized in their implementation. In no course students showed less learning growth in comparison to conventional courses. According to Hake: "*For survey classification and analysis purposes I define:'Interactive Engagement' methods as those designed at least in part to promote conceptual understanding through the interactive engagement of students in heads-on (always) and hands-on (usually) activities which yield immediate feedback through discussion with peers or instructors, all as judged by their literature descriptions*" ([10], p. 65).

Ideally, such formats stimulate learners to think at a high cognitive level, link to their prior knowledge, develop their solutions, ideas, and concepts, and explain them in the next step. When these aspects are achieved, these formats are considered cognitively activating [11–13]. Then such formats are also ideally suited to increase the quality of teaching from an educational science perspective [13,14]. Thus, the goal of reorganizing university teaching is to provide interactive and cognitively activating courses.

The 'Science education initiative' from the USA is a prime example of how courses can be redesigned on a bigger scale (two universities and seven departments) to be interactive [15]. The effectiveness of interactive teaching methods in introductory courses has also been demonstrated in Germany [16]. For the implementation of such formats in advanced courses like theoretical physics, however, there need to be more materials in German language and adapted to German university courses that are cognitively activating and interactive. At this point, this dissertation starts with the development of such materials.

The development task is to apply methods that have proven to be cognitively activating and interactive in introductory lectures or school education to theoretical physics and to produce material that other lecturers can adopt.

Only teaching methods that best fit the subject-specific requirements of theoretical physics are selected to ensure this development is as successful as possible. For example, the high degree of mathematical formulation and formalism is among the subject-specific features. Peer instruction [17], learning with worked examples [18], educational reduction [19], and verbalization and visualization [20,21] of formulas were selected as suitable teaching methods. Suppose the teaching method (e.g., reduction) is not already activating. In that case, the new material is supported by the activating element of discussion in small groups [22] or other methods.

Using the model of educational reconstruction [23], these teaching methods are adapted, and appropriate materials are developed to be used in the university teaching of theoretical physics. The developed materials can be used in lectures, practice groups, seminars, and tutorials. Later, the model of educational reconstruction for developing materials for prospective teachers is extended by an additional aspect (namely, the clarification and analysis of pedagogical content knowledge).

A further part of this thesis is the application of the developed material in training prospective physics teachers in theoretical physics. At first, the focus is on mathematical education materials for students with a second subject other than mathematics and thus receive a much lower level of mathematical education. The second central point is to link theoretical physics with school physics and pedagogical content knowledge in teacher education.

University knowledge of theoretical physics can be quickly dismissed as distant from school and irrelevant to the teaching profession later in life. However, this is a fallacy. The development of broad and deep subject knowledge is essential for the development of pedagogical content knowledge [9] and, e.g., for the explanatory performance of teachers [13]. With the material for a seminar developed here, prospective teachers should learn to be aware of the links between all three areas of knowledge (university knowledge, school knowledge, and pedagogical content knowledge) to deepen specialized knowledge in all areas. For this purpose, the model of educational reconstruction will be adapted.

1.2 Outline

Chapter 2 summarizes the theoretical and educational framing. The model of educational reconstruction, on which the developments in this thesis are based, is presented. In addition, a review of the science education research on the respective teaching methods is presented. From this state of research, consequences for the application in theoretical physics are drawn in chapter 3, and possible implementation forms are presented and classified. Chapter 4 provides an example of how to use these methods in the context of an educational reconstruction to convey complex knowledge. The chapter shows how to exclude local hidden parameters in the entanglement of quantum objects with interactive and activating teaching. Chapter 5 presents the development of math review courses to assist students with little math training before lectures on theoretical physics. Furthermore, chapter 6 presents the structure and educational rationale of accompanying seminars in theoretical physics, which link subject knowledge and pedagogical content knowledge in undergraduate teacher education.

2 | Theoretical framework

The aim of this dissertation is the development of new learning materials and environments for theoretical physics. The new learning materials shall address the most significant problems of students and support learners. Due to the personnel and time resources, the person developing the materials is the teacher and the researcher. In this case, the best research or development method is participatory action research. This development takes place in the context of the model of educational reconstruction. The following chapter describes the theoretical framework of the model of educational reconstruction, based on which new materials are developed. In addition it is explained how they were tested and observed for initial corrections and revisions.

2.1 The model of educational reconstruction

The study design of the present work is based on the model of educational reconstruction (MER). Kattmann et al. [24] introduced the model in 1997 as a framework to improve teaching and learning science [23]. This framework combines two fields in science education research to generate new teaching methods or materials. These two fields are research work restricted to issues of subject matter knowledge and empirical research trying to generate objective data on teaching and learning. At the end of the educational reconstruction, new learning environments, like instructional materials, learning activities, and learning and teaching sequences, should be designed and developed.

The MER has established itself in the educational landscape and is repeatedly mentioned as a core aspect of research in science education [25] and also in physics education (e.g., non-linear dynamics [26], general relativity [27], climate change [28], ...). The model was chosen for this thesis mainly because it focuses on a constant interrelation between input and output or between development and evaluation, which is suitable for subject development work.

2.1.1 The German interpretation of 'Bildung,' 'Didaktik' and 'Elementarisierung'

In German the MER is called 'das Modell der didaktischen Rekonstruktion'. A literal translation is 'the model of didactical reconstruction .' Because language, politics, and society influence teaching methods and educational research, specific technical terms have different traditions and meanings. The following section describes the German interpretation of the most important technical terms.

According to Duit et al., [23], behaviorist ideas had a much smaller impact on the educational system in Germany than in the USA. Instead, constructivist ideas were more important. It has become established in the research community that teaching should go beyond just presenting science content. It should be reconstructed into a content structure for instruction with consideration of additional variables such as students' pre-instructional conceptions, methods of instruction, and used media. The

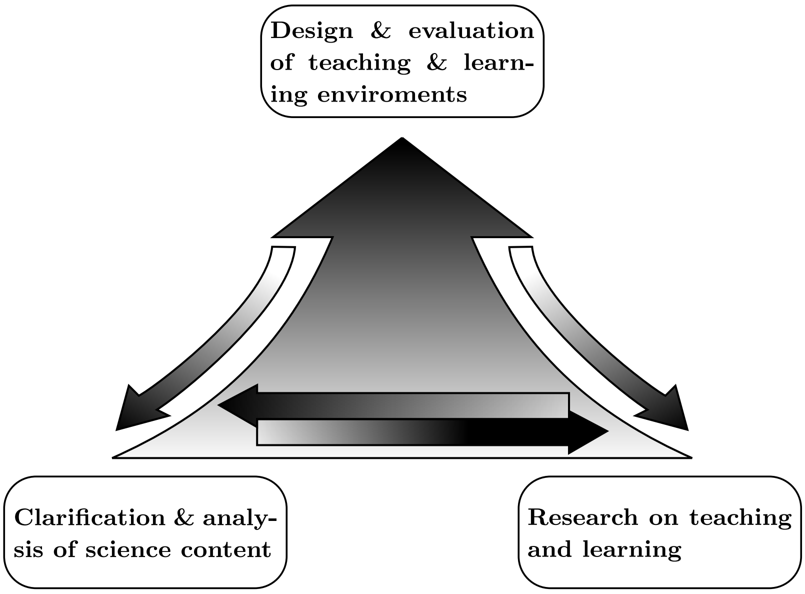


Figure 2.1. The three components of the model of educational reconstruction and their interrelations according to [23].

idea behind this is that learners must reconstruct the scientific content for themselves to understand it.

According to Duit et al. [23], the literal translation of 'Bildung' (which is formation) only includes some of the meanings of the German word in this context. 'Bildung' in German refers to the formation or training of the learner as a whole person, i.e., to develop the learner's personality. The meaning of 'Didaktik' is based on the concept of 'Bildung.' It describes the analytical process of converting domain-specific knowledge into school knowledge that contributes to the education of young people, as described above. In the German 'Didaktik,' the 'Elementarisierung' method (elementarization) is also crucial. It means breaking down a topic into its basic phenomena, principles, and general laws. After this breakdown, the topic and its effects should be reconstructed with those elements. Thus, the 'Elementarisierung' does not mean simplifying the science content. Instead, it is a delicate task of finding a balance between correctness from the scientific point of view and accessibility for students, according to Duit et al. [23].

2.1.2 The aspects of the MER

The model of educational reconstruction is classically composed of three components [24], see figure 2.1. Clarifying and analyzing science content and research on teaching and learning form the basic framework. These two components flow equally into the development of new learning materials and environments. All components are described in more detail below.

Clarification and analysis of science content

According to Kattmann et al. [24], this aspect aims at the clarification of the science content and its structure from an educational point of view. This clarification contains a critical analysis of particular topics in leading textbooks or publications to their suitability as teaching content. Ideally, the science content, processes, and views of the nature of science are included. The aim is the elementarization of science content structure to reconstruct a content structure for instruction in the next step. Therefore, science content issues and issues of students' perspectives have to be taken into account.

In general, this process is not linear but influenced by the two other aspects of the MER and a recursive procedure.

Research on teaching and learning

When a new course or new teaching material is developed, the methods used should be approved by empirical research, and students' perspectives should be drawn out. Therefore, one central aspect of the MER is a study of the teaching and learning research in this field. However, empirical research is only available on some topics. In such cases, interviews and small-scale learning studies can be helpful for researchers [29].

In this original publication, Kattmann et al. [24] called this component of the MER 'collection of pupils perspectives.' However, the MER is not only helpful in developing new teaching materials for school content. The more global approach of 'research on teaching and learning' still includes students' perspectives (e.g., their learning difficulties, interests, self-concepts, and attitudes) [23] but also research on teaching in general, instructional media and methods (maybe from related topics), and student learning.

Design and evaluation of teaching and learning environments

The last component of the MER is built on the first two. The results of subject matter clarification, the research findings on students' perspectives, and suitable teaching methods or media are incorporated into the design of the new teaching material. All components should be equally weighted in the development process. Ultimately, a teaching environment is structured by the students' specific needs and learning capabilities and stands on a solid foundation of scientific content.

Phases of testing and evaluation accompany the development. Approaches that do not resonate with students or need clarification on instructions can be sorted out or revised early in development. Therefore, the whole process can be described as recursive since all components affect each other. Ideally, the new teaching and learning environment is evaluated independently at the end of the whole process.

2.1.3 Adjustments for this development work

In its original version, the model of educational reconstruction (MER) consists of three main parts, described above. Nevertheless, the model has already been developed further. There is also a model for teacher education [23, 30] called 'educational reconstruction for teacher education' (ERTE). The authors propose an enhancement to the procedure explained in section 2.1.2. In this case, the three components are:

1. Clarification of concepts of educational structuring.

2. Research on teachers' pedagogical knowledge (PCK).

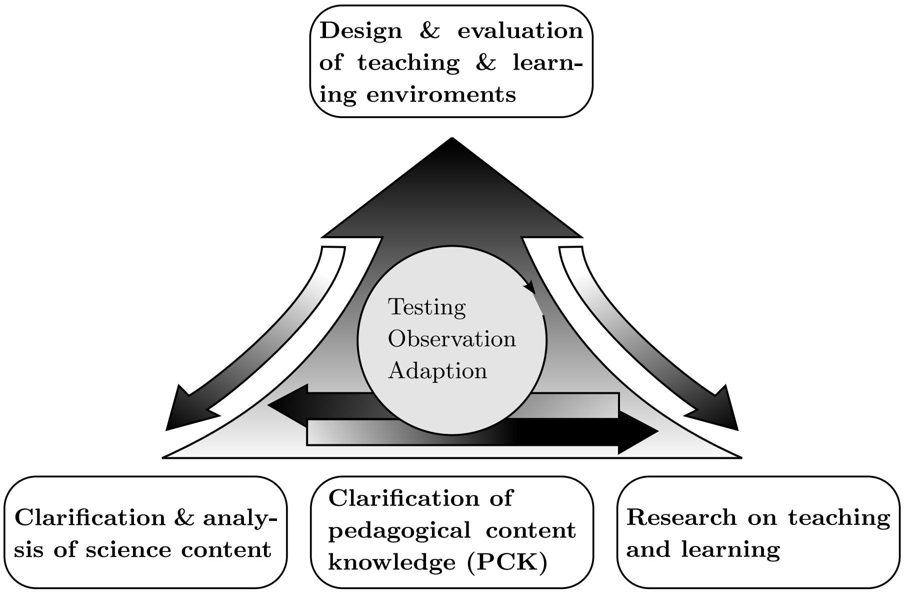


Figure 2.2. The four components of the adapted Model of Educational Reconstruction and their interrelations.

3. Guidelines for teacher education.

Component 1 includes the main ideas and results of the MER. They are extended by research on teachers' pedagogical knowledge (PCK), namely the teachers' knowledge and beliefs about students' preconceptions, representations of subject knowledge, teaching, and design of learning environments. As in the MER, these components are used recursively to develop teacher education guidelines.

Since many of the developed materials of this work are used in teachers' education, this thesis is grounded on an adapted model. The MER from section 2.1.2 is extended by the component of 'clarification of pedagogical knowledge' (PCK), see figure 2.2. Clarifying the PCK is necessary since this work focuses on theoretical physics, and there is hardly any research on suitable PCK. The material developed in this work relates to this model.

It has already been shown in quantum mechanics [31,32] that such an approach can have a positive impact on improving teacher education. A third motivation to include PCK in the MER is the impact of content knowledge (CK) and PCK on teaching quality in explaining situations [13]. Both are relevant in improving the quality of explaining. Therefore, there is a reason for three components to influence the development of teaching and learning environments in teacher education, cf. section 6.

This new component of 'clarification of pedagogical content knowledge' is not considered for every development work but in every case in which it was worthwhile.

2.1.4 Testing and observation for improvements

Of course, testing and improving various errors or inconsistencies are part of any development work. Ideally, the new materials or teaching concepts are evaluated and revised as part of the educational reconstruction. However, an extensive and as objective as possible evaluation is connected with an immense amount of work. The developers of the MER [23,24] propose interview studies to derive guidelines for redesigning learning sequences and designing learning environments. In the next step, learning processes will be investigated in teaching experiments with a few learners at a time. The learners' "ways of thinking" will be deduced and linked to the learning activities. Finally, further studies will be conducted in natural environments of science teaching. The limitations and the special design of learning processes under real teaching situations with different teachers will be considered.

Ideally, such an evaluation would also follow this development work. However, a complete evaluation as envisioned by the MER developers differs from the purpose of this thesis. The main focus is on developing new material based on clarifications of CK, PCK, and the teacher's perspective known from the literature. Nevertheless, testing and revising the materials and instructional concepts is also essential for this thesis. In order to work out guidelines for developing and revising the learning sequences, the new materials were used in different courses and tutorials and evaluated with the help of a standardized observation form. Consequently, this development work can be described as the first, and in some cases the second, cycle of participatory action research (as described by [33, 34]).

Observation as research method

Observations and participant observations have long been a scientific method in qualitative social research [35]. In systematic (scientific) observations, a distinction is made between structured and unstructured observation [35, 36]. Unstructured observations are often used to gather qualitative facts. In contrast, structured observations attempt to quantify findings in predetermined categories. Another critical factor is the role of the observer and the degree of involvement in the process. Is the observer active or passive in the process, and is it a participant observation?

In the context of this work, tutorials for theoretical physics were first observed in an unstructured way. The aim was to identify the most significant and salient problems students encountered in these lectures. Then, these problems were addressed with new learning materials and environments. These new materials were tested in tutorials and seminars and thereby observed and evaluated with a structured observation protocol by the instructor. Consequently, both cases are open and participatory observations.

The content and design of the observation protocol are based on work from chemistry didactics [37], physics didactics [38] and qualitative social research [39]. The relevant categories/questions according to which the new materials are evaluated and revised are:

1. Are the work assignments comprehensible for the students? — Task (formulation)
2. How is the work behavior of the students during the processing? — Working behavior
3. How is the communication among students and between students and teachers? — Communication
4. What is the quality of the students' final results? — Results
5. What is the teacher's subjective assessment of the learning success in general and suitability for other lecturers? — Assessment

6. Were the students cognitively activated? — Activation

The idea is to rate the material in these six categories and their subcategories (criteria) in a structured manner but also leave some space for qualitative observations and comments. Figure 2.3 shows the complete observation protocol.

Of course, these observations only partially evaluate the new learning materials. However, they do provide guidelines that allow for structured revision and improvement.

2.1.5 Comparison to other models

The model of educational reconstruction has some similarities with another model for developing learning materials and courses, which is better known in the English-speaking world. It is called 'backward design' (for an overview see [40–42]).

The principle behind this philosophy is to define the learning objectives, determine where students encounter learning challenges, and create learning environments that help learners achieve the learning objectives. These three sections can be summarized in three questions described in the science education initiative model [15].

1. What should students learn?
2. What are students learning?
3. What instructional approaches improve student learning?

These three sections compare very well with the three components of the MER. The answers to question 1 could also result from the clarification and analysis of science content as suggested by the MER. Question 2 targets the learners' current status and problems, i.e., their perspective, which is also addressed in component 2 of the MER, the research on teaching and learning. Questions 1 and 2 now influence question 3 in much the same way as in the MER. At the end of both processes are materials, courses, or learning environments oriented to the learning goals and the learners' perspective. According to both models, the new developments should be critically reviewed in various test phases and adapted or redesigned accordingly.

2.2 Status of educational research on activating teaching.

2.2.1 Learning with worked examples

Worked examples (WE) are step-by-step solutions of exercises or tasks. Well structured and installed in the learning process of novices, they improve the initial acquisition of cognitive skills compared to conventional problem-solving activities (the so-called worked example effect) [18,43,44]. Effective learning with worked examples can consist, for example, of studying several solutions and explaining them oneself before solving a similar problem without additional support (example-problem pairs [43]).

Learning with worked examples has been proven more effective than conventional problem-solving in topics such as algebra [45], statistics [46], geometry [47], in undergraduate mechanics and optics [48] as well as electricity [49], but also in text comprehension [50] and in writing of essays in English literature [51].

Observation protocol

Date	University:	Students number:	Method/Media:
Time:	Subject:	Required time:	Observer:

Criteria	Specificity				Comments
Task (formulation)	++	+	0	-	
intelligible					
extent					
practiced so far					
Working behavior	++	+	0	-	
motivation					
concentration					
diligence					
Communication	++	+	0	-	
verbal activity					
questions to the instructor					
correct use of technical language					
general debate					
Results	++	+	0	-	
technically correct					
diligence					
expectations fulfilled					
Assessment	++	+	0	-	
learning success					
suitability for other lecturers					
Activation	++	+	0	-	
general					
overdispersion					

Figure 2.3. The used observation protocol with its six rating categories and their subcategories (translation form German).

Cognitive load theory

The superiority of learning with worked examples as compared to conventional problem solving is often explained by the cognitive load theory (e.g., [52, 53]). Cognitive load can be described as the amount of working memory resources a person needs to fulfill a task. The cognitive load theory describes three types of cognitive load that are relevant during learning. The intrinsic load refers to the complexity of the learning contents concerning a learner's prior knowledge. The second type, the extraneous load, refers to irrelevant activities (in general or under certain circumstances) for learning. For example, searching for sources or new information generates extraneous load if it is not a particular requested learning goal. In well-designed worked examples, this type of cognitive load is reduced as much as possible and, therefore, a starting point for improvements. The third type, germane load, refers to cognitive resources bound by learning-relevant activities. Self-explanations of the step-by-step solution is an example of such cognitive load. In contrast to extraneous load, germane load is desired and crucial when a deep understanding of the learning contents is requested.

The worked example effect can be effective in learning because when learners solve problems at the beginning of an unknown topic, most cognitive resources are usually bound to technical aspects of solving problems other than the construction of abstract schemata, which leads to transfer and can help to solve related problems. Solving problems at the beginning is usually accomplished by a means-ends-analysis strategy. Here a substantial portion of cognitive load is needed to keep many aspects in mind, such as the current problem state, the goal state, and the differences between them, and thus is no longer available for learning processes like the construction of abstract schemata, which are crucial in theoretical physics. In contrast, learners can concentrate on gaining understanding when studying worked examples since they are freed from performance demands (cf. Ref. [54]).

These research results prove that learning with worked examples is superior to conventional problem-solving in various learning situations. So far, there are non or little systematic examples of applications in upper-division physics. However, there are no reasons why there should be no positive effect. In particular, examples from mathematics show that even complex tasks can successfully be addressed with worked examples [55]. These circumstances render them a promising tool for improving upper-division theoretical physics teaching. Thus, worked examples open a further methodical variation for the transformation of upper-division physics courses as it is pursued, e.g., in the Science Education Initiative (for an overview, see [15, 56–58]). In addition, worked examples might help students overcome typical difficulties with specific mathematical tools (see, e.g., [59–61]) when solving problems with long and complex mathematical transformations.

Designs for effective worked examples

Worked examples are not, per se, superior or more efficient in learning as compared to conventional problem-solving [62]. However, when worked examples are structured and designed such that extraneous load is decreased and germane load is increased, learners profit from them. A good design to reduce extraneous load takes several effects of the cognitive load theory into account (cf. [18, 63, 64]). These are the split-attention effect, the redundancy effect, and the expertise reversal effect. When learners have to split their attention between at least two sources of information that have been separated spatially or temporally, extraneous cognitive load is increased, and the learning efficiency is decreased (split attention effect). Information that is redundant or unnecessary has the same effect on learners' cognitive load (redundancy effect) as well as information already known by learners and stored in their long-term memory (expertise reversal effect). In order to motivate learners to study the examples and to process the information in worked examples, it is recommended to demand them to solve a similar problem after studying the worked example.

In addition to reducing extraneous cognitive load, the goal is to increase the germane load. The self-explanation effect and the studying error principle can be used for this purpose. That is, self-explanation must be actively encouraged because learners who study examples longer and explain them more actively to themselves are more successful than others [65]. However, most learners are passive or superficial self-explainers, according to Renkl [66]. This research result concludes that demanding self-explanations through instructional procedures is crucial in learning from worked examples. Self-explanation does not necessarily mean a talking-aloud procedure. There is evidence that prompting written self-explanations fosters learning outcomes [67,68]. The self-explanation effect can be increased by explaining correct and incorrect solutions [69]; especially explaining why incorrect solutions are wrong helps to avoid these errors later [70]. Correcting their mistakes in mid-term exams can help students to increase their performance in the final exam [71]. However, there is only a positive effect in finding and explaining errors in worked examples for learners with adequate prior knowledge [72]. Thus, implementing errors in worked examples too early can overwhelm weaker learners with little prior knowledge. To prohibit such overburdening weaker learners need additional support by explicitly marking errors [72] or by expert explanations and feedback on why specific steps in the solution are correct or incorrect [73].

Next to the most recommended scheme of worked examples, viz. example-problem pairs [43, 49], other approaches with more intermediate steps are possible. In many tasks, combining self-explanation prompts and fading or successively removing more and more worked-out steps in solutions can be beneficial, as Atkinson et al. [74] have shown. They found that this combination positively influences the quality of example processing for far-transfer tasks.

2.2.2 Verbalization and visualization of formulas

Multiple external representations

Multiple external representations, i.e., expressing the same thing through different representations, e.g., text, formula, and image, are essential to subject-specific communication [75, 76]. Especially in science contexts, subject-specific representations are a crucial part of communication, and thus of research and teaching [77,78]. Fundamental skills for learners in natural sciences are thus, for example, the inference of information from different representations and the interpretation, construction, and transformation of different representations [75, 79–82]. The added value of external representations can be justified by reducing cognitive resources in solving equations [83, 84]. Generally, a distinction is made between external and internal (mental) models [75,85]. A universally accepted detailed classification of external representations does not yet exist, but their value has been extensively studied [86]. Learners' challenges include switching or translating between different forms or representations [87]. Due to the often abstract formalism in lectures, the most relevant representation change for theoretical physics must be the verbalization of formulas [20, 21, 88].

Verbalization of formulas

The verbalization of formulas should be explicitly trained because students often view physical equations as nothing more than computational tools into which numbers are plugged, and results are spit out. This strongly instrumental view is also found in the literature, referred to as recursive "plug-and-chug" [89], "plug-and-chug unstructured" [90], "calculation framing" [91], "technical dimension" [92], or "formulas epistemological beliefs" [93], among other terms. It is suspected [77] that this instrumental view is also partially promoted in physics classes and in calculating tasks. A qualitative understanding is important for adequately handling formulas and their information. In educational research, there are many approaches to define what "understanding formulas" means, and there is evidence that learners have difficulties describing and reflecting mathematical expressions in their physical context [20, 94–96].

There are also some approaches to actively promote a formula understanding in courses among students and pupils [20, 77]. One approach [20] is that students can demonstrate their understanding of a formula through various activities related to a formula, such as showing an association map of the formula, describing its components in their own words, identifying special cases, applying the formula in problem-solving, and more [21]. These approaches have been taken up and extended for German-speaking countries [88, 97, 98]. The goal is to train the use of formulas with different activities. In a formula profile as a worksheet, learners can be encouraged to do activities mentioned in [20]. A questionnaire and a checklist can provide further assistance. The interpretation of a formula includes [21]

1. naming the formula symbols,
2. a unit consideration,
3. the conditions of validity,
4. drawing a graph or sketch,
5. considering special cases and limiting cases,
6. describing the physical meaning of individual terms or quotients of the formula,
7. expressing the content of the formula in own words.

The supporting questionnaire may contain the following questions [98]:

- To which physical subject area does the formula belong?
- Write down the physical quantities which occur in the formula, with their units!
- Under which conditions is the formula valid?
- Which physical process does the formula describe?
- What happens to a quantity in the formula if one decreases or increases another quantity? ("The more/less (greater smaller)..., the ..." statements)

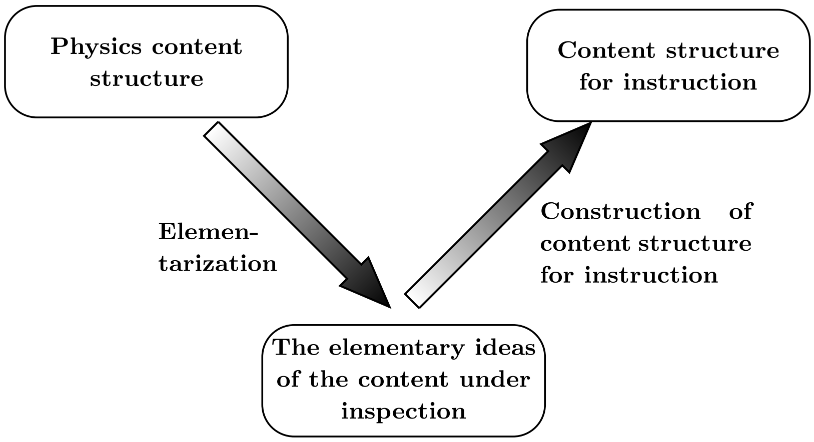


Figure 2.4. Towards a content structure for instruction in the educational reconstruction, according to [23].

- What does a graph look like that shows the relationship between two quantities in the formula?
- How was the formula introduced in class? (Does it define a new physical quantity, or was it derived with an experiment?)
- What was done with this formula in class? (For example, was something calculated with it, or was a problem from everyday life solved with its help?)
- For which applications does one need this formula?

2.2.3 Educational reduction, elementarization and reconstructions

As already described in the theoretical framework (see sections 2.1 and 2.1.1), elementarization and subsequent reconstruction for learning purposes are essential components in German-language physics education research. The general elementarization consists of educational reduction, content-related and methodological elementarization. In the following, these categories are outlined and presented. This work is oriented strongly to the summaries of [19].

Elementarization with a subsequent educational reconstruction pursues a general goal (see also figure 2.4): For the development of teaching units (or materials), the topic treated is reduced to the essential sense units and then reconstructed from them. In other words, the content structure of a particular topic is transformed into a content structure for instruction [23]. If the prior knowledge and the needs of the learners are taken into account, this reconstruction can significantly facilitate access to the new topic.

In elementarization, educational reduction (or appropriate simplification) is relevant first of all. Besides the deliberate omission of less relevant information and the concentration on the essential core statements, simplification can be achieved by experiments and pictorial and symbolic representations. Relevant for this work are especially the simplification through pictorial and symbolic representations. This relevance is given by the fact that elementarization is not only a characteristic of physics education but also of physics itself. Thus, special symbols are introduced, especially in theoretical physics, in order to be able to represent physical laws and their derivation in a simplified way. Thus, mathematical representation with such symbols is simplified in structure (with minimal signs and symbols) but is maximally informative in representing physical concepts. In addition, pictures can help to mentally process and interpret physical texts that are difficult to understand.

In parallel to the reduction, finding content-related and methodical units of sense is necessary, i.e., the essential elements of a topic. These can vary depending on the topic and the learners' level of proficiency. A subject-educational analysis of the subject structure can determine the physical elements. Content elements are fundamental concepts, principles, and guiding ideas in physics for a particular subject content.

On the one hand, methodological elementarization refers to the term "physical methods", which means the typical procedures in physics. These refer to the experimental and theoretical methods used to confirm, refute, and further develop physical hypotheses and theories. They require the greatest possible precision of the predictions derived from hypotheses and a high accuracy and reliability of the data obtained in experiments. On the other hand, methodical elementarization can also mean the decomposition into elementary teaching-learning steps.

For learners, elementarization or reconstruction can become relevant and can activate their knowledge construction if these steps are (partially) carried out by themselves. An example of this can be the creation of concept maps [99–101].

2.2.4 Peer instruction

Developed by Eric Mazur [17], peer instruction has been established in the USA for a long time and is also increasingly used in Germany in various lectures [16]. This success is unsurprising when one enumerates the various benefits and improvements [102–106] that arise as soon as learners are asked specific comprehension questions and then allowed to discuss them, namely using peer instruction.

Learners are stimulated to think about central questions of the respective topic, learn to discuss and logically argue about technical issues, can overcome known comprehension difficulties with the help of fellow students, and receive substantial feedback to assess their learning progress better. Peer instruction also offers advantages for lecturers. Within the framework of peer instruction, they receive feedback on the current level of knowledge of the learners and can enter into professional discussions with students more easily.

The basic principle of peer instruction [17] is to ask learners a question and provide multiple-choice answers. The learners' task is to understand the concept question and decide on or vote for an option of an answer. With appropriate voting systems, peer instruction becomes an activating element for all students. Everyone is asked to engage with the question and decide on an option for an answer. This joint voting is a big advantage as compared to an open question in the auditorium, where in the worst case, the usual suspects come up with an answer, and the rest does not have to deal with the question. The multiple-choice answer options and anonymous voting can lower the weaker students' inhibition threshold. Next comes the essential core element of peer instruction. Here students in small groups are supposed to convince each other of their answers or discuss the topic. Thereby students learn to discuss physically. In addition, they deal with the material even more deeply through the mutual exchange because

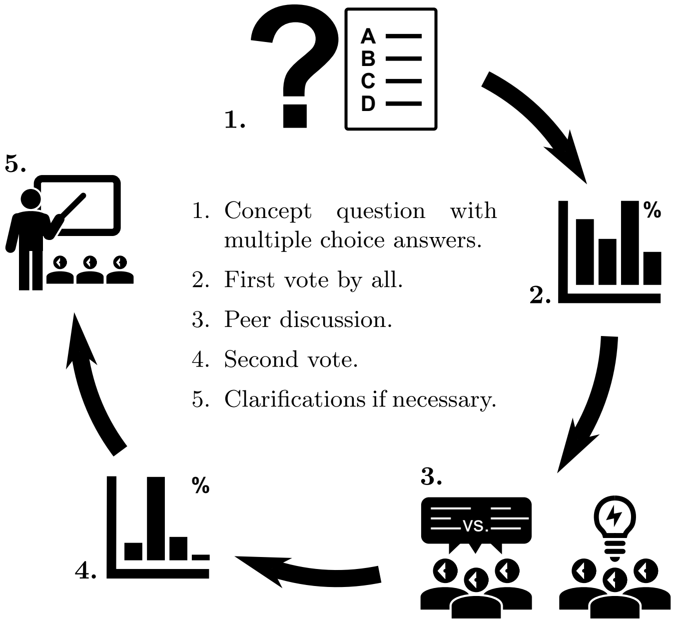


Figure 2.5. The scheme of the principle of peer instruction.

they have to understand the arguments of the others, or they have to come up with new arguments by themselves if initial persuasion attempts were not fruitful. After this peer discussion, a second vote is taken. If the question was formulated understandably and the requirement level was appropriate, the tendency in the second vote is most likely toward the correct answer. Learning progress and a tendency to answer correctly can be observed in all small groups, regardless of how students voted the first time [17]. This progress is partly due to their ability to point out thinking errors to each other. Afterward, the lecturer should again classify the technical principles and the correct answer for everyone. This scheme is illustrated in figure 2.5.

Mazur gives certain criteria for such questions. The questions should

- focus on a single concept,
- not be solvable by application of formulas,
- offer attractive multiple-choice distractors,
- be clearly worded,
- not be too easy or too difficult.

Mazur developed, revised, adapted, and sorted his questions according to these criteria [17], such questions are called *concept tests*. In addition, statistical analysis of the response behavior is an important method of verifying the criteria. In the first voting, the quota of correct answers should be $40\,\%$ to $80\,\%$. If the rate is below this span, often no discussion among learners occurs at a sufficient level. If the rate is above this span, a clear majority has already understood the concept, and a discussion phase would have little added value.

Because peer instruction has been convincing as a teaching method, it has become an export hit from physics education to other subjects such as mathematics and sciences [16, 103, 107]. A major challenge in converting traditional courses to peer instruction is the development of appropriate questions. While there is an extensive collection of questions [17] to fall back on in introductory lectures, there are hardly any questions or examples in the German language that support lectures in theoretical physics in the sense of peer instruction. However, there are some approaches to integrating peer instruction in upper-division physics (especially in quantum mechanics) in the English-speaking world (cf. [57, 108–111]).

3 | Supporting material for theoretical physics

This chapter describes the development and categorization of supporting materials using the model of educational reconstruction. Before this is done, the necessity and goals of such materials are clarified in section 3.1. Section 3.2 describes the general problems in theoretical physics observed in tutorials and accompanying seminars. The presented materials are intended to address these problems and are built on the foundation of teaching methods, of which added instructional value has been demonstrated empirically in other topics. The following sections describe the development and implementation of materials based on learning with worked examples (section 3.3), the verbalization and visualization of formulas (section3.4), educational reconstruction (section 3.5), and peer instruction (section 3.6).

3.1 Justification and objective

The elaboration and development of the materials are the first steps in a feasibility study. Which teaching methods that have proven their didactic value elsewhere can be used in theoretical physics? The newly developed teaching materials should therefore fulfill several requirements. On the one hand, the materials should activate students cognitively and stimulate self-study. On the other hand, the materials should be easy to integrate for other lecturers into their teaching or to be readily adaptable to their needs if necessary. The second step is to produce material. Over eight semesters, material on teaching methods has been developed and tested to see if they met these requirements.

Current research results on the study entrance phase and university teaching in Germany suggest the necessity of cognitive activation of students. The increasing dropout rates in physics over decades [1, 2, 112, 113] in combination with the frequent justification of subject-related difficulties [3] prove that university teaching is no longer adapted to the needs and prior-knowledge level of students in physics. The fact that the knowledge level of students has changed over the years is stressed by a study by Buschhüter et al. [4]. Students in 2013 had more difficulties solving a test than their comparison group from 1978 in almost all topics. Furthermore, many students do not reach the target higher education level after adequate progress in their studies [8]. Lectures, of course, play a significant role in increasing knowledge during studies. Richard Hake [10] was able to show (and this was confirmed for the German-speaking area in 2021 [16]) that students in classical courses show an average learning gain of only $23\,\%$ [10]. Classical courses in this context mean lectures, laboratory practicals, and the practice of arithmetic problems. Significantly better learning gains are achieved in interactive courses ($48\,\%$ on average [10]), which often ask their students to engage actively with the material themselves. Therefore, the goal for the new materials should always be to activate students for self-study and to promote engagement with the essential subject contents.

3.2 General problems to be addressed

Possible reasons for the high dropout rates and missing the target level may be the following five problems. These are general problems that can occur in theoretical physics. A correlation has not been investigated scientifically. Nevertheless, addressing or remedying these problems can improve the quality of teaching. The material developed as part of this work aims to do just that. Moreover, observations at the Stuttgart and Jena campuses show that students struggle with these problems.

1. **Lectures can just be "consumed" without active participation.**
   When lectures are structured according to the principle of front-of-class teaching, it can become a problem if the students do not actively participate [10]. However, such participation is not specifically demanded in all lectures. Furthermore, when students are prompted to ask questions, in most cases, only a small number does so or has the courage to contribute verbally. Also, if lecturers actively call on individual students to answer questions, only a few learners can be reached in a lecture. Just consuming the lecture does not have to be a sign of laziness. When students are solely consumed with copying and comprehending the content or comments of the lecturer, they do not have the time or mental resources to respond to questions or formulate their own.
   This problem can be observed in the accompanied lectures. According to students' statements, they are often challenged or overwhelmed by the content and the lectures' pace. "I did not understand that in the lecture." is a typical statement repeatedly expressed in this or a similar way. In addition, students report time difficulties in preparing for and following up on lectures; "I have not had time to look at this again." or "I have to repeat this before the exam." are frequently mentioned problems. Although students are motivated to ask questions in all lectures and accompanying events, only a few students do so due to the content-related backlog when reviewing or copying from the blackboard.

2. **Students themselves "speak" too little about physics.**
   As mentioned in the point above, even if there is the possibility to ask and answer questions in lectures, not everyone can use this offer. The same is true for exercise groups. When homework is discussed, the amount of students talking is also meager. Homework from the previous week is often presented only by individual students (again, front-of-class teaching). Questions are also possible and encouraged here, but often they are technically mathematical. In addition, there is often no time for questions and discussions on the physics topics behind the task since simply discussing the solution can take up the entire time of the exercise group. So students only talk or discuss physics themselves during study group periods when they are doing assignments. Feedback is given in the exercise groups only, and it is doubtful whether a learning effect can be expected with such a time lag.
   There was the possibility to ask questions in all courses, but not everyone did use this offer. Even in accompanying courses offered by third parties (which are not authorized to examine), very few questions are asked at the beginning. Conversely, only a small proportion of students respond to questions from the instructor on their initiative. And direct prompts to answer questions are perceived as discomforting: "I would rather say nothing than something wrong!"

3. **Lecture modules and exams allow for compartmentalization without interconnecting the different physics topics.**
   Only the specific examination of a course is relevant for passing academic studies, which are structured in modules. After passing an exam, the course contents are considered to be understood and internalized. For students, there is thus hardly any external incentive to review the contents of passed modules for new courses or to check them for similarities.
   This problem is not aware to every student. Nevertheless, it can be observed. A prime example is the dependency on forces, fields, and potentials. A topic that is always part of the mechanics lecture. However, many students cannot make use of this knowledge in the next semester in electrostatics and magnetostatics. In addition, mathematical methods covered in other lectures are often not directly present. In a short test on mathematics from [114] (complex numbers, vector calculation in 3D, 2×2 matrices, a simple Delta-function and one homogeneous differential equation) at the beginning of quantum mechanics not half of the problems was solved correctly on average.

4. **Often, exercises take more time on math problems than physics problems.**
   In exercises, mathematical techniques are a hurdle in themselves. This challenge becomes even more drastic if there is no mathematical training parallel to the study of physics due to the choice of subject in the teacher training program. Even if mathematical lectures are in the physics curriculum, these only cover some relevant knowledge or their knowledge needs to be regularly trained in other lectures. Thus, students are forced to self-study. The time required here comes at the expense of the student's engagement with the physical content. According to the principles of cognitive load theory, the mental load of finding mathematical content that has never been covered before should be immense. However, students with more mathematical training could also have a high load when they have to search for mathematical solutions.
   At the University of Stuttgart and Friedrich Schiller University Jena, the courses "Mathematical Methods in Physics" in the first semester are for teaching students without mathematics as a second subject the only mathematical training in their studies. This limited mathematical education leads to problems when mathematical formalism becomes increasingly important in theoretical physics. Addressing these problems is appreciated: "Converting these math-heavy formulas into a picture has helped me a lot." or "The statement of what Green's function describes made the whole subject more vivid for me."

5. **Ungraded semester credit may be earned if practice assignments are just copied.**
   At German universities written submissions and voting tasks as ungraded semester performances are common, i.e., tasks and their solutions are handed in for evaluation or presented by students in the exercise groups. In many cases, the students are expected to take responsibility for their work. The tasks may be worked on in groups. The individual group members' efforts cannot be adequately checked in written submissions or presentations. Written submissions by group solutions are inevitably duplicated, and the presentation of exercises are spot checks that students can actively approach. The more students are over-

loaded in such a system, the more attractive the method of copying solutions becomes to them. The demand for personal responsibility is justified regarding an exam in which the independent development of solutions is the best preparation. In oral examinations, however, the written processing of arithmetical problems is of little relevance. Even in written exams, a lack of preparation can lead to a pass if the level of an entire course is not appropriate.

Since exercise groups were not directly supervised or observed, no statements can be made on this.

3.3 Learning with worked examples

The main focus in this subsection is on the presentation (section 3.3.3) of a four-step approach that was developed as part of this thesis. This is intended to provide other faculty with a framing in the implementation of worked examples in their own teaching. In preparation for the development of this approach, general conclusions about the use of worked examples in theoretical physics are drawn from literature (section 3.3.1) and clarification is provided as to why Lagrangian mechanics is a prime example of the use of this approach (section 3.3.2). The observations and classifications that led to the development of the four-step approach are then enumerated (section 3.3.4). In addition, suggestions are made as to how this approach can be used concretely in teaching and in which subject areas further application would be useful. The conclusions contain a small outlook on additional designs for worked examples that were tested (section 3.3.5).

3.3.1 Conclusions for the design of worked examples in theoretical physics

To be most efficient in learning scenarios worked examples must be designed according to the effects and principles described in section 2.2.1. However, the more complex the problem is the more complex it is to take account of all these effects in the given solution. This relation leads to a special challenge in theoretical physics since it requires a broad spectrum of previous knowledge and skills (mathematically and physically). However, this does not mean that worked examples are not applicable. Santosa [55] implicated that worked examples can increase the learning efficiency in complex tasks of calculus such as higher-dimensional integrals, exactly the type of mathematics many physics problems demand to get solved. This result provides a strong motivation to extend this approach to exercises in theoretical physics.

In addition to physical principles (e.g. laws, boundary conditions, etc.), mathematical challenges (such as differential equations, differential & integral calculus, linear algebra and so on) also increase the difficulty of physics tasks. When students are not well versed in mathematical techniques they have to interrupt their thoughts on the intended problem with a search for (mathematical) solution methods. A search which can lead to other textbooks or lectures as sources of information creating extraneous load due to the split-attention effect. Thus, the split attention effect is most crucial in the applications we have in mind to reduce extraneous cognitive load. Reducing the spatial split attention effect in such complex exercises is challenging due to the amount of information. Nevertheless, it can be minimized to a certain degree by highlighting important information, correlations or dependencies in the problem. In addition, offering a step by step solution of a worked example should reduce the temporal split attention effect.

Since students of theoretical physics are no total novices in problem solving, our conclusion is mainly to foster self-explanations to increase germane cognitive load in the continuative examples. One possibility is to request written explanations for certain or all steps in the solution from students [67, 68]. Another possibility is to extend the examples by certain comprehension questions students are requested to answer. When students have shown adequate problem solving skills or a broad prior knowledge, finding and correcting errors should improve the effects of self-explanation even more.

3.3.2 Clarification of Lagrangian mechanics as a paradigm for worked examples in theoretical physics

The next section provides a paradigm of how worked examples could be implemented in theoretical physics. In every topic in which there is a universal solution structure worked examples are applicable. To illustrate this approach, the topic of Lagrangian mechanics of the second kind is chosen. The solution structure behind various application examples is itself a prime example of solution structures and therefore predestined to illustrate "worked examples".

1. Set up the holonomic constrains and calculate the degrees of freedom S.
2. Define generalized coordinates q_i according to the holonomic constrains.
3. Set up the kinetic (T) and potential (V) energy.
4. Set up the Lagrangian.
5. Determine the equations of motion for every generalized coordinate.
6. Reduce equations if possible.

In addition, Lagrangian mechanics is usually the first exposure of university students to a new concept in theoretical physics, and thus additional support is appropriate here. The examples used in this paradigm can be found in modified form in textbooks such as [115, 116]. An English version of the first book is also available [117].

3.3.3 Developed design: The four-step approach

The four-step approach provides a blueprint for how worked examples can be used in theoretical physics. The four steps here refer to the level of support provided to students in the process, scaling back with each level. The advantages of this approach are that classical computational problems, as they are traditionally treated in exercise series, can very easily be integrated into it and that individual stages can be skipped, depending on the performance level of the students.

The first step in our scheme is a maximally elaborate worked example with a detailed step-by-step solution. After this preparation we foster self-explanations in step two by demanding written explanations from the students along to a given mathematical solution. In step three this is extended by finding and fixing errors with an explanation.

Step four consists of a problem without any solution and it is the student's task to develop and provide a full solution.

The whole program fits into a 90 minute presence exercise course or can be dealt with by students at home. All examples are typical text book problems and can be adopted easily.

Step 1: Maximally elaborated solution - Pendulum on springs

This first step in the scheme should be a maximally elaborated worked example with a detailed step-by-step solution. The split attention effect is reduced by highlighting the important aspects in the problem description. Explanations are given right beside each task of the solution procedure. Mathematical explanations should be given where necessary depending on the mathematical capabilities of the learners.

In the context of Lagrangian mechanics the pendulum on springs (see figure 3.1) was chosen as first example since it contains many typical problems of text book exercises in Lagrangian mechanics. Thus, the detailed solutions of this problem presents the learners a broad overview. Students can learn how to deal with two coupled masses and two types of potential energy. For novices in Lagrangian mechanics we recommend a very detailed solution for example by explicitly naming all dimensions and reasoning with holonomic constraints the irrelevance of the z component. If the students are already familiar with holonomic constraints, this part is redundant and should be excluded from the worked example.

Exercise: Set up the Lagrangian (or Lagrangian function) and determine the equations of motion. Reduce them as much as possible.

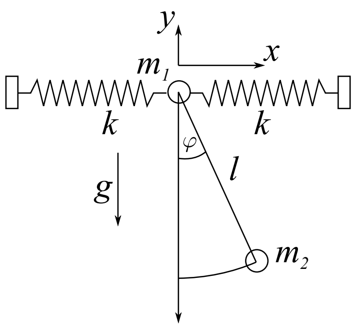


Figure 3.1. Sketch of the two masses of example 1. Mass m_1 is clamped between two springs and m_2 is a pendulum fixed to m_1 in a gravitational field.

A point mass m_1 is attached via two springs with the same spring constant k between two walls. The equilibrium point of the springs corresponds to the center position of the point mass between those walls. The restoring force is $|\mathbf{F}| = k|\mathbf{x}|$. Mass m_1 is only able to move horizontally along the x-axis. A second point mass m_2 is attached to m_1 via a massless rod with length l. The second point mass can oscillate in the xy-plane under influence of a homogeneous gravitational field with force $\mathbf{F_G} = -m_2 g \mathbf{e_y}$. The angle of twist is given by φ (see figure 3.1). The small-angle approximation applies.

Set up the holonomic constrains and calculate the degrees of freedom.	
$z_1 = 0,\ z_2 = 0$ $y_1 = 0,\ (x_1 - x_2)^2 + y_2^2 = l^2$	The masses do not move in the z_1, z_2 and y_1 coordinate. The x_2, y_2 components of mass 2 depend on x_1 and the rod's length l. The dependency is defined by the Pythagorean theorem.
$S = 6 - 4 = 2$	Two free particles have six degrees of freedom (2 times the 3 spatial dimensions). Since there are four constraints the total number of degrees of freedom reduces down to two.
Define generalized coordinates q_i according to holonomic constrains.	
$q_1 = x_1 = x, \qquad q_2 = \varphi$	A wise choice for the generalized coordinates is the x position of mass 1 and the angle of deflection.
$x_1 = x, \quad x_2 = x + l\sin\varphi$ $y_1 = 0, \quad y_2 = -l\cos\varphi$ $z_1 = 0, \quad z_2 = 0$	The coordinates are chosen such that mass m_1 is in the center in the equilibrium state. The transformation between Cartesian and the generalized coordinates is helpful for setting up the Lagrangian.
$\dot{x}_1 = \dot{x}, \quad \dot{x}_2 = \dot{x} + l\dot{\varphi}\cos\varphi$ $\dot{y}_1 = 0, \quad \dot{y}_2 = l\dot{\varphi}\sin\varphi$ $\dot{z}_1 = 0, \quad \dot{z}_2 = 0$	The derivative of time delivers the required velocity. Therefore the time dependency of all variables and the chain rule must be taken into account.
Set up the kinetic T and potential V energy.	
$T = \frac{m_1}{2}\dot{x}_1^2 + \frac{m_2}{2}\left(\dot{x}_2^2 + \dot{y}_2^2\right)$ $= \frac{m_1}{2}\dot{x}^2 + \frac{m_2}{2}(\dot{x}^2 + l^2\dot{\varphi}^2$ $+ 2l\dot{x}\dot{\varphi}\cos\varphi)$	The overall kinetic energy is the sum of all velocity components. y_1, z_1 and z_2 contribute nothing since there is no motion in these directions.
$V_1 = \frac{k}{2}(-x)^2 + \frac{k}{2}x^2 = kx^2$ $V_2 = -m_2 g l\cos\varphi$	The potential energy of the first point mass is the energy stored in the two springs. The second mass swings in a homogeneous gravitational field.
Set up the Lagrangian.	
$L = T - V$ $= \frac{m_1 + m_2}{2}\dot{x}^2 + \frac{m_2}{2}l^2\dot{\varphi}^2$ $+ m_2 l(\dot{x}\dot{\varphi} + g)\cos\varphi - kx^2$	The Lagrangian is the overall kinetic energy minus the potential energy.

Determine the equations of motion for every generalized coordinate.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0$$
$$= \frac{d}{dt}\left[(m_1 + m_2)\dot{x} + m_2 l\dot{\varphi}\cos\varphi\right] + 2kx$$
$$= (m_1 + m_2)\ddot{x} + m_2 l\ddot{\varphi}\cos\varphi - m_2 l\dot{\varphi}^2\sin\varphi + 2kx$$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\varphi}} - \frac{\partial L}{\partial \varphi} = 0$$
$$= \frac{d}{dt}\left[m_2 l^2\dot{\varphi} + m_2 l\dot{x}\cos\varphi\right] + m_2 l(\dot{x}\dot{\varphi} + g)\sin\varphi$$
$$= m_2 l\ddot{\varphi} + m_2\ddot{x}\cos\varphi + m_2 g\sin\varphi$$

Via the Euler-Lagrange equations the equations of motion can be determined. To do so, only the derivatives must be calculated in the correct order.

Reduce equations if possible.

$$(m_1 + m_2)\ddot{x} + 2kx = m_2 l(\dot{\varphi}^2\varphi - \ddot{\varphi})$$
$$l\ddot{\varphi} + g\varphi = -\ddot{x}$$

The approximation of small angles ($\sin\varphi \approx \varphi$, $\cos\varphi \approx 1$) simplifies expansions of trigonometric functions and reduces the equations of motion.

These two differential equations are strongly coupled and as the term $m_2 l\dot{\varphi}^2$ indicates not even linearly coupled but quadratically. It is not trivial to solve such equations, but it is also not part of this exercise.

Step 2: Fostering self-explanations - Mass in a cone in a gravitational field

In this example, students are encouraged to formulate their own explanations. A simpler example is sufficient for this task since the self-explanation is in the focus. This helps the students to get started.

In Lagrangian mechanics the problem 'Mass in a cone in a gravitational field' (see figure 3.2) was chosen as second example. The problem is not as difficult as example 1, since there is only one mass and one potential energy.

Exercise: Set up the Lagrangian and determine the equations of motion. Reduce them as much as possible.

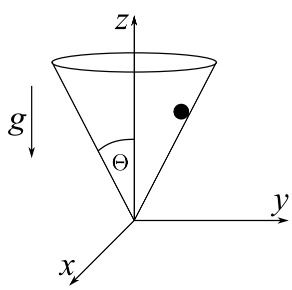


Figure 3.2. Sketch of example 2. A pellet moves under influence of gravity on the inner surface of a cone with an aperture angle Θ.

On the inner face of an upwards opened cone there is a pellet with mass m. The cone has an aperture angle of 2Θ and the pellet can move frictionless on its inner surface. It is under the influence of the gravitational force $\mathbf{F_G} = -mg\mathbf{e_z}$. The axis of the cone is identical with the z-axis and its top is located in the origin, see figure 3.2.

Set up the holonomic constrains and calculate the degrees of freedom. $\tan\Theta = \frac{\rho}{z} \quad \Leftrightarrow \quad z = \rho\cot\Theta$ $S = 3 - 1 = 2$
Define generalized coordinates q_i accordant to holonomic constrains. $q_1 = \rho, \qquad q_2 = \varphi$ $x = \rho\cos\varphi,\ y = \rho\sin\varphi,\ z = \rho\cot\Theta$
Set up the kinetic T and potential V energy. $T = \frac{m}{2}[\dot{x}^2 + \dot{y}^2 + \dot{z}^2]$ $= \frac{m}{2}\left[(1 + \cot^2\Theta)\dot{\rho}^2 + \rho^2\dot{\varphi}^2\right]$ $V = mgz = mg\rho\cot\Theta$
Set up the Lagrangian. $L = T - V$ $= \frac{m}{2}\left[(1 + \cot^2\Theta)\dot{\rho}^2 + \rho^2\dot{\varphi}^2\right]$ $- mg\rho\cot\Theta$
Determine the equations of motion for every generalized coordinate. $\frac{d}{dt}\frac{\partial L}{\partial\dot{\rho}} - \frac{\partial L}{\partial\rho} = 0$ $m(1 + \cot^2\Theta)\ddot{\rho} - m(\rho\dot{\varphi}^2 - g\cot\Theta) = 0$ $\frac{d}{dt}\frac{\partial L}{\partial\dot{\varphi}} - \frac{\partial L}{\partial\varphi} = 0$ $m(\rho^2\ddot{\varphi} + 2\rho\dot{\rho}\dot{\varphi}) = 0$
Reduce the equations if possible. $(1 + \cot^2\Theta)\ddot{\rho} - \rho\dot{\varphi}^2 + g\cot\Theta = 0$ $\rho^2\ddot{\varphi} + 2\rho\dot{\rho}\dot{\varphi} = 0, \qquad \rho \neq 0$

Step 3: Finding and fixing errors - Two masses on a wedge coupled by a spring

To foster self-explanation the third example has errors implemented. This task should be more challenging than step 2, therefore the chosen example should be more difficult again. In this approach for Lagrangian mechanics an example with two generalized coordinates (figure 3.3) was chosen in a way that the errors cannot be found easily. In the incorrect solution a holonomic constraint is wrong. This error was chosen because students often have problems with the transition between mathematics and physics, but it does not influence the further solution. The second error is located in the potential energy of the spring. Only the extension or compression compered to the stress-free length is relevant for the potential energy. The missing stress-free length in the potential is a typical mistake students make and is easily overlooked in a given solution.

Exercise: Set up the Lagrangian and determine the equations of motion of the problem given in Fig. 3.3.

Two masses m_1 and m_2 move on a wedge. There is no friction but a gravitational field given by the force $\mathbf{F_G} = -m_{1/2}g\mathbf{e_y}$. The masses are connected via a massless spring with a spring rate k and a stress-free length l.

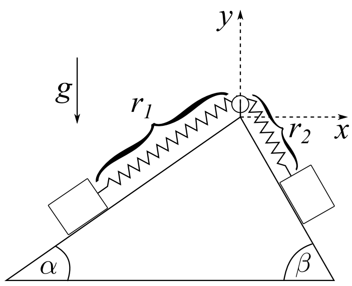


Figure 3.3. Sketch of example 3. Two masses on a wedge connected via a massless spring in a gravitational field.

