Comparison of Local and Global Approaches for Parametric Model Order Reduction for Systems with Distributed Moving Loads

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Abstract

In order to ensure a numerically efficient simulation of elastic multibody systems, model order reduction has to be employed for reducing the complexity of the underlying Finite-Element-Models. Elastic multibody systems with moving loads can be modeled as parameter dependent systems for which methods from parametric model order reduction have to be applied. In this contribution, two local and a global approach from parametric model order reduction are investigated. A comparison is made with respect to their approximation quality in frequency domain and time domain and their numerical cost in transient simulations. As a numerical example, a linear drive with a distributed moving load is considered.

Keywords: Parametric Model Order Reduction, Moving Load, Elastic Multibody System, Multiple Inputs and Outputs.

1. Introduction

The increasing demand for highly dynamic but nevertheless energy-efficient machines leads to the development of more and more lightweight machines. However, the reduction of the moved masses leads to a reduced stiffness of the structures inducing additional vibrations. In order to rate the vibration behavior already during the product development, these machines can be modeled as elastic multibody systems where flexible bodies are incorporated in the equation of motion of rigid multibody systems. Very often, the flexible bodies are discretized using the Finite-Element-Method. Due to fine spatial discretization even models of simple components can exceed hundreds of thousands degrees of freedom. Therefore, an essential step for a numerically efficient simulation of elastic multibody systems is the reduction of the elastic degrees of freedom by modern model order reduction (MOR). In the past years, modern input-output-based MOR-methods have been successfully applied in the context of elastic multibody systems for elastic bodies with constant system matrices, see [1]. However, in many engineering applications the system matrices are not constant due to sliding components or material removal. One approach is to treat the system matrices as time dependent and to apply reduction methods for linear time-variant systems as in [2]. Another approach is to treat the system matrices as parameter dependent and to apply reduction methods for parametric systems. An overview about methods for parametric model order reduction (PMOR) can be found in [3]. In this contribution PMOR methods for systems with spatially distributed moving loads are investigated. Two so-called local approaches from [4] and [5] and a global approach from [6] are investigated and compared. In local approaches different individually reduced models are interpolated which results in small reduced order parametric models. In contrast, global approaches try to find one representative projection matrix capturing all the parameter dependent dynamics tending to deliver rather large reduced order models.

The novel contribution of this paper is a comparison of quite different global and local approaches for parametric model order reduction for systems with distributed moving loads with many inputs and outputs. The paper is structured as follows. First, the theoretical background for elastic multibody systems and linear MOR is presented in Sec. 2. An emphasis lies on the reduction of systems with many inputs and outputs. In Sec. 3, the two local and the global approaches for PMOR are depicted. As a numerical example a linear drive is investigated which is presented in Sec. 4. Then, the three approaches are compared with regard to their approximation quality in frequency domain, in time domain, and their numerical efficiency. The comparison is followed by the conclusion.

2. Background

2.1. Elastic Multibody Systems

In a first step, the elastic continuum describing the elastic body is spatially discretized with the Finite-Element-Method resulting in a system of ordinary differential equations

$$\mathbf{M}\cdot\ddot{\mathbf{q}}(t)+\mathbf{D}\cdot\dot{\mathbf{q}}(t)+\mathbf{K}\cdot\mathbf{q}(t)=\mathbf{F}(t). \tag{1}$$

Here, $\mathbf{q}(t) \in \mathbb{R}^N$ describes the nodal displacements, $\mathbf{F}(t)$ the acting forces on the elastic structure and $\mathbf{M}$, $\mathbf{D}$, $\mathbf{K} \in \mathbb{R}^{N\times N}$ are the mass, damping and stiffness matrices, respectively. Using the Floating Frame of Reference formulation [7] the small linear elastic deformations of the elastic bodies can be incorporated in the equation of motion of a multibody system. The nonlinear equation of motion then reads

$$\begin{bmatrix} m\mathbf{E} & m\tilde{\mathbf{c}}^T(\mathbf{q}) & \mathbf{C}_t^T \\ m\tilde{\mathbf{c}}(\mathbf{q}) & \mathbf{J}(\mathbf{q}) & \mathbf{C}_r^T(\mathbf{q}) \\ \mathbf{C}_t & \mathbf{C}_r(\mathbf{q}) & \mathbf{M} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{a}(t) \\ \boldsymbol{\alpha}(t) \\ \ddot{\mathbf{q}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{h}_t(t) \\ \mathbf{h}_r(t) \\ \mathbf{h}_e(t) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ -\mathbf{K}\cdot\mathbf{q}(t) - \mathbf{D}\cdot\dot{\mathbf{q}}(t) \end{bmatrix} \tag{2}$$

for a single elastic body. The translational and rotational rigid body accelerations are described by $\mathbf{a}(t)$ and $\boldsymbol{\alpha}(t)$. The global mass matrix contains the mass of the elastic body m, its inertia $\mathbf{J}(\mathbf{q})$, and its center of mass $\tilde{\mathbf{c}}(\mathbf{q})$. The terms $\mathbf{C}_t$ and $\mathbf{C}_r(\mathbf{q})$ describe the coupling between the rigid body movement and the elastic deformations. The forces and moments acting on the body are described by $\mathbf{h}_t(t)$, $\mathbf{h}_r(t)$ and $\mathbf{h}_e(t)$. A detailed description of the Floating Frame of Reference formulation can be found in [7].

2.2. Model Order Reduction in EMBS for Systems with many Inputs and Outputs

Due to a fine spatial discretization the number of elastic degrees of freedom N in Eq. (1) can easily exceed hundreds of thousands. Therefore, evaluations of Eq. (2) in transient simulations become numerically expensive. Model Order Reduction seeks to replace a high dimensional full order model as in Eq. (1) by a reduced order model preserving the input-output behavior as good as possible. For that, the linear-elastic parts of Eq. (2) are regarded as a linear time-invariant second order system with

$$\begin{aligned} \mathbf{M}\cdot\ddot{\mathbf{q}}(t) + \mathbf{D}\cdot\dot{\mathbf{q}}(t) + \mathbf{K}\cdot\mathbf{q}(t) &= \mathbf{B}\cdot\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\cdot\mathbf{q}(t). \end{aligned} \tag{3}$$

Here, $\mathbf{B} \in \mathbb{R}^{N\times l}$ and $\mathbf{C} \in \mathbb{R}^{m\times N}$ are the input and output matrices, and $\mathbf{u}(t) \in \mathbb{R}^l$ and $\mathbf{y}(t) \in \mathbb{R}^m$ the system input and system output. The solution of the full order model is then approximated with $\mathbf{q} \approx \mathbf{V}\cdot\bar{\mathbf{q}}$ in a low dimensional subspace $\mathscr{V}$ of order n spanned by the columns of the projection matrix $\mathbf{V} \in \mathbb{R}^{N\times n}$. Plugging this approximation into Eq. (3) and left multiplying with $\mathbf{W}^T$ yields the reduced order model

$$\begin{aligned} \bar{\mathbf{M}}\cdot\ddot{\bar{\mathbf{q}}}(t) + \bar{\mathbf{D}}\cdot\dot{\bar{\mathbf{q}}}(t) + \bar{\mathbf{K}}\cdot\bar{\mathbf{q}}(t) &= \bar{\mathbf{B}}\cdot\mathbf{u}(t) \\ \bar{\mathbf{y}}(t) &= \bar{\mathbf{C}}\cdot\bar{\mathbf{q}}(t) \end{aligned} \tag{4}$$

with the reduced system matrices

$$\begin{gathered} \bar{\mathbf{M}} = \mathbf{W}^T\cdot\mathbf{M}\cdot\mathbf{V}, \quad \bar{\mathbf{D}} = \mathbf{W}^T\cdot\mathbf{D}\cdot\mathbf{V}, \quad \bar{\mathbf{K}} = \mathbf{W}^T\cdot\mathbf{K}\cdot\mathbf{V}, \\ \bar{\mathbf{B}} = \mathbf{W}^T\cdot\mathbf{B}, \quad \bar{\mathbf{C}} = \mathbf{C}\cdot\mathbf{V}. \end{gathered} \tag{5}$$