Set up the holonomic constrains and determine the number S of degrees of freedom

$$z_1 = 0, \qquad z_2 = 0, \qquad \frac{y_1}{x_1} = \tan\alpha, \qquad \frac{y_2}{x_2} = \tan\beta$$

$$S = 6 - 4 = 2$$

Define generalized coordinates, which conform to the holonomic constrains

$$q_1 = r_1, \qquad q_2 = r_2$$

Transformations

$$\begin{aligned} x_1 &= -r_1 \cos\alpha, & x_2 &= r_2 \cos\beta \\ y_1 &= -r_1 \sin\alpha, & y_2 &= -r_2 \sin\beta \\ z_1 &= 0, & z_2 &= 0 \end{aligned}$$

Set up the kinetic T and potential V energy

$$\begin{aligned} T &= \frac{m_1}{2}\dot{r}_1^2(\cos^2\alpha + \sin^2\alpha) + \frac{m_2}{2}\dot{r}_2^2(\cos^2\beta + \sin^2\beta) \\ &= \frac{1}{2}(m_1\dot{r}_1^2 + m_2\dot{r}_2^2) \end{aligned}$$

$$V = -m_1 g r_1 \sin\alpha - m_2 g r_2 \sin\beta + \frac{k}{2}(r_1 + r_2)^2$$

Set up the Lagrangian

$$L = T - V = \frac{1}{2}(m_1\dot{r}_1^2 + m_2\dot{r}_2^2) + m_1 g r_1 \sin\alpha + m_2 g r_2 \sin\beta - \frac{k}{2}(r_1 + r_2)^2$$

Determine the equations of motion for every variable

$$\begin{aligned} \frac{d}{dt}\frac{\partial L}{\partial \dot{r}_1} - \frac{\partial L}{\partial r_1} &= 0 & \frac{d}{dt}\frac{\partial L}{\partial \dot{r}_2} - \frac{\partial L}{\partial r_2} &= 0 \\ m_1\ddot{r}_1 - m_1 g \sin\alpha + k(r_1 + r_2) &= 0 & m_2\ddot{r}_2 - m_2 g \sin\beta + k(r_1 + r_2) &= 0 \end{aligned}$$

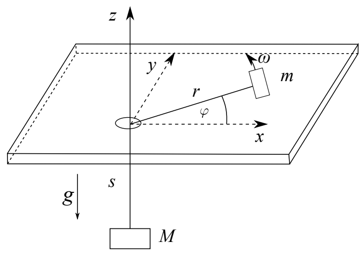


Figure 3.4. Sketch of example 4. A rotating mass on a tabletop is pulled towards a hole by a second mass in a gravitational field.

Step 4: Student's task - Rotating mass on a tabletop connected to a second hanging mass

Since learners take the self explanation more serious when they have to solve a familiar problem by their own, the last step is the most important. Here a task is suitable which contains again all typical elements but is not too easy or has a transfer part. In this scheme for Lagrangian mechanics (figure 3.4) a rotating mass on a tabletop connected to a second hanging mass as fourth example is appropriate since many elements of the first three examples can be used in this solution. The coupling of two masses can be found in examples 1 & 3, the rotary motion in example 2. However, the combination of both effects did never appear. Thus, there is a transfer students have to master solving this problem.

Exercise: Set up the Lagrangian and determine the equations of motion of the following setup. Reduce them as much as possible. A mass m rotates without friction on a tabletop. The mass is connected to a second mass M by a string of the length $l = r + s$. The second mass is below the tabletop and is pulled downwards by the influence of the gravitational force $\mathbf{F_G} = -Mg\mathbf{e_z}$.

3.3.4 Testing and enhancements

Observations

The following list is the summary of the most striking and important observations that led to, or evaluate, the four-step approach.

- **For physics problems the first step with a detailed solution is needed.**
  In previous tests of worked examples in Lagrangian mechanics of the first kind only a mathematical solution without explanations was delivered. Since the observed third semester students are no total novices to problems in physics, additional explanations were expected to be redundant. But this approach overstrained the students and no learning progress was observed.

- **Different notion can lead to confusion.**
  When existing problem sets are used for the worked examples, the notion should be the same in every example, otherwise extraneous cognitive load can be increased and the worked examples lose their efficiency.

- **The level of student ability is very critical to the pace of progress.**
  Students with a lot of prior knowledge and commitment can master the first

two steps of the described approach much faster than their fellow students. The observed performance spectrum can be very large and should be taken into account by lecturers. It may be helpful to provide additional assignments for the high-performing students or to provide varying levels of assistance to match the performance level.

- **All students, regardless of prior knowledge, can be activated.**
  According to the students' own statements, all of them, regardless of prior knowledge and progress in the seminar, felt cognitively activated by the work assignments in the four-step approach. The work behavior was concentrated and careful. The results achieved were technically correct to a large extent.

- **The errors in step 3 should differ in their type.**
  There should be 2 to 4 errors, so that the students continue to search after the first finding but the wrong solution does not become too confusing. The type of errors should depend on the learning objectives of the instructor. In the example above the focus is on physically incorrect terms but also miscalculations are possible to increase the number of errors. In general, errors in the translation of the physics problem into its mathematical description, one calculation error for a quick boost of accomplishment and errors that are typical for the used kind of problems can be recommended. It can be helpful to skip the third step in one year and to search in student solutions for patterns in their mistakes.

Embedding in instruction

A beneficial side effect of the self-explanation fostering variants are their use for lecture certificates or ungraded semester performances. In some physics study programs students have to solve a certain amount of problems every week to obtain an admission to the exam. This practice is justified with the training students get by solving problems as a preparation for the exam and it is meant to be a protection for students not to take an exam il-prepared. As mentioned above worked examples can be superior to problem solving. This is why we suggest to exchange the often used problem solving exercises by worked examples which demand self-explanations or finding and fixing errors.

However, if the students do not solve all problems by themselves and a rating of their work is required, the rating system has to be adapted. Instead of taking just the number of correct solution steps or solved problems into account, it is possible to establish a rating on the steps of the worked example scheme presented above. The quality of the students explanations in the self-explanation tasks as well as the number of detected errors in the error finding problem can legitimate the lecture certificate. In particular, the number of problems that can be utilized for the rating does not change much. We have had good experiences by exchanging two problems without assistance with four worked examples of the types described above.

The scheme presented does not need to be rigidly adopted by other instructors. Most important are the presentation of the solution structure (step 1) and the student's task (step 4). The steps in between can help to ease the transition from understanding a solution to producing a solution yourself. Therefore, this four-step approach can also be split or modified and adapted to time and structural constraints. While the four-steps approach filled 90 minutes in our case, the four steps can be split up due to time restrictions into consecutive seminars (down to 45 minute seminars). The time constraints and the level of the students must be weighed here. It is conceivable to outsource steps 1 and 2 in the homework or to omit one step. For teaching students we once skipped step 2 in favor of a educational introduction of worked examples and cognitive load theory.

Applicability of the structure in other topics

In every topic were there is a universal solution structure to problems similar to that in Lagrangian mechanics our presented scheme and worked examples in general are applicable. For example solving the time-independent Schrödinger equation

$$\hat{\mathrm{H}}\,|\Psi\rangle = E|\Psi\rangle \tag{3.1}$$

with a solution structure like:

1. Determine the dimension of the Hilbert space and set up the Hamiltonian according to the physics problem.
2. Set up the eigenvalue equation.
3. Determine the eigenvalues.
4. Determine the eigenstates according to their eigenvalues.
5. Check for boundary conditions.

Since physics problems occur in well structured domains, there are many topics suitable for worked examples. Just to inspire the readers mind we want to mention a few examples among many others. There are calculations with work integrals, or inertia tensors, Noether's theorem and Hamiltonian mechanics in classical mechanics. In quantum physics besides the time-independent Schrödinger equation there are perturbation theory and the determination of Clebsch-Gordan coefficients. In classical electromagnetism many solution methods of Poisson's equation like the method of image charges or the multipole expansion are suitable for worked examples as well as are calculations for thermodynamic cycles or the determination of partition functions in statistical mechanics.

3.3.5 Other designs and approaches

The design described in the previous subsection refers to how worked examples can be used in classical computational tasks with a specific computational scheme. However, the cognitive load theory proves that also learning in more qualitative environments can be supported with worked examples (e.g. text comprehension and essay writing [50, 51]), and thus they can be used in other aspects of physics. For example, in theoretical physics, worked examples can be used to derive formulas and solutions to link content knowledge from different topics and to train technical language.

The problem completion effect

Problem completion is a form of learning with worked examples that has already been tried and tested in complex tasks such as programming. Students are given the beginnings of a solution and must explain and understand it for themselves before solving the remaining parts. This type of worked examples is predestined for fields in physics where a solution or derivation only needs to be or can be done once. For example, in the derivation of the wave equation in vacuum from Maxwell's equations or the electrodynamic potentials. The attempt to link this with other didactic methods such as the verbalization of formulas also proves to be cognitively activating according to observations and is described later in section 3.4.

Worked examples for the introduction of unknown methods

As described in the following sections, it is also the aim of this thesis to test other educational methods in the context of theoretical physics. New methods are always a challenge for students and it takes some time to adapt. Consistent training of the technical language or the summary and reduction of physical contents and methods can overstrain the students, if this was not practiced before. However, this overload can be easily prevented if students work through a worked example at the beginning of a new unit and explain it to themselves. Many of the methods presented below, therefore, include elements of learning with worked examples.

3.4 Verbalization and visualization of formulas

The high degree of mathematization in theoretical physics makes an adequate handling of formulas indispensable. In this section, methods and approaches are presented to motivate students to actively deal with the verbalization and visualization of formulas.

3.4.1 Conclusions for theoretical physics from literature

It has been shown that it is possible to implement the methods discussed in Section 2.2.2 at the university level [20, 77]. The approach of different activities to deal with formulas seems to be suitable, if not predestined, for theoretical physics as well. The high proportion of formulas in theory lectures requires students to quickly deal with different mathematical representations in different situations and to gather information from them. A broad set of methods should be a clear advantage here. The tasks from section 2.2.2 should only be adapted to the age and level of the students. Thus, depending on the topic, more items can be added or removed in the questionnaire and checklist if physics or formalism require it. To name all occurring formula signs belongs to the standard and should also be demanded in theoretical physics (partly even this becomes a challenge for students, see section 3.4.3).

A unit approach can be useful in many places to generate new knowledge for students. As an example, consider the Langrange multipliers for constraint forces, which adapt to the respective direction vector. For example, in cylindrical coordinates two constraint forces could have the definitions of $\vec{F}_1 = \lambda_1 \vec{e}_z$ and $\vec{F}_2 = \lambda_2 \frac{1}{r} \vec{e}_\varphi$. In order to keep the unit of a force, the Langrange multiplier must adapt. For other subjects, the effort for a unit consideration can get out of hand and the benefit is no longer in proportion to the effort. On the other hand, the validity conditions should be checked for each formula just to make students aware of the limitations of the model over and over again.

Graphs and sketches are not always easy to find and suitable for every topic. Especially in quantum mechanics sketches are difficult because not always an equivalent representation to the formalism can be found. However, there remains the possibility to symbolize processes or states with logical pictures and thus to use different external representations. Graphs can also be helpful wherever they are possible without complex numerical calculations. Especially in the case of the wave function, for which there is no visual representation, graphs can be used to represent it or to show its course. Dealing with graphs in several dimensions can be difficult and should, therefore, be trained explicitly. As an example, consider the propagation of waves in space and time.

Special and boundary cases should always be reviewed by students if possible. Especially in cases in which exact plots can only be achieved with complex calculations, the consideration of special and borderline cases can provide first information about progression of the function of individual quantities.

The physical meaning of individual terms and quotients also remains relevant. Due to the presumably longer formulas and terms, this part becomes even more important for a better overview. In particular for equations, which can be solved only with approximations, the estimation about progression of individual terms is of great importance: Which terms dominate in which boundary cases and which meaning do they have (example: dipole radiation in the near and far field.)?

Of course, a summary in the students' own words remains important and the goal of the actual exercise. At the end, however, the embedding of the formula in a concrete context can be helpful. One goal of theoretical physics is finally the generalization and unification of physical laws. In order to understand these laws and to fill them with life, it can be a task for students to think about concrete application purposes and areas. Thus the whole checklist used in this work reads as follows. The interpretation of a formula includes [21]

1. naming the formula symbols,
2. (a unit consideration,)
3. the conditions of validity,
4. a graph or sketch,
5. consideration of special cases and limiting cases,
6. a description of the physical meaning (and/or the mathematical origin) of individual terms or quotients of the formula,
7. an expression of the content of the formula in own words,
8. an example or application of this formula in physics or every day life.

The list of questions according to [98] can be supplemented by the following questions:

- From which formulas/conditions was this formula (or individual parts) derived?
- What is (which quantities are) quantized here?
- Does a consideration in one dimension help?

3.4.2 Designs and material

Work in small groups

The checklists (and the list of questions) proposed and extended by Bagno et al. [20,98] can be handed out to students in seminars or exercise groups as a support with the work assignment to verbalize/visualize various formulas – in short to translate them. Depending on time and previous knowledge the approaches can be different. In a 90 minute seminar, for example, distributing formulas according to proficiency level is a good idea. In quantum mechanics, a previously unknown formalism is introduced in a short time and can overwhelm students if the familiarization period is too short. In order to work on a broad but coherent spectrum of formulas in a seminar, working in small groups (as another activating element) can help to cover more contents or topics.

The assignment is to translate and discuss the definition of the expectation value,

$$\langle \hat{A} \rangle = \langle \psi | \, \hat{A} \, | \psi \rangle \,, \tag{3.2}$$

the uncertainty relation for two non-commutative observables,

$$\Delta \hat{A} \cdot \Delta \hat{B} = \frac{1}{2} \left| \langle \big[\hat{A}, \hat{B} \big] \rangle \right|^2 , \tag{3.3}$$

the continuity equation,

$$\partial_t \varrho(\vec{r}, t) + \nabla \cdot \vec{j} = 0, \tag{3.4}$$

and for the strongest students the Ehrenfest theorem

$$\frac{d}{dt} \langle \hat{\vec{p}} \rangle = \frac{1}{m} \frac{d^2}{dt^2} \langle \hat{\vec{r}} \rangle = - \langle \Delta V \rangle \equiv \langle \hat{\vec{F}} \rangle \,, \tag{3.5}$$

in a group work. Each group of two is supposed to use the list of questions to characterize their own formula and then to present it to the entire course. In the presentation round, dependencies and connections of the formulas are worked out. Translating the above formulas turns out to be difficult in different degrees. The expectation value and the uncertainty relation represent the easier part. Through performance differentiation, learning successes can be seen in all small groups – without the strong performers always being ahead of the others.

Verbalization of formulas and worked examples

The task of verbalization with the help of the checklist is possible and the working atmosphere and the final results are usually satisfying (see section 3.4.3 for more details), but the simple working through of the checklist and the often unfamiliar task does not lead to the highest motivation among students at the beginning. The reason for this observation is assumed to be an overstraining with this open task, since here, in contrast to more common computational tasks, the procedure and the presentation of results have not been practiced earlier and are thus new and unfamiliar.

Better results and a stronger activation of the students can be achieved by a combination of the methods of verbalization of formulas and worked examples. That is, the instructor gives students a worked example to study independently where possible. Then the concept or principle is carried out by the students themselves for a similar or closely related formula. In this way, the inhibition threshold at the beginning can be overcome more easily and the students get into a productive working phase more quickly. If this method has already been introduced and is known for some time, the assistance can be gradually reduced.

This method can best be illustrated with Maxwell's equations. The instructor indicates the categories relevant to him or her and creates a profile for the first of Maxwell's equations, Gauss' theorem. Since a theory lecture for prospective teachers accompanied this work in parallel, the relevant points in the sample fact sheet are:

1. Verbalization and visualization of the differential representation.

2. Verbalization and visualization of the integral representation.

3. Representation and relation to the constitutive equations.

4. Embedding of the equation in the potential representation.

5. Application of the formula in lecture and school.

Items 1 & 2 are relevant for all the reasons stated in this section and cover items 1-4, 6 & 7 of the checklist in 3.4.1. Consequently, this is the focus of this exercise. The reference to the constitutive equations and the potential representation with calibration shows the limits of Maxwell's equations in vacuum, respectively classifies them in other forms of representation. The question about the fields of application in school and study should also let the students consciously deal with this classification: For which kind of problems is the equation the basis, respectively which topics of physics can be explained with it? The sample solution (see figure 3.5) is handed out to the students with the task to explain the structure and the contents themselves (worked example structure). Afterwards, it is the further work order for the students to create a fact sheet for each of the remaining of Maxwell's equations. Again, working in small groups can be incorporated as a cognitively activating element. From observations, however, it is recommended to let all students create at least one profile individually, such that all are activated and weaker students do not simply hang on to the stronger ones. Here, too, an allocation of the profiles according to performance level is beneficial to a homogeneous performance progress. Nevertheless, in order to use the activating character of small group work, the students can present their results to each other. However, to ensure correct results, all equations should be presented or corrected under the control of the lecturer.

Firm structures for the first handling of verbalization

As mentioned above, this new type of task (to verbalize formulas) and the open end result can lead to starting difficulties and overwhelm the students. In addition, the fear of saying the wrong thing in front of fellow students and teachers seems to be another inhibition threshold. To lower this inhibition threshold written formulations can help. In this way, initial statements can be recorded and then revised and improved until the students are satisfied and confident with their formulation in the group. In order to have these written formulations trained as broad as possible on as many specialist questions as possible, a design in table form is suitable. How this can look like will be explained here using the example of electrostatics.

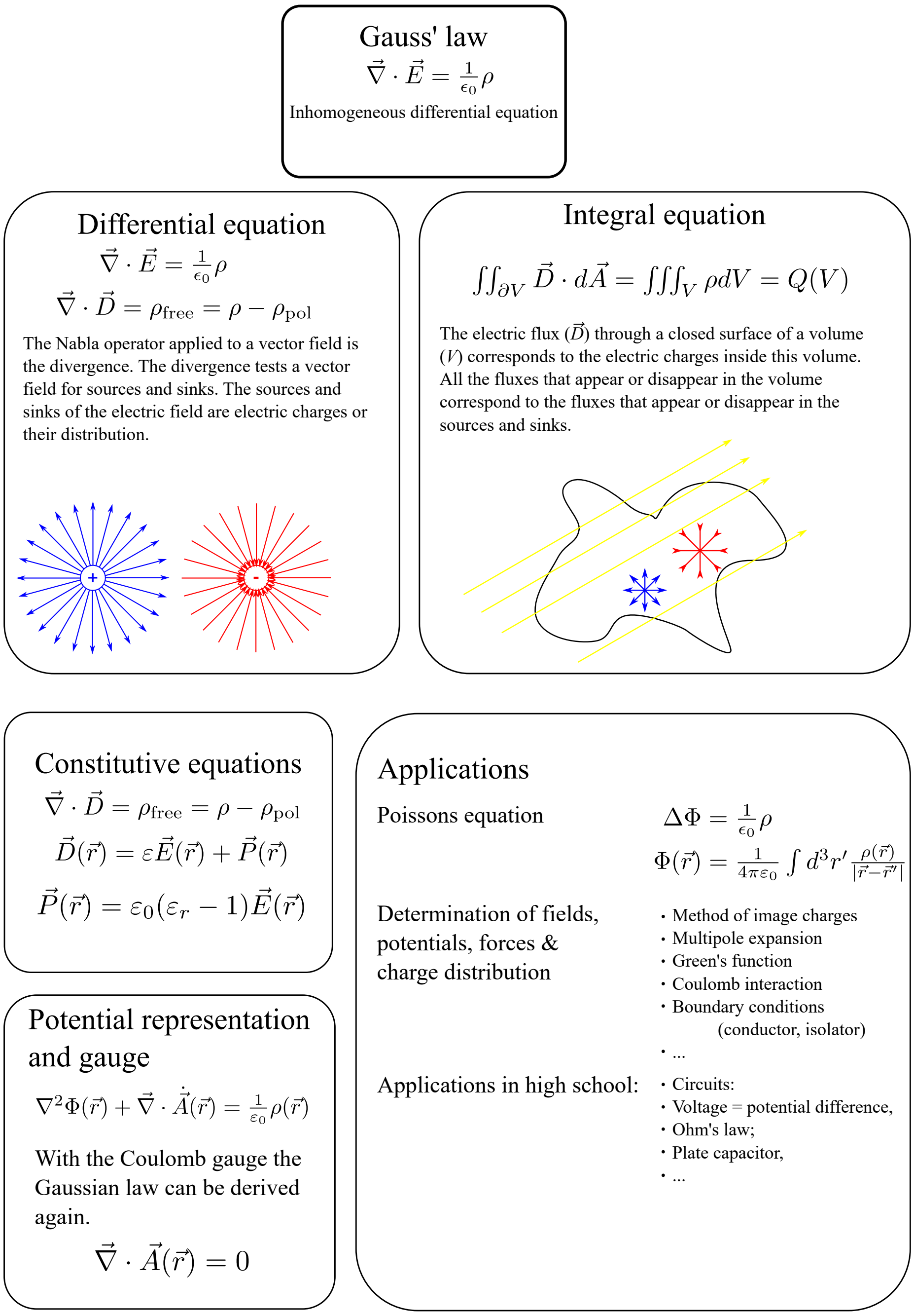


Figure 3.5. Worked example of a formula profile for the first of Maxwell's equations, the Gaussian law.

How does a \ act on a	charge	field	potential	conductor	dielectric
vacuum	creates a field or potential				
charge					
field					
potential					
conductor					
dielectric					

Figure 3.6. Table for verbalization of electrostatic interactions.

Forces (and this includes the Coulomb force) describe interactions. They depend on the charges and their distance. In case of complex charge distributions or large distances to strong charges, the description of the effect of the charges via fields or potentials is suitable. Mathematically this can be represented very compactly and compressed,

$$\vec{F}_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\vec{e}_{12}}{|\vec{r}_1 - \vec{r}_2|^2}, \tag{3.6}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\varepsilon_0} \varrho, \tag{3.7}$$

$$\Delta\Phi = \frac{1}{\varepsilon_0} \varrho. \tag{3.8}$$

When formulating dependencies or interactions, however, there are many linguistic nuances that depend on the direction of the effect. Learners of physics may not be fully aware of this from the outset. For example, the effect of an electric field on an electric charge is spoken about in a completely different way than the effect of an electric charge on an electric field. The aim of this task is to find linguistic representations for all the cases that lie behind these seemingly simple formulas and to become aware of their subtle differences. The form of a table was chosen as a compact design to highlight the linguistic differences in talking the one way or the other (see figure 3.6). The task is to formulate how one quantity acts on the other in electrostatics. Filling out the table as a work assignment can help to activate all students and get them to formulate concrete statements. Making things concrete seems to be a necessary step in the learning process, otherwise one can settle for vague notions and ideas. Without checking whether someones own ideas are coherent and can be understood by others, there is no improvement process and thus no learning progress. This elementary way of talking about simple and basic interactions should be relevant and interesting especially for teaching students whose later profession also consists of talking about and explaining physics.

In the example of figure 3.6, one rectangle is already filled in to present the expectation level to the students. In addition, rectangles are blacked out when no new knowledge acquisition is expected because nothing physically interesting happens. Also blacked out are the rectangles that go too far into solid-state physics and exceed the students' knowledge. A couple of the answers in this example are effortless. However, students may find this challenging because they expect complicated answers. The learning objective of such simple answers for students is to learn to trust or make their own judgments.

If the method of verbalizing and illustrating formulas is better known to the students and the handling of it is somewhat practiced, the assistance can be reduced. This gives the students more freedom to set their own priorities and to generate their own insights.

Peer instruction and verbalization of formulas

The methods mentioned above are very suitable for structuring the verbalization and visualization of formulas for students, and for practicing their use, such that the quality of technical discussions increases steadily. A disadvantage of these methods, however, is the large amount of time that must be invested. If all students work on the translation tasks independently or in small groups and then the results are secured or exchanged in presentation rounds or large group discussions, there is a time discrepancy between the processing and the feedback on the students' formulations. It is also not always possible in large groups to give feedback to all students individually. In the case of written submissions for verbalization, feedback can be given to each student individually, but the time discrepancy between the processing and the feedback received becomes even greater and a learning effect only sets in if the students subsequently deal with the feedback independently.

A solution for this problem can be the feedback between the students themselves. For this purpose, students can be asked to verbalize or visualize formulas and then present the results to each other and then correct them. This can take place in small discussion groups. The peer instruction (section 2.2.4) also follows such an approach. Actually, this method is intended to train conceptual understanding of physics and the handling of formulas is explicitly rejected. In the case of theoretical physics, with its abstract formalism, it can be helpful to break this rule in favor of a conceptual handling of formulas. Methods how this can be implemented in an activating and profitable way are presented in section 3.6.2.

3.4.3 Observation and enhancements

The most obvious observation at the Stuttgart and Jena sites is that the students themselves have hardly any experience with the explicit verbalization or visualization of formulas. This is shown by a slight overstraining or helplessness at the first contact with these teaching methods.

The checklist and the questionnaire can help here, but these aids do not lead to the desired working atmosphere and results for all students. The discussions in small groups are very reserved at first, probably because of the students' fear of saying something wrong. In addition, in some cases, even simple formula symbols or terms in the formulas and equations cannot be stated correctly if the content of the lecture has not been reviewed. However, with enough time and help from fellow students or the instructor, the final results are satisfactory.

With the more closed tasks such as the table and the peer questions (see section 3.6 for more details), the students initially find it easier and the motivation to work on the assignments seems to be greater. Thus, these more prescribed work orders represent the most important improvements in the verbalization and visualization of formulas in theoretical physics. However, the degree of assistance should be gradually reduced as students become more experienced in these methods.

Above all, there is the impression that there is a general need for explicit training in the verbalization of formulas. It often takes a long time in courses to build up a basis of trust, in which students dare to participate actively and verbally.

3.5 Educational reduction, elementarization and reconstructions

3.5.1 Conclusions for the design in theoretical physics

The methods of elementarization and reconstruction become activating when students perform them themselves or discuss them in small groups, as the example of concept maps shows [99–101]. This direct application should also be feasible in theoretical physics without significant problems. If the essential terms or elements for the concept map are given, the students' task is to reconstruct the different topics.

In addition, it is also possible to train elementarization by asking students to summarize textbooks or the lecture for themselves. The task behind this is to reduce the contents of the lecture or a textbook to the essentials or to develop their own category systems. In addition to content elements specific to each topic, sections from lectures and textbooks can also methodically be elementarized or grouped into recurring categories. Typically, lectures are always composed of the elements of

- definition,
- derivation or transformation,
- generalization,
- explanation,
- enumeration/listing,
- application of a formula or theory, and
- example or sample calculation.

Traditional reviewing and summarizing lectures or textbooks is thus nothing more than an elementalization of the content and methodology of the topics discussed. While new students traditionally are advised not only to attend the lectures but to prepare for and follow up on them, often no guidance or assistance is given regarding how this can be done in concrete terms. Consequently, students are left on their own when following this recommendation. It can be a hurdle, especially for weaker students, to follow this recommendation. This setting means that addressing and practicing elementarization and reconstruction with students in a concrete way could prove to be beneficial for learning.

Furthermore, if simplification and reduction are not only part of physics teaching but also a central part of physics itself, students should directly discuss this. Specifically, students should address the questions of when simplifications in physics are justified and they should discuss the limits of models based on simplifications.

3.5.2 Design and material

Summarize lectures

This unit aims at giving students the possibility to elementarize and summarize lectures. A question catalog as already introduced for the verbalization of formulas was used in the teaching design. For summarizing lectures the catalog can look like this:

1. What is the structure of the section/chapter?
    - definition, derivation or transformation, generalization, explanation, enumeration/listing, application of a formula or theory, and example or sample calculation
2. What is the topic of the section/chapter?
3. Which physical problem is treated here?
4. (optional: What is quantized here and how is it expressed?)
5. What is the central physical idea in this section/chapter?
6. How is this idea described in the script/formalism? (What is the mathematical formulation of the physics problem or the central idea?)
    - What formulas occur and what do they describe?
    - How are the formulas related?
7. What substantial physical quantities occur and how are they defined?
8. Can the central idea be represented graphically?
    - sketch, graph, plot, mindmap, ...
9. What are the essential insights from this section/chapter?
10. What is the need for the topic covered?
11. Does the content refer to previous lectures/content?

This list is presented to the students for a suggestion. Since some questions have large overlaps in content, the goal is not simply to work through these questions. The goal is to answer the questions that fit the topic or to answer several questions simultaneously. This overlap encourages students to think critically about the questions or to formulate their own questions better suited to the topic.

If this type of summary has never been practiced before, the introduction to this method can again be supported by "worked examples", cf. section 3.3. The students' task then is to explain this sample solution to themselves and to formulate their summaries elsewhere. For the quantum mechanical harmonic oscillator, a sample solution might look like the following:

1. **What is the structure of the section/chapter?**
   The whole chapter is a derivation for a new formalism to describe the quantum mechanical harmonic oscillator with the help of the energy eigenvalues without using position space.

2. **What is the topic of the section/chapter?**
   The harmonic oscillator in quantum mechanics. The quantum mechanical analog to the mechanical oscillator describes a system with a quadratic potential.

3. **Which physical problem is treated here?**
   The quantum mechanical harmonic oscillator in one dimension and its mathematical description.

4. **What is quantized here and how is it expressed?**
   The physically reasonable stationary states of a quantum particle in a harmonic ($\sim x^2$) potential define the quantization of the particle's eigenvalues to the Hamiltonian in the particular state. The energy required to transition from one eigenstate to the next is thus clearly defined and discrete.

5. **What is the central physical idea in this section/chapter?**
   All states should be describable by the ground state and a "raising operator". Thus mathematically, every state is addressable.

6. **How is this idea described in the script/formalism?**
   Stationary Schrödinger equation to describe the problem:

$$\begin{aligned} \hat{H}\ \varphi(x) &= E\varphi(x) \\ \left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{m\omega^2}{2}x^2\right)\varphi(x) &= E\varphi(x) \\ \hbar\omega\left(\hat{b}^\dagger\hat{b} + \frac{1}{2}\right)\varphi(x) &= E\varphi(x) \end{aligned}$$

   Any stationary state in harmonic potential:

$$\begin{aligned} |\varphi_n\rangle &= \frac{1}{\sqrt{n!}}(\hat{b}^\dagger)^n\,|\varphi_0\rangle \\ &= \frac{1}{\sqrt{n!}}(\hat{b}^\dagger)^n \alpha_0 e^{-\frac{\alpha^2 x^2}{2}} \\ &= \sqrt{\frac{\alpha}{\sqrt{\pi}}}\frac{1}{\sqrt{2^n n!}}\hat{H}_n(\alpha x)e^{-\frac{\alpha^2 x^2}{2}} \end{aligned}$$

7. **What important physical quantities occur and how are they defined?**
   Raising operator:

$$\hat{b}^\dagger = \frac{1}{\sqrt{2}}\left(\alpha x - \frac{1}{\alpha}\frac{d}{dx}\right), \quad \alpha^2 = \frac{m\omega}{\hbar}$$

   Lowering operator:

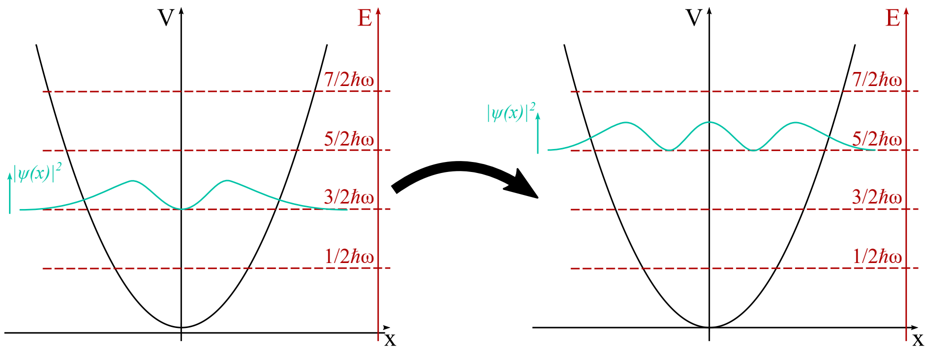
$$\hat{b} = \frac{1}{\sqrt{2}}\left(\alpha x + \frac{1}{\alpha}\frac{d}{dx}\right)$$

   Number operator:

$$\hat{N} = \hat{b}^\dagger\hat{b}$$

8. **Can the central idea be represented graphically?**

$$\hat{b}^{\dagger} |\varphi_1\rangle = \alpha_2 |\varphi_2\rangle :$$



9. **What are the essential insights from this section/chapter?**
   Mathematically, everything can be built on top of each other, and all states can be described only with the ground state and raising or lowering operators. The energy levels for stationary states in the harmonic oscillator are equidistant.

10. **What is the need for the topic covered?**
    With the formalism of raising or lowering operators, one can describe many other physical potentials as well as the interaction of particles in such potentials with the environment. The raising operator can describe the excitation of an electron in an atom to another energy level.
    Sometimes $\hat{b}^{\dagger}, \hat{b}$ also are called creation and annihilation operators. With this formulation, one can describe single quanta and how they interact in many-quantum systems as a complex whole (quantum statistics in solid-state physics).

11. **Does the content refer to previous lectures/content?**
    The potential a quantum particle feels here is the same as in the classical harmonic oscillator. In contrast to a classical equation of motion the Schrödinger equation is the describing element here. Since the same physics must be described in the new formalism with rising and lowering operators and in position representation, both representations are equivalent. This equivalence also is to be recognized by the fact that the position and momentum operator can express the rising and lowering operator.

Concept maps

The concept maps aim at building up an individual structure of a topic from essential terms and elements of this topic. The aim is to create links and interconnections between the elements to prevent the compartmentalization of particular topics or formulas. The terms and elements can be given depending on the degree of difficulty or the time available.

It has to be considered that a complete mapping of all relations and dependencies in physics is impossible. Therefore, the learning goal for students is to deal with dependencies and the structure of physical knowledge in general and, thus, to learn about the nature of science. Therefore, concept maps at different levels of abstraction

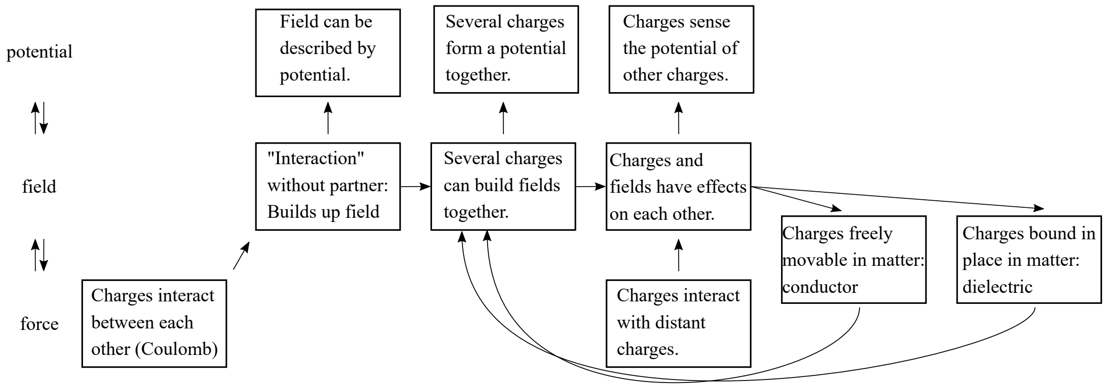


Figure 3.7. Example of a concept map of electrostatic interactions.

can be imagined for different topics, as the example of a concept map for interactions in electrostatics illustrates.

The task for students in electrostatics may be to create a concept map for interactions after they have verbalized them (see figure 3.6 in 3.4.2). This map should express the different representations at the different levels of abstraction (force vs. field vs. potential). Figure 3.7 shows a possible solution to this task.

Recurring elements in different representations

In physics, many subjects can be represented or treated in different ways. For example, Maxwell's equations can be represented in differential or integral form. Furthermore, these field equations also can be expressed entirely by potentials. The same elements of physics or its formalisms can have different representations. Towards the end of a lecture, when students have covered all the individual topics, students may be reminded of this basic structure.

The representation of the same physics by different formalisms also exists in quantum mechanics. The elements of the formalism (e.g., in the stationary Schrödinger equation: Hilbert space, state and wave function, observables, measurement process, measurement and eigenvalues, eigenstates, and measurement probabilities) come in different notations in different contexts. However, they always have the same basic meaning, e.g., in problems with discrete eigenstates (spin measurement), in problems with continuous bases (position and momentum representation of the free particle), or in problems that can be treated in different representations (ladder operator method and space representation in the harmonic oscillator). For a general understanding of quantum mechanics, it should be helpful to have students specifically search for these parallels. Compactly, these parallels can be presented in tabular form; see figure 3.8. The effects of "worked examples" can be used to make the introduction easier again. The first example for the spin measurement is given, and the task is to explain these relations by oneself. Then the students have to summarize the mentioned elements in the presentation of the position and momentum of the free particle. Another example is the ladder operator method of a particle in a harmonic oscillator. In order to train the verbalization of formulas and the interplay between mathematics and physics, a column on the mathematical meaning of the individual elements can be inserted.

QM	Mathematical	Spin	Free particle	Particle in harmonic oscillator
Hilbert space	Hilbert space $\mathcal{H}$ is a real or complex vector space with a scalar product $\langle \cdot \vert \cdot \rangle$.	• 2 dimensional: Spin "up" & Spin "down" • orthogonality: $\langle z_i \vert z_j \rangle = \delta_{ij}$	• ∞ dimensional • Orthogonality: $\langle x' \vert x \rangle = \delta(x - x')$	
State/ wave function	A vector $\psi \in \mathcal{H}$ that can be represented in different bases and notations.	Example in z-direction $\mathrm{up} = \vert z+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\mathrm{down} = \vert z-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ generall state: $\vert\psi\rangle = c_1 \vert z+\rangle + c_2 \vert z-\rangle$		
Observable Observable physical quantity	Linear operator $\hat{A}$ on $\mathcal{H}$.	Spin in x,y,z direction (if z is the preferred direction) $\hat{\boldsymbol{\sigma}}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\hat{\boldsymbol{\sigma}}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\hat{\boldsymbol{\sigma}}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$		
The actual measured value	The possible measured values result from the eigenvalues of an eigenvalue equation. $\hat{\boldsymbol{A}} \vert\psi\rangle = a \vert\psi\rangle$	Energy of the spin in the external magnetic field (in a Stern-Gerlach experiment) $H \vert\psi\rangle = E \vert\psi\rangle$ $-\mu B_z \hat{\boldsymbol{\sigma}}_z \vert z\pm\rangle = \pm \frac{\mu B_z}{2} \vert z\pm\rangle$		
Eigenstates as basis	Often the best basis to describe a state is a linear combination of eigenstates to an observable, which are orthogonal and normalizable. $\vert\psi\rangle = \sum_i c_i \vert\phi_i\rangle$	The eigenstates spin "up" & "down" to the respective measurement direction x,y or z. $\vert\psi\rangle = c_1 \vert z+\rangle + c_2 \vert z-\rangle$ $\vert x+\rangle = \frac{1}{\sqrt{2}} \vert z+\rangle + \frac{1}{\sqrt{2}} \vert z-\rangle$		
Measurement = Projection + Collapse	A state or vector is described in the basis of the observable. After the measurement, the state reduces to the eigenvector corresponding to the measured eigenvalue. $\vert\psi\rangle_{\mathrm{before}} = \sum_i c_i \vert\phi_i\rangle$ $\vert\psi\rangle_{\mathrm{after}} = 1 \cdot \vert\phi_j\rangle$	Spin "up" in x-direction is measured in z-direction. $\vert\psi\rangle_{\mathrm{before}} = \vert x+\rangle = \frac{1}{\sqrt{2}} \vert z+\rangle + \frac{1}{\sqrt{2}} \vert z-\rangle$ $\vert\psi\rangle_{\mathrm{after}} = \vert z+\rangle$ or $\vert\psi\rangle_{\mathrm{after}} = \vert z-\rangle$		
Measurement probability	The coefficients c_i are the proportion of the eigenstate $\vert\phi_i\rangle$ in the total state $\vert\psi\rangle$. $\vert c_i \vert^2 = p_i$ is the probability with which the system is measured in the $\vert\phi_i\rangle$ eigenstate if it was in the state $\vert\psi\rangle$ before the measurement.	Probability to measure $\vert z-\rangle$ when the spin was previously in the state $\vert x+\rangle$. $\vert \langle z - \vert x+\rangle \vert^2 = \left\vert \langle z- \vert \left(\frac{1}{\sqrt{2}} \vert z+\rangle + \frac{1}{\sqrt{2}} \vert z-\rangle \right) \right\vert^2$ $= \left\vert \frac{1}{\sqrt{2}} \langle z - \vert z-\rangle \right\vert^2 = \left\vert \frac{1}{\sqrt{2}} \right\vert^2$ $= \frac{1}{2}$		

Figure 3.8. The used table to help students get started summarizing the elements of the quantum mechanical formalism in different topics with different representations.

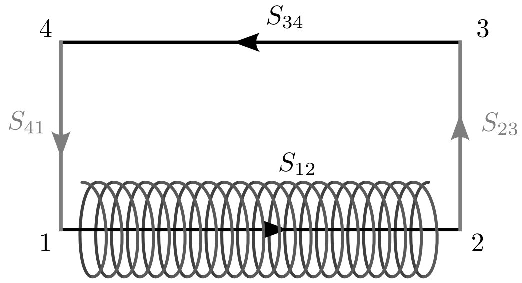


Figure 3.9. Sketch of the integration paths for Ampere's circuital law to determine the field of a long thin coil.

Explicit discussion of reductions

If simplification and reduction in physics are to be discussed with students concretely, it is a good idea to talk about topics they already know from school. When comparing the treatment of a topic with the help of "school physics" and the treatment of the same topic with the methods of theoretical physics, one can explicitly discuss the questions of when simplifications in physics are justified and where the limits of models are.

As an example, consider the field of a long thin coil. Students will be shown the derivations for the field of a long thin coil as used in school and as typically found in university textbooks.

Derivation 1:

To determine the field in a long thin coil, the integration path for Ampere's law of electrostatics is divided into four sections (see figure 3.9). The students are given the simplified calculation path and asked to discuss or answer the questions.

$$\begin{aligned} \oint_S \vec{B} \cdot d\vec{s} &= \mu_0 N \cdot I \\ \int_{S_{12}} \vec{B} \cdot d\vec{s}_{12} + \int_{S_{23}} \vec{B} \cdot d\vec{s}_{23} + \int_{S_{34}} \vec{B} \cdot d\vec{s}_{34} + \int_{S_{41}} \vec{B} \cdot d\vec{s}_{41} &= \mu_0 N \cdot I \\ \int_{S_{12}} \vec{B} \cdot d\vec{s}_{12} &= \mu_0 N \cdot I \\ B \int_{S_{12}} ds_{12} = Bl &= \mu_0 N \cdot I \\ \Rightarrow B &= \mu_0 \frac{N}{l} I \end{aligned} \tag{3.9}$$

Tasks:

1. Make a sketch showing the magnetic field of a long thin coil and note the relevant integration paths $\int_{S_{12}}, \int_{S_{23}}, \ldots$.
2. Sketch a short coil. How does the magnetic field outside the coil change compared to the long thin coil?
3. Why do some path sections contribute nothing to the integral and can be neglected?
4. Why can the field part B be taken out of the integral and be placed in front of it?

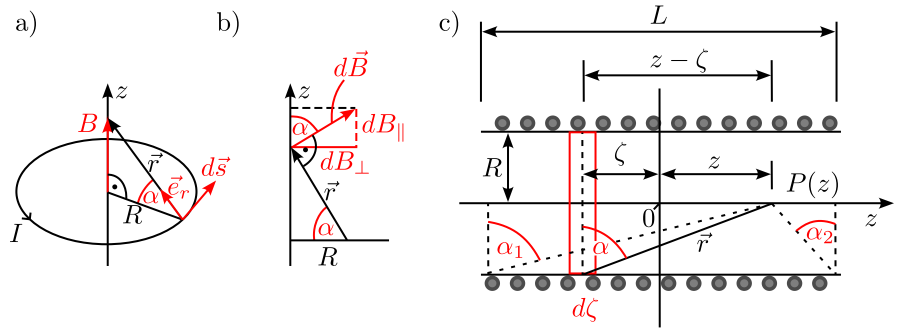


Figure 3.10. Sketches used to calculate the magnetic field of a circuit loop a) in the loop plane, b) on the symmetry axis of the loop and c) to calculate the magnetic field of a cylindrical coil on it's symmetry axis.

The aim of these questions and tasks is to let students formulate the reasons themselves. The solutions here are that the proportions of paths A and B cancel each other out. If path C is moved so far outwards that the B field approaches zero, this section also makes no contribution, and the term is omitted.

Derivation 2:

Derivation 2 is a summary of section 3.2.6 of the textbook [118]. For the second determination of the magnetic field of a long coil, first the field of a conductor loop is examined. If the current loop lies in the x-y-plane (see figure 3.10.a), then according to Biot-Savart's law, the magnetic field B reads

$$\vec{B}(\vec{r}) = -\frac{\mu_0 \cdot I}{4\pi} \cdot \int \frac{\hat{\vec{e}}_r \times d\vec{s}}{|\vec{r}|^2}. \tag{3.10}$$

On the symmetry axis (z-axis through the center, see figure 3.10.b) we obtain the contribution dB of the path element ds to the magnetic field:

$$d\vec{B} = -\frac{\mu_0 \cdot I}{4\pi} \cdot \frac{\vec{r} \times d\vec{s}}{r^3} \tag{3.11}$$

When integrating over all path elements of the circle, the components perpendicular to the symmetry axis $dB_\perp = d\vec{B} \cdot \sin\alpha$ average to zero. Only the parallel component $dB_\parallel = d\vec{B} \cdot \cos\alpha$ remains, which appears as follows when integrating because of $|\vec{r} \times d\vec{s}| = \frac{R}{\cos\alpha} \cdot ds$:

$$B_z = B_\parallel = \int dB_\parallel = \int |d\vec{B}| \cdot \cos\alpha = \frac{\mu_0 \cdot I}{4\pi r^3} \cdot \oint R \cdot ds = \frac{\mu_0 \cdot I \cdot R}{4\pi r^3} \cdot 2\pi R. \tag{3.12}$$

Because of $r^2 = R^2 + z^2$ we obtain

$$B_z = \frac{\mu_0 \cdot I \cdot R^2}{2(z^2 + R^2)^{3/2}}. \tag{3.13}$$

Now the magnetic field inside a long coil with n windings per meter shall be determined. The zero point of the coordinate system shall be in the center of the coil, whose symmetry axis is chosen as z-axis (figure 3.10.c). The part of the magnetic field in the point $P(z)$, which is generated by the $n \cdot d\zeta$ windings with the cross section $A = \pi \cdot R^2$ in the length interval $d\zeta$ is, according to the field of a conductor loop (3.13)

$$dB = \frac{\mu_0 \cdot I \cdot R^2 \cdot n \cdot d\zeta}{2\left[R^2 + (z-\zeta)^2\right]^{3/2}}. \tag{3.14}$$

The total field at the location $P(z)$ is obtained by integration over all turns from $\zeta = -L/2$ to $\zeta = +L/2$. The integral can be solved with the substitution $z - \zeta = R \cdot \tan\alpha$ and gives:

$$B(z) = \int_{-L/2}^{+L/2} dB = -\frac{\mu_0 In}{2} \int_{\alpha_1}^{\alpha_2} \cos\alpha \, d\alpha$$
$$= \frac{\mu_0 In}{2} \left\{ \frac{z + L/2}{\sqrt{R^2 + (z + L/2)^2}} - \frac{z - L/2}{\sqrt{R^2 + (z - L/2)^2}} \right\} \tag{3.15}$$

Students tasks:

1. What equation is obtained by looking at the field at the center $z = 0$ and by simplifying for a long thin coil with $L \gg R$?
2. What magnetic field is obtained at the end of the coil under the same simplification as above?
3. How much can one rely on the equation from the first derivation?
4. Which limitations do the second derivation and its solution have?

These questions aim to recognize that with appropriate simplifications in the coil center, the same formula for the magnetic field is found as with derivation 1 in equation (3.9). These simplifications can also be indicated in numbers, i.e., the flux density B at the coil edges is only half of the density at the center. One learning goal of this discussion is for students to recognize the need for simplification when solving physical problems mathematically by novices. The second learning goal is that students learn to recognize the limitations and restrictions of simplifications and simple models.

3.5.3 Observation and enhancements

It is noticeable that many students, according to their statements, do not have time to review the lectures. *"I haven't had time to review the lecture yet."* and *"I'll have to study it again before the exam."* are recurring statements in this context. If such a lecture repetition is used as a method in seminars, it is very likely that many students still need to gain systematic experience with it because they have never done it on their own. Without preparation, students may feel overwhelmed with the assignment of summarizing a lecture. This is the reason for introducing "worked examples" into this method. Suggested categories (definition, derivation, application of a formula, ...; see "What is the structure of the section" in the question catalog of summarizing lectures in section 3.5.2) into which sections from the lecture can be placed are mentioned here as particularly helpful. High-performing students, however, do not need this additional support.

The situation is similar for the concept maps. (This type of summary had not been practiced with students in university education before it was used in this thesis, neither in Stuttgart nor in Jena.) As a new method, this can have a deterrent effect. However, the introduction can be facilitated by a sample example from another subject area or by providing elements or terms. Once the first hurdle is overcome, student participation and, thus, activation are high. As an extension or improvement, the concept maps can be used very well to discuss the nature of science with students.

The table on the basic terms or elements in the formalism of quantum mechanics is a great challenge for students. This method has been tested only once so far. The elements for the free particle were not filled in satisfactorily and the last column could not be dealt with at all in the time available. Therefore, it is recommended to discuss these parallels in the different applications only after the particular topics have been covered sufficiently with the students.

In the explicit discussion of reductions, derivation 2 can overwhelm students if the processing time is too short. The calculation was unified and summarized to prevent this as compared to the first approach. However, the learning objective of critical consideration of the scope of physical formulas can be achieved very well, as was observed in the quality of the discussions following these tasks.

In general, it can be recommended to offer ”worked examples” or assistance when students are confronted with such methods and tasks for the first time in their university education. This additional aid helps to reduce initial inhibitions and to present a level of requirements to the students. Subsequently, these tasks can have a motivating and activating effect on students.

3.6 Peer instruction

3.6.1 Conclusions for the design in theoretical physics

The starting point for developing new concept tests for theoretical physics are, of course, Mazur's criteria and his examples. Naturally, there are also physics concepts that are addressed in theory lectures, for which questions can be developed with attractive distractors. The development and categorization of such questions are described in the following section.

In addition, Mazur's criteria have to be modified at point two ("The questions should not be solvable by the application of formulas.") for theoretical physics. Theoretical physics is characterized by the fact that there are points at which human imagination or simple models reach their limits and the practice of physics is, in the first place, possible with a mathematical formalism. It does not make sense to completely dispense from formulas and their use. The goal should not be to simply insert values into formulas and let learners calculate. However, it can be profitable when conceptual and qualitative handling of formulas is specifically trained. It can be a topic of its own to illustrate or train the interplay between mathematics and physics or to develop a conceptual understanding of formulas and learn to extract physical information from formulas. Thus, the adapted basic rule for concept questions for a formula understanding reads as follows: The question should

- ~~not be solvable by application of formulas,~~
- illustrate/train the interplay between mathematics and physics.

3.6.2 Design and material

This section presents the categories into which the developed questions can be classified. For each category, a didactic justification for this type of question is provided and explained using an example. In the area of classical concept questions, as suggested by Mazur, these are the categories “clarify/define/concretize technical terms,” "physical concepts," "understand/interpret representations/graphs," and for student teachers, "assess student conceptions." The new questions for conceptual understanding of formulas can be categorized as "translating physics-mathematics," "translating mathematics-physics," "interpreting formulas," and "estimating from formulas."

Classical concept tests

Clarify/define/concretize technical terms

According to observations, students can often quickly adopt the technical terms from the lectures and incorporate them into their own language. However, when asked for in detailed discussions, the impression can often be given that the meaning of the terms is only sometimes fully understood. With targeted peer questions, however, technical terms can be made more tangible, or students can be anonymously made aware of their gaps in knowledge. An appropriate question for the topic "inertial system" is:

Which example(s) is/are it (an) inertial system(s)?

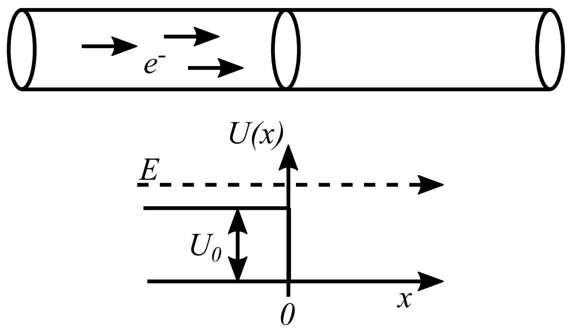
1. a chain merry-go-round
2. a skydiver in free fall
3. a school classroom
4. the earth
5. the international space station ISS
6. several examples are inertial systems
7. none of the mentioned examples is a real inertial system

The concept of the inertial system is considered important by many students. However, if they are asked to name a concrete example or to judge whether there is a good example at all, many are overwhelmed. This is also shown by the peer question on the inertial system, when here many students do not realize that no example represents a real inertial system. Following this question, however, it is a good opportunity to repeat the question and ask for the approximate best inertial system, thus stimulating a discussion about approximations and estimations.

Physical concepts

Comprehension questions about physical concepts are the most typical questions for peer instruction. In the context of theoretical physics, it is useful to transfer concepts that are calculated in lectures and exercises on very elementary examples to real applications or phenomena. This can help to get access to the high abstraction level of theoretical physics. The following example addresses the topic of a potential step in quantum mechanics, translated and modified from [119]. The drift direction of the electrons was changed to correspond to the logical reading direction and to reduce cognitive effort.

A beam of electrons, in which all have the same energy E, moves through a conductor. At the point $x = 0$, the material of the conductor changes such that the potential energy of the electrons drops from U_0 to zero. Given $E > U_0$, which statement most accurately describes the transmission and reflection of the electrons?



1. All electrons are transmitted because all have an energy of $E > U_0$.
2. Some of the electrons are transmitted and some are reflected because they actually have a range of energies.
3. Some of the electrons are transmitted and some are reflected because they behave like waves.
4. All electrons are transmitted. Because the potential decreases, there is no reason for the electrons to be reflected.

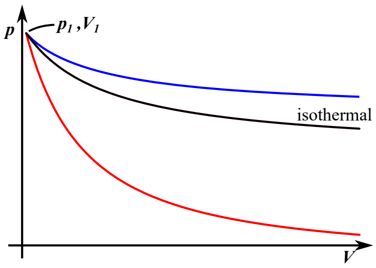
The abstract example of the potential level is applied here to a subject area of classical physics, namely electronics. Many students do not recognize the connection of the contexts and do not apply the learned new knowledge from quantum mechanics to electronics. It is also true here that a part of the electrons is reflected and in sensitive circuits attention must be paid to the impedance in order to prevent reflection phenomena. Peer discussion can be very helpful in findings on interconnections.

Of course, new phenomena and concepts can also be concretized or solidified with concept tests as in the collapse of the wave function in measurement processes, e.g., the Stern-Gerlach experiment.

Understand/interpret plots/graphs

Interpreting graphs has been a popular way to get students in the task of thinking since Mazur's own work. This can, of course, be used in theoretical physics as well. Even if certain concepts or questions have already been covered in previous experimental physics lectures, repetition may be useful or even necessary, as for example when comparing changes of state in thermodynamics.

In the $p\ V$ diagram an isothermal change of state (black) is shown from p_1, V_1 to $p_2 < p_1$ and $V_2 > V_1$. If the system was allowed to expand adiabatically from the p_1, V_1 state, what would the change of state in the graph look like?



1. The graph of the adiabatic change of state must lie above the isothermal one (blue).
2. The graph of the adiabatic change of state must lie below the isothermal one (red).
3. Isothermal and adiabatic changes of state cannot be distinguished in the $p\ V$ diagram.
4. An adiabatic change of state cannot be represented in the $p\ V$ diagram, only in the $p\ T$ diagram.

The question of graph interpretation can be used to encourage learners to think about and discuss the evolution of physical quantities under different conditions. In the example here, the pressure must fall faster in the adiabatic change of state than in the isothermal change of state since energy is supplied from outside only in the isothermal change of state.

Assessment of scientific misconceptions

Since the concepts developed in this work are mainly tested with teaching students, this fourth category was established in order to link content knowledge with pedagogical content knowledge. The students' task is to recognize typical scientific misconceptions of pupils and to classify them in a subject-related framework.

Evaluate the following student statement: 'The seasons occur because of the elliptical orbit of the Earth. We are closer to the sun in summer than in winter.'

1. The statement is true.
2. The statement is wrong. Seasons occur because of the tilted axis of the earth.
3. The statement is wrong. Seasons occur because the Coriolis force drives hot air north in summer and south in winter.
4. The statement is wrong. The elliptical path only explains why it is warmer in the southern hemisphere than in the north.

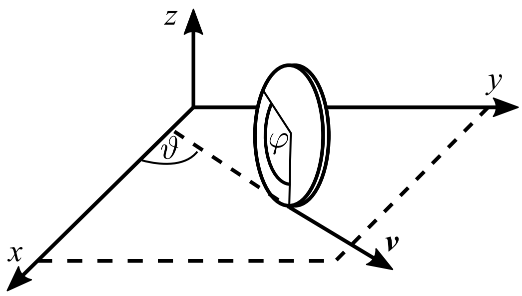
The correct answer here is 2. As discussed in section 2.1.3, for teaching students it can be profitable to treat content knowledge in a pedagogical context [31, 32]. A central point in physics education is the knowledge about scientific misconceptions. In anonymous votes during the peer instruction, prospective teachers can reflect on their own preconceptions without suffering any disadvantage. Stronger students that already have overcome the specific misconception can practice explanatory patterns in order to be able to eliminate student preconceptions.

Concept tests for formula understanding

Physics-mathematics & math-physics translation

This category trains the absolute basis which is necessary to gain information from formulas or to represent physical problems mathematically to be able to work with them at all. This skill is essential in theoretical physics when it is not possible to work on problems with conceptual ideas alone. Unfortunately, in our experience, many students have problems exactly in this area. The category should also contain questions on the circumstances certain formulas and the formalisms are valid or can be applied. The actual calculation afterwards is often not the problem. First exercises in the context of peer instruction can, e.g., be formulated for physical boundary conditions that must be formulated mathematically such as the determination of constraints in Lagrangian mechanics.

A wheel that cannot fall over and cannot slide rolls on a plane. What is the matching constraint / do the matching constraints look like?



1. $\omega \cdot t = \varphi$, $z = a$
2. $x - y \cdot \tan^{-1}(\vartheta) = 0$, $z = a$
3. $z = a$
4. The answers 1. and 2. are both correct.

When students work on exercise series, translating physical information into a mathematical formulation, and vice versa, can be more challenging than mathematical computation. With peer instruction and such questions, this translations can be practiced already in the lecture. In this example answer 2 is correct.

The translation physics-mathematics can logically be reversed. In Lagrangian mechanics, for example, students could be asked to draw conclusions about the motion from given generalized coordinates.

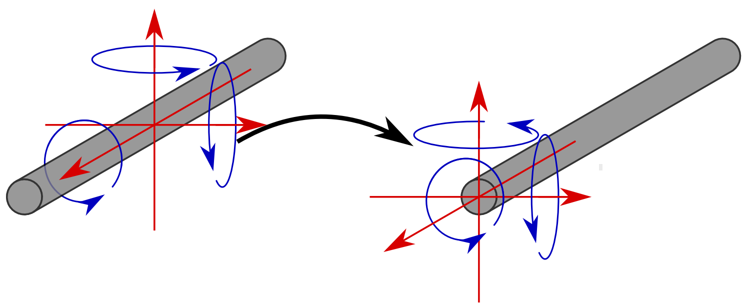
Interpretation of formulas

If one goes a step further and lets students assign solutions of the Euler-Lagrange equation to the original physical problem, one is in the category of interpretation of formulas. What is meant here is the ability to extract information that is more complex than the simple location from a formula. The effect of the parallel axis theorem of the inertia tensor of a body is an example for that.

The axes of rotation of the thin rod with the tensor of inertia

$$\frac{m}{12}l^2\begin{pmatrix}0&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

are shifted from the center to one end. Steiner's theorem of parallel axis describes the change of a moment of inertia $\Theta = \Theta_S + Ml_s^2$. What is the new inertia tensor?



1. $\begin{pmatrix}0&0&0\\0&\frac{m}{3}l^2&0\\0&0&\frac{m}{3}l^2\end{pmatrix}$
2. $\begin{pmatrix}m\left(\frac{l}{2}\right)^2&0&0\\0&\frac{m}{12}l^2&0\\0&0&\frac{m}{12}l^2\end{pmatrix}$
3. $\begin{pmatrix}-m\left(\frac{l}{2}\right)^2&0&0\\0&\frac{m}{12}l^2-m\left(\frac{l}{2}\right)^2&0\\0&0&\frac{m}{12}l^2-m\left(\frac{l}{2}\right)^2\end{pmatrix}$

Relevant for answering this question is only the knowledge that the moment of inertia in the inertia tensor disappears when rotating around the thin axis. This property does not change when the center of rotation is moved. Thus no calculation has to be done. The only tensor that retains this information is that in number 1. This information can be read out of the given inertia tensor by asking oneself the question 'Why is the first element zero?'. Therefore only a general knowledge about inertia tensors is necessary to answer this question correctly.

Estimating from formulas

This information also does not always have to be analytically exact or complete. It is often helpful to make rough predictions from formulas in order to estimate what effects a certain phenomenon or property will have in real problems. As an example, consider the complex refractive index.

When the complex refractive index

$$k = \frac{\omega \cdot \hat{n}}{c} = \frac{2\pi}{\lambda} \cdot \hat{n}\,, \quad \hat{n} = n + i\kappa,$$

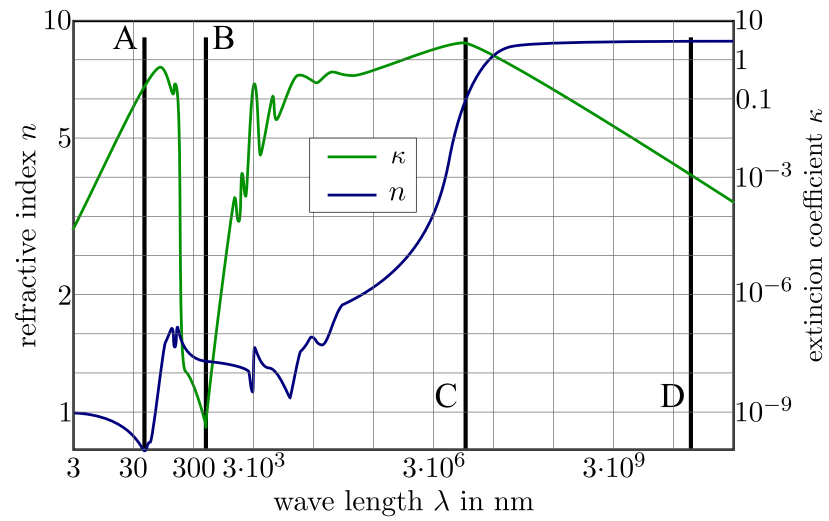
is inserted into the formula for a plane wave

$$\hat{\vec{E}} = \hat{\vec{E}}_0 \cdot \mathrm{e}^{i(\vec{k}\cdot\vec{x} - \omega\cdot t)},$$

this results in

$$\hat{\vec{E}} = \hat{\vec{E}}_0 \cdot \mathrm{e}^{-(\kappa \frac{2\pi}{\lambda} x)} \cdot \mathrm{e}^{i(n\frac{2\pi}{\lambda}x - \omega t)}.$$

In the diagram, the real (n) and imaginary (κ) components of the refractive index are given. For which of the marked wavelengths is water most transparent?



1. A: approx. $40\,\mathrm{nm}$
2. B: approx. $400\,\mathrm{nm}$
3. C: approx. $6\,\mathrm{mm}$
4. D: approx. $40\,\mathrm{m}$

The aim here is to link the absorption of light with an attenuation, i.e., the first, purely real exponential term of the plane wave. The estimation is then: the smaller the attenuation, the further the penetration depth. This is achieved with an extinction coefficient k as small as possible and a wavelength as big as possible. The combination at D is best, so this is the correct answer.

The ability to estimate can be very helpful in studies or in general when calculation or simulation results are checked. Therefore, it should be specifically promoted in lectures with peer questions of this type.

3.6.3 Testing and enhancements

Over a period from winter semester 2019/20 to summer semester 2022, the various questions were developed and tested at the University of Stuttgart to accompany the lectures “Grundlagen der Theoretischen Physik für Lehramt I & II”, which focused on classical mechanics, quantum mechanics, electrodynamics and thermodynamics. Parts of it were also tested at Friedrich Schiller University Jena accompanying the lectures “Theoretische Mechanik für Lehramt” and “Elektrodynamik für Lehramt” (These concept questions are highlighted with a star in their title in appendix A.4).

Modified quality criteria

Unfortunately, an empirical study of learning gains from the developed questions was not possible due to the small sample and the format of the seminar. Since the developed questions were only used in accompanying seminars, at least one control group would also be necessary to separate the learning gain of the regular lecture from the learning gain of the peer instruction. However, the number of participants does not allow for this. In the context of this study, the number of participants even never was in a range that would have been necessary for an adequate statistical evaluation, according to Mazur. As a consequence the quality criteria are adjusted here.

Instead of statistical evaluation, the questions are further developed, improved or discarded according to the following criteria.

- A tendency towards the correct answer can be seen in the second vote.
- The question encourages discussion among students.
- Errors or misleading questions are replaced or revised.

The peer instructions met lively interest from almost all students with good feedback. This was used to constantly refer to the current problems and suggestions of the students when new questions were developed. The procedure ensured that the new questions also addressed the problems and interests of the students.

Typical mistakes when creating/choosing concept tests

Misjudging the level of students is the most common mistake when creating or using concept questions. Using the example of the question on the transmission of light in water (estimating from formulas), it could be shown that the insertion of the refractive index into the formula of the plane wave should be given since drawing conclusions from the wave equation alone overtaxed the students in the short time available. Formulating the question as precisely as possible is another challenge that may require several revision steps to ensure that students do not misunderstand the question. At last the amount of time per question should also not be underestimated.

Time Scope

Approximately 10 minutes should be allotted in lecture per peer question with double voting and discussion in between. This is somewhat different for the concept tests for formula understanding. Here, students needed much more time for reflection and discussion (about 15 minutes total per question). Therefore, the question about the transmission of light in water (estimating from formulas) has only three given answers to reduce the needed time for reading and understanding. This study cannot clarify whether this is generally the case or whether students were simply unaccustomed to dealing with formulas in this way and therefore needed more time. Thus, the Peer Instruction requires considerably more time per topic compared to classical courses. In return, however, all students, and not only the usually active individuals, are stimulated to think.

Conclusion

Overall, the observation shows that students are very interested in embedding peer instruction in theoretical physics courses. In teaching evaluations by the student councils, this method got exclusively positive evaluations and was called for more frequently applications in the future. In addition the most active participation of all students could be observed during phases of peer instruction, in presence as well as digitally during the pandemic. The peer instruction was able to motivate all students to actively participate in times of online lectures without obligations.

4 Contradiction in 90 minutes - A teaching unit on local hidden parameters

This chapter represents a model lecture or seminar constructed from the methods of chapter 3. It is created according to the principles of educational reconstruction and should be maximally activating for students. The topic chosen is quantum mechanical entanglement and the disproof of local hidden parameters in quantum mechanics. Local constraints on development and testing (additional learning objectives, prior knowledge, and time management) have led to the choice of quantum mechanical spin, rather than polarized photons, as the entangled parameter.

Section 4.1 lists the justification and objectives for such a seminar. Section 4.2 provides the clarification of the relevant physics issues. Section 4.3 presents the research on teaching and learning in this context, and Section 4.4 describes the design and structure of the course.

4.1 Justification and objective

Entanglement is one of the most central concepts required for quantum revolution 2.0, in particular in quantum sensing and quantum computing. However, entanglement is also one of the most difficult aspects for learners to grasp. Education and teaching should thus pay special attention to this topic, so as not to leave the impression of incomplete quantum physics [120] or explanations with hidden parameters. Historically, Bell's inequality [121] and its experimental confirmation [122–126] should be mentioned here as a major milestone in quantum mechanics. In 2022 Clauser, Aspect and Zeilinger received the Nobel Prize in Physics for this work. However, Bell's inequality is difficult for students to understand and it gets worse with the CHSH inequality [122] for experimental verification. Therefore, the experiments alone may not be particularly convincing if not all steps are comprehended and understood.

Thus, here a learning unit is presented that ties in with and dispels the intuitive notion of hidden parameters. Students are activated and supported by proven methods from physics education research such as peer instruction, worked examples and working with multiple representations. The goal is to animate the students to play out a thought experiment with and without hidden parameters. In addition to the proper quantum mechanical calculus, the students' task is to illustrate quantum mechanics with a hidden parameter using analog experiments with classical parameters (material properties or programmable chips). The result should be a contradiction between the quantum mechanical solution and quantum physics with hidden parameters as proposed by Lucien Hardy [127]. The experiment according to Hardy can clarify unambiguously at the end that a local hidden parameter cannot describe quantum mechanics.

The course is developed as a companion seminar for teaching students. The accompanied lecture treated quantum mechanics conventionally through the limits of classical physics and from the free particle to the hydrogen problem. Entanglement, the Stern-Gerlach experiment, and the Dirac formalism were not covered at all, or barely. Therefore, the course proposed here focuses on the Dirac notation of spin physics before investigating entanglement and hidden local parameters. A nationwide teacher training course showed that the repetition of quantum mechanical basics using spin physics as an example is also profitable for in-service teachers before entanglement can be understood. The whole course can thus be attractive for different target groups.

4.2 Clarification and analysis of science content

4.2.1 The Einstein-Podolsky-Rosen (EPR) paradox

The phenomenon of entanglement in quantum mechanics goes back to Einstein, Podolsky and Rosen [120]. In their 1935 publication they do not mention the term yet, but they describe the phenomenon. They use it for the argument that "*[...] either (1) the quantum mechanical description of reality given by the wave function is not complete or (2) when the operators corresponding to two physical quantities do not commute the two quantifies cannot have simultaneous reality*" (p. 777, [120]).

For this hypothesis the term complete is characterized as: "*every element of the physical reality must have a counterpart in the physical theory*" (p. 777, [120]).

And the definition for reality reads: "*If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity*" (p. 777, [120]). Such a reality is given in quantum mechanics for any observable $\hat{A}$ by its eigenstates φ_n and the corresponding eigenvalues a_n,

$$\hat{A}\psi_n = a_n\psi_n. \tag{4.1}$$

The authors try to prove their point via a contradiction in the laws of quantum mechanics. They start with the common conclusion of Heisenberg's uncertainty principle that "*when the momentum of a particle is known, its coordinate has no physical reality*" (p. 778, [120]). On the other hand the authors show in the case of two entangled particles, that with measurements of position on one particle and momentum on the other particle physical reality must exist for both particles, which is a contradiction to the statement given by the uncertainty principle.

To illustrate this, a quantum mechanical system of two particles is introduced. The particles interact in finite time and are separated afterwards. The separation is needed such that the authors can suppose that there is no longer any interaction between the two parts. According to quantum mechanics the wave function of such a system can be described by

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} \mathrm{e}^{(2i\pi/h)(x_1 - x_2 + x_0)p} dp, \tag{4.2}$$

where x_0 is an arbitrary offset in space and h Plank's constant. The two particles are sent to different measurement devices. In order to describe the momentum of particle one the wave function can be expressed via the eigenfunctions

$$u_p(x_1) = \mathrm{e}^{(2i\pi/h)px_1} \tag{4.3}$$

with corresponding eigenvalue p. Then the state from (4.2) reads

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} \psi_p(x_2) u_p(x_1) dp, \tag{4.4}$$

where

$$\psi_p(x_2) = \mathrm{e}^{-(2\pi i/h)(x_2 - x_0)p}. \tag{4.5}$$

These ψ_p are the eigenfunctions of the operator

$$P = \frac{h}{2i\pi} \frac{\partial}{\partial x_2} \tag{4.6}$$

corresponding to the eigenvalue $-p$ of the momentum of the second particle.

In order to describe the coordinate of the first particle and the corresponding eigenfunctions

$$v_x(x_1) = \delta(x_1 - x), \tag{4.7}$$

with the delta-function δ, equation (4.2) can be rewritten as

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} \varphi_x(x_2) v_x(x_1) dx, \tag{4.8}$$

where

$$\varphi_x(x_2) = \int_{-\infty}^{\infty} \mathrm{e}^{(2i\pi/h)(x - x_2 + x_0)p} dp = h\delta(x - x_2 + x_0) \tag{4.9}$$

are the eigenfunctions of the operator

$$Q = x_2 \tag{4.10}$$

of the coordinate of the second particle with eigenvalue $x + x_0$.

Since the operators of momentum and coordinate do not commute

$$PQ - QP = \frac{h}{2i\pi} \tag{4.11}$$

the authors reasoning goes as follows. In case of measurement of the momentum of the first particle wave function (4.4) collapses into the eigenfunction $u_k(x_1)$ corresponding to the measured eigenvalue p_k of the momentum. Therefore, the second particle is found in the state given by the wave function $\psi_k(x_2)$. On the other hand, in the case of measurement of the first particle's coordinate, the wave function (4.8) collapses into the eigenfunction $v_r(x_1)$ corresponding to the measured eigenvalue x_r. Therefore, the second particle is found in the state given by the wave function $\varphi_r(x_2)$.

"We see therefore that, as a consequence of two different measurements performed upon the first system, the second system may be left in states with two different wave functions. On the other hand, since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system. This is, of course, merely a statement of what is meant by the absence of an interaction between the two systems. Thus, it is possible to assign two different wave functions (in our example ψ_k and φ_r) to the same reality (the second system after the interaction with the first)" (p. 779, [120]).

Because of this contradiction with the uncertainty principle the authors argue that quantum mechanics is not complete. In this argument, the authors assume a local theory that conforms to the laws of relativity, and thus excludes the influence the measurement on one particle has on the measurement of the other. A local theory is thereby characterized as: *"If two systems cannot interact, then the measurement on one system cannot change the state of the second system"* (p. 102, [116]).

4.2.2 The Bohmian-EPR thought experiment

In 1957 Bohm and Aharonov [128] introduced a special example which can present the arguments discussed above in a simplified form. Given is a wave function of two spin one-half particles in a so called (singlet) Bell-state,

$$\Psi = \frac{1}{\sqrt{2}} \left[\Psi_+(1)\Psi_-(2) - \Psi_-(1)\Psi_+(2) \right], \tag{4.12}$$

where $\Psi_+(1)$ refers to the wave function in which particle one has spin component $+\frac{1}{2}$, etc. Two particles are prepared in such a state and then separated by a process that does not affect the total spin, which is zero. After they have been separated to the point where they can no longer interact, an arbitrary component (i.e., rotated to any axis relative to the original orientation) of the spin of the first particle (A) is measured. This measurement can be done with a Stern-Gerlach experiment. Since the total spin is still zero, it can be concluded that the same component (orientation) of the spin of the other particle (B) is opposite that of (A).

If this was a classical system, there would be no difficulty interpreting these results since all components of the spin of each particle are precisely defined at each instant. In quantum theory, difficulty interpreting the above experiment arises because only one component of the spin of each particle can have a definite value at any given time. Thus, if the x-component of the spin is unique, then the y- and z-components are indeterminate, which can be described in terms of Pauli matrices with the commutation relation

$$[\sigma_i, \sigma_j] = 2i \sum_{k=1}^{3} \epsilon_{ijk}\sigma_k. \tag{4.13}$$

From these thoughts one obtains a similar contradiction as published by Einstein, Podolsky and Rosen.

4.2.3 Bell's theorem

The non-local aspect of quantum mechanics is difficult to grasp because it goes against the experience from our classical world. Hidden parameters, which are not yet known to physics, but which can precisely predict all measurement results, therefore, appear attractive. These parameters would finally be the part which, according to Einstein, Podolsky and Rosen, could render quantum mechanics complete.

It is precisely this circumstance that Bell's inequality deals with [121]. Bell formulates explicitly a quantum theory with local hidden parameters for the example proposed by Bohm and Aharonov (4.12). One can show mathematically that no theory with local hidden parameters will be able to reproduce the results of quantum mechanics. Thus, the starting point of the inequality is the quantum mechanical correlation of measured values. The product of the measured values for the compatible spin components in the directions $\vec{a}$ and $\vec{b}$ (both are unit vectors) in the entangled Bell state (4.12) with total spin zero has an expectation value of

$$E(\vec{a}, \vec{b}) \equiv \langle \Psi | \, \vec{\sigma}_1 \cdot \vec{a} \;\; \vec{\sigma}_2 \cdot \vec{b} \, | \Psi \rangle = -\vec{a} \cdot \vec{b}. \tag{4.14}$$

This correlation must be reproduced in a theory with local hidden parameters λ. For this purpose, the expectation value of such spin measurements is defined as follows

$$P(\vec{a}, \vec{b}) = \int d\lambda \rho(\lambda) A(\vec{a}, \lambda) B(\vec{b}, \lambda), \tag{4.15}$$

where $\rho(\lambda)$ is the normalized probability distribution of λ, and A, B are functions reproducing the quantum mechanical measurement results (possible eigenvalues of the Pauli matrices) of particle one in $\vec{a}$ direction and particle two in $\vec{b}$ direction. Here applies

$$\vec{a}\cdot\vec{\sigma}_1\,|\Psi_{\vec{a},A}\rangle = A(\vec{a},\lambda)\,|\Psi_{\vec{a},A}\rangle\,, \qquad \vec{b}\cdot\vec{\sigma}_2\,|\Psi_{\vec{b},B}\rangle = B(\vec{b},\lambda)\,|\Psi_{\vec{b},B}\rangle$$
$$A(\vec{a},\lambda) = \pm 1, \qquad B(\vec{b},\lambda) = \pm 1. \tag{4.16}$$

When the spin of both particles is measured in the same direction $\vec{a}$, the measured values are

$$A(\vec{a},\lambda) = -B(\vec{a},\lambda) \tag{4.17}$$

because of the properties of the singlet Bell state (4.12). With (4.15), (4.17) and a measurement in a third direction $\vec{c}$ the difference between two expectation values of measurements in different directions is

$$\begin{aligned} P(\vec{a},\vec{b}) - P(\vec{a},\vec{c}) &= -\int d\,\lambda\;\rho(\lambda)\left[A(\vec{a},\lambda)A(\vec{b},\lambda) - A(\vec{a},\lambda)A(\vec{c},\lambda)\right] \\ &= \int d\,\lambda\;\rho(\lambda)A(\vec{a},\lambda)A(\vec{b},\lambda)\left[A(\vec{b},\lambda)A(\vec{c},\lambda) - 1\right]. \end{aligned} \tag{4.18}$$

With (4.16) and the knowledge that $\int d\,\lambda\;\rho(\lambda)A(\vec{a},\lambda)A(\vec{b},\lambda) \leq 1$ one gets Bell's theorem

$$\begin{aligned} \left|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})\right| &\leq \int d\,\lambda\;\rho(\lambda)\left[1 - A(\vec{b},\lambda)A(\vec{c},\lambda)\right] \\ \left|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})\right| &\leq 1 + P(\vec{b},\vec{c}). \end{aligned} \tag{4.19}$$

For many direction vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ the quantum mechanical expectation value $E(\vec{a},\vec{b}) = -\vec{a}\cdot\vec{b}$ (4.14) satisfies this inequality (4.19). However, there are directions for which it is not compatible with quantum theory, i.e., for which the inequality

$$0 \leq 1 - \vec{b}\cdot\vec{c} - \left|\vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{c}\right| \tag{4.20}$$

is violated. Assuming the angle between $\vec{a}$ and $\vec{b}$ is $\varphi_1 = \pi/3$ and the angle between $\vec{a}$ and $\vec{c}$ is $\varphi_2 = 2\pi/3$, equation (4.20) becomes

$$\begin{aligned} 0 \leq& 1 - \cos(\varphi_2 - \varphi_1) - |\cos\varphi_2 - \cos\varphi_1| \\ & 1 - \cos\frac{\pi}{3} - \left|\cos\frac{2\pi}{3} - \cos\frac{\pi}{3}\right| = -\frac{1}{2}. \end{aligned}$$

4.2.4 The CHSH inequality

Later in 1969 Clauser, Horne, Shimony and Holt [122] presented a generalized version off Bell's theorem, the so called CHSH inequality. This inequality could be checked in an experiment, which was done by Freedman and Clauser [123]. For this inequality one starts again at the singlet Bell state (4.12). Then the correlation function

$$P(a,b) \equiv \int_\Gamma A(a,\lambda)B(b,\lambda)\rho(\lambda)d\,\lambda \tag{4.21}$$

is defined, where Γ is the total λ space. The variables a and b are adjustable apparatus parameters for measurements and $A(a,\lambda)$ and $B(b,\lambda)$ are deterministic functions representing the measurement results. The possible values are ± 1. Therefore, this correlation function is equal to Bell's expectation value (4.15).

Now also the difference between two correlation functions with different apparatus parameters are taken and rearranged according to Bell,

$$\begin{aligned}|P(a,b)-P(a,c)| &\leq \int_\Gamma |A(a,\lambda)B(b,\lambda)-A(a,\lambda)B(c,\lambda)|\,\rho(\lambda)d\,\lambda \\ &= \ldots = \\ &= 1-\int_\Gamma B(b,\lambda)B(c,\lambda)\rho(\lambda)d\,\lambda. \end{aligned} \tag{4.22}$$

To avoid Bell's experimentally unrealistic restriction that for some pair of parameters there is perfect correlation (e.g. $P(a,b)=1$), a fourth parameter d is needed and correlations are defined as

$$P(d,b)=1-\delta, \tag{4.23}$$

where $0\leq\delta\leq 1$ and the experimentally interesting cases will have δ close to but not equal to zero. By dividing the total λ space Γ into two regions with $\Gamma_\pm=\{\lambda|A(d,\lambda)=\pm B(b,\lambda)\}$ there is $\int_{\Gamma_-}\rho(\lambda)d\,\lambda=\delta/2$. At the right hand side of (4.22) follows

$$\begin{aligned}\int_\Gamma B(b,\lambda)B(c,\lambda)\rho(\lambda)d\,\lambda &= \int_\Gamma A(d,\lambda)B(c,\lambda)\rho(\lambda)d\,\lambda-2\int_{\Gamma_-}A(d,\lambda)B(c,\lambda)\rho(\lambda)d\,\lambda \\ &\geq P(d,c)-2\int_{\Gamma_-}|A(d,\lambda)B(c,\lambda)|\,\rho(\lambda)d\,\lambda=P(d,c)-\delta. \end{aligned} \tag{4.24}$$

This results with (4.22) and (4.23) in the CHSH inequality

$$\begin{aligned}|P(a,b)-P(a,c)| &\leq 2-P(d,b)-P(d,c) \\ |P(a,b)+P(d,b)+P(d,c)-P(a,c)| &\leq 2. \end{aligned} \tag{4.25}$$

The parameters a,b,c and d can also be realized again with the angle setting of Stern-Gerlach experiments. The concrete experimental implementation, as envisioned in [122] and implemented in [123], is based on the polarization of entangled photons and their detection. The influence of the polarizers and detectors on concrete measurements is also addressed in these publications and integrated into the inequality. However, in this work only the principal understanding of the physics behind these phenomena is to be worked up, and emphasis on this is given below.

4.2.5 Hardy's approach: Non-locality for two particles without inequalities

In 1993, Lucien Hardy presented an experiment that could prove the non-locality for two particles without inequalities for almost all entangled states [127]. The experiment starts with a system of two (not maximally) entangled particles, where α and β are two real constants with $\alpha^2+\beta^2=1$,

$$|\Psi\rangle=\alpha\,|+\rangle_1|+\rangle_2-\beta\,|-\rangle_1|-\rangle_2\,. \tag{4.26}$$

The plus and minus signs in this notation represent orientations of electron spins in the z-direction with directions "up" and "down", which can be measured in a

Stern–Gerlach experiment. These two particles are sent to two different researchers (Alice and Bob) with different Stern–Gerlach experiments. Alice and Bob can measure the spin orientation in the z-direction and two other directions tilted by the angles θ_1 and θ_2, where $\tan(\theta_1/2) = \sqrt{\alpha/\beta}$ and $\tan(\theta_2/2) = -(\alpha/\beta)^{3/2}$. These tilted measurements can be expressed in new bases. There is the skewed θ_1-basis with states "up", $|u\rangle$, and "down", $|v\rangle$, and the horizontal θ_2-basis with the basis states "right", $|c\rangle$, and "left" $|d\rangle$. The initial state can now, according to quantum theory, be expressed for four different measurement cases.

1. Alice and Bob both decide to measure the spin in θ_1-direction:

$$|\Psi\rangle = N(AB\,|u\rangle_1\,|v\rangle_2 + AB\,|v\rangle_1\,|u\rangle_2 + B^2\,|v\rangle_1\,|v\rangle_2) \tag{4.27}$$

2. Alice measures the spin in θ_2-direction and Bob in θ_1-direction:

$$|\Psi\rangle = N(A\,|c\rangle_1\,|u\rangle_2 + B\,|c\rangle_1\,|v\rangle_2 - A^2A^*\,|c\rangle_1\,|u\rangle_2 - A^2B\,|d\rangle_1\,|u\rangle_2) \tag{4.28}$$

3. Alice measures the spin in θ_1-direction and Bob in θ_2-direction:

$$|\Psi\rangle = N(A\,|u\rangle_1\,|c\rangle_2 + B\,|v\rangle_1\,|c\rangle_2 - A^2A^*\,|u\rangle_1\,|c\rangle_2 - A^2B\,|u\rangle_1\,|d\rangle_2) \tag{4.29}$$

4. Alice and Bob both decide to measure the spin in θ_2-direction:

$$\begin{aligned}|\Psi\rangle = N((1-|A|^4)\,|c\rangle_1\,|c\rangle_2 + A^2BA^*\,|d\rangle_1\,|c\rangle_2 + A^2A^*B\,|c\rangle_1\,|d\rangle_2 \\ - A^2B^2\,|d\rangle_1\,|d\rangle_2).\end{aligned} \tag{4.30}$$

Here, $N = \frac{1-|\alpha\beta|}{|\alpha|-|\beta|}$, $A = \frac{\sqrt{\alpha\beta}}{\sqrt{1-|\alpha\beta|}}$ and $B = \frac{|\alpha|-|\beta|}{\sqrt{1-|\alpha\beta|}}$ are prefactors.

Equation (4.30) clearly states that in some cases Alice and Bob can both measure spin "down", $|d\rangle$, in the θ_2-basis. If the measurement result is predetermined by a hidden parameter, then in such an experiment, the two particles must be prepared in such a way that if Alice and Bob measure in the θ_2-basis, both get spin "down" as a result.

If just such a spin pair were underway, but Bob had chosen to measure in the oblique θ_1-basis, his result would also be predetermined according to Equation (4.28), as Bob must measure spin "up" in the θ_1-basis because there is no other possibility in which the predetermined result "θ_2-down" for Alice is preserved. When Bob chooses the skewed θ_1-basis so late that his measurement cannot affect Alice's measurement according to the laws of realism and locality, then a hidden parameter λ, which is still unknown but is supposed to fully describe quantum mechanics, must account for the result of Equation (4.28). The same is true in case Bob remains in the θ_2-basis and Alice chooses the skewed θ_1-basis according to Equation (4.29). Here, the only possibility for Alice is to obtain the value "up" in the θ_1-basis.

Finally, if both chose the Stern–Gerlach experiment in the θ_1-direction, the two cases with a hidden parameter λ discussed above indicate that both Alice and Bob must measure spin "up" since no setup knows the orientation of the other. However, such a measurement result is forbidden according to quantum theory, cf. Equation (4.27). Here lies the contradiction, which can be checked experimentally.

4.2.6 Conclusion and relevant elements for reconstruction

According to the current state of knowledge of physics, the EPR paradox is not a paradox or contradiction in quantum mechanics at all. Rather, entanglement is an elementary part of quantum mechanical nature and cannot be described with local hidden parameters.

In the scientific community, as in literature, Bohm's variant of the EPR paradox has prevailed because it is much more descriptive and quicker to understand. For a reconstruction the achievement of Einstein, Podolsky and Rosen can be mentioned, but rather for a mediation of the historical context or for the illustration of the nature of science and knowledge gain. For the mediation of the contents, however, Bohm's example is rather recommended.

Furthermore, there are other approaches which describe and try to explain the phenomenon of quantum entanglement. In addition, Bohm's pilot wave theory [129, 130] and the many-world theory [131] have to be mentioned. However, these approaches have not become generally accepted in the scientific community, which is why they will not be discussed further here.

The situation for Bell's inequality and the CHSH inequality is similar to that of the EPR paradox. They are important historical milestones for the experimental proof to which the Nobel Prize in Physics was awarded in 2022. However, the handling of correlations, statistical expectation values, the many directions of measurement, and the estimation with inequalities make it difficult for novices in quantum mechanics to understand all statements and logical steps. In the experiment according to Hardy, a case-by-case analysis of four cases with 2 angle settings for Alice and Bob is sufficient in the end. While Hardy's concept is experimentally challenging the low number of different cases renders it well suited for a didactical approach. Therefore, in the following the topic of non-locality in quantum physics and local hidden parameters is reconstructed according to Hardy. The essential elements to fully explain the experiment are presented below. These elements refer to a set of four qualitative rules of quantum mechanics, which have been identified elsewhere [132, 133] as the basic traits of quantum physics. The following definitions are from [134]:

1. **Statistical behavior:** A result of a single event cannot be predicted, it is random! Only statistical predictions (for many repetitions) are possible in quantum physics.

2. **Ability to interfere:** Single quantum objects can contribute to an interference pattern, if there are more than one classically possible ways leading to the same experimental result. None of these ways will then "realize" in a classical sense.

3. **Unique measurement results:** Although a quantum object in a state does not have a fixed value of the measured quantity, you always find a unique measurement result. Repeating the same measurement may give a different result, though.

4. **Complementary:** "Which way" information and an information about an interference pattern mutually are exclusive. Quantum objects cannot be prepared in a defined position with a defined momentum at the same time.

The relevant elements for the reconstruction of the quantum entanglement and the exclusion of local hidden parameters are as follows.

The spin (or polarization of single photons) and its representation as a fundamental physical property to be studied.

The phenomenon of entanglement can be explained and discussed in general as in the EPR paper [120], but for a first access it is recommended to illustrate these properties by a concrete example. The model system itself should be as simple as possible and as complex as necessary. The properties of two-level systems and their relatively simple description in a two-dimensional Hilbert space perfectly are suited as a prime example. For this work the quantum mechanical spin was chosen to make additionally clear that the location space and Hilbert space are usually not identical. The Hilbert space can thus be introduced and motivated better than in the case of the polarization of photons. The quantum mechanical properties of spin and their mathematical representation are thus independent learning objectives in this unit. Through the mathematical representation of the spin state with different basis vectors, the "ability to interfere" can be demonstrated and discussed here.

The Stern-Gerlach experiment (the spin measurement) as a model for the quantum mechanical measurement process with the collapse of the wave function.

A central point in Einstein's, Podolsky's and Rosen's apparent contradiction is the collapse of the wave function during a measurement. This circumstance can be ideally illustrated by the Stern-Gerlach experiment. The collapse of the wave function in Hilbert space leads here to a rotation of the spin in location space, which can be checked afterwards again.

Here, the traits of "complementary" and "unique measurement results" can be discussed. The complementary is given in the spin settings. Each spin can be detected only in a state with parallel or anti-parallel alignment to the inhomogeneous magnetic field. These measurement results are at the same time unambiguous, because each spin must chose in the measurement one or the other spin orientation in such a magnetic field. Furthermore, the "loss of information" at the collapse of the wave function can be discussed, which can possibly be counted as the fifth (and unofficial) trait of quantum physics.

Changing the bases as a basis for probability statements.

Crucial for Hardy's experiment (as already for Bell and the CHSH inequality) are the spin orientation (or polarization) measurements in different directions. In the formalism of quantum mechanics this is expressed by different basis vectors, which represent the angular setting of the measuring apparatus. If a spin is oriented in one direction, but is measured in another direction, only probability statements of the outcome are possible. This is the basic trait "statistical behavior" according to [133]. Mathematically, this change of basis and the probability interpretation shall be identified with the projection of vectors.

The singlet Bell state as the simplest illustration of the phenomena.

The singlet Bell state (4.12) provides the simplest example that exhibits all the relevant phenomena and is thus the content element on which the course builds. This is true not only for Hardy's experiment but also for Bell's inequality or the CHSH inequality.

Linguistic elements

In order to understand the phenomenon of entanglement and its impact on our understanding of nature, fundamental terms need to be clarified and defined. The relevant terms are:

a) determinism,

b) locality,

c) causality,

d) reality,

e) complete theory.

Methodical element: cognitive contradiction – thinking through the case distinction.

The method of how hidden local parameters are excluded is a cognitive contradiction in Hardy's experiment [127]. Therefore, the aim of this teaching unit is to present a formulation of quantum mechanics with a hidden parameter and to test it in different measurement constellations. In order to reach and understand the contradiction as quickly as possible, the reformulation is guided and supported with cognitive activating elements.

4.3 Research on teaching and learning

The educational value of Hardy's proposal to refute local hidden parameters has already been recognized by others and didactically applied to everyday examples. There are attempts to illustrate the described phenomena with flashing lamps [135], Dutch doors [136] or quantum cakes [137].

This work is not intended to be oriented on these approaches. A reference to everyday life can illustrate the principles of measurement and the measurement results very well and probably make them approachable and interesting for pupils. However, quantum mechanics is a theory that is completely opposed to our everyday experience. Such pictures from everyday life could, however, bear the danger of reinforcing the classical pictures (and local hidden parameters are nothing else).

Furthermore, this work relies heavily on the fact that learning success depends primarily on the time and dedication that learners invest themselves. That means that learners are cognitively activated and the contradiction in a quantum theory with local hidden parameters is self-identified. Thus, the whole chapter 3 can be seen as the research on teaching. For the teaching unit on entanglement and hidden local parameters, the following activating methods should be valuable particularly.

Verbalization and visualization of formulas

Quantum mechanics builds on phenomena and principles without everyday experience, intuition, or analogies. The best representation that exists for it is the mathematical formalism. Therefore, learners should practice a way of dealing with it and train methods of how to read information and statements from the formalism and which other representations are still valid. The verbalization and visualization of formulas should therefore be a central component of all sections, both in teaching of content by the teacher and in self-study by the students.

Peer instruction

In large groups, peer instruction can help to ensure that everyone deals with the content at crucial points. Relevant questions that arise during more intensive discussion can also be posed to the learners in a pre-formulated manner. In this way, everyone deals with these questions and not just those who ask them of their own accord. The method of peer instruction can, of course, also be combined here with the verbalization of formulas.

Learning with worked examples

Learning with worked examples can support the gain of knowledge in the quantum mechanical consideration of the experiment. This can ensure that all relevant steps are thought through or calculated by students themselves. Thus, there remain no places where the additional knowledge of the teacher or other literature must be relied upon.

4.4 Design and Evaluation

Learning objectives of the course can be derived from the elements identified in the clarification of the science content 4.2.6. These learning objectives can be broadly divided into two categories. The first is knowing and understanding the properties and mathematical descriptions of spin and its measurement. The second category involves entanglement and hidden local parameters. The learning objectives are:

Quantum mechanical description of spin

1. Properties of spin in the Stern-Gerlach experiment.
2. Definition and mathematical description of states and observables.
3. The difference between location space and Hilbert space.
4. What happens in a measurement of Alice or Bob?

Entanglement and local hidden parameter

5. Definition of localism and realism.
6. Understanding of entanglement.
7. Definition of local hidden parameters.
8. What should be measured in physics with local hidden parameters?
9. What is measured in quantum physics?

In the following course, the learning objectives will be addressed sequentially.

4.4.1 The Stern-Gerlach experiment and the quantum mechanical spin

At the beginning, the Stern-Gerlach experiment is introduced and its measurement results are explained with the properties of the quantum mechanical spin. These properties are then related to the trait of unique measurement results.

Subsequently, these properties are translated into a mathematical formalism that accurately represents all of them. To support this, multiple external representations are relied on such that the information in the formalism can be recognized as quickly as possible. This includes arrows in Dirac notation representing the spin direction in location space and a representation of the spin right next to the mathematical representation using logical images in location space and Hilbert space.

A representation of the spin in y-direction with complex prefactors is deliberately omitted here since these are not needed for the later experiment on local hidden parameters and thus only would unnecessarily complicate the translation physics-mathematics.

An important insight for the learners is that for each complementary quantum property (to the previous ones) a dimension in Hilbert space is needed.

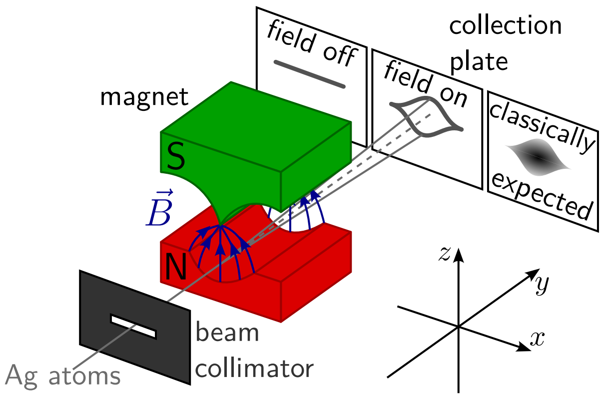


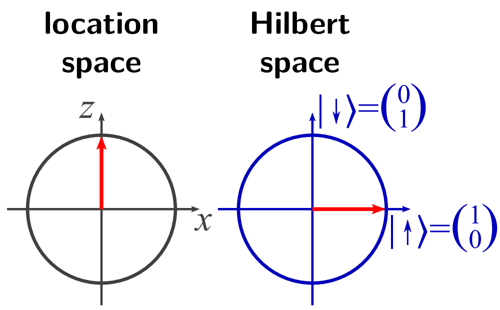
Figure 4.1. Sketch of the Stern-Gerlach experiment. Silver atoms are sent through an inhomogeneous magnetic field and are deflected parallel or antiparallel to the magnetic field depending on their spin.

The relevant measurement results from the Stern-Gerlach experiment (see figure 4.1) representing the properties of quantum mechanical spin are:

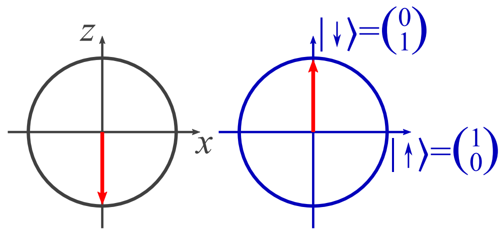
1. Every single atom is deflected up or down by a fixed amount.
2. If many atoms are measured in succession, both results are equally likely.
3. If the orientation of the magnetic field and the screen are rotated in the x,z-plane, 1 and 2 are preserved.

Spin in z-direction:

$$\text{up} = |+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |\uparrow\rangle =$$

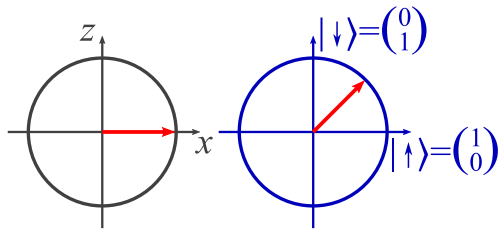


$$\text{down} = |-\rangle = \begin{pmatrix}0\\1\end{pmatrix} = |\downarrow\rangle =$$

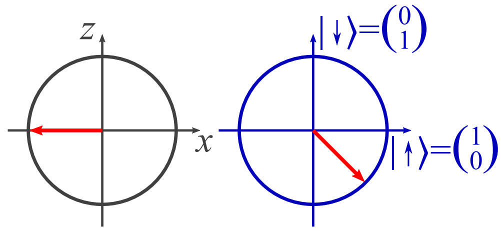


Spin in x-direction:

$$\begin{aligned}\text{right} &= \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}\\ &= \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) = |\rightarrow\rangle\end{aligned}$$

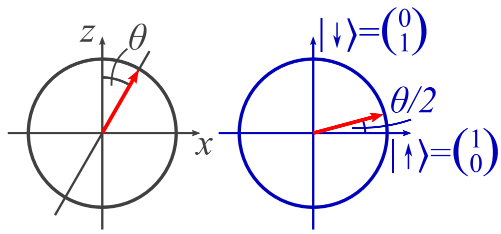


$$\begin{aligned}\text{left} &= \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} - \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}\\ &= \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle) = |\leftarrow\rangle\end{aligned}$$



Spin in x, z-plane rotated by θ:

$$\begin{aligned}\text{up}(\theta) &= \cos(\theta/2)\begin{pmatrix}1\\0\end{pmatrix} + \sin(\theta/2)\begin{pmatrix}0\\1\end{pmatrix}\\ &= \cos(\theta/2)\,|\uparrow\rangle + \sin(\theta/2)\,|\downarrow\rangle\end{aligned}$$



As an activating element, and to consolidate the mathematical formalism, it is now the students' task to formulate an appropriate formulation for the spin "down" state in the θ-direction. This can be implemented classically as a work phase or in the context of peer instruction.

Peer instruction:

What does the spin down(θ) look like?

1. $\cos\left(\frac{\theta}{2}\right)|\uparrow\rangle - \sin\left(\frac{\theta}{2}\right)|\downarrow\rangle$
2. $\sin\left(\frac{\theta}{2}\right)|\uparrow\rangle - \cos\left(\frac{\theta}{2}\right)|\downarrow\rangle$
3. $-\sin\left(\frac{\theta}{2}\right)|\uparrow\rangle + \cos\left(\frac{\theta}{2}\right)|\downarrow\rangle$
4. None of the above answers is correct.
5. Several of the above answers are correct.

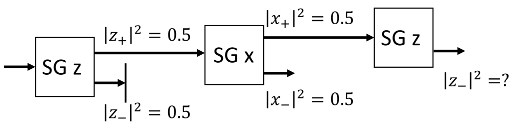
Since in these simple examples the phase information of the spin can be neglected, both 2. and 3. are legitimate representations for the spin "down" in θ-direction. Thus, the correct answer is 5.

4.4.2 Spin measurement in different orientations

If the previous section has already ended with a peer question on the Dirac notation, this section on the measurement process and the collapse of the wave function can be started with peer instruction. The question what happens in a sequence of Stern-Gerlach experiments aims at the loss of information after a measurement and is an elementary part of quantum mechanics. Subsequently, these properties of quantum mechanical measurement are translated into the formalism and it is shown how probability statements (traits "statistical behavior" and "ability to interfere") can be quantified concretely.

Peer instruction:

What is the probability that spin state down$(z) = |\downarrow\rangle$ is measured for a silver atom flying into the final Stern-Gerlach setup?



1. $100\,\%$
2. $50\,\%$
3. $25\,\%$
4. $0\,\%$
5. That depends on the time of flight and interaction with the magnetic field.

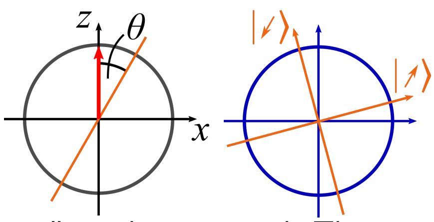
Because the information about the state before the measurement is lost during a measurement, the history of measurements of the silver atom is irrelevant for the last measurement. The probability that a silver atom with spin "up" in the x-direction results in "down" for a spin measurement in the z-direction is $50\,\%$, as described before. Therefore answer 2. is correct here.

The loss of information in a measurement about the state before is also described as the collapse of the wave function. The following section explains what happens with a spin "up" in z-direction in a measurement in θ-direction, how this can be described mathematically, and the probabilities for the respective measurement results.

The direction θ, in which the measurement is done, spans a new basis with its spin states in Hilbert space:

$$\text{up}(\theta) = |\nearrow\rangle = \cos\left(\frac{\theta}{2}\right)|\uparrow\rangle + \sin\left(\frac{\theta}{2}\right)|\downarrow\rangle$$

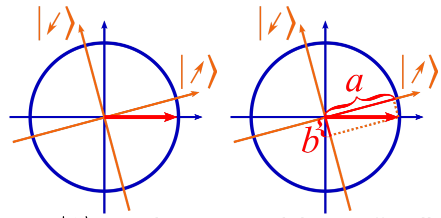
$$\text{down}(\theta) = |\swarrow\rangle = \sin\left(\frac{\theta}{2}\right)|\uparrow\rangle - \cos\left(\frac{\theta}{2}\right)|\downarrow\rangle$$



Also in the new direction only spin "up" or "down" can be measured. That is, the basis vectors $|\nearrow\rangle$ and $|\swarrow\rangle$ also describe the states into which a spin is forced when measuring the spin in θ direction. With what probability is a spin "up(z)" measured in the new basis "up" or "down"?

The spin $|\uparrow\rangle$ (red arrow) is projected in Hilbert space onto the new rotated basis:

$$|\uparrow\rangle = a\,|\nearrow\rangle + (-b)\,|\swarrow\rangle$$



The probability of measuring the original spin $|\uparrow\rangle$ in the rotated basis "up" is the magnitude square of the coefficient of this state.

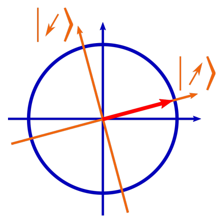
$$\begin{aligned}\textbf{prob}\{\mathsf{up}(\theta)|\,|\uparrow\rangle\} &= |\,\langle\nearrow\,|\,\uparrow\rangle\,|^2 = |\,\langle\nearrow|\cdot(a\,|\nearrow\rangle - b\,|\swarrow\rangle)|^2 \\ &= |a\underbrace{\langle\nearrow\,|\,\nearrow\rangle}_{=1} - b\underbrace{\langle\nearrow\,|\,\swarrow\rangle}_{=0}|^2 = |a|^2\end{aligned}$$

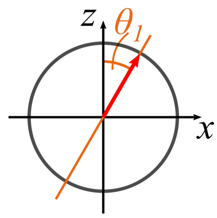
Analogously, for the probability in the rotated basis "down" holds:

$$\begin{aligned}\textbf{prob}\{\mathsf{down}(\theta)|\,|\uparrow\rangle\} &= |\,\langle\swarrow\,|\,\uparrow\rangle\,|^2 = |\,\langle\swarrow|\cdot(a\,|\nearrow\rangle - b\,|\swarrow\rangle)|^2 \\ &= |a\,\langle\swarrow\,|\,\nearrow\rangle - b\,\langle\swarrow\,|\,\swarrow\rangle\,|^2 = |b|^2\end{aligned}$$

Directly after a measurement the spin is in exactly the state corresponding to the measurement. If here in the example a spin "up(θ)" is measured in the rotated basis, the spin is also rotated in location space by the angle θ with respect to its original orientation.

$$\mathsf{up}(\theta) = |\nearrow\rangle = \cos\left(\frac{\theta}{2}\right)|\uparrow\rangle + \sin\left(\frac{\theta}{2}\right)|\downarrow\rangle$$





The projection "spin up in rotated direction" $(\hat{\nearrow})$ can be defined in Dirac notation as:

$$(\hat{\nearrow}) = |\nearrow\rangle\,\langle\nearrow|$$

The associated measured values are eigenvalues of the operator $(\hat{\nearrow})$ and take the values 0 and 1,

$$(\hat{\nearrow}) \widehat{=} \begin{cases} 1, & \text{if spin up}(\theta) \text{ was measured,} \\ 0, & \text{otherwise.} \end{cases}$$

The same applies to other directions:

$$(\hat{\swarrow}) \widehat{=} \begin{cases} 1, & \mathsf{down}(\theta), \\ 0, & \mathsf{up}(\theta), \end{cases} \qquad (\hat{\leftarrow}) \widehat{=} \begin{cases} 1, & \mathsf{down}(90^\circ), \\ 0, & \mathsf{up}(90^\circ), \end{cases} \qquad (\hat{\rightarrow}) \widehat{=} \begin{cases} 1, & \mathsf{up}(90^\circ), \\ 0, & \mathsf{down}(90^\circ). \end{cases}$$

4.4.3 Entanglement, locality and reality

In this section the phenomenon of entanglement and the necessary terminology will be introduced. Using the example of pion decay, the singlet Bell state is introduced. Afterwards the terms of determinism, causality, reality, locality and complete theory from section 4.2.6 are introduced and discussed.

Pions are the lightest mesons. Today they are not considered as elementary particles because they consist of valence quarks. Like all mesons they are bosons, i.e. they have an integer spin. There is one neutral pion and two charged pions. All three are unstable and decay by the electroweak interaction. Neutral pions decay into an electron, a positron, and a photon in $1.17\,\%$ of all decays. During the decay all known conservation laws of physics apply. The conservation of energy and momentum can be discussed here additionally, but the conservation of angular momentum can be used to justify why the electron and the positron must have an antiparallel spin. The two particles are therefore in a singlet Bell state.

$$\pi^0 \rightarrow e^+ + e^- + \gamma$$

These two new particles then can be separated and sent to different researchers with different Stern-Gerlach experiments, namely Alice (A) and Bob (B). The spin physics of electrons and positrons are equal.

$$|\psi\rangle_{\text{Bell}} = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2 \right) \tag{4.31}$$

When the positron and the electron are separated after their creation and sent to Alice and Bob (sketched in figure 4.2), the problem of Einstein Podolsky, and Rosen with their conception of reality and locality can be discussed very vividly.

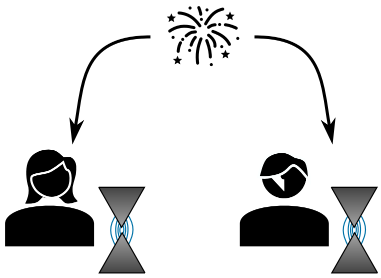


Figure 4.2. Sketch of the creation and separation of two particles in a Bell state. They are sent to Alice and Bob with a Stern-Gerlach experiment each. The Sketch should help to illustrate the problems of Einstein, Podolsky and Rosen with their assumption of reality and locality.

For a qualitatively adequate discussion of entanglement, essential terms must be defined and understood. These are terms and concepts that have been discussed in the scientific community for a very long time and in different contexts. In some cases, these discussions are on the borderline between philosophy and physics. It is therefore not to be expected that these discussions can be repeated in a lecture or class in a comparable quality and detail. Thus, it is not the claim of this thesis to represent these discussions in their completeness. The focus on this topic is strongly on the physical aspects. The explanations of the terms can be taken as working definitions to provide a solid introduction to this topic, and not as complete representations of the state of science. The relevant terms are:

a) **Determinism:** An event is uniquely predetermined if the initial situation is known with sufficient precision.
According to the Copenhagen interpretation, quantum mechanics is no longer

deterministic in the single case of measurement. In the case of the collapse of the wave function an objective coincidence prevents an unambiguous prediction when several measurement results are possible. Otherwise, quantum mechanics has many deterministic aspects, such as the Schrödinger equation and its stochastic interpretation.

b) **Locality** according to EPR [120]: If two systems cannot interact, then the measurement on one system cannot change the state of the second system.
Relevant to this definition of locality is the insight from relativity that nothing in the universe propagates faster than the speed of light. However, the view has gained acceptance in the scientific community that quantum mechanics is a non-local theory. The reasons for this are the experiments described here.

c) **Causality:** No effect without cause.
This classical interpretation of causality is closely related to determinism. If one knows the state of a system in all parameters, one can calculate a future state from it with the help of the laws of nature. Quantum mechanics, at least according to the Copenhagen interpretation, is not deterministic only in the measurement process. In a sense, causality persists in quantum mechanics. Indeed, the effect of one measurement on a second measurement mediated by entanglement is non-local. However, due to the unpredictability of the single measurement, no information can be transmitted with superluminal speed yet.

d) **Reality** according to EPR [120]: *"If we can predict the value of a quantity with certainty without disturbing the system, then there exists an element of physical reality that we associate with that quantity."*
Mapping reality or nature is an elementary part of physics. Until the development of quantum mechanics, this reality was considered independent of its description and observation. Why or how this can be the case is part of philosophical definitions and discussions and will not be addressed here. However, the observations of Clauser, Aspect and Zeilinger reveal experimental facts which prove that a simple and classical understanding of reality and locality is not sufficient to describe quantum mechanics. The stochastic interpretation of the wave function is simply part of the quantum mechanical reality.

e) **Complete theory** according to EPR [120]: *"Every element of the physical reality must have a counterpart in the physical theory."*
This very fact was challenged in quantum mechanics by Einstein, Podolsky and Rosen. Today the view has prevailed that quantum mechanics is complete and we cannot say more about a system than is possible with the wave function and its probability interpretation.

Since mainly knowledge and definitions are presented in this section, the activating part lies on a discussion in small groups and later in the large group about the views on Einstein, Podolsky and Rosen.

4.4.4 The Hardy experiment

In this section, the Hardy experiment is presented and discussed in quantum mechanical terms. To do so, the initial state is expressed in all the basis representations possible given in Alice's and Bob's random choice of angle setting. To remain consistent with the previous notation and to make the discussion of states simpler, no arbitrary new basis states are chosen. Since the initial state is represented in the z-basis, Alice and Bob have the option of setting their Stern-Gerlach experiments once oblique and once perpendicular to it.

Interactive elements here are the verbalization of the state equations and the collection of information from them. Concretely, these elements are again implemented in a peer instruction. Due to time constraints, the seminar will refrain from calculating the conversion of the initial state into the representation in different bases. Another activating element can be to work on these transformations in the context of "worked examples" as homework. Then, each step in the exclusion of hidden local parameters can be reproduced by the students themselves.

Two particles with spin 1/2 are generated in the not maximally entangled state

$$|\Psi\rangle = \alpha\,|\uparrow\rangle_1\,|\uparrow\rangle_2 - \beta\,|\downarrow\rangle_1\,|\downarrow\rangle_2 \tag{4.32}$$

and sent to different locations. There they can be measured by two parties (usually called Alice and Bob) in two bases rotated with respect to the z-direction.