The projection matrices $\mathbf{W}$ and $\mathbf{V}$ should be chosen such that the error in time domain $\|\mathbf{y}(t) - \bar{\mathbf{y}}(t)\|$ or in frequency domain $\|\mathbf{H}(s) - \bar{\mathbf{H}}(s)\|$ become small, where

$$\mathbf{H}(s) = \mathbf{C}\cdot\left(s^2\mathbf{M} + s\mathbf{D} + \mathbf{K}\right)^{-1}\cdot\mathbf{B}, \quad \mathbf{H} \in \mathbb{C}^{m\times l} \tag{6}$$

is the Laplace transform of Eq. (3). The projection matrices $\mathbf{W}$ and $\mathbf{V}$ can be obtained by different reduction methods, as modal truncation, Krylov subspace methods, or reduction with Gramian matrices. A detailed description of different model order reduction methods in elastic multibody systems can be found in [1]. Here, only the Component Mode Synthesis and Tangential Interpolation are presented since they will be used later.

2.3. Component Mode Synthesis

The basic idea of the Component Mode Synthesis (CMS) is, to combine different types of ansatz functions in the projection matrices to preserve certain properties of the full order model in the reduced order model. Very often, eigenmodes calculated by the conservative eigenvalue problem

$$\left(-\omega_{r,i}^2\mathbf{M} + \mathbf{K}\right)\cdot\boldsymbol{\Phi}_{\mathrm{r,i}} = \mathbf{0}, \quad i = 1,\ldots,N \tag{7}$$

are combined with correction modes. These additional correction modes can ensure for example an exact static solution, or an exact frequency response at certain frequencies. An overview about different types of correction modes can be

found in [8]. In this work, eigenmodes are combined with correction modes obtained by the static response due to unit forces

$$\mathbf{V}_{\text{corr}} = \mathbf{K}^{-1} \cdot \mathbf{B}. \tag{8}$$

This ensures an exact static solution of the reduced model, thus $\mathbf{H}(0) = \bar{\mathbf{H}}(0)$. The correction modes are then combined with n_{eig} eigenmodes to the projection matrix

$$\mathbf{V} = [\boldsymbol{\Phi}_{r,i} \;\; \mathbf{V}_{\text{corr}}] \in N \times n, \quad i = 1, \ldots, n_{\text{eig}}, \quad n = n_{\text{eig}} + l. \tag{9}$$

The number of correction modes is fixed to the number of columns in $\mathbf{B}$ which is equal to the number of system inputs. The number of kept eigenmodes j can be used to control the approximation quality. In comparison to pure modal truncation this CMS approach yields better results in the lower frequency range which is normally of special interest in the context of EMBS. A major drawback of this approach is, that for systems with many inputs the order of the reduced model may become large since one correction mode has to be considered for every column in $\mathbf{B}$.

2.4. Tangential Interpolation

The transfer function of Eq. (6) of the full order model can be written as a power series

$$\mathbf{H}(s) = \sum_{j=0}^{\infty} -\mathbf{T}_j^{\sigma_k} s = \sum_{j=0}^{\infty} -\frac{1}{j!} \frac{\partial^j \mathbf{H}(s)}{\partial s^j} s^j, \tag{10}$$

where $\mathbf{T}_j^{\sigma_k}$ are called the moments of j-th order around σ_k, [1]. The basic idea of interpolation-based or moment-matching approaches is to match moments of the reduced order model with moments of the full order model up to a chosen order. This ensures that the values and the derivatives of the transfer function of the reduced order model is in agreement with the transfer function of the full order model up to a desired order, thus

$$\frac{\partial^j \mathbf{H}(\sigma_k)}{\partial s^j} = \frac{\partial^j \bar{\mathbf{H}}(\sigma_k)}{\partial s^j}. \tag{11}$$

The moments $\mathbf{T}_j^{\sigma_k}$ are not calculated explicitly, but with a numerically stable Arnoldi algorithm, see [9]. Choosing the projection matrices as

$$\text{span}(\mathbf{V}) = \text{span}\left[\left(\sigma_1^2 \mathbf{M} + \sigma_1 \mathbf{D} + \mathbf{K}\right)^{-1} \cdot \mathbf{B}, \ldots, \left(\sigma_v^2 \mathbf{M} + \sigma_v \mathbf{D} + \mathbf{K}\right)^{-1} \cdot \mathbf{B} \right] \tag{12}$$

$$\text{span}(\mathbf{W}) = \text{span}\left[\left(\mathbf{C} \cdot \left(\mu_1^2 \mathbf{M} + \mu_1 \mathbf{D} + \mathbf{K}\right)^{-1}\right)^T, \ldots, \left(\mathbf{C} \cdot \left(\mu_v^2 \mathbf{M} + \mu_v \mathbf{D} + \mathbf{K}\right)^{-1}\right)^T \right] \tag{13}$$

with $\sigma_k = \mu_k$ for $k = 1, \ldots, v$ ensures

$$\mathbf{H}(\sigma_k) = \bar{\mathbf{H}}(\sigma_k) \quad \text{and} \quad \frac{\partial \mathbf{H}(\sigma_k)}{\partial s} = \frac{\partial \bar{\mathbf{H}}(\sigma_k)}{\partial s} \quad \text{for} \quad k = 1, \ldots, v. \tag{14}$$

In this basic form the order of the reduced model can only be a multiple of the number l of columns in $\mathbf{B}$ or the number m of columns in $\mathbf{C}$. For systems with many inputs and outputs this results in a rather large order of the reduced model. In [10] an extension called tangential interpolation is suggested in which additional tangential direction $\mathbf{b}_k \in \mathbb{C}^m$ and $\mathbf{c}_k \in \mathbb{C}^l$ are introduced. The projection matrices then become

$$\text{span}(\mathbf{V}) = \text{span}\left[\left(\sigma_1^2 \mathbf{M} + \sigma_1 \mathbf{D} + \mathbf{K}\right)^{-1} \cdot \mathbf{B} \cdot \mathbf{b}_1, \ldots, \left(\sigma_v^2 \mathbf{M} + \sigma_v \mathbf{D} + \mathbf{K}\right)^{-1} \cdot \mathbf{B} \cdot \mathbf{b}_v \right] \tag{15}$$

$$\text{span}(\mathbf{W}) = \text{span}\left[\left(\mathbf{c}_1^T \cdot \mathbf{C} \cdot \left(\mu_1^2 \mathbf{M} + \mu_1 \mathbf{D} + \mathbf{K}\right)^{-1}\right)^T, \ldots, \left(\mathbf{c}_v^T \cdot \mathbf{C} \cdot \left(\mu_v^2 \mathbf{M} + \mu_v \mathbf{D} + \mathbf{K}\right)^{-1}\right)^T \right] \tag{16}$$

where the order of the reduced model n is independent of the number of system inputs and outputs. However, the transfer function of the full order model is then only interpolated along the tangential directions $\mathbf{b}_i$ and $\mathbf{c}_i$ with

$$\mathbf{H}(\sigma_k) \cdot \mathbf{b}_k = \bar{\mathbf{H}}(\sigma_k) \cdot \mathbf{b}_k \quad \text{and} \quad \mathbf{c}_k^T \cdot \mathbf{H}(\mu_k) = \mathbf{c}_k^T \cdot \bar{\mathbf{H}}(\mu_k). \tag{17}$$