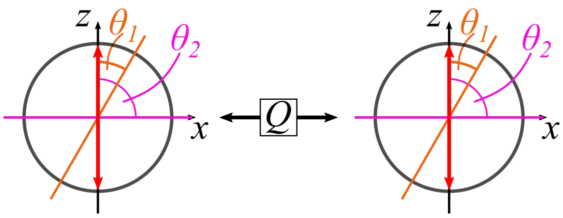
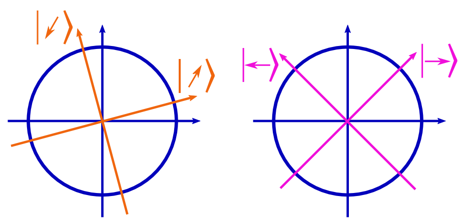


Figure 4.3. Sketch of the orientations θ_1 and θ_2, in which Alice and Bob can measure the spin of their particle.

These two measurement directions provide four new basis vectors

$$|\nearrow\rangle = \frac{1}{\sqrt{|\alpha| + |\beta|}}(\beta^{\frac{1}{2}}\,|\uparrow\rangle + \alpha^{\frac{1}{2}}\,|\downarrow\rangle)$$
$$|\swarrow\rangle = \frac{-1}{\sqrt{|\alpha| + |\beta|}}(\alpha^{\frac{1}{2}}\,|\uparrow\rangle - \beta^{\frac{1}{2}}\,|\downarrow\rangle)$$
$$|\rightarrow\rangle = \frac{1}{\sqrt{|\alpha|^3 + |\beta|^3}}(\beta^{\frac{3}{2}}\,|\uparrow\rangle + \alpha^{\frac{3}{2}}\,|\downarrow\rangle)$$
$$|\leftarrow\rangle = \frac{-1}{\sqrt{|\alpha|^3 + |\beta|^3}}(\alpha^{\frac{3}{2}}\,|\uparrow\rangle - \beta^{\frac{3}{2}}\,|\downarrow\rangle)$$



with

$$\tan\frac{\theta_1}{2} = \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}}, \ \tan\frac{\theta_2}{2} = -\left(\frac{\alpha}{\beta}\right)^{\frac{3}{2}}.$$

The state (4.32) can be rearranged with the new bases and some computational work to

$$|\Psi\rangle = N(AB\,|\nearrow\rangle_1\,|\swarrow\rangle_2 + AB\,|\swarrow\rangle_1\,|\nearrow\rangle_2 + B^2\,|\swarrow\rangle_1\,|\swarrow\rangle_2) \tag{4.33}$$
$$|\Psi\rangle = N(A\,|\rightarrow\rangle_1\,|\nearrow\rangle_2 + B\,|\rightarrow\rangle_1\,|\swarrow\rangle_2 - A^2A^*\,|\rightarrow\rangle_1\,|\nearrow\rangle_2 - A^2B\,|\leftarrow\rangle_1\,|\nearrow\rangle_2) \tag{4.34}$$

$$|\Psi\rangle = N(A\,|{\nearrow}\rangle_1\,|{\rightarrow}\rangle_2 + B\,|{\swarrow}\rangle_1\,|{\rightarrow}\rangle_2 - A^2A^*\,|{\nearrow}\rangle_1\,|{\rightarrow}\rangle_2 - A^2B\,|{\nearrow}\rangle_1\,|{\leftarrow}\rangle_2) \tag{4.35}$$

$$|\Psi\rangle = N((1-|A|^4)\,|{\rightarrow}\rangle_1\,|{\rightarrow}\rangle_2 + A^2BA^*\,|{\leftarrow}\rangle_1\,|{\rightarrow}\rangle_2 + A^2A^*B\,|{\rightarrow}\rangle_1\,|{\leftarrow}\rangle_2 - A^2B^2\,|{\leftarrow}\rangle_1\,|{\leftarrow}\rangle_2) \tag{4.36}$$

$$\text{with} \qquad A = \frac{\sqrt{\alpha\beta}}{\sqrt{1-|\alpha\beta|}}, \qquad B = \frac{|\alpha|-|\beta|}{\sqrt{1-|\alpha\beta|}}, \qquad N = \frac{1-|\alpha\beta|}{|\alpha|-|\beta|}.$$

The following peer questions can be used to encourage students to extract relevant information from these states.

Peer instruction

What does Bob measure when Alice has measured $(\hat{\leftarrow}) = 1$?

1. $(\hat{\leftarrow}) = 1$ and $(\hat{\nearrow}) = 1, 0$ with different probabilities.
2. $(\hat{\leftarrow}) = 0$ and $(\hat{\nearrow}) = 1, 0$ with different probabilities.
3. $(\hat{\nearrow}) = 1$ and $(\hat{\leftarrow}) = 1, 0$ with different probabilities.
4. $(\hat{\nearrow}) = 0$ and $(\hat{\leftarrow}) = 1, 0$ with different probabilities.

To lower the difficulty level a bit, the first question can be replaced by the solution to show the learners the principle procedure.

If Alice gets confirmation on spin "left" ($(\hat{\leftarrow})_1 = 1$), i.e., the first spin is reduced to the state $|{\leftarrow}\rangle_1$, equation (4.34) says that Bob will definitely find his spin in the state $|{\nearrow}\rangle_2$.

$$\begin{aligned}(\hat{\leftarrow})_1\,|\Psi\rangle &= {}_1|{\leftarrow}\rangle\,\langle{\leftarrow}|_1\,N(A\,|{\rightarrow}\rangle_1\,|{\nearrow}\rangle_2 + B\,|{\rightarrow}\rangle_1\,|{\swarrow}\rangle_2 - A^2A^*\,|{\rightarrow}\rangle_1\,|{\nearrow}\rangle_2 \\ &\quad - A^2B\,|{\leftarrow}\rangle_1\,|{\nearrow}\rangle_2) \\ &= |{\leftarrow}\rangle_1\,N(A\;{}_1\langle{\leftarrow}\mid{\rightarrow}\rangle_1\,|{\rightarrow}\rangle_2 + B\;{}_1\langle{\leftarrow}\mid{\rightarrow}\rangle_1\;|{\swarrow}\rangle_2 \\ &\quad - A^2A^*\;{}_1\langle{\leftarrow}\mid{\rightarrow}\rangle_1\;|{\nearrow}\rangle_2 - A^2B\;{}_1\langle{\leftarrow}\mid{\leftarrow}\rangle_1\;|{\nearrow}\rangle_2) \\ &= |{\leftarrow}\rangle_1\,N(-A^2B)\,|{\nearrow}\rangle_2 = -NA^2B\,|{\leftarrow}\rangle_1\,|{\nearrow}\rangle_2\end{aligned}$$

Equation (4.36) says that Bob can measure spin "left" and "right" ($(\hat{\leftarrow})_1 = 1, 0$) if he also chooses the horizontal base. So answer 3 is correct.

What does Alice measure when Bob has measured $(\hat{\leftarrow}) = 1$?

1. $(\hat{\leftarrow}) = 1$ and $(\hat{\nearrow}) = 1, 0$ with different probabilities.
2. $(\hat{\leftarrow}) = 0$ and $(\hat{\nearrow}) = 1, 0$ with different probabilities.
3. $(\hat{\nearrow}) = 1$ and $(\hat{\leftarrow}) = 1, 0$ with different probabilities.
4. $(\hat{\nearrow}) = 0$ and $(\hat{\leftarrow}) = 1, 0$ with different probabilities.

Equation (4.35) provides the solution for the case where Bob measures spin "left" ($(\hat{\leftarrow})_1 = 1$), and Alice measures in the oblique basis. Then Alice will measure spin "up"

$((\hat{\nearrow})_1 = 1)$ in either case. If Alice also chooses to measure in the horizontal basis, both spin orientations are possible measurement outcomes. Answer 3 is, therefore, correct.

With what probability do Alice & Bob both measure spin $(\hat{\leftarrow}) = 1$, or both spin $(\hat{\nearrow}) = 1$ in their experimental setup?

1. $|A^2B^2|^2$, and $0\,\%$ respectively.
2. $(1 - A^4)$, and $|NAB|^2$ respectively.
3. $-NA^2B^2$, and $100\,\%$ respectively.
4. $|NA^2B^2|^2$, and $0\,\%$ respectively.

For this third question, the equations (4.33) and (4.36) provide the solution. The magnitude square of the prefactors gives the measurement probability to measure the spins in the corresponding states. The last term of equation (4.36) provides the solution for the first case. In equation (4.33), the state for the measurement spin "up" of both spins in the oblique basis does not occur. This measurement result is therefore forbidden and does not occur. The probability of such a measurement is, therefore, $0\,\%$. So answer 4. is correct.

4.4.5 Quantum mechanics with local hidden parameters

In order to exclude quantum mechanics with local hidden parameters, Hardy generates a contradiction with the real quantum mechanical measurement combinations. As shown in section 4.2.5, for this purpose only those particle pairs have to be considered which guarantee the measurement combination spin "left" and "left" when Alice and Bob are simultaneously set to measurement in the horizontal basis. The task of the students is now to investigate in a distinction of cases which measurement combinations predict hidden local parameters for such spin pairs. Consequently, only four different measurement combinations of Alice and Bob are relevant. These can be supported again with logical images.

The starting point are spin pairs, for which a local hidden parameter guarantees spin "left" and "left" measurement results when Alice and Bob both measure in the horizontal basis, see figure 4.4.

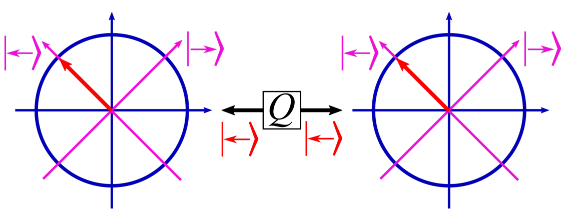


Figure 4.4. Logical image for a spin measurement for Alice (left) and Bob (right) in the horizontal basis with a spin pair leading the predetermined result.

If Bob chooses the oblique basis (illustrated in figure 4.5) so late that his measurement cannot affect Alice's measurement according to the laws of realism and locality, then a local hidden parameter, which is still unknown but is supposed to fully describe quantum mechanics, must account for the result from equation (4.34). In this case, Bob's result would also be predetermined as spin "up" in the oblique basis due to equation (4.34).

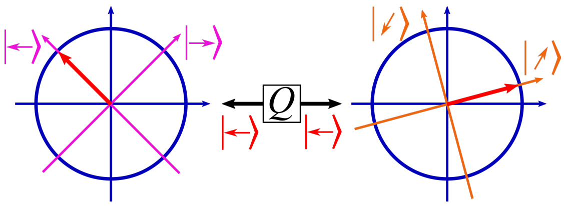


Figure 4.5. Logical image for a spin measurement for Alice (left) in the horizontal basis and Bob (right) in the oblique with a spin pair leading to the predetermined result in the horizontal basis shown in figure 4.4.

Imagine the reverse case, illustrated in figure 4.5. Alice decides to turn her Stern-Gerlach experiment around after the particles have been prepared, and measures in the oblique θ_1 basis. This is done while the particles are in transit and no information about the angle Alice changed can reach Bob's particles before they are measured. In this case, Alice's result would also be predetermined as spin "up" in the oblique basis due to the equation (4.35).

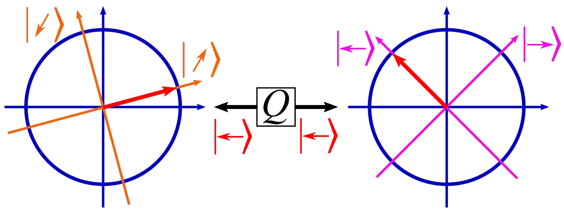


Figure 4.6. Logical image for a spin measurement for Alice (left) in the oblique basis and Bob (right) in the horizontal basis with a spin pair leading to the predetermined result in the horizontal basis shown in figure 4.4.

The same argumentation is valid for the case that both decide to change their experiment from the horizontal to the oblique basis after the generation of the particles, see figure 4.7. The decision of one must not have any influence on the measurement of the other, in case one would have stayed in the horizontal basis. Local hidden parameters would have to ensure that in this case Alice and Bob measure a spin "up" in the oblique base.

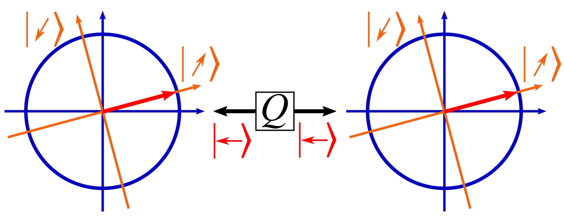


Figure 4.7. Logical image for a spin measurement for Alice (left) and Bob (right) in the oblique base with a spin pair leading to the predetermined result in the horizontal basis shown in figure 4.4 according to the intermediate steps in figures 4.5 and 4.6.

But the fact that Alice and Bob both measure spin "up" in the oblique θ_1-basis is forbidden by equation (4.33). Thus, we have a contradiction between quantum theory with and without a hidden parameter. This fact is also experimentally verified. With the right bases for spin measurement, depending on the initial state, certain spin combinations cannot be measured.

This whole elaboration should be as easy to understand as possible by using logical pictures and a quickly interpretable notation, nevertheless this thought experiment with the case distinction can be perceived as difficult and hurdle. An analog experiment can serve as a support in this case. Here, a hidden parameter is concretely implemented in the material properties of balls and the students can act out the case distinction themselves.

The hidden parameter is the magnetizability of balls, which pass through a box with two-way junctions (see figure 4.8). Magnetizable balls are deflected, and the others are not. Two Stern–Gerlach experiments each now can be selected by Alice and Bob, each representing a measurement in the θ_1 or θ_2-direction. The setup can be built to reproduce all the cases discussed above for a $|\leftarrow\rangle_1 |\leftarrow\rangle_2$-pair. Thus, the hidden parameter setup provides the theoretical result for a hidden parameter quantum physics, i.e., one possible local description of quantum physics.

The setup is designed in a way that an electron pair with the hidden parameter "magnetizable" will reproduce the measurement result spin "left" for Alice and Bob when they decide to use the horizontal Stern–Gerlach experiment. If the red handle is pointing to the right the ball will take the left path representing the Stern-Gerlach experiment in the horizontal basis. Due to the magnet at the second bifurcation, both balls will go the left path, resulting in a measurement spin "left", $|\leftarrow\rangle$. If one chooses the right path, which stands for a Stern–Gerlach experiment in the oblique basis, the magnet on the right-hand side guarantees a measurement outcome of spin "up" as is demanded in Equations (4.34) and (4.35). Thus, without communication at the bifurcation points, Alice and Bob will both measure spin "up", $|\nearrow\rangle$, when they chose the oblique Stern–Gerlach experiment, which is forbidden in quantum mechanics.

Figure 4.8. 3D-printed box with two-way connections representing a Stern-Gerlach experiment. The red handle determines the orientation of the Stern–Gerlach experiment (the right path leads to a measurement in the θ_1-direction, the left in the θ_2-direction). Magnets at the bifurcation points guarantee the predetermined result for magnetizable balls. See text for details.

5 | Refresher math courses

This chapter describes the structure and development of block courses for review of mathematical methods in theoretical mechanics and electrodynamics. In section 5.1 a short justification and the objectives are given. Section 5.2 provides a clarification of the relevant contents. In section 5.3 general and day-specific learning objectives are defined. Section 5.4 contains the material for a model day. Here, the example of differential operators and integral theorems is used to show how one day can be structured in a math review course. In section 5.5 the observations and subsequent improvements are described.

5.1 Justification and objective

The block courses are aimed primarily at student teachers. A reappraisal and an offer of mathematical methods required in physics is important for several reasons. If content related overwhelming leads to students dropping out of the course, the mathematical challenges in physics studies must be addressed as well as the physics content. The goal of the block courses here is to offer support since mathematical challenges can be even more dramatic in teacher training programs. Traditionally, in Germany two subjects and educational sciences are studied for the high school teaching profession. For physics, mathematics is very strongly recommended as the second subject. However, many students decide to take a different subject combination. Thus, only student teachers who have chosen mathematics as a second subject receive the basic mathematical training that is part of the curriculum in a pure physics program. Thus, the necessary mathematical training for a physics degree often remains subject to self-study. The aim of the block courses described here is to support student teachers in this process.

5.2 Clarification and analysis of science content

According to the module manuals of the university of Jena the lectures "Theoretical Mechanics", and "Electrodynamics" cover the following topics: Mechanics of a mass point, mass point systems, d'Alembert's principle, Lagrange's equations of 1st and 2nd kind, Hamilton's principle, rigid bodies, Hamiltonian mechanics, differential and integral representations of Maxwell's equations, microscopic and macroscopic electrodynamics, electrostatics and magnetostatics, quasi-stationary fields, generation and propagation of electromagnetic waves.

The following basic mathematical knowledge is necessary to deal with these physical topics: complex numbers, multidimensional analysis, ordinary differential equations, linear algebra, and vector analysis. A classification of which physical topics need which mathematical methods is shown in tables 5.1 (mechanics) and 5.2 (electrodynamics). The emphasis is clearly on differential and integral calculus, vector calculus, curvilinear coordinate systems, and differential equations. These mathematical methods here are needed in almost all areas of theoretical physics and, therefore, should be the

physical topic	needed mathematical topics
point mechanics	differential, integral, and vector calculus and curvilinear coordinate systems
Newtonian mechanics	complex numbers, differential, integral, and vector calculus, curvilinear coordinate systems, power series, and differential equations
balance equations	complex numbers, differential, integral and vector calculus, curvilinear coordinate systems, power series, differential equations, differential operators, and integral (Stokes) theorem
celestial mechanics	complex numbers, differential, integral, and vector calculus, curvilinear coordinate systems, differential equations, and conic sections
accelerated reference frames	complex numbers, differential, integral, vector, and matrix calculus, curvilinear coordinate systems, and differential equations
systems of mass points	complex numbers, differential, integral and vector calculus, curvilinear coordinate systems, power series, and differential equations
rigid bodies	complex numbers, differential, integral, vector, and matrix calculus, curvilinear coordinate systems, and differential equations
Langrange mechanics	complex numbers, differential, integral, and vector calculus, curvilinear coordinate systems, differential equations, and differential operators
variational problems and Hamilton's principle	complex numbers, differential, integral, and vector calculus, curvilinear coordinate systems, and differential equations
Hamiltonian mechanics	complex numbers, differential, integral, and vector calculus, curvilinear coordinate systems, and differential equations

Table 5.1. Table of required mathematical topics in theoretical mechanics sorted by physical topics of the accompanied lecture.

primary focus of compact courses. In addition to these methods, for mechanics the conic sections and in electrodynamics the Dirac delta function, the Fourier transform, Green's function, and the spherical harmonics may be relevant.

5.3 Learning objectives of individual courses

The block courses should address different learning objectives. First, there are general learning objectives that are identical in all courses. In addition, content objectives are defined for each day. In the following section all these objectives (the general and day specific ones) are defined and presented.

physical topic	needed mathematical topics
electrostatics	differential, integral, and vector calculus, curvilinear coordinate systems, differential operators, integral theorems, spherical harmonics, and Dirac delta function
magnetostatics	differential, integral, and vector calculus, curvilinear coordinate systems, differential operators, integral theorems, and spherical harmonics
slowly changing fields	differential, integral and vector calculus, curvilinear coordinate systems, differential equations, and differential operators
the general electromagnetic field	differential, integral, and vector calculus, curvilinear coordinate systems, differential equations, differential operators, integral theorems, Dirac delta function, Fourier transformation, and Green's function

Table 5.2. Table of required mathematical topics in theoretical electrodynamics sorted by physical topics of the accompanied lecture.

In general, the courses and the materials used should achieve the following objectives:

- Because of the limited time, the material should be limited to the essential computational aspects and their physical function.
- To achieve the highest possible activation of the students, the parts with student tasks and activities should occupy the most significant space.
- The entry should make as few prerequisites as possible to support especially the low-performing students.
- As many notations as possible should be presented such that students can use all textbooks or scripts for their benefit.

In preparation for mechanics and electrodynamics, the learning objectives for each studying day read as follows.

5.3.1 Mechanics

1. **Difference & integration with one variable, power series, complex numbers**
   Learning Goal: "Students will be able to independently apply the following computational methods: derivative and integration rules, substitutions and partial integration, Taylor series and calculating with complex numbers, application of Euler representation."
   In terms of content, all mathematical methods should already be known to students. The goal is that students practice the most important calculation rules and methods before these methods are needed in more complex areas in the following days.

2. **Vector and matrix calculus**
   Learning Goal: "Students will be able to apply the basics of linear algebra independently (vector calculus, scalar and vector product, Levi-Civita symbol, matrix calculus, determination of the determinant and the eigenvalue problem)."
   Main emphasis in vector calculus is on weaning the component notation. The highlight of the matrix calculus is the eigenvalue equation, which is needed for coupled differential equations and the determination of moments of inertia. For this purpose, the calculation rules and properties of matrices are repeated with emphasis on the determinant.

3. **Differential equations**
   Learning Goal: "Students will be able to solve ordinary linear differential equations. They will also know that coupled differential equations can be decoupled with an eigenvalue problem."
   Ordinary linear differential equations form the bulk of the differential equations to be solved in mechanics. In their studies, students must be able to solve them independently. Coupled differential equations do not occur so often. Thus only a qualitative understanding is required here.

4. **Differential and integral calculus with several variables**
   Learning Goal: "Students will be able to form partial and total derivatives and solve line, or volume integrals in Cartesian and curvilinear coordinates."
   Derivatives are needed, for example, in determining velocities or as preliminaries to differential operators. Line integrals form the basis of the work and the volume integrals are needed in the determination of mass or the inertia tensor.

5. **Differential operators & conic sections**
   Learning Goal: "Students will be able to perform and physically interpret the major applications of the Nabla operator. Students will be able to relate conic sections to relevant orbits in celestial mechanics."
   Differential operators become relevant when changing representations of conservative fields in terms of potentials. A physical interpretation is relevant because it often is the transition between physics and mathematics that is challenging, rather than the plain calculations before or after. Conic sections are relevant in mechanics only in celestial mechanics and thus are no longer a fundamental computational method. Nevertheless, it was repeated at the request of the instructor.

5.3.2 Electrodynamics

1. **Vectors & multidimensional integrals**
   Learning Goal: "Students will master these fundamentals for the more complex topics of the next few days."
   These mathematical methods were already part of the repetition before mechanics. Further repetition nevertheless is deemed necessary so that a lack of practice in these fundamentals will inhibit understanding of the more complex topics.

2. **Curvilinear coordinates & the Dirac delta function**
   Learning Goal: "Students will be able to switch between different coordinate systems and perform calculations in all of them. Students will be able to solve

simple integrals using the delta function."
Switching between different coordinate systems is needed more often in electrodynamics than in mechanics to simplify calculations. Consequently, a more in-depth study and more practice should be helpful for students. In addition, the delta function is a mathematical representation of point charges or the basis of Green's function. Knowledge around the delta function, its calculation rules and applications should therefore be present.

3. **Line & surface integrals**
   Learning Goal: "Students will be able to solve line and surface integrals for different geometries and in different coordinate systems."
   Line and especially surface integrals become even more important in electrodynamics than in mechanics. Because of time reasons, a repetition of the surface integrals before the mechanics was renounced. Thus the emphasis is put on this topic here.

4. **Differential operators & integral theorems**
   Learning Goal: "Students will be able to apply and physically interpret the differential operators (gradient, divergence, rotation, and Laplace). Students will also be able to apply Gauss' theorem and Stokes' theorem and interpret them physically as well."
   A full day for differential operators and integral theorems is justified by their importance to electrodynamics, e.g., in Maxwell's equations. Physical interpretation remains significant, as often switching between the physical and mathematical content is challenging more than simple arithmetics.

5. **Spherical harmonics & Green's function**
   Learning Goal: "Students will know the uses and applications of spherical harmonics and Green's function in electrodynamics."
   These two topics, as compared to the other mathematical methods, are not fundamentals but rather specific methods for particular physics topics. Students will rarely encounter these topics again, especially in teacher education courses. Therefore, the emphasis here is on a qualitative understanding of the methods and their utility rather than on formal calculations.

5.4 Design and structure of a model day in the block course

As an example of how a single day in a block course might be structured, this section considers differential operators and integral theorems for electrodynamics. The vector operator nabla forms the basis for electrodynamics' most important differential operators. Relevant to Maxwell's equations and the description of electromagnetic fields or potentials are the gradient, the divergence, and the curl. The Laplace operator can also be treated as the basis of the Poisson and wave equations. For integral theorems, Gauss' theorem and Stokes' theorem are relevant for Maxwell's equations or various boundary conditions of electromagnetic fields. A review of mathematical methods for electrodynamics should be limited to these topics.

In the case of differential operators, Gauss' theorem, and Stokes' theorem, the learning objectives from section 5.3 mean that a short introduction or lecture repeats (or, in some cases, presents for the first time) the essentials. These essentials include definitions, a physical interpretation, and possibly a computational example. The physical interpretation is vital in two ways. Firstly, in the sense of verbalization of formulas, it shows how physical information can be drawn from formulas. And secondly, students can be shown the relevance of mathematics in physical contexts if reference is made to relevant lecture content. Proofs or detailed derivations are deliberately omitted in order to convey as much content as possible in a short time. For a formally correct mathematical education, reference must be made to the mathematics lectures. In addition, students will be handed out a collection of formulas on differential operators in curvilinear coordinates.

After the content presentation, the mathematical methods are explained by the students to themselves in various example problems and then applied by themselves in further tasks. This division of assignments is motivated by the principles of "worked examples" and is intended to activate and support students in working independently. Ideally, these tasks are examples of the respective method and thus cover as wide a range of applications as possible. A reference to the physical content of the subsequent lecture should help to ensure that these methods are recognized as quickly as possible in the lecture and the associated exercise series.

5.4.1 Lecture for differential operator and integral theorems

Following the aspects for the lecture discussed above the contents of the lecture "mathematical methods of physics" (from the winter semester 11/12 at the University of Stuttgart [138]) are reduced to the following script.

Nabla operator

Definition in cartesian coordinates:

$$\nabla = \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix} = \vec{e}_x \frac{\partial}{\partial x} + \vec{e}_y \frac{\partial}{\partial y} + \vec{e}_z \frac{\partial}{\partial z} \tag{5.1}$$

Gradient

The gradient of a scalar field $\Phi(\vec{r})$ is defined as

$$\mathrm{grad}\Phi(\vec{r}) = \nabla\Phi(\vec{r}) = \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix} \Phi(\vec{r}) = \begin{pmatrix} \partial\Phi(\vec{r})/\partial x \\ \partial\Phi(\vec{r})/\partial y \\ \partial\Phi(\vec{r})/\partial z \end{pmatrix} \tag{5.2}$$

Physical interpretation:
The gradient answers the question: How does a scalar field change along a direction?

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\Phi(\vec{r}_0 + t\vec{a}) &= \frac{\mathrm{d}}{\mathrm{d}t}\Phi(r_{0,x} + ta_x, r_{0,y} + ta_y, r_{0,z} + ta_z)_{|t=0} \\ &= \frac{\partial\Phi}{\partial x}_{|t=0} \underbrace{\frac{\mathrm{d}(r_{0,x} + ta_x)}{\mathrm{d}t}}_{a_x} + \frac{\partial\Phi}{\partial y}_{|\vec{r}_0} a_y + \frac{\partial\Phi}{\partial z}_{|\vec{r}_0} a_z \end{aligned}$$

$$= (\nabla\Phi)_{|r_0} \cdot \vec{a} = \vec{a} \cdot \nabla\Phi(\vec{r}_0) \tag{5.3}$$

This is called a directional derivative. The expression $\vec{a} \cdot \nabla\Phi(\vec{r}_0)$ assumes a maximum if $\vec{a}$ and $\nabla\Phi(\vec{r}_0)$ point in the same direction. This means that $\nabla\Phi$ at point $\vec{r}_0$ always points in the direction of the strongest increase of $\nabla\Phi(\vec{r})$.
Examples:

mathematical example: $\Phi = \mathrm{e}^{x^2} \cdot y \cdot \ln(z)$

$$\nabla\Phi = \begin{pmatrix} 2x\mathrm{e}^{x^2} y \ln(z) \\ \mathrm{e}^{x^2} \ln(z) \\ \mathrm{e}^{x^2} y/z \end{pmatrix}$$

gravitational potential: $\Phi(\vec{r}) = -\frac{MmG}{r} = \frac{-MmG}{\sqrt{x^2+y^2+z^2}}$

gravitational field: $\vec{F}_G = -\nabla\Phi(\vec{r})$

$$= \frac{-MmG}{\sqrt{x^2+y^2+z^2}^3} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{-MmG}{r^3}\vec{r}$$

$$= -\frac{MmG}{r^2}\vec{e}_r$$

Divergence

$$\mathrm{div}\vec{A} = \nabla \cdot \vec{A} = \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix} \cdot \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \tag{5.4}$$

Physical interpretation:
The divergence identifies sources and sinks of a vector field. For example, positive charges are sources of the electric field (see figure 5.2),

source: $\mathrm{div}\vec{A} > 0,$ sink: $\mathrm{div}\vec{A} < 0.$

Examples:

$$\vec{A} = \begin{pmatrix} x^2 \\ 2y \\ z \end{pmatrix}, \qquad \mathrm{div}\vec{A} = 2x + 2 + 1 = 2x + 3$$

$$\vec{B} = \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}, \qquad \mathrm{div}\vec{B} = 0 + 0 + 0 = 0$$

Laplace operator

$$\Delta\Phi = \mathrm{div}\,\mathrm{grad}\Phi = \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} \cdot \begin{pmatrix} \partial_x\Phi \\ \partial_y\Phi \\ \partial_z\Phi \end{pmatrix} = \partial_x^2\Phi + \partial_y^2\Phi + \partial_z^2\Phi \tag{5.5}$$

Definition as operator without function:

$$\Delta = \frac{\partial^2}{\partial_x^2} + \frac{\partial^2}{\partial_y^2} + \frac{\partial^2}{\partial_z^2} \tag{5.6}$$

Applications in physics:

Poisson's equation: $\Delta\Phi = 4\pi G\varrho(\vec{r})$ (gravitational)

$$\Delta\Phi = \frac{1}{\varepsilon_0}\varrho(\vec{r}) \quad \text{(electrical)}$$

Wave equation: $$0 = \left(\Delta - \frac{1}{v_{ph}^2}\frac{\partial^2}{\partial t^2}\right)\Psi(\vec{r},t)$$

Schrödinger equation: $$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = \left(-\frac{\hbar^2}{2m}\Delta + V(\vec{r})\right)\Psi(\vec{r},t)$$

Curl

$$\mathrm{rot}\vec{A}(\vec{r}) = \nabla \times \vec{A}(\vec{r}) = \begin{pmatrix}\partial_x\\ \partial_y\\ \partial_z\end{pmatrix} \times \begin{pmatrix}A_x(\vec{r})\\ A_y(\vec{r})\\ A_z(\vec{r})\end{pmatrix} = \begin{pmatrix}\partial_y A_z - \partial_z A_y\\ \partial_z A_x - \partial_x A_z\\ \partial_x A_y - \partial_y A_x\end{pmatrix} \tag{5.7}$$

Physical interpretation:
The curl identifies vorticity fields,

vorticity field: $\mathrm{rot}\vec{A} \neq 0$, curl-free: $\mathrm{rot}\vec{A} = 0$.

A gradient field always has zero curl:

$$\mathrm{rot}\,\mathrm{grad}\Phi = \begin{pmatrix}\partial_y\partial_z\Phi - \partial_z\partial_y\Phi\\ \partial_z\partial_x\Phi - \partial_x\partial_z\Phi\\ \partial_x\partial_y\Phi - \partial_y\partial_x\Phi\end{pmatrix}. \tag{5.8}$$

Conversely, one can prove that the following relation holds in a simply connected field.

If $\mathrm{rot}\vec{A}(\vec{r}) = 0,$ then there is a $\Phi(\vec{r})$ such that $\vec{A}(\vec{r}) = \mathrm{grad}\Phi(\vec{r})$.

If for a force $\vec{F}(\vec{r})$ the condition $\mathrm{rot}\vec{F} = 0$ is valid, then there is a potential $V(\vec{r})$ with

$$\vec{F}(\vec{r}) = -\nabla V(\vec{r}). \tag{5.9}$$

Such forces are called conservative and the work in such force fields is independent of the path

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}(\vec{r})\,\mathrm{d}\vec{r} = -V(\vec{r}_2) + V(\vec{r}_1).$$

Examples:

$$\mathrm{rot}\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \nabla \times \begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$$

$$\mathrm{rot}\begin{pmatrix}0\\ x\\ 0\end{pmatrix} = \nabla \times \begin{pmatrix}0\\ x\\ 0\end{pmatrix} = \begin{pmatrix}0\\ 0\\ 1\end{pmatrix}$$

Interpretation as a flow field: A ball in the flow, as in the second example, starts to rotate around its own axis. The curl gives the orientation of the rotation and the rotation speed (direction and magnitude of the vector). In this example, this is the case at every point in the x-y plane. To the left and right of the y-axis the curl remains the same, the ball only gets an additional velocity in the y-direction or the negative y-direction. This interpretation can be illustrated with figure 5.1.

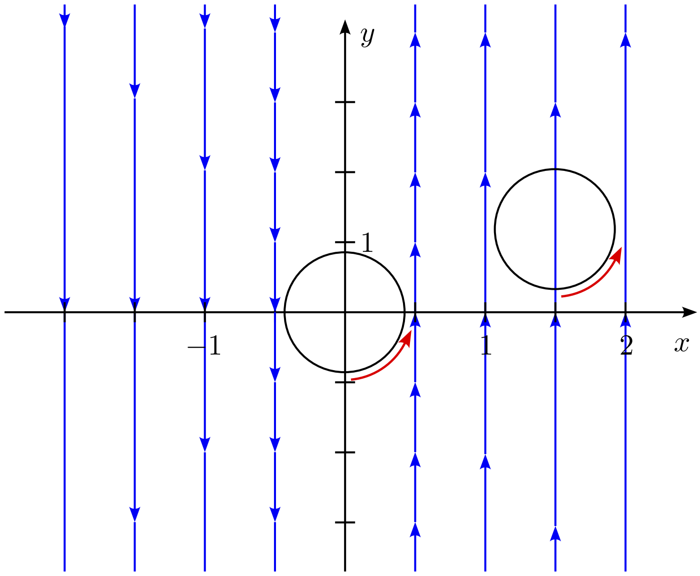


Figure 5.1. Sketch of the example of a vorticity field (blue). A ball (black circle) as a test charge would rotate (red) around its own axis at both points in such a field.

Divergence theorem

$$\int_V \mathrm{div}\vec{F}\,\mathrm{d}V = \int_{S=\partial V} \vec{F}\cdot \mathrm{d}\vec{A} \tag{5.10}$$

Here $\vec{F}$ is a vector field, V is the volume, S is the boundary of the volume, and $\mathrm{d}\vec{A}$ is a vector surface element with normal vector outward.
Physical interpretation:
The left-hand side of the theorem is a "summation" over all sources and sinks of the vector field $\vec{F}$. On the right-hand side of the equal sign the flow of the vector field $\vec{F}$ through the boundary S is determined. Together, this means that as much field $\vec{F}$ must flow out of the volume V as has flowed in plus what is created in sources in the volume V minus what disappears in sinks. This is illustrated in figure 5.2. The divergence theorem for the electric field is known as Gauss's law.

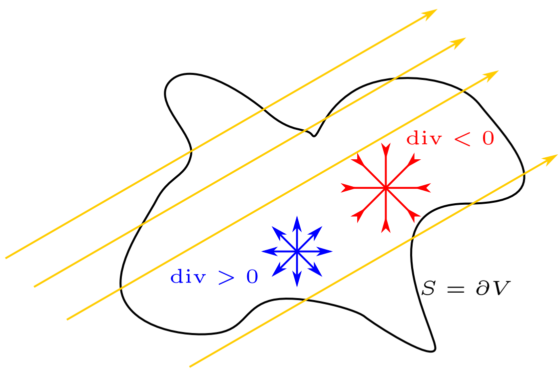


Figure 5.2. Representation of the divergence theorem. There must be as much field (yellow) flowing into a volume as flowing out plus or minus the field that is created in sources (blue) or disappears in sinks (red).

Stokes theorem

$$\int_A \mathrm{rot}\vec{F}(\vec{r}) \cdot \mathrm{d}\vec{A} = \int_{C=\partial A} \vec{F}(\vec{r}) \cdot \mathrm{d}\vec{r} \tag{5.11}$$

Here $\vec{F}$ is a vector field, A is the surface, $\mathrm{d}\vec{A}$ is a vector surface element, and C is the boundary of the surface.
Physical interpretation:
On the left-hand side of the theorem, we have an integration over the vortices of the vector field on surface A. On the right-hand side is a line integral along the boundary of surface A. The following statement can be made if this is interpreted as a working integral. On a closed path (only), work is generated, or work has to be done if there are vortices on the surface A.

Physical application of integral theorems

The divergence theorem as Gauss's law of the magnetic flux in Maxwell's equations:

$$\int_{\partial V} \vec{B} \cdot \mathrm{d}\vec{A} = \int_V \mathrm{div}\vec{B}\,\mathrm{d}V = 0. \tag{5.12}$$

The magnetic flux into a closed volume is always exactly as large as the flux out. Therefore, there are no sources and sinks of magnetic flux and thus no magnetic monopoles.

Maxwell-Faraday equation

$$\mathrm{rot}\vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\dot{\vec{B}} \tag{5.13}$$

with Stokes' theorem

$$\int_{\partial A} \vec{E} \cdot \mathrm{d}\vec{s} = \int_A \mathrm{rot}\vec{E} \cdot \mathrm{d}\vec{A} = -\int_A \dot{\vec{B}} \cdot \mathrm{d}\vec{A} \tag{5.14}$$

reads: The electric circulation over the edge curve ∂A of a surface A is equal to the negative time change of the magnetic flux through the surface. In other words: The time variation of the $\vec{B}$ field leads to an electric field, which is responsible for the induction voltage.

5.4.2 Exercises for differential operators and integral theorems

The following exercises are part of the block course that should activate the students. To cover as many aspects and problems as possible in the short time available, the "worked examples" method is chosen. This way, the students can be activated by self-explanations, and the introduction to the respective methods can be facilitated. Therefore, for each topic there is one task with a sample solution and one task without a solution for self-study. A performance differentiation also can take place here. Stronger students, who only want a little more practice, can be advised to avoid looking at the sample solutions. By working on the exercises entirely by themselves, they will be slowed down a bit in speed. Weaker students can consult the "worked examples". If the work assignment is taken seriously, these students can be activated by self-explanations.

Problem 1

For task 1, the sample solution is provided for students to familiarize themselves with the solution methods. Based on the principles of learning with "worked examples", this task is intended to provide an overview of the various uses and applications of differential operators. Task parts a and b are intended to train the formal handling of the operators. In electrodynamics many calculations or derivations are performed without concrete numerical values. A formal handling of the differential operators should prepare for this. Task part c already reaches into the integral theorems. On the one hand, the calculation of a curl can be practiced explicitly, on the other hand this task shows how much calculation effort can be saved with the integral theorems.

a) Prove the relation

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$

for the vector fields $\vec{A} = \vec{A}(\vec{r})$ and $\vec{B} = \vec{B}(\vec{r})$.

b) Determine for the vector field

$$\vec{A} = \begin{pmatrix} x(4-2a) + y(a-3) + xy(8x-4ax) \\ x(3a-5) + y(4-2a) + xy(8y-4ay) \\ 0 \end{pmatrix} \mathrm{e}^{-(x^2+y^2)}$$

the parameter a such that $\vec{A}$ becomes a gradient field. What then is the scalar field $\psi(x, y, z)$ for which $\vec{A} = \operatorname{grad} \psi$?

c) Calculate for the vector field

$$\vec{B} = \begin{pmatrix} 4y \\ x \\ 2z \end{pmatrix}$$

the surface integral

$$\int_A (\nabla \times \vec{B}) \cdot \mathrm{d}\vec{A} \,,$$

where A is the hemisphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$, directly and using Stokes' theorem.

Problem 2

Task 2 now links the differential operators to electrodynamics. In the context of dipoles and waves, students can apply and practice the differential operators themselves. The students do not have a sample solution available here.

In electrodynamics, the correlations

$$\vec{E}(\vec{r}, t) = -\nabla\phi(\vec{r}, t) - \frac{\partial}{\partial t}\vec{A}(\vec{r}, t) \,, \qquad \vec{B}(\vec{r}, t) = \nabla \times \vec{A}(\vec{r}, t)$$

can be used to obtain the electric field strength $\vec{E}(\vec{r}, t)$ and the magnetic flux density $\vec{B}(\vec{r}, t)$ from a scalar potential $\phi(\vec{r}, t)$ and a vector potential $\vec{A}(\vec{r}, t)$.

a) A static (no time dependence) electric dipole can be described by the scalar potential
$$\phi(\vec{r}) = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$$
with $r = |\vec{r}|$ and $\vec{A} = \vec{0}$. Here $\vec{p}$ is independent of $\vec{r}$. Give the corresponding electric field $\vec{E}$.

b) A static magnetic dipole can be described by the vector potential
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$
and $\phi = 0$. Here $\vec{m}$ is independent of $\vec{r}$. Calculate the magnetic flux density $\vec{B}$.

c) In vacuum, for the vector potential, the wave equation can be expressed as
$$\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} - \Delta\vec{A} = \vec{0}$$
with the speed of light c. Show that $\vec{A}(\vec{r},t) = \vec{A}_0\exp(\mathrm{i}(\vec{k}\cdot\vec{r} - \omega t))$ for $|\vec{k}| = \omega/c$ is a solution of the wave equation. For this case $\vec{E}$ and $\vec{B}$, where $\vec{A}_0$, $\vec{k}$, and ω are constants, depend neither on $\vec{r}$ nor on t. What is the angle between both fields $\vec{E}$ and $\vec{B}$?

Problem 3

In task 3 typical applications of Gauss' theorem and Stokes' theorem from electrodynamics are given. Simple geometries (sphere and cylinder) should facilitate the introduction to this topic. In addition, a sample solution is provided to the students.

a) A solid sphere carries a radially symmetric charge distribution of the total charge Q. In this case, you can assume for symmetry reasons that the electric field outside the sphere has the form $\vec{E} = E(r)\vec{e}_\mathrm{r}$ or in vacuum $\vec{D} = \varepsilon_0\vec{E} = \varepsilon_0 E(r)\vec{e}_\mathrm{r}$. Via $\operatorname{div}\vec{D} = \varrho$, the dielectric displacement $\vec{D}$ and the charge density ϱ are related. Use this knowledge and Gauss's theorem to calculate $E(r)$. Start by calculating the total charge from
$$Q = \int_V \varrho\,\mathrm{d}V$$
if the volume V contains the sphere completely.

b) A long wire has an azimuthally symmetric current I flow through it. The magnetic flux density then takes the form $\vec{B} = B(\varrho)\vec{e}_\varphi$. For the magnetic field strength applies in the vacuum $\vec{H} = \vec{B}/\mu_0$. In absence of time varying electric fields $\vec{H}$ is given by $\operatorname{rot}\vec{H} = \vec{j}$ with the current density $\vec{j}$, which is included in the calculation of the current through the area A,
$$I = \int_A \vec{j}\cdot\mathrm{d}\vec{A}\,.$$
Use Stokes' theorem to calculate $B(\varrho)$ outside the wire.

c) Specify a possible vector potential $\vec{A}$ such that the flux density from b) can be determined by $\vec{B} = \operatorname{rot}\vec{A}$.

Problem 4

Task 4 is an application of the integral theorems, which must be solved by the students themselves. This task builds on the previously discussed task. Part a does not require a calculation with the goal of stimulating logical reasoning with the integral theorems. Parts b and c use Gauss' theorem to motivate the delta function to describe point charges in electrodynamics.

Remember the vector field

$$\vec{v} = \begin{pmatrix} 0 \\ a \\ b \end{pmatrix}$$

and the cone K of yesterday's sheet 3 problem 14, of which the circular base lies on the x-y-plane with its center at the origin of the coordinate system and has the radius $R = 1$. Its tip lies on the z-axis at $z = 1$.

a) Now consider a cone which is identical except for twice the height to K. The vector field $\vec{v}$, which is constant in the whole space, flows in twice the volume as compared to K and the curved surface area is increased by a factor of $\sqrt{5/2}$, which rests on the same base surface as that of K and thus has the same boundary. What is the flux of $\vec{v}$ through its sheath surface? Give the result without reintegration, but justify your answer.

b) Calculate the volume integral

$$\int_V \Delta \frac{1}{r} \mathrm{d}V \qquad \text{with} \qquad r = \sqrt{x^2 + y^2 + z^2} \, ,$$

where the volume V is a sphere at the origin of the coordinate system with radius $R = 1$. The differential area element of its surface is in spherical coordinates

$$\mathrm{d}\vec{S} = \sin(\vartheta) \, \vec{e}_{\mathrm{r}} \, \mathrm{d}\vartheta \, \mathrm{d}\varphi \, .$$

Use Gauss' integral theorem for this and don't bother that you do not know $\Delta 1/r$ at the origin of the coordinate system yet.

c) It can be shown that

$$\Delta \frac{1}{r} = 0$$

for $\vec{r} \neq \vec{0}$ holds. Using the result from the previous part of the problem, now specify $\Delta 1/r$ including the origin in such ways that Gauss' theorem is satisfied.

5.5 Observation and enhancements

"Worked examples" instead of lecture exercises

First attempts with review courses were not yet set up in the style of "worked examples". Instead, the lecturer demonstrated some of the exercises. However, motivation and concentration in these lecture exercises are not very high. In addition, according to their own statements, students prefer a course structure with "worked examples".

Worked examples only are used adequately with an introduction

The concept of "worked examples" only is accepted slowly by the students at the beginning. Students often try to solve the first tasks themselves without assistance. It is assumed that this is due to the unfamiliarity of the method on the one hand and, on the other hand, to an incorrect performance assessment of the students. After a didactic introduction to the method of learning with "worked examples" the percentage of students who use the sample solutions appropriately can be increased.

The course cannot replace mathematical education only complement it

Students with very little prior mathematical training still have major problems with the content and with the pace in the course. For some students in the course some methods were completely new and not a repetition. However, the course is not designed to replace a mathematical education.

After a few days in a row, the concentration decreases

From the third day on, the motivation and thus also the attention of students decreases noticeably. This is all the more problematic, because the more complex topics are dealt with towards the end of the course. Therefore, sufficient and extended breaks are explicitly recommended.

Voluntary block courses reach only a small proportion of students

In order not to make the study load greater than legally permitted, these courses were offered for voluntary participation during the week before the start of lectures. This offer is only used by a small part of students. In addition to parallel university obligations (exams, homework,...), vacation or non-presence at the place of study also are cited as reasons for non-participation. The active participation does not increase with an additional online support offer.

6 | Theoretical physics in a school physics context

This chapter describes the development according to the model of educational reconstruction of seminars, which follow the demand for prospective physics teachers and are adapted to professional education. The goal is to strengthen in-depth school knowledge and pedagogical content knowledge by explicitly linking university knowledge from the lectures with pedagogical content and school knowledge. Section 6.1 explains these goals and the rationale for them in more detail. Section 6.2 describes the students' perspective. Section 6.3 provides a clarification of the science content, and section 6.4 a clarification of the pedagogical content. Section 6.5 describes the development and structure of a seminar on constraining forces and Lagrange multipliers as an example. Then, the learning objectives of the remaining units are presented in a compact form. Finally, section 6.6 describes the observations and improvements of the material.

6.1 Justification and objective

For the design of cognitively activating physics lessons, the following three areas of professional knowledge have proven to be relevant for teachers: Physics content knowledge, pedagogical content knowledge, and pedagogical knowledge (compare [139]). In addition, adaptability and support seem to be crucial for teaching quality in general and for explaining subject contents. For example, adaptation is the most important strategy for effective explanation [140]. However, adaptation to the situational needs of students is only possible if the teacher has a deep enough understanding of the scientific background and pedogigacal content principles such that the example, explanatory approach, analogy or model, and didactic method can be changed spontaneously. The seminars developed here are intended to make a contribution as to how the content knowledge and the pedagogical content knowledge can be specifically promoted through building connections within the framework of university education. What can theoretical physics contribute to this in teacher training?

In assessing and categorizing the content knowledge of prospective teachers, a division into school knowledge (SK), in-depth school knowledge (DSK), and university knowledge (UK) has proven useful (e.g. [9, 141, 142]). SK is defined from the requirement that an average high school student should be able to fulfill at the end of lower secondary school (and, in the context of this thesis, higher secondary school) and tasks that can be used in school. The reference standard is the "Bildungsstandards Physik für den mittleren Schulabschluss" (Konferenz der Kultusminister der Länder in der Bundesrepublik Deutschland, 2004 [143] and 2020 [144]). Content that represents knowledge completely detached from school is UK, in the definition, according to Woitkowski [145]. Tasks from this level cannot be solved even by excellent high school students. (For this work, this definition is only partially correct. Although high school students cannot solve the content and tasks from this level, the content covered is only partially detached from school knowledge. Often this separation exists only in the formalism and the degree of mathematization but not in the physical contents). The DSK forms the mediator between the SK and the UK and should, at the appropriate level, clear up the apparent irrelevance of the SK in the subject studies and, likewise, the apparent irrelevance of the UK in the later teaching profession. The following list of characteristics of this level of knowledge can help define what constitutes DSK ([145], p. 303).

1. "*Explicit combination of school and university knowledge.*
2. *Application of university thinking to typical school physics problems.*
3. *Systematizing and making connections of school knowledge against the background of university physics; a comprehensive view of physics as a whole.*
4. *Practice (or capture) elaborate ways of thinking, speaking, and behaving about physics using objects from school physics.*
5. *Reflection on the meaning, genesis, and use of concepts in school physics.*"

This knowledge is considered particularly relevant for the later professional life of teachers. The goal of teacher training should therefore be the formation and promotion of DSK, in addition to gaining insight into the working methods of the subject (UK).

Theoretical Physics, with its high degree of abstractness and a strong focus on mathematical formalism, arguably represents university knowledge in the education of prospective teachers like no other lecture. Students repeatedly question the relevance of this part of education for the later teaching profession. This questioning could be explained by a lack of linkage of theoretical physics with school physics (equally by lecturers and students). It would follow that a sufficient amount of DSK is not or cannot be built up at these points in the education.

However, current research results in pedagogical content knowledge allow the conclusion that an interlocking and interconnection of content knowledge on the university level with school physics and pedagogical content knowledge can increase prospective teachers' competence in all areas. Thus, linking these areas offer the chance to discuss the relevance for the later profession and thus to deepen the pedagogical content knowledge [9, 139] and to improve the explanatory power [13], to deepen the content knowledge [9, 145] (maybe especially of the weaker students [31, 32]).

This thesis aims at the development of accompanying seminars for student teachers that support and complement classical lectures in theoretical physics (in this thesis with focus on theoretical mechanics and electrodynamics). For this purpose, selected contents of the lectures are reworked with different pedagogical content knowledge methods and placed in the context of school physics to increase university knowledge, in-depth school knowledge, and pedagogical content knowledge.

6.2 Prospective teachers' perspective

6.2.1 Content knowledge

Current subject didactic measurement instruments to determine content knowledge and the pedagogical content knowledge of student teachers (e.g. [9, 142]) confirm an increase in content knowledge at all levels (SK, DSK, and UK) and pedagogical content knowledge [9] during the course of study, but the target level in content knowledge is not reached by some students [8]. According to Enkrott [9], some students do not even reach the prior knowledge level of the average students by the 5th semester. This is doubly critical, as there is a relation between the acquisition of pedagogical content knowledge and existing content knowledge [9] and both facets of knowledge are relevant for acting in the classroom [141, 146]. However, quite generally, content knowledge increases at all levels (SK, DSK, UK) during the course of study, even if SK is not taught directly [9]. For example, knowing UK does not directly influence teachers' actions in critical situations [139,141] but does influence SK and DSK, which in turn influence creative situational action. Thus, exposure to content at the higher levels apparently also benefits school knowledge if an appropriate level is achieved by students.

6.2.2 Pedagogical content knowledge

While pedagogical content knowledge can generally be seen as relevant for classroom action [141], the data is not so clear in the specific case. For example, pure knowledge of misconceptions does not seem to have a positive impact on teaching. In a comparison of a lesson on electrical energy, Finnish students learn more than German students, with the Finnish teachers knowing less about typical misconceptions than their German colleagues. For example, they cannot explicitly name these misconceptions [147, 148]. Controlled factors in this study included students' cognitive abilities and teachers' pedagogical content knowledge. On the other hand, pedagogical content knowledge seems to be essential for teachers' explanatory performance [13].

6.2.3 Linking content knowledge and pedagogical content knowledge

Subject knowledge seems to be a basic prerequisite for acquiring pedagogical content knowledge, at least these measures correlate very strongly [9]. Also in the study on teachers' explanatory performance [13] it could be shown that both CK and PCK were important to improve the quality of explaining. PCK has the key role here in the transfer from CK to explanatory performance. A direct relationship between CK and explanatory performance could not be established. However, the content knowledge remains extremely relevant in the background. I.e. with a lack of content knowledge the quality of the explanations decreases, but content knowledge alone is not sufficient for good explanatory performance. A linking of subject knowledge and pedagogical content knowledge in explicit courses can be conducive to learning for the pedagogical content knowledge [13, 32]. Linking these subject areas could even have positive effects for content knowledge, possibly even to a special extent for the lower-performing students [31, 32]. However, these findings have to be taken with caution due to the small number of participants.

6.2.4 Perspective at one's own location

A full-scale investigation, as indicated in the previous sections cannot be accomplished within the scope of this thesis as the focus is on the development of targeted instructional materials. However, in group discussions and in observation of students as they work through the materials developed, trends can be discerned that are consistent with the research findings mentioned above. Typical statements such as: "What good will this [note: theoretical physics] do me for school later?" or, "Why are we kept busy with all the math problems? The school doesn't need specialized idiots as teachers." suggest that the link between UK and SK is not adequately taught in the courses or that a broad DSK has not yet been formed. Also, when asked where links with school physics are in the lecture content, many students cannot answer spontaneously.

6.3 Clarification and analysis of pedagogical content knowledge

The pedagogical content knowledge can be divided into four areas: instructional strategies, scientific misconceptions, experiments and pedagogical concepts [149, 150]. For theoretical physics, the aspects concerning experiments can be classified as less relevant and thus excluded. All other aspects of pedagogical content knowledge can be treated in the context of theoretical and school physics.

Since the goal of the planned seminars is to look at the contents of the lectures from different pedagogical content angles and to establish links to school physics, there will be a strong focus on instructional strategies. Basically, the question "With which alternative instructional strategies could the contents of theoretical physics also be worked up?" is to be answered. This should not only be a principled and theoretical discussion, but the students should experience these instructional strategies themselves and subsequently transfer the university knowledge and the pedagogical content knowledge to contents of school physics. Not all instructional strategies are suitable for this purpose, but only those that can also be applied in theoretical physics with added value. These are

- peer instruction,
- the verbalization and visualization of formulas,
- learning with worked examples,
- and the reduction and elementarization of contents.

Why these instructional strategies are relevant for theoretical physics and how different implementations in theoretical physics can look like is discussed in detail in chapter 3.

Furthermore, some pedagogical concepts can be combined with the lectures in theoretical physics, e.g.,

- the nature of science
- and the educational reconstruction.

The comparison of historical knowledge gain and the general structure of textbooks or lectures provides a good basis to discuss the nature of science as a pedagogical concept. In addition, the students themselves can use the concept of educational reconstruction to link the content of theoretical physics and school physics with a clarification of content knowledge and the determination of the perspective of students.

Finally, scientific misconceptions as an aspect of pedagogical content knowledge should be mentioned. While these only have a limited concrete connection to university content knowledge, content knowledge can help to identify such misconceptions in all subject areas. Furthermore, training explanations (e.g., in peer instruction) can help prospective teachers to develop methods to support students in overcoming such conceptions, e.g., for situational action in the classroom.