Choosing additionally $\sigma_k = \mu_k$ for $k = 1, \ldots, v$ ensures

$$\mathbf{c}_k^T \cdot \frac{\partial \mathbf{H}(\sigma_k)}{\partial s} \cdot \mathbf{b}_k = \mathbf{c}_k^T \cdot \frac{\partial \bar{\mathbf{H}}(\sigma_k)}{\partial s} \cdot \mathbf{b}_k \quad \text{for} \quad k = 1, \ldots, v. \tag{18}$$

3. Parametric Model Order Reduction for Moving Loads

3.1. General

In many engineering applications load positions are not constant, but parameter dependent. Such kinds of systems arise for example in the simulation of gear trains, cranes, or sliding components. The equation of motion for such systems then becomes

$$\begin{aligned} \mathbf{M}\cdot\ddot{\mathbf{q}}(t)+\mathbf{D}\cdot\dot{\mathbf{q}}(t)+\mathbf{K}\cdot\mathbf{q}(t) &= \mathbf{B}(p)\cdot\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}(p)\cdot\mathbf{q}(t), \end{aligned} \tag{19}$$

where in this contribution only the special case with $\mathbf{C}(p)=\mathbf{B}^T(p)$ is investigated. Since the input and output matrices of such systems are not constant anymore, applying standard input-output-based MOR techniques yields only very poor results. In the past years advances have been made in parameter preserving reduction of parameter dependent systems. A summary of methods suitable for parametric model order reduction in elastic multibody systems can be found in [11]. In [3] a distinction between local and global approaches is suggested. In local approaches the parameter dependent system is reduced for different parameter values individually, then the reduced system matrices are interpolated for arbitrary parameter values. In contrast, global approaches try to find one representative subspace capturing the entire parameter dependent dynamics of the parameter dependent system. In this contribution only orthogonal projection with $\mathbf{W}=\mathbf{V}$ is investigated.

3.2. Local Parametric Model Order Reduction

In local parametric model order reduction d parameter sample points $p_i,\ i=1,\ldots,d,\ p_i\in\mathscr{P}$ are defined. Then, the parametric system from Eq. (19) is evaluated for all parameter samples p_i and reduced to an individual subsystem. The reduction method can be chosen arbitrarily, but the reduced order n has to be equal for all reduced subsystems. This delivers a set of projection matrices $\mathbf{V}_i(p_i)$ and a set of corresponding reduced system matrices

$$\begin{aligned} \bar{\mathbf{M}}(p_i)=\mathbf{V}_i^T(p_i)\cdot\mathbf{M}\cdot\mathbf{V}_i(p_i),\quad \bar{\mathbf{D}}(p_i)=\mathbf{V}_i^T(p_i)\cdot\mathbf{D}\cdot\mathbf{V}_i(p_i),\quad \bar{\mathbf{K}}(p_i)=\mathbf{V}_i^T(p_i)\cdot\mathbf{K}\cdot\mathbf{V}_i(p_i), \\ \bar{\mathbf{B}}(p_i)=\mathbf{V}_i^T(p_i)\cdot\mathbf{B}(p_i),\quad \bar{\mathbf{C}}(p_i)=\mathbf{C}(p_i)\cdot\mathbf{V}_i(p_i). \end{aligned} \tag{20}$$

Since the resulting set of reduced state vectors $\bar{\mathbf{q}}_i$ lies in different subspaces $\mathscr{V}_i$ a direct interpolation between the reduced system matrices from Eq. (20) is not meaningful.

In [4] an additional coordinate transformation $\bar{\mathbf{q}}_i=\mathbf{T}_i\cdot\tilde{\mathbf{q}}$ is suggested to describe all subsystems within the same reduced coordinates $\tilde{\mathbf{q}}$. For that, all individual projection matrices are concatenated into one projection matrix $\mathbf{V}_{\text{all}}=[\mathbf{V}_1,\ldots,\mathbf{V}_d]$. A reference projection matrix can be obtained with

$$\mathbf{U}\cdot\Sigma\cdot\mathbf{N}^T=\mathbf{V}_{\text{all}} \tag{21}$$

$$\mathbf{V}_{\text{ref}}=\mathbf{U}(:,1:n) \tag{22}$$

from the first n left singular vectors of $\mathbf{V}_{\text{all}}$. The additional transformation matrices $\mathbf{T}_i$ are then

$$\mathbf{T}_i=\left(\mathbf{V}_{\text{ref}}^T\cdot\mathbf{V}_i\right)^{-1} \tag{23}$$

and the adjusted projection matrices become

$$\tilde{\mathbf{V}}_i=\mathbf{V}_i\cdot\mathbf{T}_i. \tag{24}$$

Finally, the reduced system matrices described in the same set of coordinates are

$$\begin{aligned} \tilde{\mathbf{M}}_i=\tilde{\mathbf{V}}_i^T\cdot\mathbf{M}\cdot\tilde{\mathbf{V}}_i,\quad \tilde{\mathbf{D}}_i=\tilde{\mathbf{V}}_i^T\cdot\mathbf{D}\cdot\tilde{\mathbf{V}}_i,\quad \tilde{\mathbf{K}}_i=\tilde{\mathbf{V}}_i^T\cdot\mathbf{K}\cdot\tilde{\mathbf{V}}_i, \\ \tilde{\mathbf{B}}_i=\tilde{\mathbf{V}}_i^T\cdot\mathbf{B},\quad \tilde{\mathbf{C}}_i=\mathbf{C}\cdot\tilde{\mathbf{V}}_i \end{aligned} \tag{25}$$

and can be interpolated with the weight functions $w_i(p)$ for arbitrary parameters as

$$\begin{aligned} \tilde{\mathbf{M}}(p)=\sum_{i=1}^{d}w_i(p)\tilde{\mathbf{M}}_i,\quad \tilde{\mathbf{D}}(p)=\sum_{i=1}^{d}w_i(p)\tilde{\mathbf{D}}_i,\quad \tilde{\mathbf{K}}(p)=\sum_{i=1}^{d}w_i(p)\tilde{\mathbf{K}}_i, \\ \tilde{\mathbf{B}}(p)=\sum_{i=1}^{d}w_i(p)\tilde{\mathbf{B}}_i,\quad \tilde{\mathbf{C}}(p)=\sum_{i=1}^{d}w_i(p)\tilde{\mathbf{C}}_i. \end{aligned} \tag{26}$$

However, even though all system matrices are described within the same set of coordinates, it is not guaranteed that the system matrices have full rank after the interpolation.

In [5] a different transformation is suggested to overcome this problem. Here, one of the individual projection matrices $\mathbf{V}_i$ is selected as reference projection matrix $\mathbf{V}_{\text{ref}}$. The transformation matrices are the solution of the minimization problem

$$\text{find } \mathbf{T}_i \quad \text{such that} \quad \|\mathbf{V}_i \cdot \mathbf{T}_i - \mathbf{V}_{\text{ref}}\|_F^2 \stackrel{!}{=} \min. \tag{27}$$

An analytical solution for this minimization problem is given by $\mathbf{T}_i = \mathbf{U}_i \cdot \mathbf{N}_i^T$, where $\mathbf{U}_i$ and $\mathbf{V}_i$ arise from the singular value decomposition of $\mathbf{U} \cdot \Sigma \cdot \mathbf{N}^T = \mathbf{V}_i^T \cdot \mathbf{V}_{\text{ref}}$. As in [4] the reduced system matrices are obtained with Eqs. (24) and (25). In contrast, in [5] the system matrices are not interpolated directly but on a matrix manifold. First, the matrices $\mathbf{Y}_i$ are mapped with a logarithmic mapping $\mathbf{\Gamma} = \text{Log}_{\mathbf{X}_{\text{ref}}}(\mathbf{Y}_i)$ onto a tangent space $\mathscr{T}_{\text{ref}}\mathscr{M}$. Here, $\mathbf{X}_{\text{ref}}$ describes the system matrices of the reference configuration from the reduction with $\mathbf{V}_{\text{ref}}$. The interpolation of all system matrices takes place on their corresponding tangent spaces and are then mapped back onto their original manifold with the exponential mapping $\mathbf{Y} = \text{Exp}_{\mathbf{X}_{\text{ref}}}(\mathbf{\Gamma})$. The logarithmic mapping can be done already during the reduction process, but the exponential mapping has to be executed in every time step for all system matrices. The advantage of the method proposed in [5] is, that the positive definiteness of the interpolated system matrices can be guaranteed for arbitrary parameters and interpolation methods. A drawback is the numerical cost for the exponential mapping during transient simulations.