6.4 Clarification and analysis of science content

Since school physics often only covers the special cases and maximum simplifications of physics, all topics of school physics are covered in theoretical physics. The challenge is to reconcile these different formalisms or to identify the simplifications and the resulting transformations. A clarification of the science content, covering all interfaces between these disciplines, would thus encompass the complete school curriculum. To implement this in the course of study would go beyond the scope of any one course and can therefore only be carried out selectively. Since the seminars accompany the lectures, topics matching the focus of the lecture are sought on a selective basis. These can be seen for mechanics in table 6.1 and for electrodynamics in table 6.2.

lecture topic	**example topics relevant for school**
Newtonian mechanics	kinematics, dynamics, statics, and kinetics (This chapter deals exclusively with school knowledge and thus provides in-depth school knowledge by itself.)
balance equations	conservation laws, conservative fields and potential energy, everyday experience (e.g. angular momentum)
celestial mechanics	nature of science (historical gain of knowledge), everyday experience/observation, Kepler's laws of planetary motion
accelerated reference frames	fictitious forces, everyday experience of the pupils (e.g. bus ride) and thus source of many scientific misconceptions, weather phenomena
systems of mass points	Newtonian mechanics for multiple masses - this represents mostly in-depth school knowledge.
rigid bodies	everyday experience, first step towards a more realistic physics (expansion = less idealization, nature of sciences)
Langrangian mechanics (1st kind)	reaction forces for comparison of force addition & decomposition.
Langrangian mechanics (2nd kind)	nature of science

Table 6.1. Table of linking selected topics in mechanics at university level and school level.

lecture topic	example topics relevant for school
electrostatics	electrical field and parallel plate capacitor
magnetostatics	magnetism and magnetic fields of coils
slowly changing fields	electricity (electric circuits, voltage as potential difference, and Ohm's law), induction, and particles in fields
the general electromagnetic field	optics, electromagnetic-waves (Hertzian dipole and antennas, interference, radiation physics, and climate)

Table 6.2. Table of linking selected topics in electrodynamics at university level and school level.

6.5 Design and Evaluation

6.5.1 Conclusions and development tasks

From the professional clarifications and the students' perspective, there is a need for targeted training in in-depth school knowledge. This knowledge (see definition section 6.1 or [145]) is based on a linkage of the UK and the SK. Therefore, for a goal-oriented development of DSK, the UK must be consolidated first, and then this UK must be linked to the SK. Step one is to be achieved by cognitively activating selective repetitions of the UK. The instructional strategy is also discussed with the students to deepen pedagogical content knowledge. Step two, the link to the SK, subsequently motivates and consolidates the subject knowledge and pedagogical content knowledge. The following development tasks result from these conclusions:

1. Selected contents of theoretical physics are to be worked up with changing cognitive activating instruction strategies.
2. If the pedagogical content knowledge is not a cognitively activating instructional strategy, but a pedagogical concept, the work in small groups should activate the students.
3. The science and pedagogical contents should be discussed based on an example from school physics.

The basic structure of the seminars is quite simple and usually follows a similar pattern: First, the pedagogical methods are summarized in a short presentation. Then, one or more topics from the lecture are reviewed using this method. Finally, the method and the contents are discussed with the course regarding their use in school.

In the following, this process will be exemplified and justified using the example of verbalization and visualization of formulas. However, much of it can also be transferred to the other seminars.

6.5.2 Structure and learning objectives of the seminars

The verbalization and visualization of formulas suit the students in a double sense. First, it provides them a tool for extracting information from mathematical formalism. Second, verbalization and visualization of formulas are skills needed in physics classes' explanations. The ability to translate formulas into language and images are expected to enable teachers to be situationally responsive to students' needs or to adapt explanations and images when necessary, as called for in [13, 141].

In the seminar on theoretical mechanics, the equation of motion in the Lagrange 1 formalism, the energy, and the virial theorem were chosen. In the equation of motion with constrained freedom of motion, a new level of abstraction, which goes beyond Newtonian mechanics, is reached for the first time in the lecture. With the illustration of reaction forces, the equation of motion and the actual motion of masses shall be made more vivid. (Since in the concrete implementation, the method of the verbalization of formulas was not known to the students up to now, the example of the equation of motion was worked out as a "worked example", see section 3.3). A sample solution of the verbalization is handed out for the equation of motion. This worked example is based on the checklist in section 3.4.1:

Equation of constrained motion

$$m\ddot{\vec{r}} = \vec{F}^a + \sum_{i=1}^{M} \vec{F}_i^z = \vec{F}^a + \sum_{i=1}^{M} \lambda_i \nabla g_i \tag{6.1}$$

1. **naming the formula symbols**
2. **unit consideration**
   The first and second part of the verbalization are sketched in figure 6.1

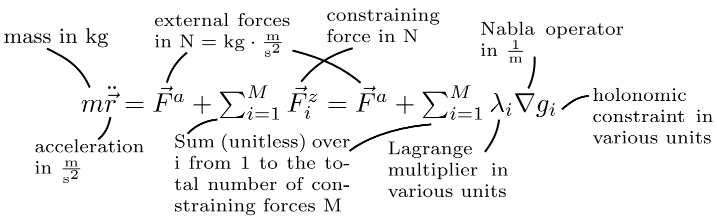


Figure 6.1. Sketch of naming all formula symbols in the equation of constraining motion with an unit consideration.

3. **the conditions of validity**
   This equation of motion is generally valid for point masses.
4. **drawing a sketch**

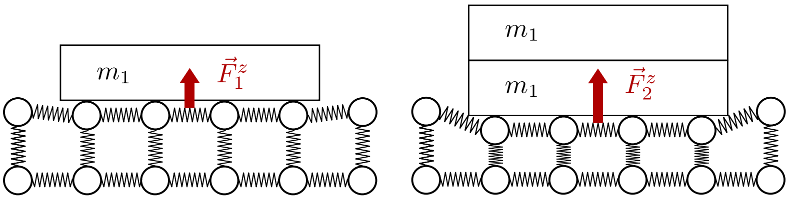


Figure 6.2. Sketch of constraining force for two different masses.

5. **consider special cases and limiting cases**
   This is not needed here.

6. **describe the physical meaning (and/or the mathematical origin) of individual terms or quotients of the formula**
   $\vec{F}_i^z = \lambda_i \cdot \nabla g_i$: The constraining force results from the Lagrange multiplier and the gradient of the constraint. The gradient of the constraint gives the direction and the Lagrange multiplier the magnitude, depending on the mass m and the environment.

7. **express the content of the formula in our own words**
   The equation of motion is composed of mass times acceleration equals force (Newton's second axiom). The force has several components when the freedom of motion is limited. The external forces were also relevant before the restriction. New are the constraining forces. Since there can be several of them in general (here expressed by M the total number of constraints), they are summarized as a sum. A constraining force results from its magnitude (the Lagrange multiplier) and its direction (the gradient of the constraint).

8. **give an example or application of this formula in physics or every day life**
   An everyday example is a roller coaster that keeps a car on track by applying constraining forces to it. In school physics, this equation of motion can help in the analysis of forces on the inclined plane. Figure 6.3 shows the force decomposition on the left and the force addition on the right. What does a force analysis according to Langrange of the first kind look like?

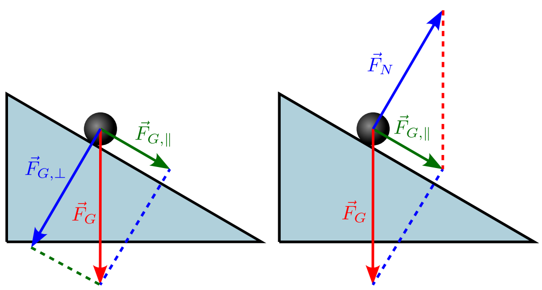


Figure 6.3. Sketch of the force analysis on the inclined plane. On the left there is the force decomposition and on the right the force addition.

To encourage self-explanation, the students' task is to answer the following questions:

- How are the Lagrange multipliers used in two examples of the lecture? (Example 1 is a mass sliding frictionlessly on a rod rotating about the coordinate origin. Example 2 is a mass in a gravity pendulum.)
- What units do these multipliers have, and where do the differences come from?
- In which direction do the constraining forces point?

Then, using the checklist from section 3.4.1, the students' work assignment is to verbalize and visualize the energy theorem $\frac{\mathrm{d}}{\mathrm{d}t}(T+U) = \sum_i \vec{F}_i^{\text{diss}} \cdot \dot{\vec{r}}_i$ and the virial theorem $\langle T \rangle = \left\langle \sum_i \frac{m_i}{2} \dot{\vec{r}}_i^2 \right\rangle = \frac{1}{2} \left\langle \sum_i \vec{r}_I \nabla_i U \right\rangle$.

To stimulate the connection to school physics, in the case of the constraining forces, the above-given sketches can be used to analyze forces on the inclined plane.

In school physics, the decomposition of forces and the addition of forces have become established as methods of force analysis. Both representations contain educational reductions and are not complete analyses in themselves. With the knowledge repeated on the equation of motion with constraints, these procedures will be discussed and a complete force analysis will be performed.

The energy theorem is easy to relate to school physics since it is treated as the conservation of energy in spring or gravity pendulums in school when frictional forces are neglected. The virial theorem is a generalization of energy conservation since it can be applied to all potential energies, not only to the gravitational potential. Here, the connection between the energy theorem in the school physics is more prominent than with the Lagrange formalism.

The other seminars are also structured according to the scheme described above. Tables 6.3 (mechanics) and 6.4 (electrodynamics) list the individual seminars with the topics covered and learning objectives. Many of the methods used here are discussed in detail in section 3 and the materials are given in appendix A.

6.6 Testing and enhancements

Students recognize the links between electrodynamics and school physics more quickly and efficiently than the links between theoretical mechanics and school physics.

Students themselves need explicit hints to recognize the relevance of theoretical mechanics to school physics. In electrodynamics, the parallels are recognized much more quickly. This can be explained by the often more complex mathematical operations in mechanics. To make the links even clearer, active naming (e.g., inclined plane and constraining forces) was therefore introduced after a first round of testing.

The treatment of instructional strategies is assessed as relevant for the later profession.

The presumed relevance is mentioned in conversations and is also reflected in the high motivation of the participating students.

A high density of the material can lead to excessive demands.

Combining three knowledge areas (UK, SK, and PCK) to develop a fourth (DSK) naturally creates a high material density in seminars. If too much content is packed into a seminar, this can lead to some students being overtaxed. This can be observed particularly when the UK was hardly understood before the repetition.

Close cooperation with colleagues is recommended in the implementation.

The repetition of the UK works best if direct reference is made to the lecture or the exercises. As mentioned in the previous point, subject difficulties are the biggest challenges in the seminars. Different notations and writing styles should be separate from these problems. If colleagues are interested in implementing, e.g., peer instruction in their own lecture, these can be left out in the accompanying seminars.

seminar	university knowledge	pedagogical knowledge	school knowledge	educational objective
concept map mechanics	Newton mechanics, balance equations, celestial mechanics	elemtarization, nature of science	Newton mechanics	reconstruction (concept map) of mechanics, comparison of historical knowledge gain vs. structure in textbook
peer instruction in theoretical physics	all topics (see section 3.6)	peer instruction, scientific misconceptions	all topics	content knowledge, peer instruction as instruction strategy, quality of explaining
Elementarization for lecture revision	rigid bodies, angular momentum, energy theorem	reduction and elementarization of content, nature of science	kinetic energy (translation vs. rotation), conservation of energy	elemtarization as studying technique, kinetic energy is more then translation energy
explaining quality	first kind Lagrange, energy, and virial theorem	verbalization and visualization of formulas	inclined plane, energy conservation	knowledge gain from formulas, quality of explaining
worked examples	Lagrange of second kind	learning with worked examples, nature of science	problem solving in school physics	worked examples as teaching method, Lagrange function as preparation for more complex topics (quantum mechanics, standard model,...)
educational reconstruction	harmonic oscillator	model of educational reconstruction	pendulum on spring or thread	physical justification of simplifications and estimation of the range of validity

Table 6.3. Summary and learning objectives of the developed seminars for mechanics.

seminar	university knowledge	pedagogical knowledge	school knowledge	educational objective
profiles of Maxwell's equations	Maxwell's equations in differential, integral and potential form	verbalization of formulas, elementarization, educational reduction, nature of science	electricity, em-waves, optics	identification of Maxwell's equations as core element of all topics in electrodynamics with concrete and practical examples
peer instruction in theoretical physics	all topics (see section 3.6)	peer instruction, scientific misconceptions	all topics	content knowledge, peer instruction as instruction strategy, quality of explaining
Fields of long coils	Biot-Savart law	educational reduction	magnetic field of a long thin coil	Limits and limitations of school formulas
reconstruction of typical school assignments	Faraday's law of induction and Lenz's law	model of educational reconstruction	induction law	critical examination of typical school assignments
dipole antenna	multipole radiation	verbalization and visualization of formulas	dipole antenna/ Hertzian dipole	differentiation near and far field, quality of explanation

Table 6.4. Summary and learning objectives of the developed seminars for electrodynamics.

7 | Conclusion and outlook

Interactive and activating teaching can significantly improve student learning. In comparison with conventional courses without activating methods (lecture in front-of-class teaching, exercise series with simple problem-solving tasks as homework, and laboratory practicals with a focus on data generation and analysis) students learn more. A transformation of conventional courses is, therefore, appropriate. In a first step it was the topic of this thesis to search for methods that are interactive and cognitively activating and can be adapted to courses in theoretical physics. The second step was to develop learning materials according to these methods. The developed materials were then tested in accompanying courses, parallel to the lectures, and improved and adapted if necessary. With the help of a standardized observation sheet, errors and approaches that did not elicit the hoped-for reactions from students were systematically revised or discarded.

Since theoretical physics is characterized by its mathematical formalism, learning with 'worked examples', verbalization and visualization of formulas, educational reconstruction, and peer instruction were chosen as appropriate instructional approaches. Researchers have already demonstrated the pedagogical added value of all these methods, but there has yet to be an implementation of these methods in theoretical physics for German universities.

Overall, materials were developed for theoretical mechanics, electrodynamics, quantum mechanics, and thermodynamics. The developments and implementations were categorized for each method and individual examples of implementation were demonstrated. Subsequently, the most striking problems and observations during the implementation were described in each case.

For learning with worked examples, a four-step approach was developed to reduce assistance to students gradually. This approach is intended to facilitate the introduction of new calculation methods but also to quickly encourage independent calculation. It has also proven helpful to hand out a worked example for new learning (or teaching) methods unknown to students, e.g., for verbalization of formulas and educational reconstruction. Then students could adopt the new learning method more quickly, and work assignments were easier to understand. With an ideal sample solution, unfamiliar tasks could lead to better activation and high participation.

In the verbalization and visualization of formulas, different designs were developed to help students to switch between representations and extract information from formulas. The designs differ in their degree of how much guidance is given or how narrow the tasks are. Observations have shown that open-ended and free-form tasks tend to inhibit rather than activate untrained students in unfamiliar tasks like a detailed verbalization of a formula.

Similar observations can be made with the methods of educational reduction and reconstruction. Again, different designs were tested and presented in this thesis. For example, it was examined how students can be animated to perform or evaluate physical and methodological reductions themselves. If the handling of educational reduction and reconstruction was still unpracticed, given worked examples or small-step tasks turned out to be helpful.

Peer instruction is the method of choice for courses with large numbers of participants. This thesis presented several categories of questions that can be implemented in the multiple-choice format in theoretical physics lectures. Classically following Mazur's guidelines, examples were described for clarifying/defining/concretizing technical terms, physics concepts, understanding/interpreting graphs or plots, and addressing student ideas. In contrast to Mazur's recommendation not to use formulas, questions were developed and presented to promote conceptual understanding of formulas. These were categorized as physics-mathematics translation, mathematics-physics translation, interpretation of formulas, and estimates from formulas.

An educational reconstruction for a teaching unit was carried out using the example of quantum mechanical entanglement and the exclusion of local hidden parameters. The goal of this reconstruction was to implement the previously described instructional approaches in an exemplary manner in a teaching unit. One conclusion of the clarification of the relevant physical contents was that the experiment proposed by Lucien Hardy is better suited for teaching than the famous inequalities of John Stewart Bell and John Clauser, Michael Horne, Abner Shimony, and Richard Holt. Subsequently, the design of the teaching unit describes how the teaching of this content could be implemented in the most interactive and activating way possible.

While most of the materials described above are suitable for all theoretical physics students, materials were developed specifically for the education of student teachers. Mathematics refresher courses have been created to be taught in advance of lectures in theoretical mechanics and electrodynamics. A focus has been on the mathematical literacy of students who do not study mathematics as a second subject and thus receive significantly less mathematics education. The materials are built primarily on verbalizing formulas and learning with worked examples.

In addition, seminars have been developed to link university knowledge from lectures with school knowledge and pedagogical content knowledge. These seminars aim at the deepening of all these areas of knowledge and to develop and strengthen school-relevant skills such as explanatory capabilities. In these seminars, the previously described teaching methods were also covered in order to provide future teachers with additional tools and methods. The development of these seminars was, again, carried out using the model of educational reconstruction. However, since the relevant knowledge is both the physics content knowledge and the pedagogical content knowledge, both components were included in the clarification of the science or knowledge content. This thesis describes exemplarily how this linkage could be implemented in teaching units, as well as the observation of the first tests of the seminars.

After the development of new teaching materials and teaching units, two logical continuations seem to be evident. First, the developed materials should be tested and verified for their effectiveness against the usual teaching methods (conventional lectures and computational exercise series). Although the implementation has already been tested in individual seminars and tutorials to develop further and improve the materials, this is not a formal evaluation. Parallel to or following an evaluation, programs and campaigns would be desirable to introduce and make available the new materials and the methods behind them to faculty throughout the country. Therefore, a general debate about university teaching in physics seems to be desirable. Hopefully, the materials developed here will provide an exciting basis for this discussion.

The possible applications of the developed materials are manifold. The material is primarily designed for use in undergraduate physics courses. The basic methods described in Chapter 3 can be used in the subject studies, Bachelor of Science, as well as in study programs for future teachers. Depending on the method, implementation is possible in lectures, exercises, or tutorials. The focus on the educational method or implementation in combination with the materials, see Chapter 6, can support teacher training programs in particular.

Beyond that, however, other areas of application are possible. For example, the teaching unit on hidden local parameters has been used in teacher training courses with much positive feedback. Educational reconstructions in other subject areas, such as atomic and solid state physics, optics, and relativity, which cognitively activate students similarly, seem promising. It should easily possible to apply the developed materials and methods to a minor course of study or other STEM subjects. In addition, teaching these materials to career changers who want to become school teachers in fast-track programs can be recommended with the results of this work.

The materials produced as part of this dissertation can be accessed and downloaded online at https://doi.org/10.18419/darus-3972. The materials are available in German.

A | Original version of the material in German

The following appendices are the German versions of the material described in detail and didactically processed in the previous thesis. All further materials, especially the elaborated seminars described in chapters 5 and 6, can be found online on the following page: https://doi.org/10.18419/darus-3972.

A.1 Worked examples

In the first section of the appendix the worksheets for learning with worked examples in Lagrangian mechanics of the second kind are presented. The first version consists of the worksheets in the described (see section 3.3) 4-step approach. The second version is a shortened version. Here, the second step, the student's written explanations, is omitted. The reason for this is to embed the worked examples in the seminar on theoretical physics in a school context (Chapter 6). Cognitive load theory and learning with worked examples are introduced at the beginning of the seminar. The time needed for this is saved by skipping the second stage.

Lagrange 2. Art - Worked Examples

Bildungswissenschaftliche Studien der „Cognitive Load Theory“ legen nahe, dass das Arbeiten mit Lösungsbeispielen (Worked Examples) dem schlichten Lösen von Übungsaufgaben überlegen ist und Lernende so effizienter lernen. Es gibt aber gewisse Vorgaben, die erfüllt sein müssen, damit das der Fall ist. So müssen sich die Lernenden die Musterlösung aktiv durchlesen und die einzelnen Lösungsschritte nachvollziehen bzw. sich selbst erklären. Dies ist auch hier Ihre Aufgabe:

1. Die erste Aufgabe und ihre Lösung aufmerksam durchlesen und sich klar machen, warum wann welcher Schritt gemacht wird.
2. Die zweite Aufgabe und ihre mathematische Lösung durchlesen und in der Tabelle daneben schriftlich die einzelnen Rechenschritte beschreiben und erklären.
3. Die dritte Aufgabe ebenfalls aufmerksam lesen und die Fehler (mind. 2) in der „Musterlösung“ finden.
4. Die vierte Aufgabe selbst bearbeiten und lösen.

1 Aufgabe: Pendel zwischen zwei Federn

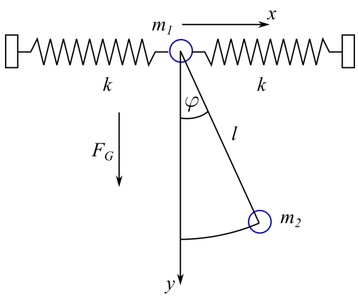


Abbildung 1: *Skizze von zwei Punktmassen aus Aufgabe 1. Masse m_1 ist zwischen zwei Federn eingespannt und Masse m_2 schwingt an m_1 fixiert als Pendel im Gravitationsfeld.*

Eine Punktmasse m_1 ist mit zwei Federn mit der identischen Federkonstante k und derselben Länge zwischen zwei Wänden befestigt. Die Ruhelage der Federn entspricht der zentralen Position der Masse m_1 zwischen den Wänden. Für die Federkraft gilt $|\mathbf{F}| = k|\mathbf{x}|$. Masse m_1 kann sich nur horizontal entlang der x-Achse bewegen. Eine zweite Punktmasse m_2 ist an m_1 befestigt über einen masselosen Faden der Länge l. Diese zweite Masse schwingt in der x-y-Ebene unter Einfluss eines homogenen Gravitationsfeldes mit der Kraft $\mathbf{F}_G = -m_2 g \mathbf{e}_y$. Der Auslenkwinkel des Pendels ist gegeben durch φ (siehe Abb. 1). Es gilt die Kleinwinkelnäherung.

Aufgabe: Stelle die Lagrange-Funktion auf und bestimme die Bewegungsgleichungen für die beiden Massen. Vereinfache das Ergebnis so weit wie möglich.

Bestimme die Zwangsbedingungen und berechne die Freiheitsgrade S.

$z_1 = 0, \quad z_2 = 0$ $y_1 = 0, \quad (x_1 - x_2)^2 + y_2^2 = l^2$	Die Massen können sich nicht frei in den z_1, z_2 und y_1 Koordinaten bewegen. Die x_2,y_2-Komponenten von Masse 2 hängen von x_1 und der Fadenlänge l ab. Diese Abhängigkeit ist über den Satz von Pythagoras beschrieben.
$S = 6 - 4 = 2$	Zwei freie Teilchen haben 6 Freiheitsgrade (2 mal die 3 Raumrichtungen). Da es 4 Zwangsbedingungen gibt, reduziert sich die Zahl der Freiheitsgrade auf 2.

Definiere die generalisierten Koordinaten, passend zu den Zwangsbedingungen.

$q_1 = x_1 = x, \qquad q_2 = \varphi$	Eine clevere Wahl der general. Koordinaten ist: 1. Die x-Position von Masse 1 2. Der Auslenkwinkel φ
$x_1 = x, \qquad x_2 = x_1 + l\sin\varphi$ $y_1 = 0, \qquad y_2 = -l\cos\varphi$ $z_1 = 0, \qquad z_2 = 0$	Die Koordinaten sind so gewählt, dass die Punktmasse m_1 im Ruhezustand im Ursprung liegt. Die Koordinatentransformation zwischen kartesischen & general. Koord. hilft bei der Bestimmung der Lagrange-Funktion.
$\dot{x}_1 = \dot{x}, \qquad \dot{x}_2 = \dot{x} + l\dot{\varphi}\cos\varphi$ $\dot{y}_1 = 0, \qquad \dot{y}_2 = l\dot{\varphi}\sin\varphi$ $\dot{z}_1 = 0, \qquad \dot{z}_2 = 0$	Die Ableitung nach der Zeit gibt die Geschwindigkeit. Die Zeitabhängigkeit aller Variablen ist zu berücksichtigen (Kettenregel).

Stelle die kinetische (T) und die potentielle Energie (V) auf.

$T = \frac{m_1}{2}\dot{x}^2 + \frac{m_2}{2}(\dot{x}^2 + l^2\dot{\varphi}^2 + 2l\dot{x}\dot{\varphi}\cos\varphi)$	Die gesamte kinetische Energie ist die Summe aller Geschwindigkeitskomponenten. y_1, z_1 und z_2 tragen dazu aber nichts bei, da es keine Bewegung in diese Richtung gibt.
$V_1 = \frac{k}{2}(-x)^2 + \frac{k}{2}(x)^2 = k(x)^2$ $V_2 = -m_2 g l\cos\varphi$	Die potentielle Energie der ersten Punktmasse ist die Energie in den beiden Federn. Die zweite Punktmasse bewegt sich im homogenen Gravitationfeld hoch und runter.

Stelle die Lagrange-Funktion auf.

$L = T - V = \frac{m_1 + m_2}{2}\dot{x}^2 + \frac{m_2}{2}l^2\dot{\varphi}^2 + m_2 l(\dot{x}\dot{\varphi} + g)\cos(\varphi) - kx^2$	Die Lagrange-Funktion ist die gesamte kinetische Energie minus die potentielle Energie.

1 Aufgabe: Pendel zwischen zwei Federn 2

Bestimme die Bewegungsgleichungen für jede einzelne general. Koordinate.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0$$
$$= \frac{d}{dt}\left[(m_1 + m_2)\dot{x} + m_2 l \dot{\varphi}\cos\varphi\right] + 2kx$$
$$= (m_1 + m_2)\ddot{x} + m_2 l \ddot{\varphi}\cos\varphi - m_2 l \dot{\varphi}^2 \sin\varphi + 2kx$$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\varphi}} - \frac{\partial L}{\partial \varphi} = 0$$
$$= \frac{d}{dt}\left[m_2 l^2 \dot{\varphi} + m_2 l \dot{x}\cos\varphi\right] + m_2 l(\dot{x}\dot{\varphi} + g)\sin\varphi$$
$$= l\ddot{\varphi} + \ddot{x}\cos\varphi + g\sin\varphi$$

Mit Hilfe der Euler-Lagrange-Gleichung können die Bewegungsgleichungen für die Koordinaten x und φ abgeleitet werden. Die richtige Reihenfolge der Ableitungen ist zu beachten.

Vereinfache die Bewegungsgleichungen so weit wie möglich.

$$(m_1 + m_2)\ddot{x} + 2kx = m_2 l(\dot{\varphi}^2\varphi - \ddot{\varphi})$$

$$l\ddot{\varphi} + g\varphi = -\ddot{x}$$

Die Näherung für kleine Winkel ($\sin\varphi \approx \varphi$, $\cos\varphi \approx 1$) vereinfacht trigonometrische Ausdrücke und die Bewegungsgleichungen lassen sich näherungsweise ohne trigonometrische Funktionen darstellen.

Diese zwei Differentialgleichungen sind gekoppelt und das auch noch in quadratischer Form, wie der Term $m_2 l\dot{\varphi}^2$ beweist. Es ist nicht trivial, solche gekoppelten Differentialgleichungen zu lösen, aber das ist auch nicht Teil dieser Aufgabe.

2 Aufgabe: Kugel auf der Innenseite eines Kegels

Auf der Innenseite eines nach oben geöffneten Kegels befindet sich eine Kugel mit Masse m. Der Kegel hat einen Öffnungswinkel von 2Θ und die Kugel bewege sich reibungsfrei auf der Innenseite des Kegels. Als äußere Kraft wirke die Gravitation mit $\mathbf{F}_G = -mg\mathbf{e}_z$. Die Symmetrieachse des Kegels falle mit der z-Achse zusammen und die Spitze liege im Ursprung wie in Abb. 2 dargestellt.
Aufgabe: Stelle die Lagrange-Funktion auf und bestimme die Bewegungsgleichung für das Problem.

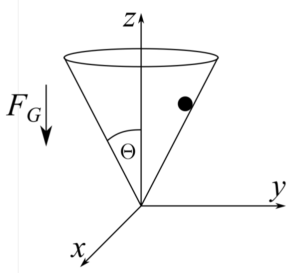


Abbildung 2: *Skizze für das 2. Beispiel. Eine Kugel, die sich unter Einfluss der Schwerkraft auf der Innenseite eines Kegels mit Winkel Θ frei bewegen kann.*

Bestimme die Zwangsbedingungen und berechne die Freiheitsgrade S

$$\tan\Theta = \frac{\rho}{z} \quad \Leftrightarrow \quad z = \rho\cot\Theta$$

$$S = 3 - 1 = 2$$

Definiere die generalisierten Koordinaten, passend zu den Zwangsbedingungen.

$$q_1 = \rho, \qquad q_2 = \varphi$$

$$x = \rho\cos\varphi,\ y = \rho\sin\varphi,\ z = \rho\cot\Theta$$

Stelle Formeln für die kinetische (T) und die potentielle Energie (V) auf.

$$T = \frac{m}{2}[\dot{x}^2 + \dot{y}^2 + \dot{z}^2]$$

$$= \frac{m}{2}\left[(1 + \cot^2\Theta)\dot{\rho}^2 + \rho^2\dot{\varphi}^2\right]$$

$$V = mgz = mg\rho\cot\Theta$$

Stelle die Lagrange-Funktion auf.

$$L = T - V$$

$$= \frac{m}{2}\left[(1 + \cot^2\Theta)\dot{\rho}^2 + \rho^2\dot{\varphi}^2\right]$$

$$- mg\rho\cot\Theta$$

Bestimme die Bewegungsgleichungen für jede einzelne general. Koordinate.

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\rho}} - \frac{\partial L}{\partial\rho} = 0$$

$$m(1 + \cot^2\Theta)\ddot{\rho} - m(\rho\dot{\varphi}^2 - g\cot\Theta) = 0$$

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\varphi}} - \frac{\partial L}{\partial\varphi} = 0$$

$$m(\rho^2\ddot{\varphi} + 2\rho\dot{\rho}\dot{\varphi}) = 0$$

Vereinfache die Bewegungsgleichungen so weit wie möglich.

$$(1 + \cot^2\Theta)\ddot{\rho} - \rho\dot{\varphi}^2 + g\cot\Theta = 0$$

$$\rho^2\ddot{\varphi} + 2\rho\dot{\rho}\dot{\varphi} = 0, \qquad \rho \neq 0$$

3 Aufgabe: Zwei Massen auf einem Keil über Federn verbunden

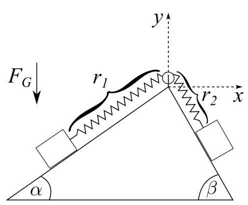


Abbildung 3: *Skizze für Aufgabe 3. Zwei Massen sind über eine Feder verbunden und auf zwei Seiten eines Keils angebracht. Von außen wirkt lediglich die Gravitation.*

Zwei Massen m_1 und m_2 bewegen sich auf einem Keil. Die Reibung sei zu vernachlässigen, aber die Massen befinden sich in einem Gravitationsfeld mit $\mathbf{F}_G = -m_{1/2}g\mathbf{e}_y$. Die beiden Massen sind über eine Feder mit der Federkonstante k verbunden, die im entspannten Zustand die Länge l besitzt.

Aufgabe: Stelle die Lagrange-Funktion auf und bestimme die Bewegungsgleichung für das Problem, dargestellt in Abb. 3.

Bestimme die Zwangsbedingungen und berechne die Freiheitsgrade S.

$$z_1 = 0, \quad z_2 = 0, \qquad S = 6 - 4 = 2$$
$$\frac{y_1}{x_1} = \tan\alpha, \quad \frac{y_2}{x_2} = \tan\beta$$

Definiere die generalisierten Koordinaten, passend zu den Zwangsbedingungen.

$$q_1 = r_1, \quad q_2 = r_2 \qquad x_1 = -r_1\cos\alpha, \quad x_2 = r_2\cos\beta$$
$$y_1 = -r_1\sin\alpha, \quad y_2 = -r_2\sin\beta$$
$$z_1 = 0, \quad z_2 = 0$$

Stelle die kinetische (T) und die potentielle Energie (V) auf.

$$T = \frac{m_1}{2}\dot{r}_1^2(\cos^2\alpha + \sin^2\alpha) + \frac{m_2}{2}\dot{r}_2^2(\cos^2\beta + \sin^2\beta) = \frac{1}{2}(m_1\dot{r}_1^2 + m_2\dot{r}_2^2)$$
$$V = -m_1 g r_1 \sin\alpha - m_2 g r_2 \sin\beta + \frac{k}{2}(r_1 + r_2)^2$$

Stelle die Lagrange-Funktion auf.

$$L = T - V = \frac{1}{2}(m_1\dot{r}_1^2 + m_2\dot{r}_2^2) + m_1 g r_1 \sin\alpha + m_2 g r_2 \sin\beta - \frac{k}{2}(r_1 + r_2)^2$$

Bestimme die Bewegungsgleichungen für jede einzelne general. Koordinate.

$$0 = \frac{d}{dt}\frac{\partial L}{\partial \dot{r}_1} - \frac{\partial L}{\partial r_1}; \qquad 0 = \frac{d}{dt}\frac{\partial L}{\partial \dot{r}_2} - \frac{\partial L}{\partial r_2}$$
$$0 = m_1\ddot{r}_1^2 - m_1 g \sin\alpha - k(r_1 + r_2); \qquad 0 = m_2\ddot{r}_2^2 - m_2 g \sin\beta - k(r_1 + r_2)$$

4 Aufgabe: Eine rotierende Masse und eine im Schwerefeld

Eine Masse m rotiere reibungsfrei auf einer Tischplatte. Über einen Faden der Länge l ($l = r + s$) sei durch ein Loch in der Platte m mit einer anderen Masse M verbunden. Wie bewegt sich M unter dem Einfluss der Schwerkraft?
Aufgabe: Stelle die Lagrange-Funktion auf und bestimme die Bewegungsgleichung für das Problem.

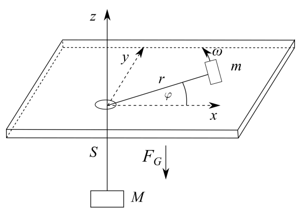


Abbildung 4: *Skizze für das 4. Beispiel. Eine rotierende Masse auf einer Tischplatte ist verbunden mit einer zweiten Masse, die im homogenen Schwerefeld hängt.*

Lagrange 2. Art - Worked Examples

Bildungswissenschaftliche Studien der „Cognitive Load Theory“ legen nahe, dass das Arbeiten mit Lösungsbeispielen (Worked Examples) dem schlichten Lösen von Übungsaufgaben überlegen ist und Lernende so effizienter lernen. Es gibt aber gewisse Vorgaben, die erfüllt sein müssen, damit das der Fall ist. So müssen sich die Lernenden die Musterlösung aktiv durchlesen und die einzelnen Lösungsschritte nachvollziehen bzw. sich selbst erklären. Dies ist auch hier Ihre Aufgabe:

1. Die erste Aufgabe und ihre Lösung aufmerksam durchlesen und sich klar machen, warum wann welcher Schritt gemacht wird.

2. Die zweite Aufgabe ebenfalls aufmerksam lesen und die Fehler (mind. 2) in der „Musterlösung“ finden.

3. Die dritte Aufgabe selbst bearbeiten und lösen.

1 Aufgabe: Pendel zwischen zwei Federn

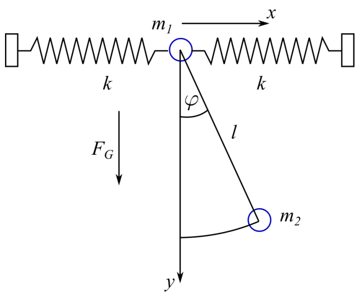


Abbildung 1: *Skizze von zwei Punktmassen aus Aufgabe 1. Masse m_1 ist zwischen zwei Federn eingespannt und Masse m_2 schwingt an m_1 fixiert als Pendel im Gravitationsfeld.*

Eine Punktmasse m_1 ist mit zwei Federn mit der identischen Federkonstante k und derselben Länge zwischen zwei Wänden befestigt. Die Ruhelage der Federn entspricht der zentralen Position der Masse m_1 zwischen den Wänden. Für die Federkraft gilt $|\mathbf{F}| = k|\mathbf{x}|$. Masse m_1 kann sich nur horizontal entlang der x-Achse bewegen. Eine zweite Punktmasse m_2 ist an m_1 befestigt über einen masselosen Faden der Länge l. Diese zweite Masse schwingt in der x-y-Ebene unter Einfluss eines homogenen Gravitationsfeldes mit der Kraft $\mathbf{F}_G = -m_2 g \mathbf{e}_y$. Der Auslenkwinkel des Pendels ist gegeben durch φ (siehe Abb. 1). Es gilt die Kleinwinkelnäherung.

Aufgabe: Stelle die Lagrange-Funktion auf und bestimme die Bewegungsgleichungen für die beiden Massen. Vereinfache das Ergebnis so weit wie möglich.

Bestimme die Zwangsbedingungen und berechne die Freiheitsgrade S.

$z_1 = 0, \quad z_2 = 0$ $y_1 = 0, \quad (x_1 - x_2)^2 + y_2^2 = l^2$	Die Massen können sich nicht frei in den z_1, z_2 und y_1 Koordinaten bewegen. Die x_2,y_2-Komponenten von Masse 2 hängen von x_1 und der Fadenlänge l ab. Diese Abhängigkeit ist über den Satz von Pythagoras beschrieben.
$S = 6 - 4 = 2$	Zwei freie Teilchen haben 6 Freiheitsgrade (2 mal die 3 Raumrichtungen). Da es 4 Zwangsbedingungen gibt, reduziert sich die Zahl der Freiheitsgrade auf 2.

Definiere die generalisierten Koordinaten, passend zu den Zwangsbedingungen.

$q_1 = x_1 = x, \qquad q_2 = \varphi$	Eine clevere Wahl der general. Koordinaten ist: 1. Die x-Position von Masse 1 2. Der Auslenkwinkel φ
$x_1 = x, \qquad x_2 = x_1 + l\sin\varphi$ $y_1 = 0, \qquad y_2 = -l\cos\varphi$ $z_1 = 0, \qquad z_2 = 0$	Die Koordinaten sind so gewählt, dass die Punktmasse m_1 im Ruhezustand im Ursprung liegt. Die Koordinatentransformation zwischen kartesischen & general. Koord. hilft bei der Bestimmung der Lagrange-Funktion.
$\dot{x}_1 = \dot{x}, \qquad \dot{x}_2 = \dot{x} + l\dot{\varphi}\cos\varphi$ $\dot{y}_1 = 0, \qquad \dot{y}_2 = l\dot{\varphi}\sin\varphi$ $\dot{z}_1 = 0, \qquad \dot{z}_2 = 0$	Die Ableitung nach der Zeit gibt die Geschwindigkeit. Die Zeitabhängigkeit aller Variablen ist zu berücksichtigen (Kettenregel).

Stelle die kinetische (T) und die potentielle Energie (V) auf.

$T = \frac{m_1}{2}\dot{x}^2 + \frac{m_2}{2}(\dot{x}^2 + l^2\dot{\varphi}^2 + 2l\dot{x}\dot{\varphi}\cos\varphi)$	Die gesamte kinetische Energie ist die Summe aller Geschwindigkeitskomponenten. y_1, z_1 und z_2 tragen dazu aber nichts bei, da es keine Bewegung in diese Richtung gibt.
$V_1 = \frac{k}{2}(-x)^2 + \frac{k}{2}(x)^2 = k(x)^2$ $V_2 = -m_2 g l\cos\varphi$	Die potentielle Energie der ersten Punktmasse ist die Energie in den beiden Federn. Die zweite Punktmasse bewegt sich im homogenen Gravitationfeld hoch und runter.

Stelle die Lagrange-Funktion auf.

$L = T - V = \frac{m_1 + m_2}{2}\dot{x}^2 + \frac{m_2}{2}l^2\dot{\varphi}^2 + m_2 l(\dot{x}\dot{\varphi} + g)\cos(\varphi) - kx^2$	Die Lagrange-Funktion ist die gesamte kinetische Energie minus die potentielle Energie.

2 Aufgabe: Zwei Massen auf einem Keil über Federn verbunden 2

Bestimme die Bewegungsgleichungen für jede einzelne general. Koordinate.

$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0$ $= \frac{d}{dt}\left[(m_1 + m_2)\dot{x} + m_2 l \dot{\varphi}\cos\varphi\right] + 2kx$ $= (m_1 + m_2)\ddot{x} + m_2 l \ddot{\varphi}\cos\varphi - m_2 l \dot{\varphi}^2 \sin\varphi + 2kx$ $\frac{d}{dt}\frac{\partial L}{\partial \dot{\varphi}} - \frac{\partial L}{\partial \varphi} = 0$ $= \frac{d}{dt}\left[m_2 l^2 \dot{\varphi} + m_2 l \dot{x}\cos\varphi\right] + m_2 l(\dot{x}\dot{\varphi} + g)\sin\varphi$ $= l\ddot{\varphi} + \ddot{x}\cos\varphi + g\sin\varphi$	Mit Hilfe der Euler-Lagrange-Gleichung können die Bewegungsgleichungen für die Koordinaten x und φ abgeleitet werden. Die richtige Reihenfolge der Ableitungen ist zu beachten.

Vereinfache die Bewegungsgleichungen so weit wie möglich.

$(m_1 + m_2)\ddot{x} + 2kx = m_2 l(\dot{\varphi}^2\varphi - \ddot{\varphi})$ $l\ddot{\varphi} + g\varphi = -\ddot{x}$	Die Näherung für kleine Winkel ($\sin\varphi \approx \varphi$, $\cos\varphi \approx 1$) vereinfacht trigonometrische Ausdrücke und die Bewegungsgleichungen lassen sich näherungsweise ohne trigonometrische Funktionen darstellen.

Diese zwei Differentialgleichungen sind gekoppelt und das auch noch in quadratischer Form, wie der Term $m_2 l \dot{\varphi}^2$ beweist. Es ist nicht trivial, solche gekoppelten Differentialgleichungen zu lösen, aber das ist auch nicht Teil dieser Aufgabe.

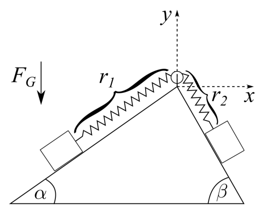


Abbildung 2: *Skizze für Aufgabe 2. Zwei Massen sind über eine Feder verbunden und auf zwei Seiten eines Keils angebracht. Von außen wirkt lediglich die Gravitation.*

2 Aufgabe: Zwei Massen auf einem Keil über Federn verbunden

Zwei Massen m_1 und m_2 bewegen sich auf einem Keil. Die Reibung sei zu vernachlässigen, aber die Massen befinden sich in einem Gravitationsfeld mit $\mathbf{F}_G = -m_{1/2} g \mathbf{e}_y$. Die beiden Massen sind über eine Feder mit der Federkonstante k verbunden, die im entspannten Zustand die Länge l besitzt.
Aufgabe: Stelle die Lagrange-Funktion auf und bestimme die Bewegungsgleichung für das Problem, dargestellt in Abb. 2.
Bestimme die Zwangsbedingungen und berechne die Freiheitsgrade S.

$z_1 = 0, \quad z_2 = 0, \qquad S = 6 - 4 = 2$

$\frac{y_1}{x_1} = \tan\alpha, \quad \frac{y_2}{x_2} = \tan\beta$

2 Aufgabe: Zwei Massen auf einem Keil über Federn verbunden 3

Definiere die generalisierten Koordinaten, passend zu den Zwangsbedingungen.

$$q_1 = r_1, \quad q_2 = r_2 \qquad \begin{aligned} x_1 &= -r_1 \cos\alpha, & x_2 &= r_2 \cos\beta \\ y_1 &= -r_1 \sin\alpha, & y_2 &= -r_2 \sin\beta \\ z_1 &= 0, & z_2 &= 0 \end{aligned}$$

Stelle die kinetische (T) und die potentielle Energie (V) auf.

$$T = \frac{m_1}{2}\dot{r}_1^2(\cos^2\alpha + \sin^2\alpha) + \frac{m_2}{2}\dot{r}_2^2(\cos^2\beta + \sin^2\beta) = \frac{1}{2}(m_1\dot{r}_1^2 + m_2\dot{r}_2^2)$$
$$V = -m_1 g r_1 \sin\alpha - m_2 g r_2 \sin\beta + \frac{k}{2}(r_1 + r_2)^2$$

Stelle die Lagrange-Funktion auf.

$$L = T - V = \frac{1}{2}(m_1\dot{r}_1^2 + m_2\dot{r}_2^2) + m_1 g r_1 \sin\alpha + m_2 g r_2 \sin\beta - \frac{k}{2}(r_1 + r_2)^2$$

Bestimme die Bewegungsgleichungen für jede einzelne general. Koordinate.

$$0 = \frac{d}{dt}\frac{\partial L}{\partial \dot{r}_1} - \frac{\partial L}{\partial r_1}; \qquad 0 = \frac{d}{dt}\frac{\partial L}{\partial \dot{r}_2} - \frac{\partial L}{\partial r_2}$$
$$0 = m_1\ddot{r}_1^2 - m_1 g \sin\alpha - k(r_1 + r_2); \qquad 0 = m_2\ddot{r}_2^2 - m_2 g \sin\beta - k(r_1 + r_2)$$

3 Aufgabe: Eine rotierende Masse und eine im Schwerefeld

Eine Masse m rotiere reibungsfrei auf einer Tischplatte. Über einen Faden der Länge l ($l = r + s$) sei durch ein Loch in der Platte m mit einer anderen Masse M verbunden. Wie bewegt sich M unter dem Einfluss der Schwerkraft?
Aufgabe: Stelle die Lagrange-Funktion auf und bestimme die Bewegungsgleichung für das Problem.

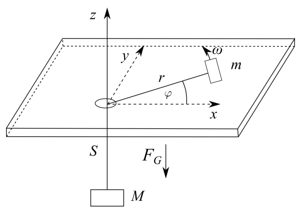


Abbildung 3: *Skizze für das 3. Beispiel. Eine rotierende Masse auf einer Tischplatte ist verbunden mit einer zweiten Masse, die im homogenen Schwerefeld hängt.*

A.2 Verbalization of formulas

Several materials have been created in this thesis on the topic of verbalization of formulas. The description and didactic classification of the materials can be found in section 3.4 of this thesis. In this appendix, one can find the checklist with the questionnaire, which is supposed to support students with the verbalization and visualization of formulas. Afterwards, an example for verbalizing the equation of motion under constraints and constraining forces is presented. It can be handed out to students in the sense of "worked examples" before they verbalize other formulas themselves. They are followed by the formula profile for Gauss's theorem in Maxwell's equations and templates for the remaining Maxwell's equations. Again, the example was handed out to the students for illustration before they were asked to independently compose the wanted posters for the remaining Maxwell's equations. The table for the interactions in electrostatics is at the end of this section.

Versprachlichung von Formeln

1 Interpretation einer Formel

1. Formelzeichen benennen.
2. Einheitenbetrachtung.
3. Gültigkeitsbedingungen.
4. Graphen oder Skizze zeichnen.
5. Spezialfälle und Grenzfälle betrachten.
6. Physikalische Bedeutung (und/oder mathematischer Ursprung) einzelner Terme oder Quotienten der Formel beschreiben.
7. Formelinhalt mit eigenen Worten wiedergeben.
8. Ein konkretes Beispiel aus dem Alltag/der realen Welt nennen, das sich mit der Formel beschreiben/erklären lässt.

2 Möglicher Fragenkatalog

1. Unter welchen Bedingungen gilt die Formel?
2. Aus welchen Formeln/Bedingungen wurde diese Formel (oder einzelne Teile) hergeleitet?
3. Welchen physikalischen Vorgang beschreibt die Formel?
4. Was passiert mit einer Größe der Formel, wenn ich die andere Größe verkleinere oder vergrößere? (Je-desto-Aussagen)
5. Wie sieht ein Graph/Plot aus, der den Zusammenhang zwischen Größen in der Formel darstellt?
6. Wie wurde die Formel in der Vorlesung eingeführt? (Definiert sie z.B. eine neue physikalische Größe? Oder woraus wurde sie abgeleitet?)
7. Für welche Anwendung wird diese Formel gebraucht?

Bewegungsgleichung mit Einschränkungen

Mögliche Antworten zur Versprachlichung

$$m\ddot{\vec{r}} = \vec{F}^a + \sum_{i=1}^{M} \vec{F}_i^z = \vec{F}^a + \sum_{i=1}^{M} \lambda_i \nabla g_i \quad (1)$$

1. **Formelzeichen benennen**

2. **Einheitenbetrachtung**
   Der erste und zweite Teil der Versprachlichung sind in der Abbildung 1 skizziert

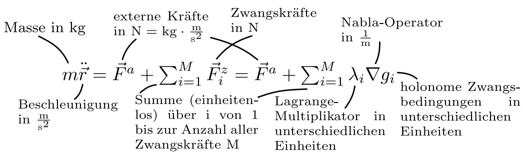


Abbildung 1: Skizze zur Benennung aller Formelzeichen in der Gleichung der eingeschränkten Bewegungsgleichung mit einer Einheitenbetrachtung.

3. **Gültigkeitsbedingungen**
   Diese Bewegungsgleichung gilt allgemein aber nur für Punktmassen.

4. **Graphen oder Skizze zeichnen**

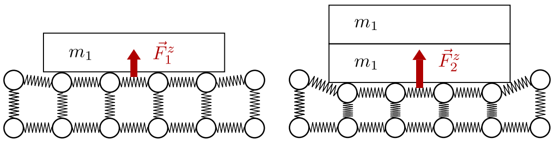


Abbildung 2: Skizze der Zwangskraft für zwei verschiedene Massen auf einer Unterlage.

5. **Spezialfälle und Grenzfälle betrachten**
   Dieser Abschnitt wird bei dieser Formel nicht benötigt.

6. **Physikalische Bedeutung (und/oder mathematischer Ursprung) einzelner Terme oder Quotienten der Formel beschreiben**
   $\vec{F}_i^z = \lambda_i \cdot \nabla g_i$: Die Zwangskraft ergibt sich aus dem Lagrange-Multiplikator und dem Gradienten der Zwangsbedingung. Der Gradient der Zwangsbedingung gibt die Richtung und der Lagrange-Multiplikator zusammen mit dem Gradienten den Betrag des Kraftvektors an, abhängig von der Masse m und der Umgebung.

7. **Formelinhalt mit eigenen Worten wiedergeben**
Die Bewegungsgleichung baut auf dem zweiten Newtonschen Axiom auf mit „Masse mal Beschleunigung". Die Kraft besteht dabei aus mehreren Komponenten, wenn die Bewegungsfreiheit eingeschränkt ist. Die äußeren Kräfte waren auch vor den Zwangsbedingungen Teil des zweiten Newtonschen Axioms und werden dort einfach als „Kräfte" bezeichnet. Neu sind also die Zwangskräfte. Da es im Allgemeinen mehrere von ihnen geben kann (hier ist die Anzahl M), werden sie als Summe zusammengefasst. Eine Zwangskraft ist definiert über ihren Betrag (dem Lagrange-Multiplikator) und ihrer Richtung (der Gradient der Zwangsbedingung).

8. **Ein konkretes Beispiel aus dem Alltag/der realen Welt nennen, das sich mit der Formel beschreiben/erklären lässt**
Ein alltägliches Beispiel für eine eingeschränkte Bewegung ist eine Achterbahn, die einen Wagen auf der Strecke hält, indem sie Zwangskräfte auf ihn ausübt. In der Schulphysik kann diese Bewegungsgleichung bei der Analyse der Kräfte auf einer schiefen Ebene helfen. Abbildung 3 zeigt die Kräftezerlegung auf der linken Seite und die Kräfteaddition auf der rechten Seite. Die Zwangskraft, die auch in diesem Beispiel auftritt, wird lediglich bei der Kräfteaddition als Normalkraft $\vec{F}_N$ sichtbar. In der Kräftezerlegung wird die Zwangskraft stillschweigend vorausgesetzt.

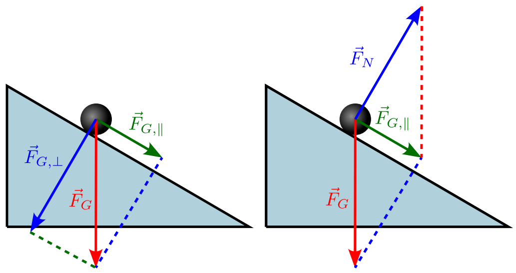


Abbildung 3: Skizze der Kräfteanalyse auf der schiefen Ebene. Links befindet sich die Kräftezerlegung und rechts die Kräfteaddition. Die Zwangskraft wird nur in der Kräfteaddition als Normalkraft $\vec{F}_N$ sichtbar.

Fragen zur Aktivierung

- Wie werden die Lagrange-Multiplikatoren in zwei Beispielen aus der Vorlesung verwendet? (Beispiel 1 ist eine Masse, die reibungslos auf einem Stab gleitet, der sich um den Koordinatenursprung dreht. Beispiel 2 ist eine Masse in einem Schwerkraftpendel.)
- Welche Einheiten haben diese Multiplikatoren und woher kommen die Unterschiede?
- In welche Richtung zeigen die Zwangskräfte?

Gaußsches Gesetz

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0}\rho$$

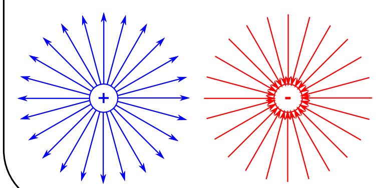
Inhomogene Differentialgleichung

Differentiell

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0}\rho$$

$$\vec{\nabla} \cdot \vec{D} = \rho_{\text{frei}} = \rho - \rho_{\text{pol}}$$

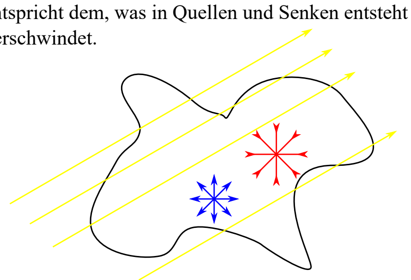
Nabla auf ein Vektorfeld ist die Divergenz.
Die Divergenz testet ein Vektorfeld auf Quellen und Senken.
Die Quellen und Senken des elektrischen Feldes sind elektrische Ladungen oder ihre Verteilung.



Integral

$$\iint_{\partial V} \vec{D} \cdot d\vec{A} = \iiint_V \rho dV = Q(V)$$

Der elektrische Fluss (D) durch eine geschlossene Oberfläche eines Volumens V entspricht den elektrischen Ladungen im Inneren dieses Volumens.
Alles was im Volumen an Fluss ensteht oder verschwindet, entspricht dem, was in Quellen und Senken entsteht oder verschwindet.



Materialgleichung

$$\vec{\nabla} \cdot \vec{D} = \rho_{\text{frei}} = \rho - \rho_{\text{pol}}$$

$$\vec{D}(\vec{r}) = \varepsilon \vec{E}(\vec{r}) + \vec{P}(\vec{r})$$

$$\vec{P}(\vec{r}) = \varepsilon_0(\varepsilon_r - 1)\vec{E}(\vec{r})$$

Potentialdarstellung

$$\nabla^2\Phi(\vec{r}) + \vec{\nabla} \cdot \dot{\vec{A}}(\vec{r}) = \frac{1}{\varepsilon_0}\rho(\vec{r})$$

Mit der Coulomb Eichung

$$\vec{\nabla} \cdot \vec{A}(\vec{r}) = 0$$

lässt sich das Gaußsche Gesetz wieder herleiten.

Anwendungen

Poissongleichung

$$\Delta\Phi = \frac{1}{\epsilon_0}\rho$$

$$\Phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int d^3r' \frac{\rho(\vec{r})}{|\vec{r} - \vec{r}'|}$$

Bestimmung von Feldern, Potentialen, Kräften & Ladungsverteilung

- Spiegelladungsmethode
- Multipolentwicklung
- Greensche Funktion
- Coulombwechselwirkung
- Randwertprobleme (Leiter, Isolator)
- ...

Anwendung in der Schule:

- Stromkreise:
- Spannung = Potential,
- Ohmsches Gesetz;
- Plattenkondensator,
- ...

Gaußsches Gesetz der Magnetfelder
$\vec{\nabla} \cdot \vec{B} = 0$

Differentiell
$\vec{\nabla} \cdot \vec{B} = 0$

Integral
$\iint_{\partial V} \vec{B} \cdot d\vec{A} = 0$

Materialgleichung

Anwendungen

Potentialdarstellung

Induktionsgesetz

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Differentiell

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Integral

$$\int_{\partial A} \vec{E} \cdot d\vec{s} = -\iint_A \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$$

Materialgleichung

Anwendungen

Potentialdarstellung

Erweitertes Durchflutungsgesetz

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

Differentiell

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{j}_{\text{frei}} + \frac{\partial \vec{D}}{\partial t}$$

Integral

$$\int_{\partial A} \vec{H} \cdot d\vec{s} = \iint_A \vec{j} \cdot d\vec{A} + \iint_A \frac{\partial \vec{D}}{\partial t} \cdot d\vec{A}$$

Materialgleichung

Potentialdarstellung

Anwendungen

wie wirkt / auf	Ladung	Feld	Potential	Leiter	Dielektrikum
Vakuum	erzeugt ein Feld/Potential				
Ladung					
Feld					
Potential					
Leiter					
Dielektrikum					

A.3 Educational reconstruction

As for the other parts, for elementary and educational reconstruction many materials are described in this work, which are presented in this section in their material form. In the beginning, the checklist for summarizing lectures and examples of such summaries can be found. In addition to the example described in detail in this thesis on the quantum mechanical harmonic oscillator the summary on rigid bodies is presented. This summary was handed out to students in the seminar "Theoretical Physics in a school context" before they were asked to summarize sections from theoretical mechanics themselves. Next, the "Concept map" on electrostatics is presented, summarizing the verbalization of electrostatic interactions. This is followed by the table on the elements of quantum mechanics as handed out to students. The first example from spin physics serves as an illustration and for self-explanation by students before they are asked to identify the same elements in other contexts. Finally, the task sheet on the critical use of simplifications for magnetic fields in long thin coils can be found.