Note that in the equation of motion (19) of the parametric full order model only the input and output matrix are parameter dependent. Both approaches from [4] and [5] result in reduced parametric systems of order n where all system matrices are parameter dependent since they are derived from different parameter dependent projection matrices $\mathbf{V}_i(p_i)$.

3.3. Global Parametric Model Order Reduction

Global Parametric Model Order Reduction aims to find one representative projection matrix $\mathbf{V}_{\text{gl}}$ for all the parameter dependent system dynamics. This global projection matrix is constructed as

$$\mathbf{V}_{\text{gl}} = [\mathbf{V}_1, \ldots, \mathbf{V}_i, \ldots, \mathbf{V}_d]. \tag{28}$$

Here, $\mathbf{V}_i$ are local projection matrices obtained by reducing the full order model at the parameter samples p_i with arbitrary reduction methods to the reduced order n_i. With $\mathbf{V}_{\text{gl}}$ the globally reduced order model becomes

$$\begin{aligned} \bar{\mathbf{M}}_{\text{gl}} \cdot \ddot{\bar{\mathbf{q}}}_{\text{gl}}(t) + \bar{\mathbf{D}}_{\text{gl}} \cdot \dot{\bar{\mathbf{q}}}_{\text{gl}}(t) + \bar{\mathbf{K}}_{\text{gl}} \cdot \bar{\mathbf{q}}_{\text{gl}}(t) &= \bar{\mathbf{B}}_{\text{gl}}(p) \cdot \mathbf{u}(t) \\ \bar{\mathbf{y}}_{\text{gl}}(t) &= \bar{\mathbf{C}}_{\text{gl}}^T(p) \cdot \bar{\mathbf{q}}_{\text{gl}}(t) \end{aligned} \tag{29}$$

with the reduced system matrices

$$\begin{gathered} \bar{\mathbf{M}}_{\text{gl}} = \mathbf{V}_{\text{gl}}^T \cdot \mathbf{M} \cdot \mathbf{V}_{\text{gl}}, \quad \bar{\mathbf{D}}_{\text{gl}} = \mathbf{V}_{\text{gl}}^T \cdot \mathbf{D} \cdot \mathbf{V}_{\text{gl}}, \quad \bar{\mathbf{K}}_{\text{gl}} = \mathbf{V}_{\text{gl}}^T \cdot \mathbf{K} \cdot \mathbf{V}_{\text{gl}}, \\ \bar{\mathbf{B}}_{\text{gl}}(p) = \mathbf{V}_{\text{gl}}^T \cdot \mathbf{B}(p), \quad \bar{\mathbf{C}}_{\text{gl}}(p) = \mathbf{C}(p) \cdot \mathbf{V}_{\text{gl}} \end{gathered} \tag{30}$$

of order $n_{\text{gl}} = \sum_1^d n_i$.