Vorlesungen Nacharbeiten

Fragenkatalog

1. Wie ist der Abschnitt/das Kapitel aufgebaut?
   - Herleitung
   - Erklärung
   - Auflistung
   - ...
2. Was ist das Thema des Abschnitts/Kapitels?
3. Welches Physikalische Problem wird hier behandelt?
4. Was ist die zentrale physikalische Idee in diesem Abschnitt/Kapitel?
5. Wie wird diese Idee im Skript/Formalismus beschrieben?
6. Was ist die mathematische Formulierung des physikalischen Problems und/oder der zentralen Idee?
   - Welche Formeln kommen vor und was beschreiben sie?
   - Wie hängen die Formeln zusammen?
7. Lässt sich die zentrale Idee auch graphisch darstellen?
   - Skizze
   - Graph/Plot
   - Mindmap(?)
8. Wofür benötigt man das behandelte Thema?
9. Nehmen die Inhalte Bezug auf vorherige Vorlesungen/Inhalte?

Harmonischer Oszillator

1 Fragenkatalog

1. Was ist das Thema des Abschnitts/Kapitels?
2. Welche physikalischen Probleme werden hier behandelt?
3. Was ist hier quantisiert und wie drückt sich das aus?
4. Welche zentralen physikalischen Ideen gibt es in diesem Abschnitt/Kapitel?
5. Wie werden diese Ideen im Skript/Formalismus beschrieben? (Was ist die mathematische Formulierung der physikalischen Probleme und/oder der zentralen Ideen?)
6. Welche wichtigen physikalischen Größen kommen vor und wie sind diese definiert?
7. Lässt sich die zentrale Idee auch graphisch darstellen?
8. Was sind wichtige Erkenntnisse aus diesem Abschnitt/Kapitel?
9. Wofür benötigt man das behandelte Thema?

2 Antworten

1. **Was ist das Thema des Abschnitts/Kapitels?**
   Der harmonische Oszillator in der Quantenmechanik. Es ist das quantenmechanische Analogon zum mechanischen Oszillator. In der klassischen Mechanik ist dieser als ein schwingungsfähiges System mit einer linearen Rückstellkraft definiert. In der Quantenmechanik beschreibt der harmonische Oszillator ein System mit einem quadratischen Potential.

2. **Welche physikalischen Probleme werden hier behandelt?**
   Der quantenmechanische harmonische Oszillator in einer Dimension und dessen mathematische Beschreibung.

3. **Was ist hier quantisiert und wie drückt sich das aus?**
   Die physikalisch sinnvollen stationären Zustände eines quantalen Teilchens in einem harmonischen ($\sim x^2$) Potential definieren die Quantisierung der Eigenwerte zum Hamilton-Operator des Teilchens im jeweiligen Zustand. Die benötigte Energie für den Übergang von einem Eigenzustand zum nächstgelegenen ist somit klar definiert und diskret.

4. **Welche zentralen physikalischen Ideen gibt es in diesem Abschnitt/Kapitel?**
   Alle Zustände sollen durch den Grundzustand und einen „Aufsteigeoperator“beschrieben werden können. Damit ist mathematisch jeder Zustand adressierbar.

5. **Wie werden diese Ideen im Skript/Formalismus beschrieben? (Was ist die mathematische Formulierung der physikalischen Probleme und/oder der zentralen Ideen?)**

Stationäre Schrödingergleichung zur Beschreibung des Problems:

$$\hat{H}\ \varphi(x) = E\varphi(x)$$
$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{m\omega^2}{2}x^2\right)\varphi(x) = E\varphi(x)$$
$$\hbar\omega\left(\hat{b}^\dagger\hat{b} + \frac{1}{2}\right)\varphi(x) = E\varphi(x)$$

Jeder stationäre Zustand (abhängig davon, der wievielte es ist) im harmonischen Potential:

$$|\varphi_n\rangle = \frac{1}{\sqrt{n!}}(\hat{b}^\dagger)^n\,|\varphi_0\rangle = \frac{1}{\sqrt{n!}}(\hat{b}^\dagger)^n\alpha_0 e^{-\frac{\alpha^2x^2}{2}} = \sqrt{\frac{\alpha}{\sqrt{\pi}}}\frac{1}{\sqrt{2^n n!}}\hat{H}_n(\alpha x)e^{-\frac{\alpha^2x^2}{2}}$$

6. **Welche wichtigen physikalischen Größen kommen vor und wie sind diese definiert bzw. hängen diese zusammen?**

Aufsteigeoperator:

$$\hat{b}^\dagger = \frac{1}{\sqrt{2}}\left(\alpha x - \frac{1}{\alpha}\frac{d}{dx}\right), \quad \alpha^2 = \frac{m\omega}{\hbar}$$

Absteigeoperator:

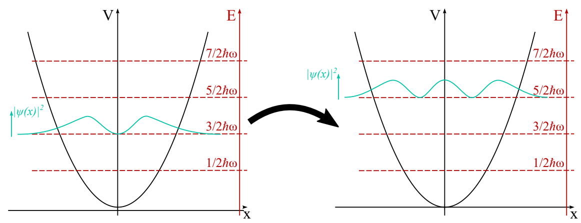
$$\hat{b} = \frac{1}{\sqrt{2}}\left(\alpha x + \frac{1}{\alpha}\frac{d}{dx}\right)$$

Besetzungszahloperator:

$$\hat{N} = \hat{b}^\dagger\hat{b}$$

7. **Lässt sich die zentrale Idee auch graphisch darstellen?**

$$\hat{b}^\dagger\,|\varphi_2\rangle = \alpha_3\,|\varphi_3\rangle :$$



8. **Was sind wichtige Erkenntnisse aus diesem Abschnitt/Kapitel?**

Man kann mathematisch alles aufeinander aufbauen und alle Zustände nur mit dem Grundzustand und Auf- bzw. Absteigeoperatoren beschreiben.
Die Energieniveaus für stationäre Zustände im harmonischen Oszillator sind äquidistant.

9. **Wofür benötigt man das behandelte Thema?**

Mit dem Formalismus von Auf-, bzw. Absteigeoperatoren kann man viele andere physikalische Potentiale beschreiben genauso wie die Wechselwirkung von Teilchen in solchen Potentialen mit der Umgebung. Die Anregung eines Elektrons im Atom auf ein anderes Energieniveau kann mit dem Aufsteigeoperator beschrieben werden.
Manchmal werden $\hat{b}^\dagger, \hat{b}$ auch als Erzeuger und Vernichter bezeichnet. Mit dieser Formulierung kann man einzelne Quanten beschreiben und wie sie in Vielquantensystemen als komplexes Ganzes zusammenwirken (Quantenstatistik in der Festkörperphysik).

2 Antworten 2

Starre Körper - Drehung um eine feste Achse

Mögliche Antworten auf den Fragenkatalog

1. **Wie ist der Abschnitt/das Kapitel aufgebaut?**

   Bei diesem Abschnitt handelt es sich um eine Herleitung und die Definition der Grundbegriffe und Prinzipien der starren Körper mit Beispiel.

2. **Was ist das Thema des Abschnitts/Kapitels?**

3. **Welches physikalische Problem wird hier behandelt?**

   Starre Körper werden als Massen mit räumlicher Ausdehnung eingeführt, wobei sich die einzelnen Massepunkte, die diesen Körper gemeinsam bilden, relativ zueinander nicht bewegen können (starr sind). Die Auswirkungen dieser räumlichen Ausdehnung auf Bewegungen werden beschrieben. Diese Prinzipien werden am Beispiel des rollenden Zylinders verdeutlicht sowie die Analogien zur schlichten Translationsbewegung aufgezeigt.

4. **Was ist die zentrale physikalische Idee in diesem Abschnitt/Kapitel?**

   In der Drehung von ausgedehnten Körpern können Energie und Impuls gespeichert werden. Wie viel, hängt von der Geschwindigkeit der Drehung $\boldsymbol{\omega}$, der Drehachse (Richtung von $\boldsymbol{\omega}$) und der räumlichen Verteilung der Masse um diese Achse (Θ) ab.

5. **Wie wird diese Idee im Skript/Formalismus beschrieben? (Was ist die mathematische Formulierung des physikalischen Problems und/oder der zentralen Idee?)** Die kinetische Energie eines ausgedehnten Körpers setzt sich aus der Translationsenergie seines Schwerpunktes und der Rotationsenergie zusammen.

   $$T = \frac{1}{2}\Theta\omega^2 + \frac{1}{2}M\dot{s}^2 \tag{1}$$

   Analog zur Translationsbewegung kann auch eine Bilanzgleichung für den Drehimpuls aufgestellt werden (hier in z-Richtung)

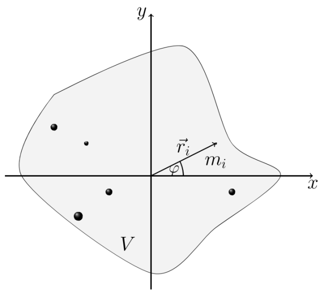
   $$\frac{d}{dt}L_z = \Theta\dot{\omega} = \sum_i (\boldsymbol{r}_i \times \boldsymbol{F}_i^a)_z. \tag{2}$$

6. **Welche wichtigen physikalischen Größen kommen vor und wie sind diese definiert?**

   Trägheitsmoment für eine Drehung um die z-Achse:

   $$\begin{aligned} \Theta &= \sum_i m_i(x_i^2 + y_i^2), && \text{für diskret verteilte Massen} \\ &= \int_V \rho(x,y,z)(x^2+y^2)\,dV && \text{für kontinuierliche Massenverteilungen} \end{aligned}$$

7. **Lässt sich die zentrale Idee auch graphisch darstellen?**



8. **Was sind wichtige Erkenntnisse aus diesem Abschnitt/Kapitel?**

   Ausgedehnte Körper verhalten sich bei Drehungen komplexer als Massepunkte!

9. **Wofür benötigt man das behandelte Thema?**

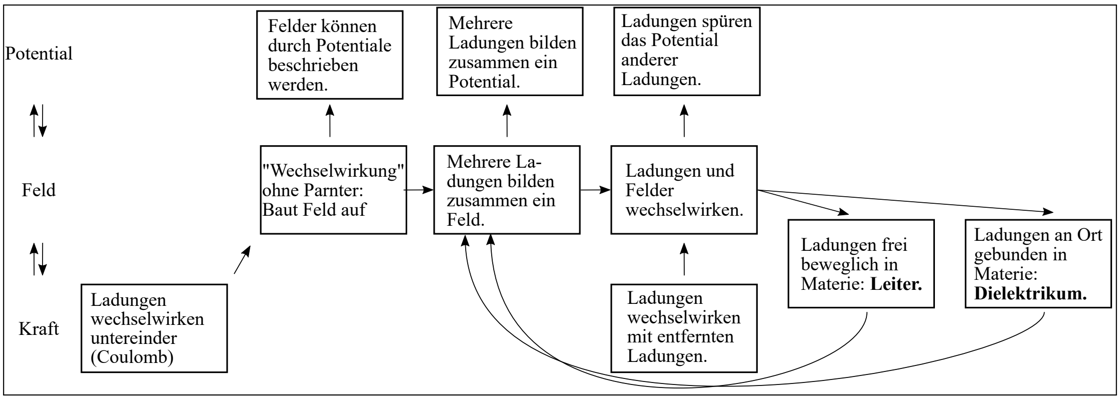
   Die starren Körper sind das einfachste Modell für das Verhalten von Massen mit Ausdehnung bei Drehungen. Dieses neue Modell beschreibt die Natur genauer als das Modell des Massepunkts. Die Speicherung von kinetischer Energie in Rotationen ist ein elementares Phänomen in der Mechanik und Natur (Machinenbau, Sport und Energiespeicherung in Gasen/Molekülen).

10. **Nehmen die Inhalte Bezug auf vorherige Vorlesungen/Inhalte?**

    - Starre Körper sind Systeme von Massenpunkten.
    - Analogie von kinetischer Energie bei Rotation und Translation sowie Impuls und Drehimpuls.

$T_{\mathsf{tra}} = \frac{1}{2}M\dot{s}^2$	$\dot{p} = m\dot{r}$
$T_{\mathsf{rot}} = \frac{1}{2}\Theta\omega^2$	$\dot{L} = \Theta\dot{\omega}$

2


Potential
Feld
Kraft
Felder können durch Potentiale beschrieben werden.
Mehrere Ladungen bilden zusammen ein Potential.
Ladungen spüren das Potential anderer Ladungen.
"Wechselwirkung" ohne Parnter: Baut Feld auf
Mehrere Ladungen bilden zusammen ein Feld.
Ladungen und Felder wechselwirken.
Ladungen frei beweglich in Materie: **Leiter.**
Ladungen an Ort gebunden in Materie: **Dielektrikum.**
Ladungen wechselwirken untereinder (Coulomb)
Ladungen wechselwirken mit entfernten Ladungen.

QM	Mathematisch	Spin	Freies Teilchen	Teilchen im Potentialtopf
Hilbertraum	Der Hilbertraum $\mathcal{H}$ ist ein reeller oder komplexer Vektorraum mit einem Skalarprodukt $\langle \cdot \vert \cdot \rangle$.	• 2 dimensional: Spin "up" & Spin "down" • Orthogonalität: $\langle z_i \vert z_j \rangle = \delta_{ij}$	• ∞-dimensional • Orthogonalität: $\langle x' \vert x \rangle = \delta(x - x')$	
Zustand/ Wellenfunktion	Vektor $\psi \in \mathcal{H}$.	Beispiel in z-Richtung $\mathrm{up} = \vert z+\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ $\mathrm{down} = \vert z-\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ allgemein: $\vert\psi\rangle = c_1 \vert z+\rangle + c_2 \vert z-\rangle$		
Observable Beobachtbare physik. Größe	Linearer Operator $\hat{A}$ auf $\mathcal{H}$.	Spin in x,y,z-Richtung (wenn z die Vorzugsrichtung ist) $\hat{\sigma}_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$ $\hat{\sigma}_x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ $\hat{\sigma}_y = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix}$		
Der eigentliche Messwert	Aus den Eigenwerten einer Eigenwertgleichung ergeben sich die möglichen Messwerte. $\hat{A} \vert\psi\rangle = a \vert\psi\rangle$	Energie des Spins im externen Magnetfeld (bei Stern-Gerlach) $H \vert\psi\rangle = E \vert\psi\rangle$ $-\mu B_z \hat{\sigma}_z \vert z\pm\rangle = \pm \frac{\mu B_z}{2} \vert z\pm\rangle$		
Eigenzustände als Basis	Die beste Basis ist eine Linear-kombination aus Eigenzuständen zu einer Observablen, die orthogonal und normierbar sind. $\vert\psi\rangle = \sum_i c_i \vert\phi_i\rangle$	Die Eigenzustände Spin "up" & "down" zur jeweiligen Messrichtung x,y oder z. $\vert\psi\rangle = c_1 \vert z+\rangle + c_2 \vert z-\rangle$ $\vert x+\rangle = \frac{1}{\sqrt{2}} \vert z+\rangle + \frac{1}{\sqrt{2}} \vert z-\rangle$		
Messung = Projektion + Kollaps	Ein Zustand wird in der Basis der Observablen formuliert. Nach der Messung reduziert sich der Zustand auf den Eigenvektor, der dem gemessenen Eigenwert entspricht. $\vert\psi\rangle_{\mathrm{vor}} = \sum_i c_i \vert\phi_i\rangle$ $\vert\psi\rangle_{\mathrm{nach}} = 1 \cdot \vert\phi_j\rangle$	Spin "up" in x-Richtung wird in z-Richtung gemessen. $\vert\psi\rangle_{\mathrm{before}} = \vert x+\rangle = \frac{1}{\sqrt{2}} \vert z+\rangle + \frac{1}{\sqrt{2}} \vert z-\rangle$ $\vert\psi\rangle_{\mathrm{after}} = \vert z+\rangle$ or $\vert\psi\rangle_{\mathrm{after}} = \vert z-\rangle$		
Messwahr-scheinlichkeit	Die Koeffizienten c_j sind der Anteil des Eigenzustands $\vert\psi_j\rangle$ am Gesamtzustand $\vert\psi\rangle$. $\vert c_j \vert^2 = p_j$ ist die Wahrscheinlichkeit, mit der das System im Eigenzustand $\vert\psi_j\rangle$ gemessen wird, wenn es vor der Messung im Zustand $\vert\psi\rangle$ war.	Wahrscheinlichkeit $\vert z-\rangle$ zu messen, wenn der Spin vorher im Zustand $\vert x+\rangle$ war. $\vert \langle z - \vert x+\rangle \vert^2 = \left\vert \langle z- \vert \left(\frac{1}{\sqrt{2}} \vert z+\rangle + \frac{1}{\sqrt{2}} \vert z-\rangle \right) \right\vert^2$ $= \left\vert \frac{1}{\sqrt{2}} \langle z - \vert z-\rangle \right\vert^2 = \left\vert \frac{1}{\sqrt{2}} \right\vert^2$ $= \frac{1}{2}$		

Magnetfelder durch Spulen

1 Lange dünne Spule

Eine lange dünne Spule erzeugt in ihrem Inneren ein statisches Magnetfeld. Der Stromfluss soll von hinten betrachtet im Uhrzeigersinn erfolgen.

$$\begin{aligned}
\oint_S \vec{B} \cdot d\vec{s} &= \mu_0 N \cdot I \\
\int_{S_{12}} \vec{B} \cdot d\vec{s}_{12} + \int_{S_{23}} \vec{B} \cdot d\vec{s}_{23} + \int_{S_{34}} \vec{B} \cdot d\vec{s}_{34} + \int_{S_{41}} \vec{B} \cdot d\vec{s}_{41} &= \mu_0 N \cdot I \\
\int_{S_{12}} \vec{B} \cdot d\vec{s}_{12} &= \mu_0 N \cdot I \\
B \int_{S_{12}} ds_{12} = Bl &= \mu_0 N \cdot I \\
\Rightarrow B &= \mu_0 \frac{N}{l} I \qquad (1)
\end{aligned}$$

1. Welches Gesetz (1. Zeile) ist Grundlage der Herleitung?
2. Fertige eine Skizze an, die das Magnetfeld einer langen dünnen Spule zeigt und notiere die relevanten Integrationswege $\oint_{S_{13}}, \oint_{S_{34}}, \ldots$.
3. Fertige eine Skizze von einer kurzen Spule, wie ändert sich das Magnetfeld außerhalb der Spule im Vergleich zur langen dünnen Spule?
4. Warum tragen manche Wegabschnitte nichts zum Integral bei und können deshalb vernächlässigt werden?
5. Warum kann der Feldanteil B aus dem Integral herausgenommen und davor gestellt werden?

2 Lange dünne Spule II

Machen Sie sich mit der zweiten ausführlicheren Herleitung für das Magnetfeld im Inneren einer langen dünnen Spule vertraut und beantworten Sie anschließend folgende Fragen:

1. Welche Gleichung erhält man, wenn man das Feld in der Mitte $z = 0$ betrachtet und das Ergebnis für eine lange dünne Spule mit $L \gg R$ vereinfacht?
2. Wie groß ist das B-Feld am Ende der Spule unter der Vereinfachung $L \gg R$?
3. Wie sehr kann man sich auf die Gleichung aus der ersten Herleitung verlassen?
4. Welche Einschränkungen haben die zweite Herleitungen und ihre Lösung?

Für die zweite Bestimmung des Magnetfeldes einer langen Spule wird zunächst das Feld einer Leiterschleife untersucht. Liegt die Stromschleife in der x-y-Ebene (siehe Abbildung 1.a), so gilt für das

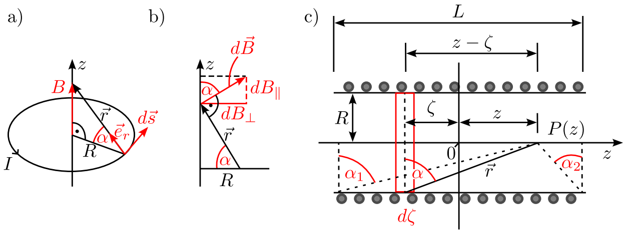


Abbildung 1: Skizzen der Kreisschleife a), b) der Symmetrieachse der Schleife und c) der Zylinderspule.

Magnetfeld B nach dem Gesetz von Biot-Savart

$$\vec{B}(\vec{r}) = -\frac{\mu_0 \cdot I}{4\pi} \cdot \int \frac{\hat{\vec{e}}_r \times d\vec{s}}{|\vec{r}|^2}. \tag{2}$$

Auf der Symmetrieachse (z-Achse durch den Mittelpunkt, siehe Abbildung 1.a & b) erhalten wir den Beitrag dB des Pfadelements ds zum Magnetfeld:

$$d\vec{B} = -\frac{\mu_0 \cdot I}{4\pi} \cdot \frac{\vec{r} \times d\vec{s}}{r^3}. \tag{3}$$

Bei der Integration über alle Bahnelemente des Kreises heben sich die Komponenten senkrecht zur Symmetrieachse $dB_\perp = d\vec{B} \cdot \sin\alpha$ gegenseitig auf und ergeben in Summe den Wert Null. Nur die parallele Komponente $dB_\parallel = d\vec{B} \cdot \cos\alpha$ verbleibt. Für diese gilt, wegen $|\vec{r} \times d\vec{s}| = \frac{R}{\cos\alpha} \cdot ds$, bei der Integration:

$$B_z = B_\parallel = \int dB_\parallel = \int |d\vec{B}| \cdot \cos\alpha = \frac{\mu_0 \cdot I}{4\pi r^3} \cdot \oint R \cdot ds = \frac{\mu_0 \cdot I \cdot R}{4\pi r^3} \cdot 2\pi R. \tag{4}$$

Aufgrund von $r^2 = R^2 + z^2$ erhalten wir

$$B_z = \frac{\mu_0 \cdot I \cdot R^2}{2(z^2 + R^2)^{3/2}}. \tag{5}$$

Nun soll das Magnetfeld im Inneren einer langen Spule mit n Windungen pro Meter bestimmt werden. Der Nullpunkt des Koordinatensystems soll in der Mitte der Spule liegen, deren Symmetrieachse als z-Achse gewählt wird (Abbildung 1.c). Der Anteil des Magnetfeldes im Punkt $P(z)$, der von den $n \cdot d\zeta$ Windungen mit dem Querschnitt $A = \pi \cdot R^2$ im Längenintervall $d\zeta$ erzeugt wird, ist nach dem Feld einer Leiterschleife (5) auf der Symmetrieachse

$$dB = \frac{\mu_0 \cdot I \cdot R^2 \cdot n \cdot d\zeta}{2\left[R^2 + (z-\zeta)^2\right]^{3/2}}. \tag{6}$$

Das Gesamtfeld an der Stelle $P(z)$ erhält man durch Integration über alle Windungen von $\zeta = -L/2$ bis $\zeta = +L/2$. Das Integral kann mit der Substitution $z - \zeta = R \cdot \tan\alpha$ gelöst werden und ergibt:

$$\begin{aligned} B(z) &= \int_{-L/2}^{+L/2} dB = -\frac{\mu_0 I n}{2} \int_{\alpha_1}^{\alpha_2} \cos\alpha \, d\alpha \\ &= \frac{\mu_0 I n}{2} \left\{ \frac{z + L/2}{\sqrt{R^2 + (z+L/2)^2}} - \frac{z - L/2}{\sqrt{R^2 + (z-L/2)^2}} \right\}. \end{aligned} \tag{7}$$

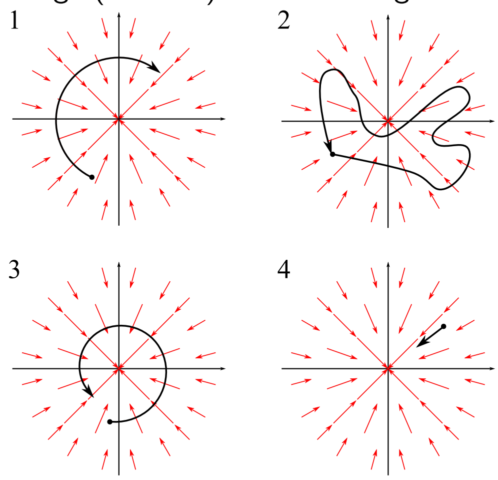
A.4 Peer Instruction

The following section lists all of the quizzes created as part of a peer instruction for this thesis. These are first thematically assigned to classical mechanics, quantum mechanics, electrodynamics, and thermodynamics/statistics. They are then sorted alphabetically by the titles of the questions. Questions marked with a star in the title were used and at Friedrich Schiller University Jena accompanying the lectures “Theoretische Mechanik für Lehramt” and “Elektrodynamik für Lehramt”.

Klassische Mechanik

A.4.1 Arbeit und Weg*

Eine Masse wird in einem Gravitationsfeld um folgende Wege verschoben. Auf welchem der Wege (schwarz) muss Arbeit geleistet werden?

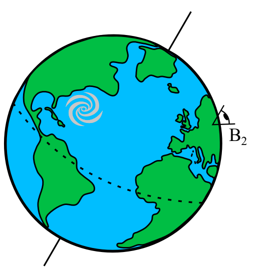


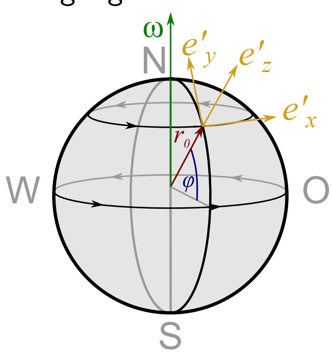
1.
2.
3.
4.
5. Auf mehreren
6. Auf keinem

Antwort: 2.

A.4.2 Beschleunigte Bezugssysteme*

Ein Satellit fliegt von Nord nach Süd um die Erde, welche Bewegung nimmt Beobachter B_2 wahr und welche Scheinkraft erklärt die Bewegung?





$$m\ddot{\boldsymbol{r}}' = \boldsymbol{F} - m\ddot{\boldsymbol{r}}_0 \underbrace{-2m\omega \times \dot{\boldsymbol{r}}'}_{\text{Corioliskraft}} - m\dot{\omega} \times \boldsymbol{r}' \underbrace{-m\omega \times (\omega \times \boldsymbol{r}')}_{\text{Fliehkraft}}$$

1. Die Fliehkraft sagt voraus, dass der Satellit für B_2 nach Osten abdriftet.
2. Die Fliehkraft sagt voraus, dass der Satellit für B_2 nach Westen abdriftet.

3. Die Corioliskraft sagt voraus, dass der Satellit für B_2 nach Osten abdriftet.
4. Die Corioliskraft sagt voraus, dass der Satellit für B_2 nach Westen abdriftet.

Antwort: 4.

A.4.3 Bewegungsgleichung für Zentralkräfte

Leitet man den Bahnvektor einer Masse m unter Einfluss einer Zentralkraft $\boldsymbol{F}(\boldsymbol{r}, \dot{\boldsymbol{r}}, t) = f(\boldsymbol{r}, \dot{\boldsymbol{r}}, t)\frac{\boldsymbol{r}}{r}$ zweimal ab, erhält man folgende Beschleunigung:

$$\begin{aligned}
\boldsymbol{r} &= r \begin{pmatrix} \cos(\Phi) \\ \sin(\Phi) \\ 0 \end{pmatrix} = r(\cos(\Phi)\boldsymbol{e}_x + \sin(\Phi)\boldsymbol{e}_y + 0 \cdot \boldsymbol{e}_z) \\
\ddot{\boldsymbol{r}} &= \left(\ddot{r} - r\dot{\Phi}^2\right) \begin{pmatrix} \cos(\Phi) \\ \sin(\Phi) \\ 0 \end{pmatrix} + \left(2\dot{r}\dot{\Phi} + r\ddot{\Phi}\right) \begin{pmatrix} -\sin(\Phi) \\ \cos(\Phi) \\ 0 \end{pmatrix} \\
&= \left(\ddot{r} - r\dot{\Phi}^2\right) \cdot \boldsymbol{e}_r \qquad\quad + \left(2\dot{r}\dot{\Phi} + r\ddot{\Phi}\right) \cdot \boldsymbol{e}_\Phi .
\end{aligned}$$

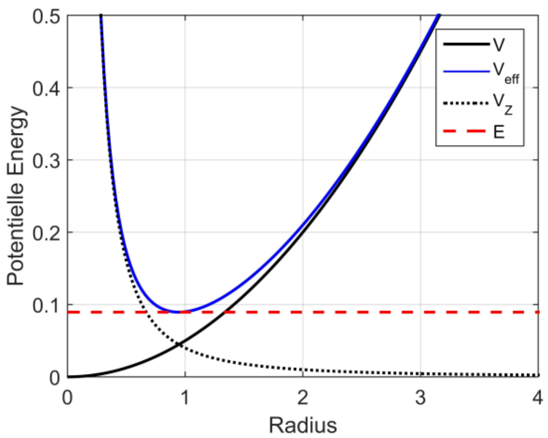
Wie ist der zweite Beitrag in azimutaler (Φ) Richtung in Bezug auf die Newtonsche Bewegungsgleichung für Zentralkräfte zu interpretieren?

1. Aufgrund der Drehimpulserhaltung verschwindet dieser Beitrag.
2. Es handelt sich um eine Scheinkraft, da alle real auftretenden Zentralkräfte in Richtung des Kraftzentrums zeigen müssen.
3. Dieser Beitrag zwingt das Teilchen auf eine Ellipsen- statt Kreisbahn.

Antwort: 1.

A.4.4 Effektives Potential*

Gegeben ist folgendes effektives Potential für ein Teilchen mit fester Energie E. Welche Aussage stimmt?



1. Die kinetische Energie ist Null.

2. Die Bahnkurve des Teilchens ist nicht geschlossen.
3. Die Bahnkurve des Teilchens ist ungebunden.
4. Das Teilchen bewegt sich auf einer Kreisbahn.

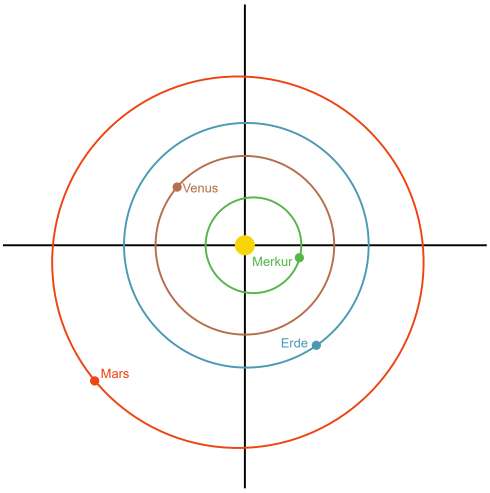
Antwort: 4.

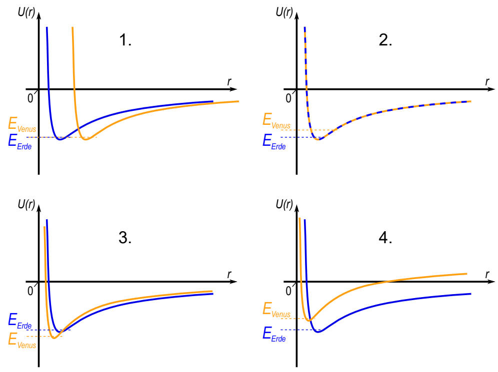
A.4.5 Effektives Potential - Erde und Venus*

Gegeben sei das effektive Potential

$$U_{\text{eff}} = \frac{L^2}{2m}\frac{1}{r^2} - \frac{\gamma M_S m}{r}$$

der Erde und ihre Energie. Welche Energie passt zur leichteren Venus ($m_v < m_E$)?

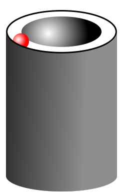




Antwort: 3.

A.4.6 Erhaltungsgrößen 1

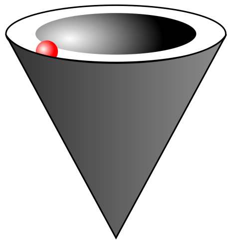
Gegeben ist ein Teilchen auf einer Zylinderoberfläche ohne äußere Kräfte:



Welche Größen sind erhalten?

1. Energie, Drehimpuls- und Impulskomponenten entlang der Zylinderachse
2. Energie, Drehimpuls und Impuls
3. Drehimpuls und Impuls
4. Drehimpuls

Antwort: 1.



A.4.7 Erhaltungsgrößen 2

Gegeben ist ein Teilchen auf einer Kegeloberfläche ohne äußere Kräfte:
Welche Größen sind erhalten?

1. Energie, Drehimpuls- und Impulskomponenten entlang der Kegelachse
2. Energie, Drehimpuls und Impuls
3. Drehimpuls und Impuls
4. Drehimpuls

Antwort: 1.

A.4.8 Erhaltungsgröße 3

Gegeben ist der eindimensionale harmonische Oszillator (Bewegung in x-Richtung). Welche Größen sind erhalten?

1. Energie und $x-$Komponente des Drehimpulses
2. Energie und Drehimpuls
3. Nur die Energie
4. Energie und x-Komponente des Impulses

Antwort: 1.

A.4.9 Erhaltungsgröße 4

In einem Ein-Teilchen-System herrsche Erhaltung der Drehimpulskomponente in z-Richtung sowie der Impulskomponenten in x- und y-Richtung. Welches System passt dazu?

1. Teilchen in der xy-Ebene.
2. Teilchen auf Kugeloberfläche.
3. Teilchen in der xz-Ebene.
4. Teilchen in der yz-Ebene.

Antwort: 1.

A.4.10 Fortbewegung PKW

In welche Richtung übt ein PKW eine Kraft auf die Straße aus, um sich fortzubewegen?

1. In Fahrtrichtung
2. Entgegen der Fahrtrichtung

Antwort: 2.

A.4.11 Generalisierte Koordinate*

Gegeben sei eine generalisierte Koordinate q für ein Teilchen mit Bahnkurve

$$x = r\cos(q)$$
$$y = r\sin(q)$$
$$z = \alpha q.$$

Welcher Bahnkurve entspricht das?

1. Einer Kreisbahn mit Höhe αq
2. Einer Helixbahn.
3. Einer Bahnkurve auf einem Kegel.
4. Einer Bahnkurve auf einem Torus.

Antwort: 2.

A.4.12 Gravitation ohne Drehimpuls*

Gegeben sei ein Teilchen im Gravitationspotential mit verschwindendem Drehimpuls, d.h. $L = 0$. Welche Bahn beschreibt das Teilchen?

1. Es stürzt ins Kraftzentrum.
2. Es stürzt auf einer Spiralbahn ins Kraftzentrum.
3. Es bewegt sich auf einer Kreisbahn.
4. Es fliegt am Gravitationszentrum vorbei.

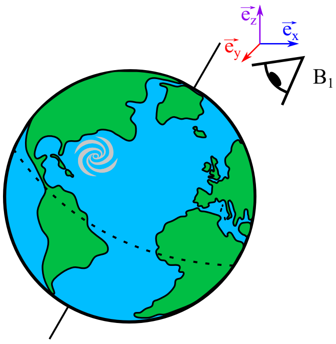
Antwort: 1.

A.4.13 Hurrikan*

Wie herum dreht sich der Hurrikan auf der Nordhalbkugel korrekterweise?

1. Im Uhrzeigersinn.
2. Gegen den Uhrzeigersinn.
3. Abhängig von den Anfangs- und Randbedingungen.

Antwort: 2.



A.4.14 Inertialsysteme*

Bei welchem/welchen Beispiel/en handelt es sich um ein Inertialsystem?

1. Ein Kettenkarussell
2. Ein Labortisch in einem Forschungsinstitut
3. Ein Fallschirmspringer
4. Ein Klassenzimmer
5. Die Erde
6. Die internationale Raumstation ISS
7. Keines der Beispiele ist ein Inertialsystem

Antwort: 7.

A.4.15 Inertialsysteme 2*

Bei welchem Beispiel handelt es sich in erster Näherung am ehesten um ein Inertialsystem?

1. Ein Kettenkarussell
2. Ein Labortisch in einem Forschungsinstitut
3. Ein Fallschirmspringer
4. Ein Klassenzimmer
5. Die Erde
6. Keines der Beispiele ist ein Inertialsystem

Antwort: 3.

A.4.16 Jahreszeiten*

Bewerten Sie folgende Schüleraussage: „Die Jahreszeiten entstehen aufgrund der Ellipsenbahn der Erde. Im Sommer befinden wir uns näher an der Sonne als im Winter.“

1. Die Aussage ist korrekt.
2. Die Aussage ist falsch. Jahreszeiten entstehen aufgrund der gekippten Rotationsachse.
3. Die Aussage ist falsch. Jahreszeiten entstehen durch die Corioliskraft, die im Sommer heiße Luft nach Norden und im Winter nach Süden treibt.

Antwort: 2.

A.4.17 Keplersche Gesetze

Welches Keplersche Gesetz ist mathematisch vollständig exakt?

1. Jeder Planet bewegt sich auf auf einer Ellipsenbahn, in deren Brennpunkt die Sonne steht.
2. In gleichen Zeiten überstreicht die gedachte Verbindungslinie zwischen Sonne und Planeten gleiche Flächen.
3. Die Quadrate der Umlaufzeiten verhalten sich wie die Kuben der großen Halbachsen zweier Planeten.

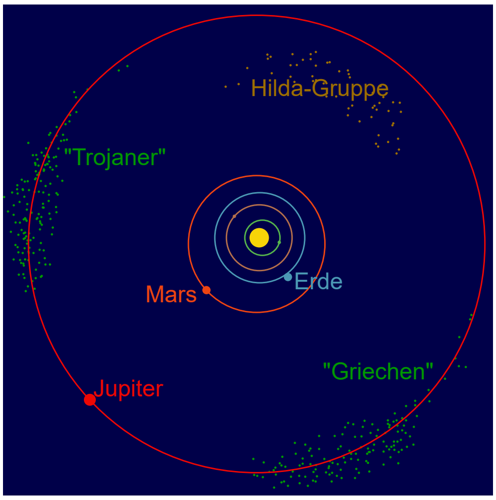
Antwort: 2.

A.4.18 Kepler und die Trojaner*

Welche Bewertung der folgenden Schlussfolgerung zum dritten Keplerschen Gesetz

$$\frac{T^2}{a^3} = \frac{4\pi^2}{M_S\gamma} \overset{\text{genauer}}{=} \frac{4\pi^2}{(M_S + m_P)\gamma}$$

hat das höchste fachliche Niveau?



„In keinem der Keplersche Gesetze kommt die Planetenmasse vor. Die Bahn eines Planeten hängt daher nicht von seiner Masse ab. Dasselbe sollte für alle Bahnen von Objekten unter dem Einfluss der Gravitation gelten.
Ein schönes Beispiel dafür sind die Trojaner. Diese sind eine Gruppe von Asteroiden, die auf der gleichen Bahn um die Sonne laufen wie der Planet Jupiter. Nach dem dritten Keplerschen Gesetz ist ihre Umlaufzeit um die Sonne die gleiche wie die von Jupiter.“

1. Die Schlussfolgerung ist absolut korrekt.

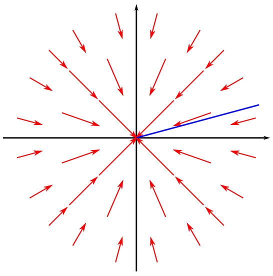
2. Die Schlussfolgerung ist in grober Näherung korrekt, d.h. wenn die Planetenmasse m_P (oder Objektmasse) im Vergleich zur Sonnenmasse sehr klein ist und vernachlässigt werden kann (siehe Formel).

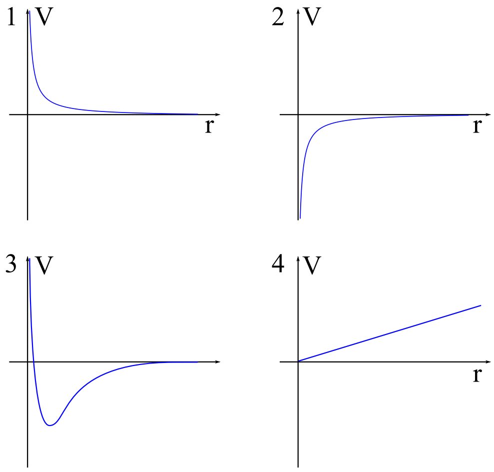
3. Die Schlussfolgerung ist zu kurz gedacht. Wenn die Aussage korrekt wäre, müssten die Trojaner oder Griechen auf der gesamten Bahn zu finden sein.

Antwort: 3.

A.4.19 Kraftfeld und Potential*

Gegeben sei das folgende rote Kraftfeld. Welches Potential würde eine Testladung entlang der blauen Linie spüren?





Antwort: 2.

A.4.20 Kreisbahn

Um den Bahnradius einer Kreisbahn zu bestimmen, muss man das Minimum des effektiven Potentials bestimmen. Im Gravitationspotential erhält man dann durch Ableitung nach r folgende notwendige Bedingung:

$$\frac{L^2}{mr^3} = \frac{\gamma m_1 m_2}{r^2}$$

Interpretieren Sie die beiden Ausdrücke. Welchen Kräften entsprechen diese?

1. Zentrifugalkraft und Gravitationskraft

2. Corioliskraft und Gravitationskraft

3. Federkraft und Gravitationskraft

4. Linearkraft und Zentrifugalkraft

Antwort: 1.

A.4.21 Lagrange-Funktion

Gegeben ist folgende Lagrange-Funktion 2 Art:

$$L(x, \dot{x}, t) = \frac{m}{2}\dot{x}^2 - \frac{k}{2}x^2$$

Welches physikalische System verbirgt sich dahinter?

1. Der eindimensionale harmonische Oszillator.
2. Ein Teilchen im Gravitationspotential.
3. Ein Teilchen auf einer schiefen Ebene mit konstanter Gewichtskraft.
4. Ein freies Teilchen.

Antwort: 1.

A.4.22 Lagrange-Gleichung harmonischer Oszillator

Wie lautet die richtige Lagrange-Gleichung 2. Art für den eindimensionalen harmonischen Oszillator?

1. $m\ddot{x} - kx = 0$
2. $m\ddot{x} + kx = 0$
3. $m\dot{x}^2 + kx^2 = 0$
4. $m\ddot{x} + kx^2 = 0$

Antwort: 2.

A.4.23 Pirouette*

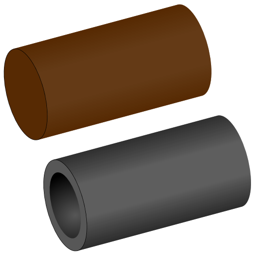
Ein Eiskunstläufer macht eine Pirouette. Währenddessen zieht er seine Arme zum Oberkörper heran und wird dadurch schneller. Warum?

1. Der Drehimpuls wird beim heranziehen der Arme größer.
2. Das Heranziehen der Arme führt zu einem Drehmoment, welches den Eiskunstläufer beschleunigt.
3. Der Drehimpuls und die Drehrichtung ist erhalten, aber der Trägheitstensor wird kleiner. Somit muss die Winkelgeschwindigkeit größer werden.

Antwort: 3.

A.4.24 Rollender Zylinder 1*

Gegeben seien zwei gleich schwere Zylinder Welcher Zylinder hat das größere Trägheitsmoment?

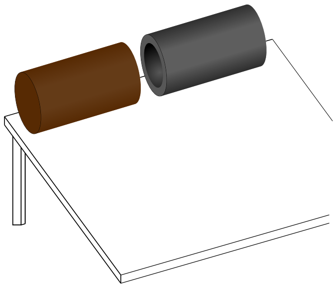


1. Der Vollzylinder.
2. Der Hohlzylinder.

Antwort: 2.

A.4.25 Rollender Zylinder 2*

Ein Voll- und ein Hohlzylinder befinden sich auf einer schiefen Ebene. Sie lassen beide



gleichzeitig losrollen. Welcher Zylinder kommt zuerst unten an?

1. Der Vollzylinder.
2. Der Hohlzylinder.

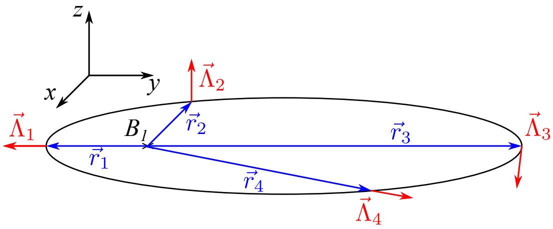
Antwort: 1.

A.4.26 Runge-Lenz-Vektor

Für den Runge-Lenz-Vektor ist definiert als

$$\vec{\Lambda} = \vec{p} \times \vec{L} - mk\vec{e}_r.$$

Welcher Runge-Lenz-Vektor ist folglich richtig eingezeichnet?



1. Λ_1

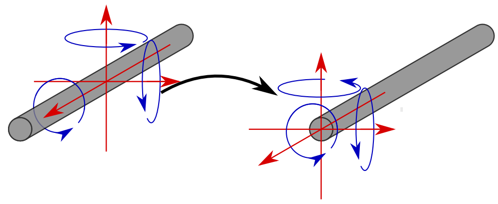
2. Λ_2
3. Λ_3
4. Λ_4

Antwort: 1.

A.4.27 Satz von Steiner*

Die Drehachse eines dünnen Stabs mit dem Trägheitstensor

$$\frac{m}{12}l^2 \begin{pmatrix} 0 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$



wird nun auf ein Ende verschoben. Der Satz von Steiner beschreibt die Änderung des Trägheitsmoments

$$\Theta = \Theta_S + Ml_S^2.$$

Wie sieht der neue Trägheitstensor aus?

1. $\begin{pmatrix} 0 & & \\ & \frac{m}{3}l^2 & \\ & & \frac{m}{3}l^2 \end{pmatrix}$
2. $\begin{pmatrix} ml^2 & & \\ & \frac{m}{12}l^2 & \\ & & \frac{m}{12}l^2 \end{pmatrix}$
3. $\begin{pmatrix} m\left(\frac{l}{2}\right)^2 & & \\ & \frac{m}{12}l^2 & \\ & & \frac{m}{12}l^2 \end{pmatrix}$
4. $\begin{pmatrix} -m\left(\frac{l}{2}\right)^2 & & \\ & \frac{m}{12}l^2 - m\left(\frac{l}{2}\right)^2 & \\ & & \frac{m}{12}l^2 - m\left(\frac{l}{2}\right)^2 \end{pmatrix}$

Antwort: 1.

A.4.28 Trägheitstensor*

Gegeben sei folgender Trägheitstensor

$$\frac{m}{12}l^2 \begin{pmatrix} 0 & & \\ & 1 & \\ & & 1 \end{pmatrix}.$$

Welches Objekt wird hier beschreiben?

1. Ein dünner Kreisring in der xy-Ebene.

2. Ein schlanker Stab auf der x-Achse.

3. Ein massiver Ellipsoid in Zigarrenform auf der x-Achse.

4. Eine dünne rechteckige Platte in der xy-Ebene.

5. Keines der Beispiele.

Antwort: 2.

A.4.29 Unwucht*

Ein Rad habe eine Unwucht. Was bedeutet dies mathematisch?

1. Der Trägheitstensor ist nicht isotrop.

2. Die Diagonalelemente des Trägheitstensors sind Null.

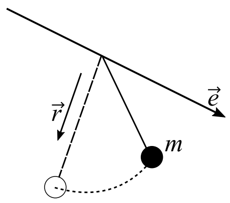
3. Die Rotationsachse zeigt nicht entlang einer Hauptachse (Eigenvektor) des Trägheitstensors.

4. Der Trägheitstensor darf nicht symmetrisch sein.

Antwort: 3.

A.4.30 Zwangsbedingung 1

Die Bewegung eines Massepunkts wird von einer durch eine Achse geführten Pendelstange auf eine Kreisbahn gezwungen. Wie sieht/sehen die zugehörige(n) Zwangsbedingung(en) aus?



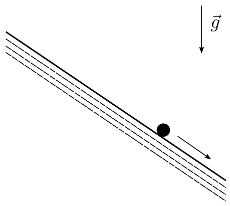
1. $\vec{r} = \begin{pmatrix} 0 \\ \sin\varphi \\ \cos\varphi \end{pmatrix}, \quad \vec{r}\cdot\vec{e} = 0$

2. $\vec{r}\cdot\vec{r} = l^2, \quad \vec{r}\cdot\vec{e} = 0$

3. $\vec{r}\times\vec{e} = 0$

4. $\vec{r} = \begin{pmatrix} 0 \\ \sin\varphi \\ \cos\varphi \end{pmatrix}, \quad |\vec{r}| = l$

Antwort: 2.



A.4.31 Zwangsbedingung 2

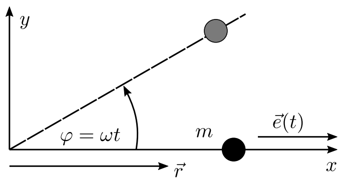
Ein Massepunkt bewegt sich auf der Oberseite einer geneigten ebenen Platte, die unendlich ausgedehnt ist. Wie sieht/sehen die zugehörige(n) Zwangsbedingung(en) aus?

1. $\vec{r} \cdot \vec{n} = 0, \quad y = 0$
2. $z \geq -\cos\alpha \cdot \vec{e}_x$
3. $\vec{r} \cdot \vec{n} \geq 0$
4. Mehr als eine Lösung ist richtig.

Antwort: 3.

A.4.32 Zwangsbedingung 3

Ein Draht mit aufgefädelter Perle rotiert mit konstanter Geschwindigkeit in einer Ebene um ein festes Zentrum. Wie sieht/sehen die zugehörige(n) Zwangsbedingung(en) für die Perle mit $\vec{e}(t) = \cos(\omega t) \cdot \vec{e}_x + \sin(\omega t) \cdot \vec{e}_y$ aus?



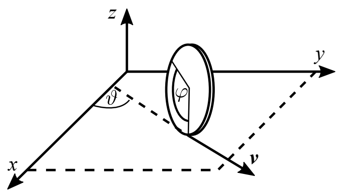
1. $\vec{r} \cdot \vec{e}(t) = 0$
2. $\vec{r} \times \vec{e}(t) = 0$
3. $\vec{r} \cdot \vec{e}(t) = 0, \quad |\vec{r}| = \omega t \cdot m$
4. $\vec{r} \times \vec{e}(t) = 0, \quad |\vec{r}| = \omega t \cdot m$

Antwort: 2.

A.4.33 Zwangsbedingung 4

Ein Rad, das nicht umfallen und nicht rutschen kann, rollt auf einer Ebene. Wie sieht die passende Zwangsbedingung/sehen die passenden Zwangsbedingungen aus?

1. $\omega \cdot t = \varphi$, $z = a$
2. $x - y \cdot \tan^{-1}\theta = 0$, $z = a$



3. $z = a$

4. 1. und 2. sind richtig.

Antwort: 2.

Quantenmechanik

A.4.34 Allgemeiner Hamilton-Operator

Welche der folgenden Aussagen ist für einen beliebigen Quantenzustand ψ immer wahr?
A: Die Anwendung des Hamilton-Operators ist das mathematische Äquivalent einer Messung der Energie des Zustandes.
B: Die Anwendung des Hamilton-Operators auf ψ gibt einem Information darüber, wie sich der Zustand über die Zeit entwickeln wird.

1. Nur A ist immer richtig.

2. Nur B ist immer richtig.

3. A und B sind immer richtig.

4. Keine Aussage ist immer richtig.

Antwort: 2.

A.4.35 Asymptotisches Verhalten des Radialteils

Der Radialteil der Lösung einer radialsymmetrischen Schrödingergleichung verhält sich für kleine Radien wie ar^{l+1}. Ist dies global eine sinnvolle Lösung der Schrödingergleichung?

1. Natürlich. Sonst könnte es ja nicht das Verhalten für kleine Radien korrekt wiedergeben.

2. Nein, nur für bestimmte Drehimpulseigenwerte l divergiert die Lösung nicht für große Radien.

3. Nein, für alle Drehimpulseigenzustände divergiert die Lösung für große Radien.

Antwort: 3.

A.4.36 Delta-Verteilung des Ortes

Gegeben sei eine Ortsverteilung in Form eines Kronecker-Deltas. Wie sieht die Impulsverteilung aus?

1. Maximal lokalisiert um einen Impulswert.
2. Es gibt keinen Zusammenhang zwischen Orts- und Impulsverteilung.
3. Maximal breit.

Antwort: 3.

A.4.37 Detektion am Spalt

Man messe im Doppelspaltexperiment, durch welchen Spalt die Teilchen jeweils fliegen. Was passiert mit dem Interferenzmuster?

1. Statt des Musters des Doppelspalts erhält man dasjenige des Einzelspalts.
2. Das Teilchen kommt gar nicht erst am Schirm an, da es schon vorher detektiert wurde.
3. Man sieht nach wie vor das Muster des Doppelspalts.

Antwort: 1.

A.4.38 Doppelspaltversuche mit Materie

Mit welcher Größe skalieren die Abstände der Intensitätsminima des Doppelspaltversuchs für Elektronen?

1. Mit dem Durchmesser der Elektronen.
2. Mit der inversen Geschwindigkeit.
3. Mit keiner, da es bei Materie kein Interferenzmuster gibt.

Antwort: 2.

A.4.39 Drehimpulskomponente in x-Richtung

Gegeben ist ein Drehimpulseigenzustand $|j, m\rangle$. Welchen Zustand erhält man bei Anwendung der x-Komponente des Drehimpulsoperators?

1. $\hat{J}_x |j, m\rangle = \frac{\lambda_+}{2i} |j, m+1\rangle - \frac{\lambda_-}{2i} |j, m-1\rangle$
2. $\hat{J}_x |j, m\rangle = \frac{\lambda_+}{2i} |j, m+1\rangle + \frac{\lambda_-}{2i} |j, m-1\rangle$
3. $\hat{J}_x |j, m\rangle = m |j, m\rangle$
4. $\hat{J}_x |j, m\rangle = 0$

Antwort: 2.

A.4.40 Eigenzustände gedrehter linearer Polarisatoren

Gegeben ist der um ϕ gedrehte lineare Polarisator mit der Projektionsmatrix

$$\begin{pmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{pmatrix}.$$

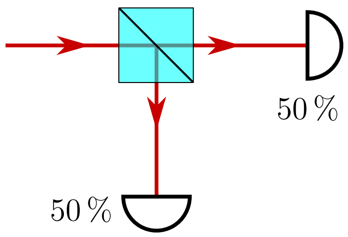
Welcher Polarisationszustand bleibt beim Durchgang durch den Polarisator erhalten?

1. $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
2. $\begin{pmatrix} \sin\phi \\ \cos\phi \end{pmatrix}$
3. $\begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}$
4. Mehrere Lösungen sind möglich.

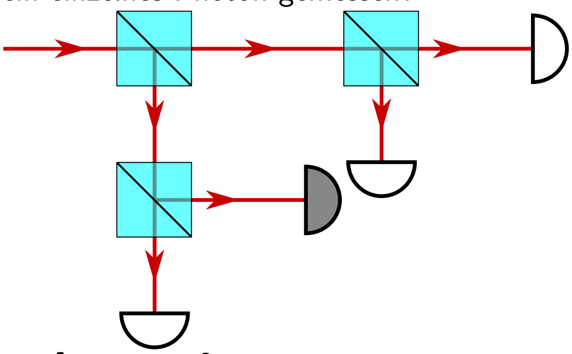
Antwort: 3.

A.4.41 Einführung in die QM mit Einzelphotonen

An einem perfekten Strahlteiler ist die Wahrscheinlichkeit für ein einzelnes Photon transmittiert oder reflektiert zu werden jeweils genau $50\,\%$.



Mit welcher Wahrscheinlichkeit wird beim nächsten Experiment im grauen Detektor ein einzelnes Photon gemessen?



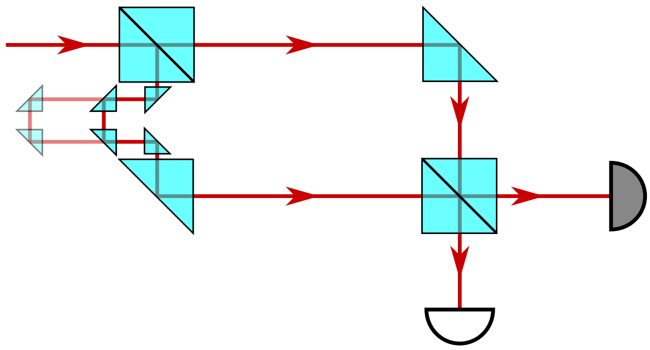
1. $100\,\%$
2. $50\,\%$
3. $25\,\%$
4. $0\,\%$

Antwort: 3.

A.4.42 Einführung in die QM mit Einzelphotonen 2

Mit Hilfe von zwei Strahlteilern wird ein Mach-Zehnder-Interferometer aufgebaut, wobei die optische Weglänge des zweiten Armes variiert werden kann. Wie groß ist die Wahrscheinlichkeit, dass ein einzelnes Photon, nachdem es in dieses System geschickt wird, am grauen Detektor registriert wird?

1. immer $100\,\%$
2. immer $50\,\%$



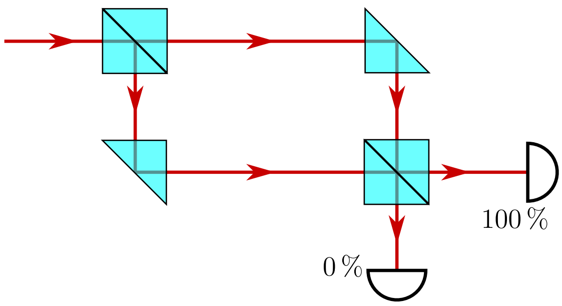
3. immer $0\,\%$

4. Das ist von der Länge des zusätzlichen Weges im zweiten Arm abhängig.

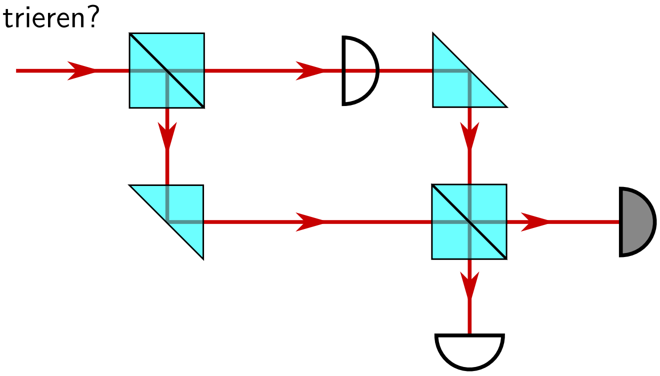
Antwort: 4.

A.4.43 Einführung in die QM mit Einzelphotonen 3

Ein Mach-Zehnder-Interferometer ist so justiert (siehe rechts), dass alle Einzelphotonen am rechten Detektor registriert werden.



Nun wird ein weiterer Detektor in den optischen Aufbau eingebracht. Wie groß ist anschließend die Wahrscheinlichkeit ein einzelnes Photon am grauen Detektor zu registrieren?



1. $100\,\%$
2. $50\,\%$
3. $25\,\%$
4. $0\,\%$

Antwort: 3.

A.4.44 Elektrisches Dipolmoment

Das elektrische Dipolmoment eines Eigenzustands des Wasserstoffatoms ist proportional zu

$$\langle n,l,m|\,\hat{\boldsymbol{x}}\,|n,l,m\rangle = \int d^3r |\psi_{nlm}|^2 \boldsymbol{x}$$

. Welche Aussage stimmt?

1. Das Dipolmoment ist immer Null.

2. Das Dipolmoment ist nur dann Null, wenn $n = 0$.

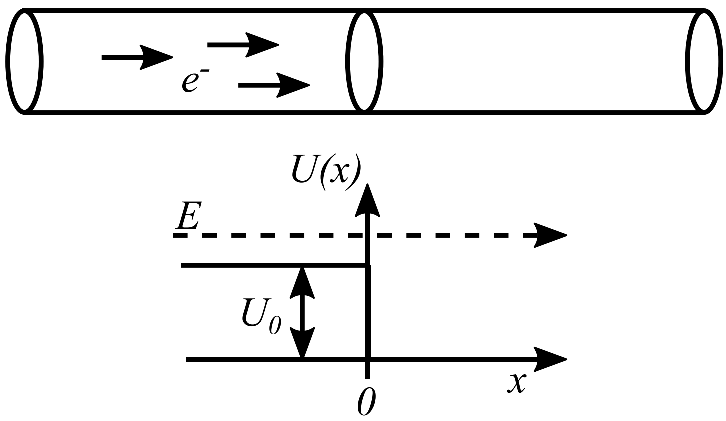
3. Das Dipolmoment ist nur dann Null, wenn $m = 0$.

4. Das Dipolmoment ist nur dann Null, wenn $l = 0$.

Antwort: 1.

A.4.45 Elektronenstrom

Ein Elektronenstrom fließe durch einen leitenden Draht. Bei $x = 0$ ändere sich das Metall so, dass das zugehörige Potential einen Sprung nach unten macht, siehe Abbildung. Welche Aussage stimmt?



1. Alle Elektronen werden reflektiert.
2. Manche Elektronen werden transmittiert, manche reflektiert, da sie tatsächlich einen gewissen Energiebereich abdecken.
3. Manche Elektronen werden transmittiert, manche reflektiert, da sie sich wie Materiewellen verhalten.
4. Alle Elektronen werden transmittiert, da sie sich immer in Bereichen mit tieferem Potential aufhalten.

Antwort: 3.

A.4.46 Erwartungswert Drehimpulskomponente in x-Richtung

Was erhält man als Erwartungswert der x-Komponente des Drehimpulsoperators für einen Drehimpulseigenzustand in z-Richtung?

1. Der Erwartungswert ist Null.
2. Der Erwartungswert hängt vom Eigenzustand ab.
3. Man erhält dasselbe Ergebnis wie für die z-Komponente.

Antwort: 1.

A.4.47 Erwartungswert Würfel

Was ist der Erwartungswert beim Würfeln mit einem handelsüblichen quadratischen Würfel mit den Augenzahlen von 1 bis 6?

1. 3
2. 6
3. 3,5
4. 4,5

Antwort: 3.

A.4.48 Gedrehter linearer Polarisator

Gegeben sei ein um $45°$ gedrehter linearer Polarisator. Durch welchen Operator wird er beschrieben?

1. $\frac{1}{2}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$
2. $\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$
3. $\frac{1}{2}\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$
4. $\frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Antwort: 2.

A.4.49 Halbzahliger Drehimpuls

Gegeben sei ein Drehimpulszustand mit $j = 3/2$. Was sind die möglichen Einstellmöglichkeiten der z-Komponente?

1. 3/2, -3/2
2. 3/2, 1, 1/2, 0, -1/2, 1 -3/2
3. 3/2, 1/2, -1/2, -3/2
4. Es gibt keine halbzahligen Drehimpulse.

Antwort: 3.

A.4.50 Hintereinanderschaltung optischer Elemente

Gegeben sei ein linearer Polarisator

$$\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

und ein $\lambda/2$-Plättchen

$$\frac{1}{2}\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}.$$

Welche Aussage stimmt?

1. Linear polarisiertes Licht kann durch beide Elemente unverändert passieren.
2. Die Reihenfolge der optischen Elemente ist relevant.
3. Beide besitzen exakt einen gemeinsamen Eigenzustand.
4. Die Reihenfolge der optischen Elemente spielt keine Rolle.

Antwort: 2.

A.4.51 Horizontaler Polarisator

Polarisationszustände von Licht lassen sich durch zwei-dimensionale Jones-Vektoren darstellen:
horizontale Polarisation $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
vertikale Polarisation $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Welcher Operator beschreibt nun einen linearen Polarisator in horizontaler Richtung?

1. $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
2. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
3. $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
4. $\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

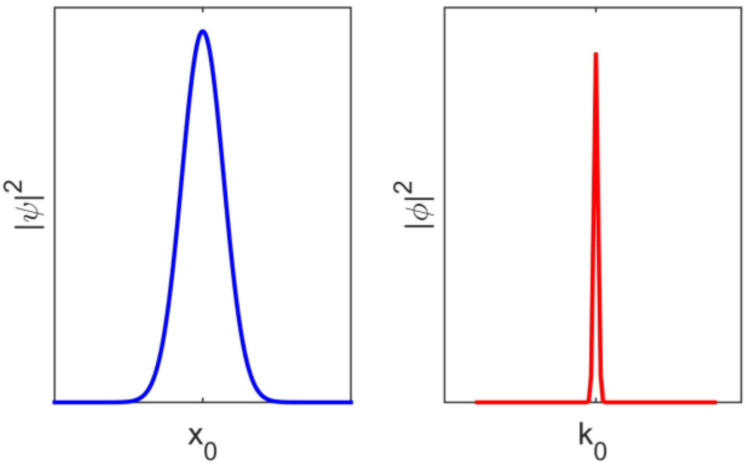
Antwort: 3.

A.4.52 Impulserwartungswert

Gegeben ist folgendes symmetrisches Wellenpaket: Was erwarten Sie für kleine Zeiten als Impulserwartungswert?

1. Kann man nicht genau sagen.
2. Masse mal Geschwindigkeit.
3. Masse mal Phasengeschwindigkeit.

Antwort: 2.



A.4.53 Lichtquelle im Doppelspalt

Was passiert, wenn man statt eines Lasers eine Glühbirne als Lichtquelle im Doppelspaltversuch nimmt?

1. Das Interferenzmuster verschwimmt oder ist gar nicht mehr sichtbar.
2. Die Abstände der Intensitätsminima werden kleiner, da die Wellenlänge des Lichts aus der Glühbirne kleiner ist.
3. Man sieht nach wie vor dasselbe Muster.

Antwort: 1.

A.4.54 Magnetisches Dipolmoment

Das magnetische Dipolmoment eines Eigenzustandes ist proportional zu

$$\langle n, l, m | \hat{\boldsymbol{L}} | n, l, m \rangle = \int d^3 r \psi^*_{nlm} \hat{\boldsymbol{L}} \psi_{nlm}.$$

Welche Aussage stimmt?

1. Das Dipolmoment ist immer Null.
2. Das Dipolmoment ist Null, wenn $n = 0$.
3. Das Dipolmoment ist Null, wenn $m = 0$.
4. Das Dipolmoment ist Null, wenn $l = 0$.

Antwort: 3.

A.4.55 Makroskopische Objekte im Doppelspaltversuch

Was ist zu beachten, damit man bei größeren Teilchen ein Interferenzmuster im Doppelspaltversuch erhält?

1. Die Teilchen müssen alle dieselbe Geschwindigkeit haben.

2. Es müssen immer mehrere Teilchen zwischen Quelle und Detektor sein, damit Interferenz auftreten kann.

3. Die Teilchen müssen kleiner als die Wellenlänge des sichtbaren Lichts sein.

Antwort: 1.

A.4.56 Matrixkommutator

Gegeben seien zwei 3×3-Matrizen A und B. Welche Aussage stimmt?

1. Der Kommutator von zwei Matrizen ist im Allgemeinen ungleich Null.

2. Der Kommutator von zwei Matrizen ist immer gleich Null.

3. Für Matrizen kann man keinen Kommutator berechnen.

Antwort: 1.

A.4.57 Matrixoperator

Eine 3×3-Matrix bildet einen dreidimensionalen Vektor auf einen anderen dreidimensionalen Vektor ab. Welche Aussage stimmt?

1. Matrizen sind keine Operatoren.

2. Matrizen sind lineare Operatoren.

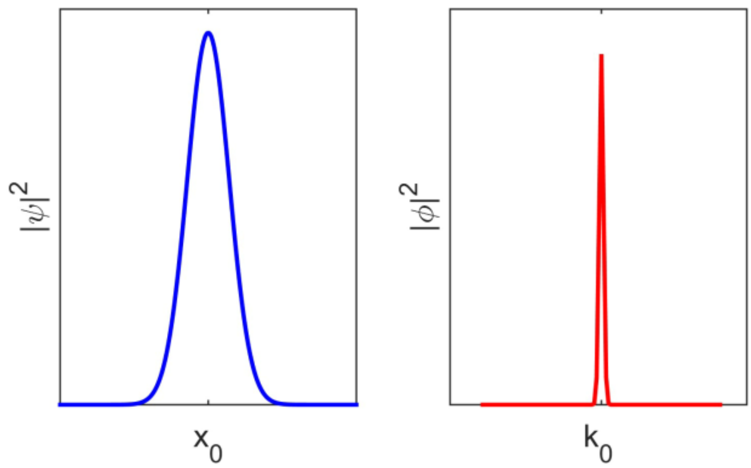
3. Matrizen sind hermitsche Operatoren.

4. Matrizen sind unitäre Operatoren.

Antwort: 2.

A.4.58 Ortserwartungswert

Gegeben sei folgendes symmetrisches Wellenpaket:



Was erwarten Sie für kleine Zeiten als Ortserwartungswert?

1. Kann man nicht genau sagen.
2. Verschiebung des Maximums mit der Gruppengeschwindigkeit.
3. Verschiebung des Maximums mit der Phasengeschwindigkeit.

Antwort: 2.

A.4.59 Ortsmessung im harmonischen Oszillator

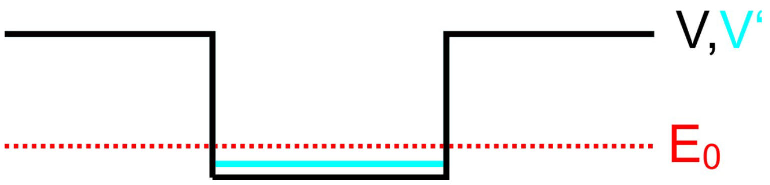
Gegeben sei ein Teilchen im Grundzustand des harmonischen Oszillators. Es wird eine Ortsmessung durchgeführt und das Teilchen am Ort $x < 0$ gefunden. Wie verhält sich das Teilchen nach der Messung?

1. Es befindet sich nach wie vor im Grundzustand.
2. Das Maximum der Aufenthaltswahrscheinlichkeit oszilliert zwischen positiven und negativen Ortskoordinaten hin und her.
3. Das Teilchen befindet sich in einem höheren Zustand, da bei der Messung dem System Energie zugeführt wurde.
4. Das Maximum der Aufenthaltswahrscheinlichkeit verharrt am Ort der Messung.

Antwort: 2.

A.4.60 Potential mit Störung 1

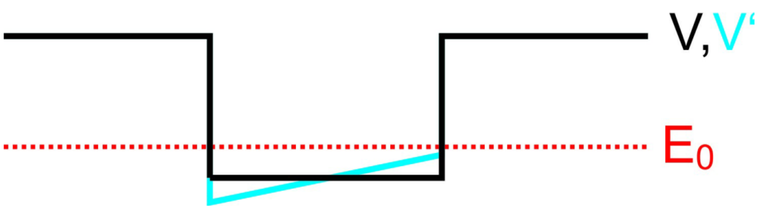
Gegeben sei ein Potential V mit der Grundzustandsenergie E_0. Dieses Potential wird gestört und ein neues Potential V' gebildet:



Was passiert mit dem Grundzustand?

1. Der Grundzustand wird energetisch erhöht.
2. Die neue Grundzustandswellenfunktion hat kein Maximum mehr im Zentrum.
3. Die Grundzustandsenergie bleibt näherungsweise gleich.
4. Die Grundzustandswellenfunktion ist nicht länger symmetrisch.

Antwort: 1.



A.4.61 Potential mit Störung 2

Gegeben sei ein Potential V mit der Grundzustandsenergie E_0. Dieses Potential wird gestört und ein neues Potential V' gebildet:

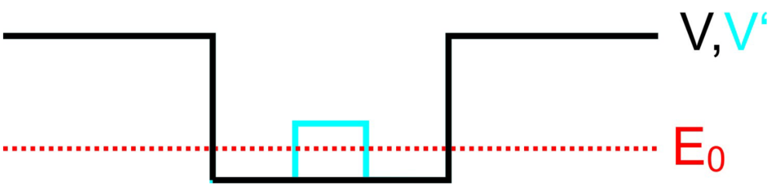
Was passiert mit dem Grundzustand?

1. Der Grundzustand wird energetisch erhöht.
2. Die neue Grundzustandswellenfunktion hat kein Maximum mehr im Zentrum.
3. Die Grundzustandsenergie bleibt näherungsweise gleich.
4. Die Grundzustandswellenfunktion ist nicht länger symmetrisch.

Antwort: 4.

A.4.62 Potential mit Störung 3

Gegeben sei ein Potential V mit der Grundzustandsenergie E_0. Dieses Potential wird gestört und ein neues Potential V' gebildet:



Was passiert mit dem Grundzustand?

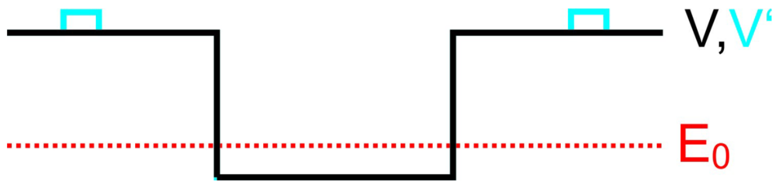
1. Der Grundzustand wird energetisch erhöht.
2. Die neue Grundzustandswellenfunktion hat kein Maximum mehr im Zentrum.
3. Die Grundzustandsenergie bleibt näherungsweise gleich.
4. Die Grundzustandswellenfunktion ist nicht länger symmetrisch.

Antwort: 2.

A.4.63 Potential mit Störung 4

Gegeben sei ein Potential V mit der Grundzustandsenergie E_0. Dieses Potential wird gestört und ein neues Potential V' gebildet: Was passiert mit dem Grundzustand?

1. Der Grundzustand wird energetisch erhöht.
2. Die neue Grundzustandswellenfunktion hat kein Maximum mehr im Zentrum.



3. Die Grundzustandsenergie bleibt näherungsweise gleich.

4. Die Grundzustandswellenfunktion ist nicht länger symmetrisch.

Antwort: 3.

A.4.64 Präparation Doppelspaltversuch

Was müssen Sie bei der Präparation des Doppelspaltversuchs beachten?

1. Je kleiner die Ortsunschärfe an der Quelle, desto besser sieht man das Interferenzmuster.

2. Die Impulsunschärfe an der Quelle muss mindestens so klein sein, dass die Breite des Wellenpakets am Doppelspalt größer als der Spaltabstand ist.

3. Die Impulsunschärfe an der Quelle muss mindestens so klein sein, dass die Breite des Wellenpaktes am Doppelspalt größer als die Spaltbreite ist.

Antwort: 2.

A.4.65 Produktansatz Radialanteil

Für große Radien muss sich die Wellenfunktion verhalten wie e^{-kr}. Der Ansatz $g(r)\mathrm{e}^{-kr}$ erscheint sinnvoll, um das Gesamtverhalten der Wellenfunktion für normierbare Eigenzustände wiederzugeben. Sie erhalten als Lösung einer Aufgabe

$$g(r) = \sum_{n=0}^{\infty} \frac{(k'r)^n}{n!}.$$

Wann ist dies physikalisch sinnvoll, wenn $k, k' \in \mathbb{R}$?

1. Für beliebige k'.

2. Für $k = k'$.

3. Für $k > k'$.

4. Für $k < k'$.

Antwort: 3.

A.4.66 Radialimpuls

Warum ist es wichtig, dass der Operator des Radialimpulses reelle Erwartungswerte liefert?

1. Weil die Eigenzustände sonst im Allgemeinen keine reellen Energieeigenwerte besitzen dürfen.
2. Weil die Wellenfunktion am Ursprung aus Symmetriegründen immer verschwinden muss.
3. Weil die Wellenfunktion am Ursprung sonst divergieren würde.

Antwort: 1.

A.4.67 Radialanteil der Wellenfunktion

Welche Aussage stimmt nicht für den Radialanteil der Wellenfunktion?

1. Der Radialanteil der Wellenfunktion verschwindet im Ursprung.
2. Der Radialanteil der Wellenfunktion darf am Ursprung nicht divergieren.
3. Der Radialanteil der Wellenfunktion geht für große Radien gegen Null.

Antwort: 1.

A.4.68 Rationale Zahlen

Betrachten Sie für rationale Zahlen die Folge

$$f_N(x) = \sum_{n=0}^{N} \frac{x^n}{n!}.$$

Welche Aussage können Sie daraus ableiten?
Hinweis: Überlegen Sie, gegen welche Funktion obige Folge konvergiert und setzen Sie ganze Zahlen ein.

1. Rationale Zahlen bilden keinen Vektorraum über dem Körper der ganzen Zahlen.
2. Der Vektorraum der rationalen Zahlen ist nicht vollständig.
3. Der Vektorraum der rationalen Zahlen ist vollständig.

Antwort: 2.

A.4.69 Rydberg-Atom

Ein Rydberg-Atom besitzt ein hoch angeregtes Elektron $(n \gg 1)$, dessen Aufenthaltswahrscheinlichkeit auf etwa einer Ellipse um den Kern beschränkt ist. Was beschreibt den Zustand des Elektrons richtig?

1. Das Elektron bewegt sich dann wie im Bohrschen Atommodell postuliert auf einer Ellipsenbahn um den Kern.
2. Messungen einer Drehimpulskomponente liefern scharfe Werte.
3. Messung der Impulses liefern scharfe Werte.

Antwort: 2.

A.4.70 Schrödingergleichung

Gegeben ist die folgende Schrödingergleichung:

$$\frac{\hat{\boldsymbol{p}}^2}{2m}\psi + \frac{\kappa}{2}\boldsymbol{x}^2 = i\hbar\partial_t\psi$$

Welche Aussage stimmt nicht?

1. Die Schrödingergleichung ist zeitabhängig.

2. Die Schrödingergleichung ist zeitunabhängig.

3. Die Schrödingergleichung beschreibt ein Teilchen in einem Potential.

Antwort: 2.

A.4.71 Standardabweichung Drehimpulskomponente in x-Richtung

Was erhält man als Standardabweichung der x-Komponente des Drehimpulsoperators für einen Drehimpulseigenzustand?

1. Die Standardabweichung ist immer Null, da der Drehimpuls eine Erhaltungsgröße ist.

2. Die Standardabweichung ist im Allgemeinen ungleich Null, da der Drehimpuls auch Einstellmöglichkeiten senkrecht zur z-Achse einnehmen kann.

3. Die Standardabweichung ist nie gleich Null, da die Wellenfunktion über den ganzen Raum verteilt Werte ungleich Null annehmen kann.

Antwort: 2.

A.4.72 Tunneleffekt

Bei welcher Anwendung spielt der Tunneleffekt keine Rolle?

1. Laserschutzbrille

2. Mikroskopie

3. Kernfusion

4. Röntgenkristallstrukturanalyse

Antwort: 4.

A.4.73 Vergleich Bohrsches Atommodell

Im Bohrschen Atommodel ist ein grundlegendes Problem, dass bewegte Ladung eigentlich Energie abstrahlen müsste. Deshalb dürfte ein Elektron nicht auf einer bestimmten Bahn verweilen. Wie wird dieses Problem quantenmechanisch behandelt?

1. Das Problem wird im Rahmen der nichtrelativistischen Quantenmechanik nicht gelöst.

2. Das Elektron vollführt in der Quantenmechanik lediglich eine radiale Bewegung durch, weshalb es nicht abstrahlt.

3. Das Elektron befindet sich in einer stationären Lösung, weswegen es nicht abstrahlt.

Antwort: 3.

A.4.74 Wellenfunktion im harmonischen Oszillator

Welche Aussage stimmt für die Wellenfunktion eines Eigenzustandes im harmonischen Oszillator?

1. Die Wellenfunktion beschreibt, wie das Teilchen im harmonischen Oszillator hin und her oszilliert.

2. Eigenzustände beschreiben stationäre Zustände im harmonischen Oszillator.

3. In klassisch verbotenen Bereichen hat die Wellenfunktion keine physikalische Relevanz.

Antwort: 2.

A.4.75 Zeitunabhängige Schrödingergleichung

Gegeben ist folgende zeitunabhängige Schrödingergleichung:

$$\frac{\hat{\boldsymbol{p}}^2}{2m}\psi + \frac{\kappa}{2}\boldsymbol{x}^2\psi = E\psi.$$

Eine allgemeine Lösung können Sie aus der Superposition von Eigenzuständen der zeitunabhängigen Schrödingergleichung bestimmen. Was müssen Sie dafür kennen?

1. Nur die Wellenfunktion zu einem bestimmten Zeitpunkt.

2. Die Wellenfunktion und ihre Ableitung zu einem bestimmten Zeitpunkt.

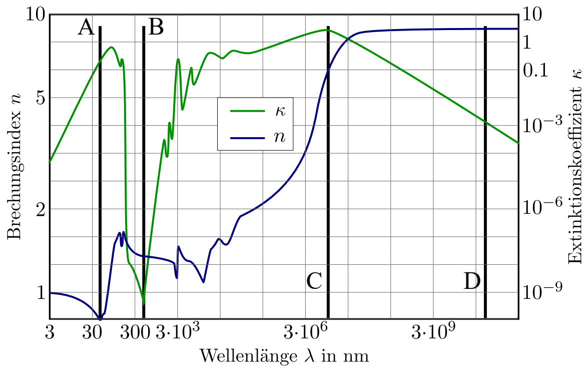
3. Die Wellenfunktion zu beliebigen Zeiten.

Antwort: 1.

Elektrodynamik

A.4.76 Absorption/Transmission*

Wenn man die Dispersionsrelation und den komplexen Brechungsindex



$$k = \frac{\omega \cdot \hat{n}}{c} = \frac{2\pi}{\lambda} \cdot \hat{n}\,, \quad \hat{n} = n + i\kappa,$$

in die Formel für eine ebene Welle

$$\hat{\vec{E}} = \hat{\vec{E}}_0 \cdot \mathrm{e}^{i(\vec{k}\cdot\vec{x} - \omega\cdot t)}$$

einsetzt, ergibt sich der folgende Zusammenhang für die E-Feldkomponente:

$$\hat{\vec{E}} = \hat{\vec{E}}_0 \cdot \mathrm{e}^{-(\kappa \frac{2\pi}{\lambda} x)} \cdot \mathrm{e}^{i(n \frac{2\pi}{\lambda} x - \omega t)}.$$

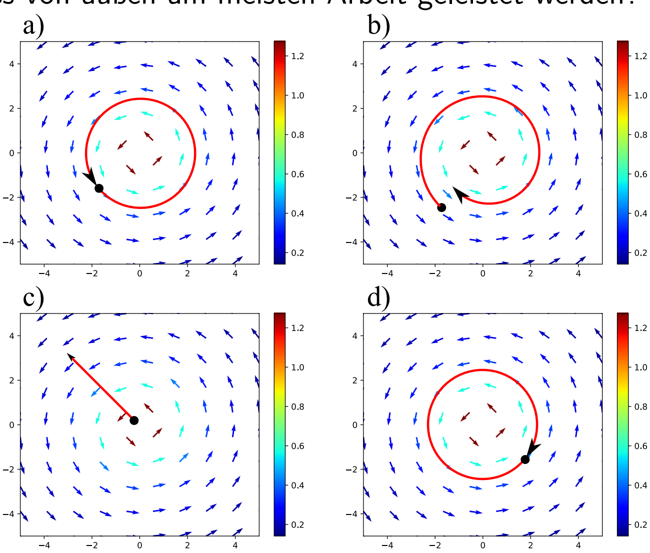
Im Diagramm sind der Real- (n) und der Imaginärteil (κ) des Brechungsindex gegeben. Für welche der markierten Wellenlängen ist Wasser am transparentesten?

1. A: $40\,\mathrm{nm}$
2. B: $400\,\mathrm{nm}$
3. C: $7\,\mathrm{mm}$
4. D: $40\,\mathrm{m}$

Antwort: 4.

A.4.77 Arbeit in Wirbelfeldern*

Eine Ladung im dargestellten Kraftfeld $\vec{B} = \frac{B_0}{r} \cdot \vec{e}_\varphi$ wird auf verschiedenen Wegen verschoben. Auf welchem Weg muss von außen am meisten Arbeit geleistet werden?



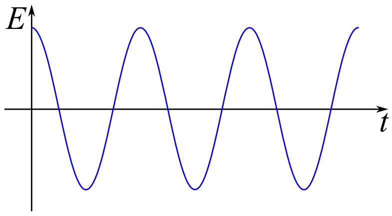
1. a
2. b
3. c
4. d
5. b & d

Antwort: 5.

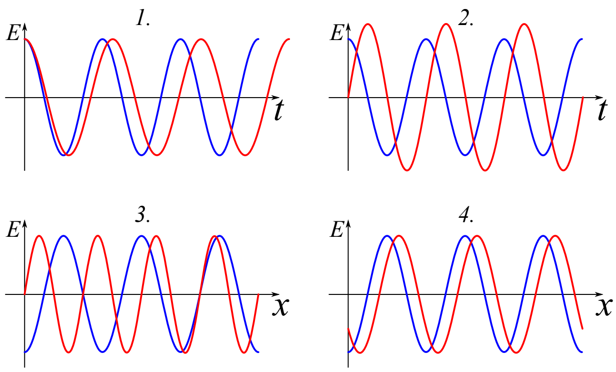
A.4.78 Darstellung von elektromagnetischen Wellen*

Eine elektromagnetische, ebene Welle (blau)

$$\vec{E} = Re\left[\hat{\vec{E}}_0 \cdot e^{i(\vec{k}\cdot\vec{r}-\omega\cdot t)}\right]$$



kann wie folgt dargestellt werden.
Welcher Graph zeigt eine mögliche Darstellung einer zweiten Welle (rot) mit $\omega_1 > \omega$?
Antwort: 3.

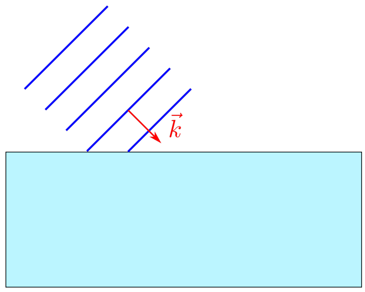


A.4.79 Dispersionsrelation*

Eine elektromagnetische ebene Welle mit Frequenz ω_0 und Wellenlänge λ_0 trifft aus der Luft kommend auf ein Medium mit dem Brechungsindex $n = 1,4$. Auf welche Größe in der Dispersionsrelation

$$|\vec{k}|^2 = \frac{\omega^2}{c^2} \cdot n^2$$

hat der Wechsel keinen Einfluss?



1. Die Richtung von $\vec{k}$.
2. Den Betrag von $\vec{k}$.
3. Die Frequenz ω.
4. Die Ausbreitungsgeschwindigkeit c.
5. Alle Größen werden sich anpassen.

Antwort: 3.

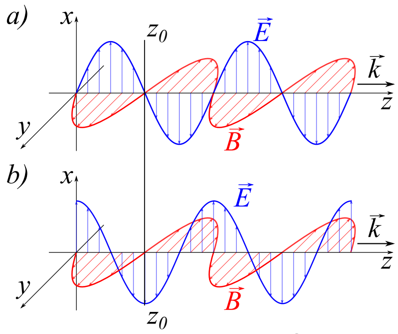
A.4.80 Elektromagnetische Felder*

Wie viele Komponenten (Speicherwerte an jedem Ort, z.B. E_x, E_y, E_z) benötigt man mindestens, um elektromagnetische Felder im Vakuum vollständig und exakt zu beschreiben?

1. 2
2. 8
3. 6
4. 4

Antwort: 4.

A.4.81 Elektromagnetische Wellen - Energieerhaltung*



Die Ausbreitung von EM-Wellen wird häufig wie in Schaubild a) dargestellt, außerdem ist die Energiedichte einer elektromagnetischen Welle mit

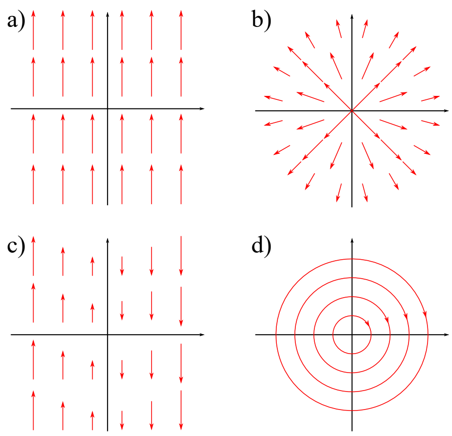
$$\overline{\omega}_{em} = \frac{1}{2}\epsilon_r\epsilon_0|\vec{E}|^2 = \frac{1}{2}\frac{1}{\mu_r\mu_0}|\vec{B}|^2$$

definiert.
Führt dies im Punkt z_0 zu einem Widerspruch mit der Energieerhaltung?

1. Ja und die Energie muss erhalten bleiben, deshalb ist Schaubild b) die physikalisch richtige Darstellung.
2. Nein, die Energieerhaltung gilt nicht in einzelnen Punkten, sondern nur in Volumina.
3. Nein, denn es steckt auch Energie in der Änderungsrate des E- und B-Feldes $(\frac{\partial\vec{E}}{\partial t}, \frac{\partial\vec{B}}{\partial t})$
4. Nein, für das E-Feld gilt $\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{x}-\omega\cdot t)}$. In z_0 ist das Feld imaginär und somit auch die Energie.

Antwort: 2.

A.4.82 Felder*



Welches der Beispiele ist ein Wirbelfeld?

1. a
2. b
3. c
4. d
5. Es wird mehr als ein Wirbelfeld gezeigt.

Antwort: 5. Die Rotation verschwindet nicht bei Feld c) und d).

A.4.83 Felder und Dichte

Im Poynting-Theorem haben alle Terme die Einheit einer Leistungsdichte.

$$\int_V \vec{j} \cdot \vec{E} \, \mathrm{d}V = -\int_V \left(\vec{\nabla} \cdot \vec{S} + \frac{\partial u}{\partial t} \right) \mathrm{d}V$$

- Die Energiedichte des EM-Feldes $u = \frac{1}{2}(\vec{E} \cdot \vec{D} + \vec{B} \cdot \vec{H}) = \frac{1}{2}(\varepsilon_0 \vec{E}^2 + \mu_0 \vec{H}^2)$.
- Der Poynting-Vektor als Energiestromdichte.
- Die Joulsche Wärme als elektrische Leistung pro Volumen.

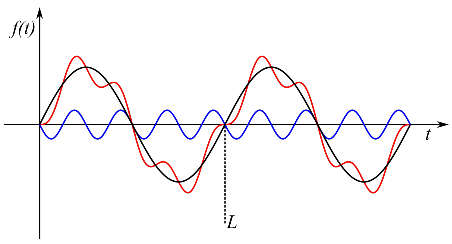
Welche Aussage kann darin über Felder und Energie treffen?

1. Die Energieerhaltung gilt nur in Volumen und nicht an einzelnen Punkten.
2. Die Energiedichte in einer EM-Welle ist immer homogen.
3. Wenn ein elektrisches Feld mit Materie wechselwirkt, geht Energie immer in Form von elektrischem Strom verloren.
4. Die absolute Energie in einem Feld kann nur auf ein Volumen bezogen werden.

Antwort: 3.

A.4.84 Fourierreihe

Wie lautet die passende Fourierreihe der roten Funktion im Schaubild?



1. $f(t) = \sin(t) - \frac{1}{4}\sin(3t)$
2. $f(t) = \sin(t) + \frac{1}{2}\sin(4t)$
3. $f(t) = \sin(t) + \frac{1}{4}\sin(2t)$
4. $f(t) = \sin(t) - \frac{1}{5}\sin(4t)$

Antwort: 4.

A.4.85 Impuls des EM-Feldes

Wenn es einen Impuls des EM-Feldes in Medien gibt

$$\vec{P}_v^{\text{Feld}} = \int_V (\vec{D} \times \vec{B}) d^3 r$$

und elektromagnetische Wellen in Medien eine langsamere Ausbreitungsgeschwindigkeit haben als im Vakuum

$$c_{\text{Medium}} = \frac{c}{n} = \frac{c}{\sqrt{\epsilon_r \mu_r}} = \frac{1}{\sqrt{\epsilon_r \epsilon_0 \mu_r \mu_0}},$$

wie breiten sich dann solche Wellen in Medien aus?

1. Durch die Anwesenheit von Ladungen werden die Änderungsraten $\frac{\partial \vec{E}}{\partial t}, \frac{\partial \vec{B}}{\partial t}$ beeinflusst.

2. Die Lorentzkraft der Ladungen auf die Felder erhöht die optische Weglänge im Medium.

3. EM-Wellen bringen Ladungen zum Schwingen, schwingende Ladungen strahlen aber EM-Wellen ab mit einem zeitlichen Versatz.

4. Dieser Umstand lässt sich erst mit der Relativitätstheorie erklären.

Antwort: 3.

A.4.86 Kugelwelle

Eine ebene Welle kann mathematisch wie folgt ausgedrückt werden.

$$\vec{E} = Re\left[\hat{\vec{E}}_0 \cdot e^{i(\vec{k}\cdot\vec{r}-\omega\cdot t)}\right]$$

Davon ausgehend, welcher Ausdruck beschreibt mathematisch eine Kugelwelle?

1. $E(r,z) = E_0\, \frac{w_0}{w(z)} \cdot \mathrm{e}^{-\left(\frac{r}{w(z)}\right)^2} \cdot \mathrm{e}^{-ik\frac{r^2}{2R(z)}} \cdot \mathrm{e}^{-i(kz-\zeta(z))}$

2. $\vec{E} = Re\left[\frac{1}{r^2\cdot\cos\vartheta}\hat{\vec{E}}_0 \cdot e^{i(\vec{k}\cdot\vec{r}-\omega\cdot t)}\right]$

3. $\vec{E} = Re\left[\frac{E_0}{r} \cdot e^{i(k\cdot r-\omega\cdot t)}\right]$

   Antwort: 3.

A.4.87 Lorenz-Eichung

Setzt man die Lorenz-Eichung

$$\vec{\nabla}\cdot\vec{A} + \mu_0\epsilon_0\dot{\Phi} = 0$$

in die Ampere-Maxwell Gleichung

$$\left(\nabla^2 - \mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\right)\vec{A} - \vec{\nabla}\left(\vec{\nabla}\cdot\vec{A} + \mu_0\epsilon_0\frac{\partial\Phi}{\partial t}\right) = -\mu_0\vec{j}$$

ein, so erhält man ...

1. ... die Poisson-Gleichung für das elektrische Feld.

2. ... die Poisson-Gleichung für Magnetfelder.

3. ... die Wellengleichung für elektromagnetische Wellen.

4. ... nichts, nur die Coulomb Eichung $(\nabla\cdot\vec{A} = 0)$ liefert ein physikalisch sinnvolles Ergebnis.

Antwort: 3.

A.4.88 Messgrößen*

Welche physikalische(n) Größe(n) aus der theoretischen Elektrodynamik kann(können) ohne den Umweg über eine der anderen elektrodynamischen Größen gemessen werden?

1. Die Lorentzkraft $\vec{F}$
2. Das Elektrische Feld $\vec{E}$
3. Das Vektorpotenial $\vec{A}$
4. Antwort 1. & 2. sind richtig.

Antwort: 1.

A.4.89 Polarisation

Was passiert, wenn sich eine rechtszirkular polarisierte Welle mit

$$\vec{E} = E_x(\cos(kz - \omega t + \varphi)) \cdot \hat{\vec{e}}_x + E_y(\sin(kz - \omega t + \varphi)) \cdot \hat{\vec{e}}_y$$

und eine linkszirkular polarisierte Welle mit

$$\vec{E} = E_x(\cos(kz - \omega t + \varphi)) \cdot \hat{\vec{e}}_x - E_y(\sin(kz - \omega t + \varphi)) \cdot \hat{\vec{e}}_y$$

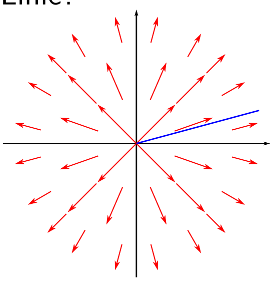
überlagern?

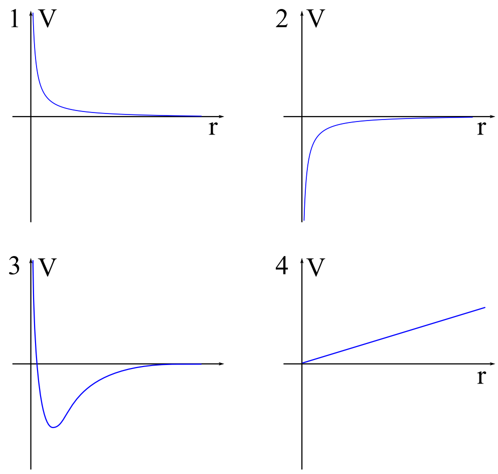
1. Es entsteht eine links-zirkular polarisierte Welle.
2. Es entsteht eine links-elliptisch polarisierte Welle mit verminderter Amplitude.
3. Gar nichts, die Wellen überlagern sich nur, man kann sie aber noch unabhängig voneinander messen.
4. Es entsteht eine linear polarisierte Welle.

Antwort: 4.

A.4.90 Potential und Feld*

Das rote Kraftfeld sei ein elektrisches Feld und die Testladung elektrisch positiv. Welches Schaubild beschreibt das Potential der Testladung entlang der blauen Linie?





Antwort: 1.

A.4.91 Poynting-Vektor

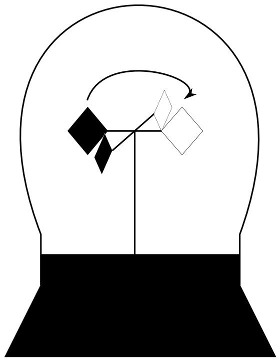
Welche Information steckt nicht im Poynting-Vektor?

$$\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right)$$

1. Die Ausbreitungsrichtung von elektromagnetischen Wellen.
2. E- und B-Felder müssen immer senkrecht aufeinander stehen.
3. Die Dichte des Energietransportes eines elektromagnetischen Felds.
4. Die Leistungsdichte eines elektromagnetischen Feldes.

Antwort: 2.

A.4.92 Satz von Poynting



Welchem Term im Poynting-Theorem

$$\frac{\partial W}{\partial t} + \vec{\nabla} \cdot \vec{S} = -\vec{j} \cdot \vec{E}$$

kann man das Aufheizen der schwarzen Flächen der Lichtmühle zuordnen?

1. $\frac{\partial W}{\partial t}$
2. $\vec{\nabla} \cdot \vec{S}$
3. $-\vec{j} \cdot \vec{E}$
4. Keiner der Terme, da die Absorption von elektromagnetischer Strahlung nicht im Poynting-Theorem behandelt wird.

Antwort: 3.

A.4.93 Telegraphengleichung

Der komplexe Wellenvektor $\vec{k}$ macht Teile der Lösung der Telegraphengleichung

$$\vec{E}(\vec{r}, t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

reell, was als Dämpfung der ebenen Welle interpretiert werden kann. Welche realen physikalischen Phänomene beschreibt eine solche Dämpfung?

1. Die Absorption von elektromagnetischer Energie.
2. Die Reflexion von EM-Wellen.
3. Die verminderte Ausbreitungsgeschwindigkeit von EM-Wellen in Medien.
4. Eine Streckung der Wellenlänge λ.

Antwort: 1.

A.4.94 Wellengleichung

Was muss an den Maxwellgleichungen gleich Null gesetzt werden, damit die Ausbreitung von Feldern im Vakuum beschrieben werden kann?

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho(\vec{r})}{\epsilon_0}$$
$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

1. $\vec{\nabla} \cdot \vec{D} = 0$
2. $\vec{j}_f(\vec{r}, t) = 0$
3. $\dot{\vec{D}} = 0$
4. $\vec{\nabla} \times \vec{E} = 0$
5. Mehrere der oben genannten Antworten.

Antwort: 5. (1. und 2.)

A.4.95 Wellengleichung 2

Die Wellengleichungen können für das E- und B-Feld, wie in der Vorlesung hergeleitet, entkoppelt werden.

$$\left(\vec{\nabla}^2 - \frac{1}{\sqrt{\epsilon_r \epsilon_0 \mu_r \mu_0}} \frac{\partial^2}{\partial t^2}\right) \cdot \vec{E} = 0$$
$$\left(\vec{\nabla}^2 - \frac{1}{\sqrt{\epsilon_r \epsilon_0 \mu_r \mu_0}} \frac{\partial^2}{\partial t^2}\right) \cdot \vec{B} = 0$$

Warum ist diese Entkopplung nur ein Rechentrick und hat keine reale physikalische Bedeutung?

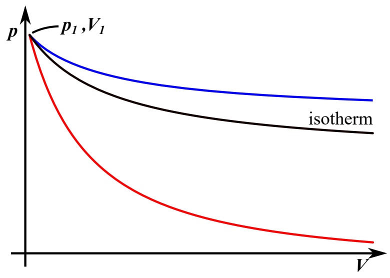
1. Weil die Entkopplung aus Ableitungen der Maxwell-Gleichungen entstehen.
2. Weil die Maxwell-Gleichungen auch einzeln gelten müssen.
3. Weil es nie ein elektrisches Feld ohne Magnetfeld geben kann.
4. Weil man die Wellengleichung für die Potentiale (Φ und $\vec{A}$) nicht entkoppeln kann.

Antwort: 2.

Thermodynamik

A.4.96 adiabatisch vs. isotherm

Im p-V-Diagramm ist eine isotherme Zustandsänderung aufgetragen (schwarz). Diese beginnt bei Druck p_1 und Volumen V_1 und endet irgendwo bei $p_2 < p_1$ und $V_2 > V_1$. Wenn das System aus dem selben Ausgangszustand adiabatisch expandiert, wie verhält sich die Kurve der Zustandsänderung im Vergleich zur isothermen?



1. Die Kurve der adiabatischen Zustandsänderung muss über der isothermen liegen liegen (blau).
2. Der Graph der adiabatischen Zustandsänderung muss unterhalb des isothermen liegen (rot).
3. Isotherme und adiabatische Zustandsänderungen lassen sich im p-V-Diagramm nicht unterscheiden.
4. Eine adiabatische Zustandsänderung kann nicht im p-V-Diagramm dargestellt werden, sondern nur im p-T-Diagramm.

Antwort: 2.

A.4.97 Differentiale angewandt

Ist $A(x,y)\,dx + B(x,y)\,dy$ ein vollständiges Differential $\left(\frac{\partial A}{\partial y}\right)_x = \left(\frac{\partial B}{\partial x}\right)_y$, so gibt es eine Funktion $f(x,y)$ mit $df = A(x,y)\,dx + B(x,y)\,dy$. Das thermodynamische Potential der inneren Energie ist eine solche Funktion

$$dE = T\,dS - p\,dV.$$

Welche Aussage lässt sich demnach über die Änderung der inneren Energie abhängig von der Entropie S treffen?
$\left(\frac{\partial E}{\partial S}\right)_V = \ldots$

1. ...$\left(\frac{\partial V}{\partial S}\right)_T$
2. ...$\left(\frac{\partial p}{\partial T}\right)_V$
3. ...$-p$
4. ...T
5. Für eine qualitiv korrekte Aussage fehlen Randbedingungen.

Antwort: 4.

A.4.98 Extensive Zustandsgrößen

Extensive Zustandsgrößen sind Zustandsgrößen, deren Maß mit der Größe des Systems skaliert. Welche Zustandsgrößen der idealen Gasgleichung

$$p \cdot V = N \cdot k_B \cdot T$$

sind extensive Größen?

1. T
2. p
3. k_b
4. V
5. Mehr als eine.

Antwort: 4.

A.4.99 Kreisprozesse

Was definiert, ob ein Kreisprozess als Motor oder als Wärmepumpe genutzt werden kann?

1. Kreisprozesse mit konstanten extensiven Größen betreiben Wärmepumpen, Kreisprozesse mit konstanten intensiven Größen Motoren.
2. Lediglich die Nutzung der abgegebenen Energie. Genutzte Wärme bedeutet Wärmepumpe, genutzte Arbeit Motor.
3. Die Reihenfolge der Zustandsänderungen oder die Laufrichtung im Zustandsdiagramm.
4. Reversible Kreisprozesse werden für Motoren benötigt, irreversible für Wärmepumpen.

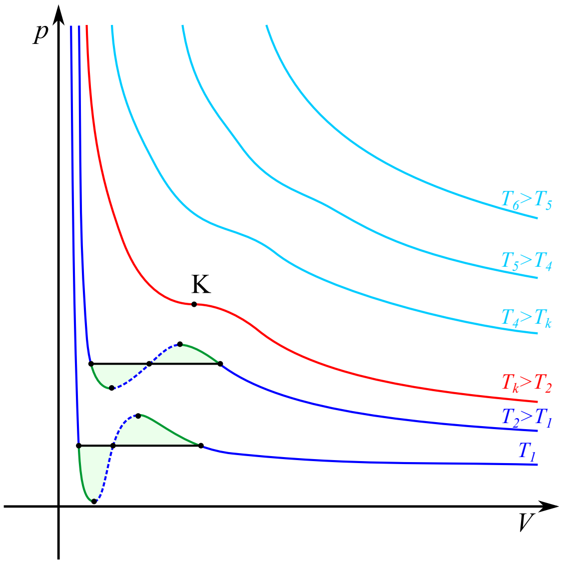
Antwort: 2.

A.4.100 Reale Gase

Im pV-Zustandsdiagramm für reale Gase sind verschiedene Isothermen skizziert. Unter der kritischen Temperatur werden die Graphen mit der Maxwellkonstruktion korrigiert, um unphysikalische Lösungen (Volumen und Druck steigen oder fallen gleichzeitig) zu kaschieren.

An den waagrechten Abschnitten der Isothermen liegen (experimentell nachweisbar) Teilchen in flüssiger und gasförmiger Phase vor. Das Diagramm scheint also zwei physikalische Aggregatzustände darzustellen. Gibt es auch Bereiche im Diagramm, in denen ausschließlich Flüssigkeiten beschrieben werden?

1. Ja, unterhalb von T_k und bei Volumina kleiner als die waagrechten Abschnitte.



2. Ja, alle Isothermen unterhalb von T_k liegen in flüssiger Phase vor.

3. Ja, alle Isothermen beschreiben irgendwann eine Flüssigkeit, der Druck muss nur entsprechend hoch und das Volumen klein sein.

4. Nein, Stoff in flüssiger Form ist nur bei der Maxwellkonstruktion zu finden. Die Grenze verläuft an den Enden der Waagrechten.

Antwort: 1.

A.4.101 Reversible Prozesse

Bei welchem Beispiel handelt es sich um einen wahren reversiblen Prozess?

1. Einer Gasflasche entweicht ihr Gas und wird wieder befüllt.

2. Der Wagen einer Achterbahn fährt eine Runde und stoppt wieder am Startpunkt.

3. Ein Weinglas zerbricht, wird eingeschmolzen und neu geblasen.

4. Ein Tasse Tee kühlt ab und wird wieder erhitzt.

5. Mehrere der oben genannten Vorgänge beschreiben reversible Prozesse.

6. Keine der oben genannten Vorgänge.

Antwort: 6.

A.5 Local hidden parameters

The following section contains the handout to the students after the teaching unit on quantum mechanical entanglement and the contradiction that local hidden parameters would cause in quantum mechanics. The educational reconstruction on which this teaching unit is based is described in Chapter 4.

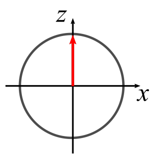
Widerlegung eines versteckten lokalen Parameters

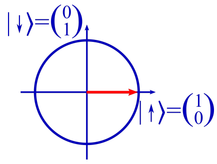
1 Spins und ihre Zustandsbasis

Spin in z-Richtung:

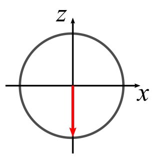
Ortsraum (x,y,z) **Hilbertraum**

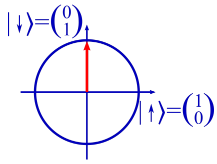
„up" $= |+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |\uparrow\rangle =$





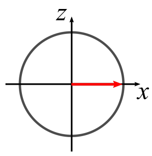
„down" $= |-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |\downarrow\rangle =$

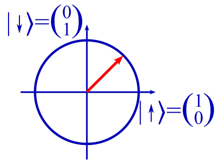




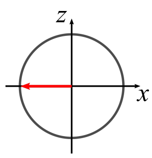
Spin in x-Richtung:

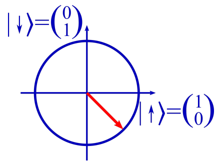
$$„\text{up}" = |+_x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) = |\rightarrow\rangle$$





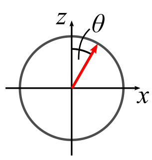
$$„\text{down}" = |-_x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle) = |\leftarrow\rangle$$

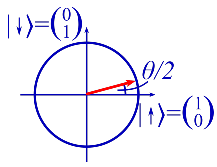




Spin in xz-Ebene um θ gedreht:

$$„\text{up}"(\theta) = |+_\theta\rangle = \cos\left(\frac{\theta}{2}\right)\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \sin\left(\frac{\theta}{2}\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \cos\left(\frac{\theta}{2}\right)|\uparrow\rangle + \sin\left(\frac{\theta}{2}\right)|\downarrow\rangle$$



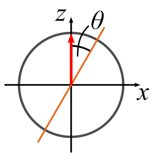


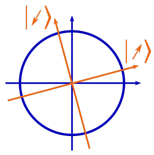
2 Messung

Die Richtung θ, in der gemessen wird, spannt eine neue Basis auf:

$$\text{„up"} = |\nearrow\rangle = \cos\left(\frac{\theta}{2}\right)|\uparrow\rangle + \sin\left(\frac{\theta}{2}\right)|\downarrow\rangle$$

$$\text{„down"} = |\swarrow\rangle = \sin\left(\frac{\theta}{2}\right)|\uparrow\rangle - \cos\left(\frac{\theta}{2}\right)|\downarrow\rangle$$

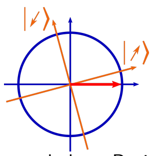


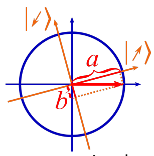


Auch in der neuen Richtung können nur Spin „up" oder „down" gemessen werden.
D.h. die Basisvektoren $|\nearrow\rangle$ und $|\swarrow\rangle$ beschreiben auch die Zustände, in die ein Spin bei einer Spinmessung in θ-Richtung gezwungen wird.
Mit welcher Wahrscheinlichkeit wird ein Spin „up" in z-Richtung in der neuen Basis „up" oder „down" gemessen?

Der Spin $|\uparrow\rangle$ (roter Pfeil) wird im Hilbertraum auf die neue gedrehte Basis projiziert:

$$|\uparrow\rangle = a\,|\nearrow\rangle + (-b)\,|\swarrow\rangle$$





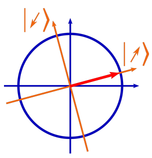
Die Wahrscheinlichkeit den ursprünglichen Spin $|\uparrow\rangle$ in der gedrehten Basis „up" zu messen, ist das Betragsquadrat des Koeffizienten dieses Zustands.

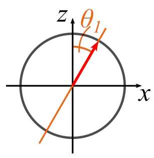
$$\textbf{prob}\{\text{up}_\theta|\,|\uparrow\rangle\} = |\langle\nearrow|\uparrow\rangle|^2 = |\langle\nearrow|\cdot(a\,|\nearrow\rangle - b\,|\swarrow\rangle)|^2 = |a\underbrace{\langle\nearrow|\nearrow\rangle}_{=1} - b\underbrace{\langle\nearrow|\swarrow\rangle}_{=0}|^2 = |a|^2$$

Analog gilt für die Wahrscheinlichkeit in der gedrehten Basis „down" zu messen:

$$\textbf{prob}\{\text{down}_\theta|\,|\uparrow\rangle\} = |\langle\swarrow|\uparrow\rangle|^2 = |\langle\nearrow|\cdot(a\,|\nearrow\rangle - b\,|\swarrow\rangle)|^2 = |a\langle\nearrow|\nearrow\rangle - b\langle\nearrow|\swarrow\rangle|^2 = |b|^2$$

Direkt nach einer Messung ist der Spin in genau dem Zustand, der der Messung entspricht. Wurde hier im Beispiel ein Spin „up" in der gedrehten Basis gemessen, wurde der Spin im Ortsraum um den Winkel θ gegenüber seiner ursprünglichen Ausrichtung gedreht.





Die Observable „Spin up in gedrehter Richtung" $(\hat{\nearrow})$ kann in Dirac-Schreibe definiert sein als:

$$(\hat{\nearrow}) = |\nearrow\rangle\langle\nearrow|$$

Die dazugehörigen Messwerte sind Eigenwerte zum Zustand $|\nearrow\rangle$ und nehmen die Werte 0 und 1 an.

$$(\hat{\nearrow}) = \begin{cases} 1, & \text{wenn der Spin „up" in der gedrehten Richtung gemessen wurde} \\ 0, & \text{sonst} \end{cases}$$

2 Messung 2

3 Ein Zustand - Zwei Spins

Ein einzelnes Spin-1/2-Teilchen konnte mit einem 2-dimensionalen Hilbertraum beschrieben werden. Zwei Spin-1/2-Teilchen spannen einen 4-dimensionalen Hilbertraum auf, da jeder Spin für sich wieder nur „up“ oder „down“ sein kann. Der allgemeine Zustand eines solches Systems ist:

$$|\Psi\rangle = |\Psi_1\rangle \otimes |\Psi_2\rangle = a\,|\uparrow\rangle_1\,|\uparrow\rangle_2 + b\,|\uparrow\rangle_1\,|\downarrow\rangle_2 + c\,|\downarrow\rangle_1\,|\uparrow\rangle_2 + d\,|\downarrow\rangle_1\,|\downarrow\rangle_2$$

Da $|\ \rangle_1$ und $|\ \rangle_2$ zwei verschiedene Teilchen beschreiben, befinden sich diese Hilberträume in unterschiedlichen Dimensionen. Die Vektoren $|\ \rangle_1$ und $|\ \rangle_2$ stehen also immer orthogonal aufeinander und das Skalarprodukt ist immer Null.

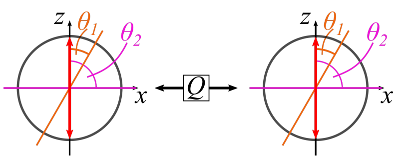
$\cdot$	$\|\uparrow\rangle_1$	$\|\downarrow\rangle_1$	$\|\uparrow\rangle_2$	$\|\downarrow\rangle_2$
$\langle\uparrow\|_1$	1	0	0	0
$\langle\downarrow\|_1$	0	1	0	0
$\langle\uparrow\|_2$	0	0	1	0
$\langle\downarrow\|_2$	0	0	0	1

4 Widerlegung des versteckten Parameters

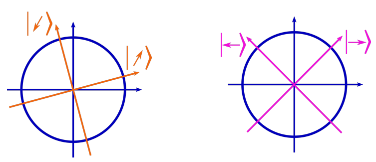
Um eine unvollständige Quantenmechanik und damit die Notwendigkeit eines versteckten Parameters auszuschließen, wird ein Zustand mit zwei Spins präpariert und anschließend in zwei verschiedenen Richtungen gemessen. Eine Quantenmechanik mit verstecktem Parameter würde zu einem Widerspruch in den Messergebnissen führen.

4.1 Das Experiment

Zwei Teilchen mit Spin-1/2 in z-Richtung werden erzeugt und an unterschiedliche Orte geschickt. Dort können sie von zwei Parteien (meist Alice und Bob genannt) in zwei Basen zur z-Richtung gedreht gemessen werden.



Die zwei Messrichtungen liefern zwei neue Basen:



$$|\nearrow\rangle = \frac{1}{\sqrt{|\alpha| + |\beta|}}(\beta^{\frac{1}{2}}\,|\uparrow\rangle + \alpha^{\frac{1}{2}}\,|\downarrow\rangle)$$

$$|\swarrow\rangle = \frac{-1}{\sqrt{|\alpha| + |\beta|}}(\alpha^{\frac{1}{2}}\,|\uparrow\rangle - \beta^{\frac{1}{2}}\,|\downarrow\rangle)$$

$$|\rightarrow\rangle = \frac{1}{\sqrt{|\alpha|^3 + |\beta|^3}}(\beta^{\frac{3}{2}}\,|\uparrow\rangle + \alpha^{\frac{3}{2}}\,|\downarrow\rangle)$$

$$|\leftarrow\rangle = \frac{-1}{\sqrt{|\alpha|^3 + |\beta|^3}}(\alpha^{\frac{3}{2}}\,|\uparrow\rangle - \beta^{\frac{3}{2}}\,|\downarrow\rangle)$$

mit $\tan\frac{\theta_1}{2} = \left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}}, \quad \tan\frac{\theta_2}{2} = -\left(\frac{\alpha}{\beta}\right)^{\frac{3}{2}}$

4.2 Verschränkter Zustand

Zwei Spin-1/2-Teilchen werden in folgendem Zustand erzeugt:

$$|\Psi\rangle = \alpha\,|\uparrow\rangle_1\,|\uparrow\rangle_2 - \beta\,|\downarrow\rangle_1\,|\downarrow\rangle_2 \tag{1}$$

Dieser Zustand kann mit den neuen Basen und etwas Rechenarbeit umgestellt werden.

$$|\Psi\rangle = N(AB\,|\nearrow\rangle_1\,|\swarrow\rangle_2 + AB\,|\swarrow\rangle_1\,|\nearrow\rangle_2 + B^2\,|\swarrow\rangle_1\,|\swarrow\rangle_2) \tag{2}$$
$$|\Psi\rangle = N(A\,|\rightarrow\rangle_1\,|\nearrow\rangle_2 + B\,|\rightarrow\rangle_1\,|\swarrow\rangle_2 - A^2A^*\,|\rightarrow\rangle_1\,|\nearrow\rangle_2 - A^2B\,|\leftarrow\rangle_1\,|\nearrow\rangle_2) \tag{3}$$
$$|\Psi\rangle = N(A\,|\nearrow\rangle_1\,|\rightarrow\rangle_2 + B\,|\swarrow\rangle_1\,|\rightarrow\rangle_2 - A^2A^*\,|\nearrow\rangle_1\,|\rightarrow\rangle_2 - A^2B\,|\nearrow\rangle_1\,|\leftarrow\rangle_2) \tag{4}$$
$$|\Psi\rangle = N((1-|A|^4)\,|\rightarrow\rangle_1\,|\rightarrow\rangle_2 + A^2BA^*\,|\leftarrow\rangle_1\,|\rightarrow\rangle_2 + A^2A^*B\,|\rightarrow\rangle_1\,|\leftarrow\rangle_2 - A^2B^2\,|\leftarrow\rangle_1\,|\leftarrow\rangle_2) \tag{5}$$

mit

$$A = \frac{\sqrt{\alpha\beta}}{\sqrt{1-|\alpha\beta|}}, \qquad B = \frac{|\alpha|-|\beta|}{\sqrt{1-|\alpha\beta|}}, \qquad N = \frac{1-|\alpha\beta|}{|\alpha|-|\beta|}$$

4.3 Fallunterscheidung

Bei der folgenden Durchführung des Experiments werden Alice und Bob jeweils unabhängig voneinander entscheiden, ob sie ihr Teilchen auf Spin „up“ in der schrägen Basis messen,

$$(\hat{\nearrow})_{1/2} = {}_{1/2}|\nearrow\rangle\,\langle\nearrow|_{1/2} = 0, 1,$$

oder Spin „down“ in der waagrechten Basis,

$$(\hat{\leftarrow})_{1/2} = {}_{1/2}|\leftarrow\rangle\,\langle\leftarrow|_{1/2} = 0, 1.$$

4.3.1 Alice misst $(\hat{\leftarrow})_1$ und Bob $(\hat{\nearrow})_2$

Wenn Alice bei dieser Messung Spin „down“ eine Bestätigung bekommt ($(\hat{\leftarrow})_1 = 1$), d.h. der erste Spin in den Zustand $|\leftarrow\rangle_1$ reduziert wird, sagt Gleichung (3), dass Bob auf jeden Fall seinen Spin im Zustand $|\nearrow\rangle_2$ finden wird.

$$\begin{aligned}
(\hat{\leftarrow})_1\Psi &= {}_1|\leftarrow\rangle\,\langle\leftarrow|_1\,N(A\,|\rightarrow\rangle_1\,|\nearrow\rangle_2 + B\,|\rightarrow\rangle_1\,|\swarrow\rangle_2 - A^2A^*\,|\rightarrow\rangle_1\,|\nearrow\rangle_2 - A^2B\,|\leftarrow\rangle_1\,|\nearrow\rangle_2) \\
&= |\leftarrow\rangle_1\,N(A\;{}_1\langle\leftarrow|\rightarrow\rangle_1\,|\rightarrow\rangle_2 + B\;{}_1\langle\leftarrow|\rightarrow\rangle_1\;|\swarrow\rangle_2 - A^2A^*\;{}_1\langle\leftarrow|\rightarrow\rangle_1\;|\nearrow\rangle_2 \\
&\quad - A^2B\;{}_1\langle\leftarrow|\leftarrow\rangle_1\;|\nearrow\rangle_2) + |\leftarrow\rangle_1\,N(A\,|\rightarrow\rangle_1\;{}_1\langle\leftarrow|\nearrow\rangle_2 + B\,|\rightarrow\rangle_1\;{}_1\langle\leftarrow|\swarrow\rangle_2 \\
&\quad - A^2A^*\,|\rightarrow\rangle_1\;{}_1\langle\leftarrow|\nearrow\rangle_2 - A^2B\,|\leftarrow\rangle_1\;{}_1\langle\leftarrow|\nearrow\rangle_2) \\
&= |\leftarrow\rangle_1\,N(-A^2B)\,|\nearrow\rangle_2 = -NA^2B\,|\leftarrow\rangle_1\,|\nearrow\rangle_2
\end{aligned}$$

4.3.2 Alice misst $(\hat{\nearrow})_1$ und Bob $(\hat{\leftarrow})_2$

Wenn Bob in der waagrechten Basis einen Spin „down“ misst ($(\hat{\leftarrow})_2 = 1$), d.h. der zweite Spin in den Zustand $|\leftarrow\rangle_2$ reduziert wird, sagt Gleichung (4), dass Alice auf jeden Fall ihren Spin im Zustand $|\nearrow\rangle_2$

finden wird.

$$
\begin{aligned}
(\hat{\leftarrow})_2\Psi &= {}_2|\leftarrow\rangle\,\langle\leftarrow|_2\,N(A\,|\nearrow\rangle_1\,|\rightarrow\rangle_2 + B\,|\swarrow\rangle_1\,|\rightarrow\rangle_2 - A^2A^*\,|\nearrow\rangle_1\,|\rightarrow\rangle_2 - A^2B\,|\nearrow\rangle_1\,|\leftarrow\rangle_2)\\
&= |\leftarrow\rangle_2\,N(-A^2B)\,|\nearrow\rangle_1 = -NA^2B\,|\nearrow\rangle_1\,|\leftarrow\rangle_2
\end{aligned}
$$

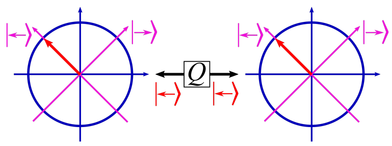
4.3.3 Alice misst $(\hat{\leftarrow})_1$ und Bob $(\hat{\leftarrow})_2$

Wenn Alice und Bob beide in der waagrechten Basis messen, werden sie nach Gleichung (5) mit einer Wahrscheinlichkeit von $|NA^2B^2|^2$ beide einen Spin „down“, also $(\hat{\leftarrow})_1 = 1$ und $(\hat{\leftarrow})_2 = 1$ messen.

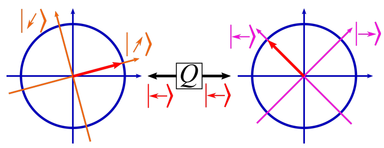
4.4 Experiment mit verstecktem Parameter

Um die Quantenmechanik mit dem Realismus und der Lokalität in Einklang zu bringen, kann ein versteckter Parameter eingeführt werden, der ein Messergebnis vorherbestimmt. Auch wenn dieser Parameter unbestimmt bleibt, kann er widerlegt werden. Dazu betrachten wir den Fall von Abschnitt 4.3.3 genauer.

Gleichung (5) sagt eindeutig, dass in manchen Fällen Alice und Bob beide Spin „down“ in der waagrechten (also θ_2) Basis messen können. Wenn das Messergebnis vorherbestimmt ist, so müssen bei einem solchen Experiment die beiden Teilchen so präpariert sein, dass, wenn Alice und Bob in der θ_2-Basis messen, beide Spin „down“ als Ergebnis bekommen.



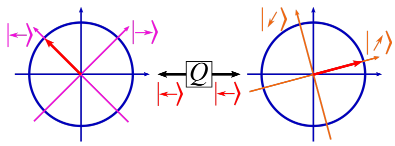
Wenn ein ebensolches Spinpaar unterwegs ist und Bob sich aber dafür entschieden hätte, in der schrägen θ_1-Basis zu messen, so wäre sein Ergebnis nach Abschnitt 4.3.1 ebenfalls vorherbestimmt, dass Bob in der schrägen Basis den Spin „up“ messen muss.



Wenn Bob sich so spät für die schräge Basis entschieden hat, dass seine Messung nach den Gesetzen des Realismus und der Lokalität nicht mehr Alice Messung beeinflussen kann, so muss ein verborgener Parameter λ, der noch unbekannt ist, aber die Quantenmechanik vollständig beschreiben soll, das Ergebnis aus 4.3.1 berücksichtigen. Die Funktion, wann Alice einen Spin „up“ in der schrägen Basis misst und von λ abhängig ist, muss in diesem Fall ergeben:

$$(\hat{\nearrow})_2(\lambda) = 1$$

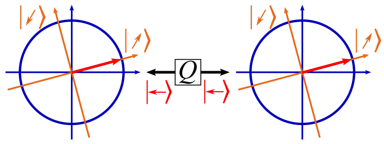
Genau dasselbe gilt für ein Experiment mit solch einem Spinpaar, wenn Alice beschließt, in der schrägen Basis zu messen. Hier muss sie ihren Spin im Zustand $|\nearrow\rangle_1$ messen.



Für die Funktion Spin „up“ mit verstecktem Parameter in der schrägen Basis gilt folglich

$$(\hat{\nearrow})_1(\lambda) = 1.$$

Wenn nun Alice und Bob sich beide kurz vor der Messung dieses Paares entscheiden in der schrägen θ_1-Basis zu messen, so müssen sie beide Spin „up“ messen.



Alle möglichen Fälle sind in der folgenden Tabelle dargestellt.

Bob \ Alice	θ_2	θ_1
θ_2	$(\hat{\leftarrow})_1 = 1,\ (\hat{\leftarrow})_2 = 1$	$(\hat{\nearrow})_1 = 1,\ (\hat{\leftarrow})_2 = 1$
θ_1	$(\hat{\leftarrow})_1 = 1,\ (\hat{\nearrow})_2 = 1$	$(\hat{\nearrow})_1 = 1,\ (\hat{\nearrow})_2 = 1$

Dass Alice und Bob in der schrägen θ_1-Basis beide Spin „up“ messen, ist nach Gleichung (2) aber verboten. Somit haben wir einen Widerspruch in der Quantentheorie mit einem lokalen versteckten Parameter.
Tatsächlich ist dieser Umstand auch experimentell überprüft. Mit den richtigen Basen zur Spinmessung, abhängig vom Ausgangszustand, können bestimmte Spinkombinationen nicht gemessen werden.

A.6 Worked out solution for the students in the mathematics course on electrodynamics.

This appendix contains the sample solution given to students in the Math Review course on Electrodynamics on Day 4. The topic covered is differential operators and integral theorems. This sample solution is for self-study before other problems of similar nature are to be solved by students for practice.

Problem 1: Vektoranalysis (Aufgabe mit Musterlösung)

a)

$$\begin{aligned}
\nabla\cdot(\boldsymbol{A}\times\boldsymbol{B}) &= \begin{pmatrix}\partial_x\\ \partial_y\\ \partial_z\end{pmatrix}\cdot\begin{pmatrix}A_yB_z-A_zB_y\\ A_zB_x-A_xB_z\\ A_xB_y-A_yB_x\end{pmatrix}\\
&=(\partial_xA_y)B_z+A_y\partial_xB_z-(\partial_xA_z)B_y-A_z\partial_xB_y\\
&\ +(\partial_yA_z)B_x+A_z\partial_yB_x-(\partial_yA_x)B_z-A_x\partial_yB_z\\
&\ +(\partial_zA_x)B_y+A_x\partial_zB_y-(\partial_zA_y)B_x-A_y\partial_zB_x\\
&=B_x(\partial_yA_z-\partial_zA_y)+B_y(\partial_zA_x-\partial_xA_z)\\
&\ +B_z(\partial_xA_y-\partial_yA_x)+A_x(\partial_zB_y-\partial_yB_z)\\
&A_y(\partial_xB_z-\partial_zB_x)+A_z(\partial_yB_x-\partial_xB_y)\\
&=\boldsymbol{B}\cdot\underbrace{\begin{pmatrix}\partial_yA_z-\partial_zA_y\\ \partial_zA_x-\partial_xA_z\\ \partial_xA_y-\partial_yA_x\end{pmatrix}}_{\nabla\times\boldsymbol{A}}+\boldsymbol{A}\cdot\underbrace{\begin{pmatrix}\partial_zB_y-\partial_yB_z\\ \partial_xB_z-\partial_zB_x\\ \partial_yB_x-\partial_xB_y\end{pmatrix}}_{-\nabla\times\boldsymbol{B}}\\
&=\boldsymbol{B}(\nabla\times\boldsymbol{A})-\boldsymbol{A}(\nabla\times\boldsymbol{B})
\end{aligned}$$

b)

$$\boldsymbol{A}=\begin{pmatrix}x(4-2a)+4(a-3)+xy(8x-4ax)\\ x(3a-5)+y(4-2a)+xy(8y-4ay)\\ 0\end{pmatrix}\mathrm{e}^{-x^2-y^2}$$

$\mathrm{rot}A$ hat nur eine z-Komponente, weil $\boldsymbol{A}$ nicht von z abhängig ist und keinen Eintrag in der z-Komponente hat.

$$\begin{aligned}
(\mathrm{rot}\boldsymbol{A})_z=(\nabla\times\boldsymbol{A})_z &= \Big\{3a-5+8y^2-4ay^2-2x\left[x(3a-5)+y(4-2a)+xy(8y-4ay)\right]\\
&\quad -(a-3)-8x^2+4ax^2+2y\left[x(4-2a)+y(a-3)+xy(8x-4ax)\right]\Big\}\,\mathrm{e}^{-x^2-y^2}\\
&=\Big\{3a-5+8y^2-4ay^2-6x^2+10ax^2-8xy+4axy\\
&\ -16x^2y^2+8ax^2y^2-a+3-8x^2+4ax^2\\
&\ +8xy-4axy+2ay^2-6y^2+16x^2y^2-8ax^2y^2\Big\}\,e^{-x^2-y^2}\\
&=\Big\{2(a-1)-14(a-1)x^2+10(a-1)y^2\Big\}\,e^{-x^2-y^2}
\end{aligned}$$

$\rightarrow$verschwindet für $a=1$.

Dann ist $\boldsymbol{A}$:

$$A=\begin{pmatrix}2x-2y+4x^2y\\ -2x+2y+4xy^2\\ 0\end{pmatrix}e^{-x^2-y^2}$$

Der Gradient des Skalerfelds muss $\boldsymbol{A}$ ergeben, also

$$\begin{aligned}\partial_x\psi &= \left(2x - 2y + 4x^2y\right)e^{-x^2-y^2}\\ \partial_y\psi &= \left(-2x + 2y + 4xy^2\right)e^{-x^2-y^2}\\ \partial_z\psi &= 0\end{aligned}$$

Man kann leicht erraten, dass

$$\psi(x,y,z) = -(1+2xy)e^{-x^2-y^2} + c$$

das erfüllt. Wenn man es nicht sieht, berechnet man

$$\begin{aligned}\int \partial_x\psi\,\mathrm{d}x &= \int \left(2x - 2y + 4x^2y\right)e^{-x^2-y^2}\mathrm{d}x\\ &= \underbrace{\int 2xe^{-x^2-y^2}\mathrm{d}x}_{\substack{u=e^{-x^2-y^2}\\ \mathrm{d}u=-2xe^{-x^2-y^2}\mathrm{d}x}=-\int du=u=-e^{-x^2-y^2}} -2y\int e^{-x^2-4}\mathrm{d}x + \int 4x^2ye^{-x^2-y^2}\mathrm{d}x\\ &= -e^{-x^2-y^2} - 2y\underbrace{\int e^{-x^2-4}\mathrm{d}x}_{\overset{\text{Part.}}{\underset{\text{Int.}}{=}}\left[xe^{-x^2-y^2}\right]+\int 2x^2e^{-x^2-y^2}} + \int 4x^2ye^{-x^2-y^2}\mathrm{d}x\\ &= -e^{-x^2-y^2} + -2xye^{-x^2-y^2} - 4y\int x^2e^{-x^2-y^2}\mathrm{d}x + 4y\int x^2e^{-x^2-y^2}\mathrm{d}x\\ &= -(1+2xy)e^{-x^2-y^2} + c(y)\end{aligned}$$

$$\begin{aligned}\frac{\partial\psi}{\partial y} &= -2xe^{-x^2-y^2} + 2ye^{-x^2-y^2} + 4xy^2e^{-x^2-y^2} + c'(y)\\ &\overset{!}{=} \left(-2x + 2y + 4xy^2\right)e^{-x^2-y^2}\\ \rightarrow\quad & c'(y) = 0 \qquad \text{und daraus folgt das } \psi \text{ von oben.}\end{aligned}$$

In drei Dimensionen entsprechen die Integrationsbedingungen $\frac{\partial f_i}{\partial x_j} = \frac{\partial f_j}{\partial x_i}$ oder $\frac{\partial f_i}{\partial x_j} - \frac{\partial f_j}{\partial x_i} = 0$ gerade $\mathrm{rot}\boldsymbol{A} = 0$ wenn man den Vector $\boldsymbol{A} = \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix}$ bildet. Die Aussagen „$\mathrm{rot}\boldsymbol{A} = 0$“ oder „$\mathrm{d}f = f_x\mathrm{d}x + f_y\mathrm{d}y + f_z\mathrm{d}z$ ist ein vollständiges Differential“, sind somit äquivalent.

- Die Funktion f, aus der sich das Differential bilden lässt, ist gerade mit dem Skalarfeld ψ identisch, für das $\boldsymbol{A} = \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} = \nabla\psi$ gilt.
- Das Differential kann auch geschrieben werden als $\mathrm{d}f = \nabla f \cdot \mathrm{d}\boldsymbol{r} = \begin{pmatrix} \partial_x f \\ \partial_y f \\ \partial_z f \end{pmatrix} \cdot \begin{pmatrix} \mathrm{d}x \\ \mathrm{d}y \\ \mathrm{d}z \end{pmatrix}$

2

c)

$$\boldsymbol{B} = \begin{pmatrix} 4y \\ x \\ 2z \end{pmatrix}, \qquad \int_A (\nabla \times \boldsymbol{B}) \cdot \mathrm{d}\boldsymbol{A}$$

Direkt: Die Fläche ist eine Halbkugel mit Radius $a,\ z \geq 0$. Parametrisierung in Kugelkoordinaten

$$\boldsymbol{r} = \begin{pmatrix} r\sin\vartheta\cos\varphi \\ r\sin\vartheta\sin\varphi \\ r\cos\vartheta \end{pmatrix}, \qquad r = a, \qquad \vartheta = 0\ldots\frac{\pi}{2}, \qquad \varphi = 0\ldots 2\pi$$

$$\mathrm{d}\boldsymbol{A} = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}\vartheta} \times \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}\varphi}\,\mathrm{d}\vartheta\,\mathrm{d}\varphi = r^2\sin\vartheta\boldsymbol{e}_\vartheta \times \boldsymbol{e}_\varphi\,\mathrm{d}\vartheta\,\mathrm{d}\varphi = r^2\sin\vartheta\boldsymbol{e}_r\,\mathrm{d}\vartheta\,\mathrm{d}\varphi$$

$$\mathrm{rot}\boldsymbol{B} = \begin{pmatrix} 0 \\ 0 \\ 1-4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}$$

$$\begin{aligned}
\int_A \mathrm{rot}\,\boldsymbol{B} \cdot \mathrm{d}\boldsymbol{A} &= \int_{\vartheta=0}^{\pi/2}\int_{\varphi=0}^{\pi} \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} \cdot r^2\sin\vartheta \underbrace{\begin{pmatrix} \sin\vartheta\cos\varphi \\ \sin\vartheta\sin\varphi \\ \cos\vartheta \end{pmatrix}}_{\boldsymbol{e}_r} \mathrm{d}\vartheta\,\mathrm{d}\varphi \\
&= \int_{\vartheta=0}^{\pi/2}\mathrm{d}\vartheta\int_{\varphi=0}^{2\pi}\mathrm{d}\varphi a^2\sin\vartheta\cos\vartheta \\
&= -6\pi a^2\int_0^{\pi/2}\sin\vartheta\cos\vartheta\mathrm{d}\vartheta \underset{\mathrm{d}u=\cos\vartheta\mathrm{d}\vartheta}{\overset{u=\sin\vartheta}{=}} -6\pi a^2\int_0^1 u\mathrm{d}u \\
&= -6\pi a^2\left[\frac{1}{2}u^2\right]_0^1 = -3\pi a^2
\end{aligned}$$

Nach dem Satz von Stokes:

$$\int_A \nabla \times \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{A} = \oint_{\partial A} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{r}$$

$\partial A =$ Kreis mit dem Radius a in der x-y-Ebene um den Ursprung (Rand von A)

$$\boldsymbol{r} = \begin{pmatrix} a\cos\varphi \\ a\sin\varphi \\ 0 \end{pmatrix}, \quad \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}\varphi} = a\begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix}, \quad \varphi = 0\ldots 2\pi$$

$$\begin{aligned}
\oint \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{r} &= \int_{\varphi=0}^{2\pi} \begin{pmatrix} 4a\sin\varphi \\ a\cos\varphi \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix} a\mathrm{d}\varphi \\
&= \int_0^{7\pi} -4a^2\sin^2\varphi\mathrm{d}\varphi + \int_0^{2\pi} a^2\cos^2\varphi\mathrm{d}\varphi \\
&= -4a^2\pi + a^2\pi = -3a^2\pi
\end{aligned}$$

3

Problem 3: Anwendungen der Integralsätze (Aufgabe mit Musterlösung)

a) Das Feld einer geladenen Kugel mit Ladung Q, berechnet sich aus

$$\begin{aligned} Q &= \int_V \rho \mathrm{d}V = \int_V \operatorname{div} \boldsymbol{D} \mathrm{d}V \\ &= \int_{\partial V} \boldsymbol{D} \cdot \mathrm{d}\boldsymbol{A} = \varepsilon_0 \int_{\partial V} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{A} = \varepsilon_0 E(r) \int_{\partial V} \boldsymbol{e}_r \cdot \mathrm{d}\boldsymbol{A}. \end{aligned}$$

Integration über eine Kugeloberfläche in Kugelkoordinaten:

$$\begin{aligned} \mathrm{d}\boldsymbol{A} &= \left(\frac{\partial r}{\partial \vartheta} \times \frac{\partial r}{\partial \varphi} \right) \mathrm{d}\vartheta \mathrm{d}\varphi, \quad \text{wobei} \quad r = \begin{pmatrix} r \sin\vartheta \cos\varphi \\ r \sin\vartheta \sin\varphi \\ r \cos\vartheta \end{pmatrix} \\ &= \begin{pmatrix} r \cos\vartheta \cos\varphi \\ r \cos\vartheta \sin\varphi \\ -r \sin\vartheta \end{pmatrix} \times \begin{pmatrix} -r \sin\vartheta \sin\varphi \\ r \sin\vartheta \cos\varphi \\ 0 \end{pmatrix} \mathrm{d}\vartheta \mathrm{d}\varphi \\ &= \begin{pmatrix} r^2 \sin^2\vartheta \cos\varphi \\ r^2 \sin^2\vartheta \sin\varphi \\ r^2 \sin\vartheta \cos\varphi \end{pmatrix} \mathrm{d}\vartheta \mathrm{d}\varphi = r^2 \sin\vartheta \boldsymbol{e}_r \, \mathrm{d}\vartheta \mathrm{d}\rho \end{aligned}$$

Also ist

$$\begin{aligned} Q &= \varepsilon_0 E(r) r^2 \int_0^\pi \mathrm{d}\vartheta \int_0^{2\pi} \mathrm{d}\varphi \underbrace{\boldsymbol{e}_r \cdot \boldsymbol{e}_r}_{1} \sin\vartheta \\ &= \varepsilon_0 E(r) r^2 4\pi \end{aligned}$$

und damit

$$E(r) = \frac{Q}{4\pi\varepsilon_0 r^2}$$

b) Der Strom durch einen Draht berechnet sich aus

$$\begin{aligned} I &= \int_A \boldsymbol{j} \cdot \mathrm{d}\boldsymbol{A} = \int_A \operatorname{rot} \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{A} = \int_{\partial A} \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{r} \\ &= \int_{\partial A} H(\rho) \boldsymbol{e}_\varphi \mathrm{d}\boldsymbol{r} = \frac{1}{\mu_0} \int_{\partial_A} B(\rho) \boldsymbol{e}_\varphi \mathrm{d}\boldsymbol{r} \end{aligned}$$

Die Parametrisierung für ein konstantes ρ ist ein Kreis mit Radius ρ um den Mittelpunkt des Drahtes:

$$\begin{aligned} \boldsymbol{r} = \rho \begin{pmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{pmatrix} \rightarrow \mathrm{d}\boldsymbol{r} &= \rho \begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix} \mathrm{d}\varphi \\ &= \rho \boldsymbol{e}_\varphi \mathrm{d}\varphi \end{aligned}$$

Also ist

$$I = \frac{1}{\mu_0} B(\rho) \rho \int_{\varphi=0}^{2\pi} \mathrm{d}\varphi \boldsymbol{e}_\varphi \cdot \boldsymbol{e}_\varphi = \frac{2\pi}{\mu 0} \rho B(\rho)$$

und damit

$$B(\rho) = \frac{\mu_0 I}{2\pi\rho}$$

4

c)

$$\begin{aligned}\boldsymbol{B} = \frac{\mu_0 I}{2\pi\rho}\boldsymbol{e}_\varphi &= \operatorname{rot}\boldsymbol{A}\\ &= \boldsymbol{e}_\rho\left[\frac{1}{\rho}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right] + \boldsymbol{e}_\rho\left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right]\\ &+ \boldsymbol{e}_z\frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho A_\varphi) - \frac{\partial A_\rho}{\partial\varphi}\right]\end{aligned}$$

Dieses System lässt sich zum Beispiel erfüllen von

$$A_\rho = \frac{\mu_0 I z}{2\pi\rho}, \qquad A_z = A_\varphi = 0$$

oder $$A_z = -\frac{\mu_0 I \ln\rho}{2\pi}, \qquad A_\rho = A_\varphi = 0.$$

5

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Zusammenfassung

Motivation

Ein genauerer Blick auf die Situation der derzeitigen Physikstudierenden in Deutschland offenbart einen besonderen Handlungsbedarf. Seit mehreren Jahren steigen die Abbrecherquoten im deutschen Physikstudium von $26\,\%$ für den Abschlussjahrgang 1999 [1] auf über $50\,\%$ für den Abschlussjahrgang 2020 [2]. Viele Studierende begründen dabei den Abbruch ihres Studiums mit den fachlichen Anforderungen [3]. Darüber hinaus zeigen Studienergebnisse von Buschhüter et al. [4], dass das physikalische Fachwissen von Studienanfängern im Jahr 2013 im Vergleich zu Studienanfängern im Jahr 1978 abgenommen hat. Hier wurde das Wissen von über 2000 Studierenden mit einem älteren bundesweiten Studieneingangstest gemessen [5, 6]. Dem gegenüber stehen die Mathematikkenntnisse der Studienanfängerinnen und -anfänger, die sich im Laufe der Jahrzehnte je nach Themengebiet der Mathematik verbessert oder verschlechtert haben [7] und damit das Niveau im Mittel gehalten wurde. Bei den Physikfragen schnitten die Studierenden von 2013 aber durchweg schlechter ab als ihre Vergleichsgruppe von 1978 [4]. Zudem hat sich gezeigt, dass ein Teil der Studierenden nach angemessenen Studienfortschritten nicht das erforderliche Zielniveau erreicht [8]. Darüber hinaus erreichen einige Studierende nach fünf Semestern nicht das Niveau, das ihre Kommilitonen, die den Jahresdurchschnitt bilden, bereits im ersten Semester hatten [9].

Diese Ergebnisse belegen, dass in vielen Fällen Studium und Studierende nicht mehr gut zusammenpassen. Von Hochschulseite kann man aber einiges tun und die Lehre an die Bedürfnisse der Studierenden anpassen, um die Abbrecherquoten zu senken und gleichzeitig den Studierenden das Wissen besser und nachhaltiger zu vermitteln. Der erste Anhaltspunkt für Verbesserungen in der Hochschullehre sind die Arbeiten von Richard Hake [10]. Im Jahr 1998 wurden an mehreren Hochschulen und Universitäten die Lernzuwächse ganzer Kohorten (mit über 6000 Studierenden) in Mechanik-Vorlesungen gemessen.

Der durchschnittliche Lernzuwachs in konventionellen Lehrveranstaltungen liegt bei weniger als $25\,\%$. Als konventionelle Lehrveranstaltungen gelten Vorlesungen im Frontalunterricht, Übungsserien mit Problemlösungsaufgaben als Hausaufgaben und Laborpraktika mit Schwerpunkt auf der Datengenerierung und -analyse.

Wird das Kursformat jedoch auf interaktive Elemente umgestellt, bei denen die Lernenden gezielt auf ihrem Leistungsniveau aufgefordert werden, selbst etwas zu tun, und zeitnah zu den eigenen Leistungen Feedback erhalten, steigt der Lernzuwachs auf durchschnittlich $48\,\%$. In keinem Kurs zeigten die Studierenden einen geringeren Lernzuwachs als in herkömmlichen Kursen.

Im Idealfall regen solche Formate die Lernenden an, auf einem hohen kognitiven Niveau zu denken, an ihr Vorwissen anzuknüpfen, eigene Lösungen, Ideen und Konzepte zu entwickeln und diese im nächsten Schritt zu erklären. Wenn diese Aspekte erreicht werden, werden diese Formate als kognitiv aktivierend bezeichnet. [11–13].

Die Entwicklungsaufgabe dieser Dissertation besteht darin, Methoden, die sich in Einführungsvorlesungen oder in der Schule als kognitiv aktivierend und interaktiv erwiesen haben, auf die Theoretische Physik anzuwenden und Materialien zu erstellen, die andere Lehrende ohne viel Aufwand übernehmen können.

Theoretischer Rahmen

Um die Anwendungen dieser Methoden auf die Theoretische Physik möglichst erfolgreich und interessant zu gestalten, wird das Modell der didaktischen Rekonstruktion [23] bei der Entwicklung der Materialien verwendet. Die entwickelten Materialien können in Vorlesungen, Übungsgruppen, Seminaren und Tutorien eingesetzt werden. Anschließend wird in der Arbeit das Modell der didaktischen Rekonstruktion für die Entwicklung von Materialien für Lehramtsstudierende angepasst. Ziel der didaktischen Rekonstruktion ist es, für Entwicklungsarbeiten zwei Strömungen der Fachdidaktik sinnvoll und konstruktiv miteinander zu verbinden [23]. Auf der einen Seite steht die (empirische) Bildungsforschung, auf der anderen die fachlichen Inhalte und ihre Aufbereitung für die Lehre. In der didaktischen Rekonstruktion werden beide Teile als „Forschung zu Lehre und Lernen (Perspektive der Lernenden)“ und „Fachliche Klärung“ gleichberechtigt in die Entwicklung neuer Curricula oder Lehreinheiten einbezogen. Für diese Arbeit wurde nach Vorlagen das Modell um die Komponente der „Fachdidaktischen Klärung“ erweitert. Damit soll ein Rahmen geschaffen werden, der die Entwicklung von Lehrkonzepten und -materialien für Lehramtsstudierende optimiert.

Für den Erfolg dieser Entwicklungen werden nur Lehrmethoden verwendet, die nachweislich kognitiv aktivierend sind und den fachspezifischen Anforderungen der Theoretischen Physik am besten entsprechen. Zu den fachspezifischen Merkmalen gehört zum Beispiel der hohe Anteil mathematischer Formulierungen und Formalismen. Als Lehrmethoden eignen sich die Peer Instruction [17], das Lernen mit Lösungsbeispielen [18], die didaktische Reduktion [19] und die Verbalisierung und Visualisierung [20, 21] von Formeln. Diese Methoden werden in der vorliegenden Arbeit detailliert vorgestellt und für die Theoretische Physik adaptiert (Kapitel 3). Zur Qualitätssicherung werden die behandelten Materialien in Unterrichtskontexten getestet und mit einem standardisierten Beobachtungsbogen bewertet.

Interaktive und aktivierende Materialien für die Theoretische Physik

Im Rahmen der Testung neuer Materialien wurden zusätzlich die Herausforderungen der Studierenden in der Theoretischen Physik beobachtet. Folgende allgemeine Problemfelder können aus den Beobachtungen abgeleitet werden:

- Mathematische Methoden der Physik (häufig unzureichende Ausbildung),
- Wechsel zwischen Repräsentationen (mathematisch, bildlich, sprachlich),
- Anknüpfung an Vorwissen (Experimentalphysik und Schulphysik),
- Identifikation und Trennung wichtiger und weniger wichtiger Inhalte,
- fachlich und zielgruppengerechtes Argumentieren und Diskutieren.

Das neu entwickelte Material hat den Anspruch, auch diese Probleme anzugehen und zu vermindern bzw. zu beheben.

Lernen mit Lösungsbeispielen

Das Studium von Lösungsbeispielen, sogenannte „worked examples“, hat sich in umfangreichen Untersuchungen als wirksame Methode zum Erlernen des Lösens gut strukturierter Probleme in Physik und Mathematik erwiesen [18, 54]. In dieser Arbeit wird ein neuer 4-Stufen-Ansatz für das Lernen mit Lösungsbeispielen vorgestellt, der die Effekte der „cognitive load theory“ berücksichtigt, die für die Theoretische Physik am relevantesten sind. Diese Lehrmethode kann sogar in Kursen eingesetzt werden, in denen die Bearbeitung von Übungsaufgaben Teil des Benotungssystems, z.B. bei Scheinkriterien, ist. Dieser 4-Stufen Ansatz wird im Kontext der Lagrange-Mechanik veranschaulicht, die sich aufgrund ihres universellen Ansatzes zur Problemlösung ideal für die Anwendung von „worked examples“eignet.

Der Mehrwert beim Lernen mit Lösungsbeispielen kann mit der „cognitive load theory“ [18] erklärt werden. Durch die Reduktion von unnötigen kognitiven Belastungen („extraneous load“) und der gezielten Förderung von kognitiven Belastungen, die den Lernzuwachs fördern („germane load“), kann das Lernen mit Lösungsbeispielen effektiver sein als das konventionelle Bearbeiten und Präsentieren von Problemlöseaufgaben. Die Aufgaben in den vier Stufen beruhen auf klassischen Problemlöseaufgaben, der Arbeitsauftrag der Studierenden wird aber jeweils so angepasst, dass diese nach der „cognitive load theory“ möglichst lernförderlich sind.

So gelten Selbsterklärungen als lernförderlich und werden deshalb in Stufe 1 und 2 von den Studierenden aktiv eingefordert, indem z.B. schriftliche Erklärungen verlangt werden. Ebenso kann das Suchen und Korrigieren von Fehlern in einer Musterlösung Lernende kognitiv aktivieren, was in Stufe 3 genutzt wird. Die Aufgabe in Stufe 4 soll zur Motivation für die Stufen 1-3 und zur Kontrolle von den Studierenden selbst gelöst werden.

Versprachlichung und Verbildlichung von Formeln

Die Beobachtungen im Rahmen dieser Arbeit zeigen, dass viele Studierende häufig weniger Probleme mit den konkreten Rechnungen haben als mit dem Aufstellen von Lösungsansätzen. Dies beinhaltet z.B. das Übersetzen von physikalischen Rahmenbedingungen in eine mathematische Form zur Bearbeitung der Aufgabe oder die physikalische Interpretation von Lösungen oder Ansätzen.

Die Versprachlichung und Verbildlichung von Formeln sollten folglich gezielt geübt werden. In dieser Arbeit werden verschiedene Ansätze vorgestellt, wie solche Übungen aussehen können. Die einfachste Form ist das Abarbeiten einer Checkliste (auf die Theoretische Physik abgewandelt nach Bagno et al. [20, 21]), um einzelne Formeln zu versprachlichen. Ist diese Art von Aufgaben bei Studierenden noch unbekannt, können Lösungsbeispiele zu anderen Formeln die Hemmschwelle bei Studierenden senken. Klare Arbeitsaufträge oder feste Strukturen in Arbeitsblättern können ebenfalls die Hemmschwelle bei schwächeren Studierenden senken und diesen helfen Beziehungen, Abhängigkeiten und Argumente in der Physik konkret zu formulieren. Mit der Vorlage von Steckbriefen für die Maxwell-Gleichungen oder Tabellen zur Versprachlichung von Wechselwirkungen in der Elektrostatik können gute Ergebnisse über alle Leistungsgruppen hinweg erzielt werden.

Didaktische Rekonstruktion und Elementarisierung

Die Trennung von mehr oder weniger wichtigen Inhalten ist ein zentraler Punkt beim Lernen. Um Studierenden Methoden und Werkzeuge an die Hand zu geben, wie sie neue Inhalte und Themen für sich strukturieren können, kann man sich an den Vorgaben der didaktischen Rekonstruktion [23] orientieren. Ganz prinzipiell fallen solche Techniken unter das Selbststudium, wenn Studierende z.B. Vorlesungen nacharbeiten. Die Beobachtungen zeigen aber, dass viele Studierende dazu keine Zeit haben oder ihnen nie gezeigt wurde, wie sie dabei effizient vorgehen können. In dieser Arbeit werden deshalb Methoden vorgestellt, wie Studierende anfangs beim Selbststudium unterstützt und angeleitet werden können.

Zu den entwickelten und getesteten Materialien gehören zum Beispiel ein Leitfaden, wie Vorlesungen systematisch zusammengefasst und nachgearbeitet werden können. Des Weiteren können Konzeptkarten (engl. mind maps oder concept maps) Studierenden helfen, ganze Themengebiete für sich zu strukturieren und Verbindungen/Vernetzungen zwischen einzelnen Inhalten oder ganzen Themen zu verinnerlichen oder in manchen Punkten überhaupt erst zu erkennen. Im Sinne der „worked examples“ können wiederkehrende Elemente von Studierenden in verschiedenen Kontexten (Messungen in der Quantenmechanik in verschiedenen Kontexten wie freien oder gebundenen Teilchen und die Dirac-Notation beim 2-Niveau-System) identifiziert und in Zusammenhang gebracht werden. Der letzte Punkt, zu dem Material vorgestellt wird, ist die kritische Auseinandersetzung mit Vereinfachungen oder didaktischen Reduktionen, die in der Schulphysik verwendet werden. So wird an das Vorwissen der Studierenden angeknüpft und dieses Wissen systematisch erweitert. Dieses Vorgehen wird am Beispiel von Magnetfeldern im Inneren von langen Spulen demonstriert.

Peer Instruction

Die Peer Instruction ist eine Lehrmethode, die sich eignet, Studierende in Vorlesungen mit kleinen und großen Teilnehmerzahlen zu aktivieren, zu Diskussionen anzuregen und ihnen direkt Feedback über den eigenen Leistungsstand zu geben. Diese Methode wurde von Eric Mazur für Einführungsvorlesungen in die Physik entwickelt [17]. Der Ablauf erfolgt immer nach einem ähnlichen Schema. Zunächst wird den Studierenden eine Verständnisfrage mit multiple-Choice Antwortmöglichkeiten präsentiert. In (idealerweise anonymen) Abstimmungen müssen sich alle Studierenden auf eine Antwort festlegen und können sich nicht darauf verlassen, dass andere Studierende die Frage beantworten und die Vorlesung fortgesetzt wird. Nach der Abstimmung wird das Ergebnis dem Kurs präsentiert, die Lösung aber noch nicht verraten, denn nun schließt sich das Herzstück der Peer Instruction an: Die Studierenden sollen in Kleingruppen ihre Sitznachbarn von der gewählten Antwort argumentativ überzeugen. Dabei erhalten sie automatisch Feedback von ihren „Peers“ oder lernen ihre Argumente logisch und überzeugend vorzutragen. Nach der Peer Diskussion erfolgt eine zweite Abstimmung. Waren das Niveau der Frage und die Diskussionszeit angemessen, ist hier ein Trend zur richtigen Antwort zu erkennen. Im Anschluss kann die Lehrkraft die falschen Antworten einordnen und auf Fragen eingehen, falls diese vorhanden sind.

Für Einführungsveranstaltungen an Hochschulen oder Universitäten gibt es große Sammlungen an Verständnisfragen [17]. Für fortgeschrittenere Kurse wie die Theoretische Physik gibt es im deutschsprachigen Raum jedoch kaum bis keine Fragen. In dieser Arbeit werden verschiedene Fragentypen für die Theoretische Physik entwickelt und getestet. Alle entwickelten Fragen lassen sich grob in folgende Kategorien einteilen:

- Fachbegriffe zu klären/definieren/konkretisieren
- Physikalische Konzepte
- Darstellungen/Graphen verstehen/interpretieren
- Schülervorstellungen beurteilen
- Übersetzung Physik-Mathematik
- Interpretation von Formeln
- Abschätzungen aus Formeln

In ihrer ursprünglichen Form verzichtet man in den Verständnisfragen auf Formeln. Für die Theoretische Physik ist ein adäquater Umgang mit Formeln und dem Formalismus aber essentiell. Deshalb befasst sich diese Arbeit auch mit der Entwicklung neuer Verständnisfragen zum Formelverständnis. Beobachtungen zum Lagrange-Formalismus zweiter Art zeigen z.B., dass viele Studierende weniger Schwierigkeiten mit den Rechenschritten haben, dafür aber mit dem Aufstellen mathematischer Formulierungen der Zwangsbedingungen oder generalisierten Koordinaten. Mit der Peer Instruction können in kurzer Zeit verschiedene Szenarien von Studierenden behandelt und durchdacht werden. Zusätzlich erhalten sie sofort Feedback und können anschließend selbstständig weitere Aufgaben komplett lösen.

Widerspruch in 90 Minuten - Eine Lehreinheit über lokale versteckte Parameter

Das Kapitel 4 stellt eine didaktische Rekonstruktion für das Thema der quantenmechanischen Verschränkung und dem Ausschluss von lokalen versteckten Parametern dar. In der fachlichen Klärung werden die historisch wichtigen Schritte [120–124], die zur Widerlegung von versteckten lokalen Parametern geführt haben, auf die wesentlichen Elemente reduziert. All diese Elemente können auch mit einem von L. Hardy vorgeschlagenen Experiment [127] beschrieben und erklärt werden.

Für einen Lehrkontext bietet dieses Experiment von Hardy aber einige Vorteile. Beobachtungen zeigen, dass Lernende mit der komplexen, langen Herleitung der Bell'schen Ungleichung und mit der Ungleichung selbst Schwierigkeiten haben. Mit Hardys Experiment können lokale versteckte Parameter über einen Widerspruch in einer Fallunterscheidung von nur vier Fällen als unvereinbar mit der Quantenphysik identifiziert werden.

Das Ziel dieser Lehreinheit ist folglich, den Widerspruch in dieser Fallunterscheidung verständlich zu machen. Die vorher beschriebenen, kognitiv aktivierenden Lehrmethoden werden im Rahmen der didaktischen Rekonstruktion so in diese Lehreinheit eingebaut, dass möglichst alle Studierende alle logischen Schritte in der Argumentationskette selbständig durchdenken und verstehen.

In einem ersten Abschnitt werden die Eigenschaften des quantenmechanischen Spins besprochen, um wichtige Erkenntnisse der quantenmechanischen Messung und des Formalismus zu festigen, die für die spätere Argumentation benötigt werden. Folgende Lernziele werden dabei adressiert:

1. Eigenschaften des Spins im Stern-Gerlach Experiment

2. Definition und mathematische Beschreibung von Zuständen und Observablen

3. Der Unterschied zwischen Ortsraum und Hilbertraum

4. Was passiert bei quantenmechanischen Messungen von Alice und Bob?

Diese Lernziele sollen mit der Verbildlichung von Formeln und der Peer Instruction erreicht werden.

Der zweite und wesentliche Teil dieser Einheit behandelt die Verschränkung und versteckte lokale Parameter. Die Lernziele dafür lauten:

5. Definitionen von Realität und Lokalität verstehen

6. Verschränkung am Beispiel von Bell-Zuständen verstehen

7. Definition von versteckten lokalen Parametern verstehen

8. Welche Messergebnisse würde eine Quantenmechanik mit versteckten lokalen Parametern vorbestimmen?

9. Was sind die realen Messergebnisse der Quantenphysik?

Für das Erreichen der Lernziele 5-6 werden in dieser Dissertation neue Arbeitsdefinitionen für angehende Lehrkräfte und den Schulkontext formuliert. In Kleingruppen sollen diese Definitionen diskutiert und verstanden werden. Für Lernziel 8 wird nach Hardys Experiment von den Studierenden eine Quantenmechanik mit versteckten lokalen Parametern durchgespielt. Daneben werden die realen quantenmechanischen Messergebnisse in der Peer Instruction vom Kurs bestimmt. Damit jeder einzelne Schritt in der Argumentationskette von den Studierenden nachvollzogen werden kann, können die mathematischen Umformulierungen des Ausgangszustandes des Experiments mit Hilfe von „worked exmaples“ als Hausaufgabe nachgerechnet werden.

Beobachtungen erster Tests haben gezeigt, dass das Durchspielen einer Quantenmechanik mit versteckten lokalen Parametern vor allem in der Fallunterscheidung mit der langen Argumentationskette Probleme bereiten kann. Um Studierende oder (nach einer weiteren Reduktion) Schülerinnen und Schüler zu unterstützen, kann diese Fallunterscheidung durch ein Analogmodel unterstützt werden. In diesem Analogexperiment werden die Spinteilchen von Kugeln repräsentiert. Der „versteckte“ lokale Parameter der Kugel ist folglich ihre Fähigkeit magnetisierbar zu sein und damit in ihrer Bahn abgelenkt zu werden oder eben nicht.

Mathematik Wiederholungskurse für die Theoretische Physik

In der Lehramtsausbildung kann, neben den allgemeinen Herausforderungen der Theoretischen Physik, die gewählte Fachkombination weitere Hürden verursachen. Aufgrund der besonderen Relevanz der mathematischen Formalismen wird die Fachkombination Physik-Mathematik empfohlen. Dies entspricht oftmals aber nicht dem Wunsch der Studierenden. Wird eine andere Fachkombination gewählt, erhalten Studierende eine wesentlich verkürzte mathematische Ausbildung im Vergleich zu den Fachstudierenden und den Studierenden mit Mathematik als Zweitfach.

An den Universitäten in Stuttgart und Jena reduziert sich die mathematische Bildung in einem solchen Fall auf das Modul „Mathematische Methoden der Physik“, das im ersten bzw. im ersten und zweiten Semester angeboten wird. Bei diesen Modulen handelt es sich um stark komprimierte Einführungen um Rechenmethoden einzuüben. Im Fachstudium werden diese Module durch weitere Mathematikveranstaltungen unterstützt oder später ergänzt, in denen die mathematischen Inhalte vertieft und ausführlicher behandelt werden. Damit werden diese Inhalte später im Studium aufgefrischt und wiederholt. Im Lehramtsstudium ohne das Fach Mathematik fällt eine solche Wiederholung in die Eigenverantwortung der Studierenden.

In dieser Arbeit werden Blockkurse vorgestellt, die entwickelt wurden, um Studierende bei dieser Wiederholung systematisch zu unterstützen. Inhaltlich beziehen sich diese Kurse auf die Mathematik der klassischen Mechanik und der Elektrodynamik. In einer fachlichen Klärung werden zunächst die relevanten mathematischen Inhalte für die Physik bestimmt. Die Kurse bestehen aus einem 90-minütigen Vorlesungsblock mit anschließender Übungszeit für die Studierenden. Der Schwerpunkt der Vorlesung liegt auf den Rechenmethoden und der Versprachlichung von Formeln. Die Übungen der Rechenmethoden werden als „worked examples“ aufgebaut, damit die Studierenden sich möglichst selbstständig die Inhalte aneignen. In Kapitel 5 werden die Lernziele der einzelnen Kurstage ausgeführt, bevor am Beispiel der Differentialoperatoren der Aufbau und das Design der Vorlesung und Übungen vorgestellt werden.

Theoretische Physik im schulischen Kontext

Für die Gestaltung kognitiv aktivierender Physiklehre haben sich die folgenden drei Bereiche des professionellen Wissens als relevant für Lehrkräfte erwiesen: Fachwissen, fachdidaktisches Wissen und pädagogisches Wissen (vgl. [139]). Darüber hinaus scheinen Anpassungsfähigkeit und Unterstützung entscheidend für die Unterrichtsqualität im Allgemeinen und für die Vermittlung von Fachinhalten zu sein. Eine Anpassung an die situativen Bedürfnisse der Schülerinnen und Schüler ist jedoch nur dann möglich, wenn die Lehrkraft den wissenschaftlichen Hintergrund und die pädagogischen Grundlagen so gut kennt, dass Beispiel, Erklärungsansatz, Analogie oder Modell und didaktische Methode situativ verändert werden können. Die hier entwickelten Seminare sollen einen Beitrag dazu leisten, wie das Fachwissen und das fachdidaktische Wissen im Rahmen der universitären Ausbildung durch Verknüpfung gezielt gefördert werden kann.

Eine Verknüpfung dieser Wissensbereiche ist vielversprechend, denn dies bietet die Möglichkeit, die Relevanz des universitären Wissens für den späteren Beruf zu erörtern, um damit die Motivation bei Studierenden zu erhöhen. Außerdem deuten Untersuchungen darauf hin, dass solche Verknüpfungen die Kompetenzen angehender Lehrkräfte in mehreren Wissensbereichen (universitäres und schulisches Fachwissen sowie fachdidaktisches Wissen) fördern können, z.B. bei der Erklärleistung [13], dem fachdidaktischen Wissen [9, 139] und dem Fachwissen [9, 145] (hier könnten eventuell sogar die schwächeren Studierenden in besonderem Maße profitieren [31, 32]).

Für die bereits behandelten Lehrmethoden (worked examples, Versprachlichung von Formeln, didaktische Rekonstruktion und Peer Instruction) werden entwickelte Seminare vorgestellt. Das Grundschema ist dabei immer ähnlich. Zu Beginn gibt es eine kleine Einführung in die fachdidaktische Methode. Anschließend wird ein Thema aus der Theoretischen Physik mit dieser Methode von den Studierenden selbst behandelt. Zum Abschluss wird das behandelte Thema mit der Schulphysik in Verbindung gesetzt. Wie im vorherigen Kapitel beziehen sich die Seminare auf die Theorie-Vorlesungen der klassischen Mechanik und der Elektrodynamik.

Das Beispiel der Versprachlichung von Formeln zum Thema der Lagrange-Multiplikatoren wird in dieser Arbeit detailliert beschrieben. Zunächst sollen die Bewegungsgleichung einer eingeschränkten Bewegung versprachlicht und die Zwangskräfte verbildlicht werden. Anschließend können die Zwangskräfte im Kontext der Kräfteaddition und Kräftezerlegung an der schiefen Ebene diskutiert werden bzw. die Reduktionen, die für die Schulphysik dazu vorgenommen werden. Abschließend werden alle entwickelten Seminare und ihre jeweiligen Lernziele in Kapitel 6 aufgelistet.

Fazit

Kognitiv aktivierende Lehrmethoden wie das Lernen mit Lösungsbeispielen nach der „cognitive load theory“, die Versprachlichung und Verbildlichung von Formeln, die didaktische Rekonstruktion und die Peer Instruction lassen sich in verschiedenen Varianten sehr gut auch in der Theoretischen Physik umsetzen. Alle vorgestellten Umsetzungsformen werden von Studierenden mit Interesse und Engagement aufgenommen, sofern sie an das Leistungsniveau der Studierenden angepasst sind und diese nicht überfordern. Alle vorgestellten Varianten können aber dahingehend angepasst werden. Die Methoden eignen sich ebenfalls, um komplexe und abstrakte physikalische Phänomene wie die quantenmechanische Verschränkung und den Ausschluss von versteckten lokalen Parametern zu behandeln.

Auch an das Lehramt angepasste Varianten wie die Seminare zur Theoretischen Physik im schulischen Kontext stoßen auf reges Interesse bei den Studierenden und sind nach ersten Tests konstruktiv in die Lehre einzubinden. Das entwickelte Material kann anderen Dozierenden eindeutig zur Übernahme und Weiterentwicklung oder Anpassung an die eigene Lehre empfohlen werden.

Die Materialien, die im Rahmen dieser Dissertation entstanden sind, können online unter dem Link https://doi.org/10.18419/darus-3972 aufgerufen und heruntergeladen werden.

Erklärung der Selbstständigkeit

Ich erkläre, dass ich diese Dissertation, abgesehen von den ausdrücklich bezeichneten Hilfsmitteln und den Ratschlägen von den jeweils namentlich aufgeführten Personen, selbstständig verfasst habe.

Stuttgart, den 05. September 2023

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