As stated before, arbitrary reduction methods can be used to construct the global projection matrix $\mathbf{V}_{\text{gl}}$. However, according to [6] using tangential interpolation for constructing the local projection matrices yields some advantages which will be presented. The following is shown in [6] for first order systems, but it holds true for second order systems as well. Here, only the special case for

$$\mathbf{C}(p) = \mathbf{B}^T(p), \quad \mathbf{M} = const., \quad \mathbf{D} = const., \quad \mathbf{K} = const., \quad \mathbf{W}_i = \mathbf{V}_i \tag{31}$$

is presented. If a local projection matrix is chosen as

$$\text{span}(\mathbf{V}_i) = \text{span}\left(\left(\sigma_i^2 \mathbf{M} + \sigma_i \mathbf{D} + \mathbf{K} \right)^{-1} \cdot \mathbf{B}(p_i) \cdot \mathbf{b}_i \right), \tag{32}$$

then

$$\mathbf{H}(\sigma_i, p_i) \cdot \mathbf{b}_i = \bar{\mathbf{H}}(\sigma_i, p_i) \cdot \mathbf{b}_i = \bar{\mathbf{H}}_{\text{gl}}(\sigma_i, p_i) \cdot \mathbf{b}_i. \tag{33}$$

This means tangentially interpolating the transfer function in a local subsystem at σ_i and p_i ensures that the transfer function at σ_i and p_i will be tangentially interpolated in the globally reduced system, too. Next, if $\mathbf{B}(p)$ is continuously differentiable in a neighborhood of p_i and Eqs. (31) and (32) hold true, then

$$\nabla p \mathbf{b}_i^T \cdot \mathbf{H}(\sigma_i, p_i) \cdot \mathbf{b}_i = \nabla p \mathbf{b}_i^T \cdot \bar{\mathbf{H}}_{\text{gl}}(\sigma_i, p_i) \cdot \mathbf{b}_i \tag{34}$$

meaning the gradient with respect to the parameter value p will be matched in the globally reduced system as well. In contrast to local approaches, the order of globally reduced models n_{gl} is not independent of the number of local subsystems. Therefore, global approaches tend to deliver larger reduced models. However, when applying global approaches for moving load problems only the input and output matrices are parameter dependent while the mass, damping, and stiffness matrices stay constant. This means that there is no need to interpolate $\bar{\mathbf{M}}_{\mathrm{gl}}$, $\bar{\mathbf{D}}_{\mathrm{gl}}$, and $\bar{\mathbf{K}}_{\mathrm{gl}}$ during transient simulations. It is also possible to apply an additional state transformation after the reduction process in order to diagonalize $\bar{\mathbf{M}}_{\mathrm{gl}}$, $\bar{\mathbf{D}}_{\mathrm{gl}}$, and $\bar{\mathbf{K}}_{\mathrm{gl}}$ which reduces the computational effort during time integration significantly.

4. Results

4.1. Investigated Model

As a numerical example a rotating linear drive as shown in Fig. 1 is investigated. The linear drive consists of a carrier made of aluminum in which a steel rack is mounted. A slide driven by a toothed belt can be moved along the rack. Linear drives are typically used in automation industry for pick and place tasks or simple assembling tasks. Since short cycle times are desired during fabrication there is an increasing demand for highly dynamic moving linear drives. In order to rate the vibration behavior already during the product development, a simulation model considering the linear drive as an elastic body is necessary. The geometry of the carrier is discretized using *ANSYS17* resulting in a Finite-Element model with $N = 37359$ degrees of freedom. The pressure forces on the carrier caused by static and dynamic loads are modeled by 20 spatially distributed single forces. The applied forces are rewritten as a product of the parameter dependent input matrix $\mathbf{B}(p(t))$ and the input vector $\mathbf{u}(t)$. The parameter $p(t) \in [0,1]$ describes the position, and therefore, the trajectory of the slide, where for $p = 0$ the slide is at the left and for $p = 1$ at the right end of the carrier, respectively. In the following, the local and global parametric model order reduction approaches from Sec. 3.2 and Sec. 3.3 are applied and compared with respect to their approximation quality in frequency domain and time domain and their numerical efficiency.

4.2. Error Measures

Four error measures are introduced to rate the approximation quality of the reduced order models. The relative approximation error in frequency domain is measured with

$$\varepsilon(p) = \sqrt{\frac{\int_{-\omega_{\max}}^{\omega_{\max}} \|\mathbf{H}(\mathrm{i}\omega, p) - \bar{\mathbf{H}}(\mathrm{i}\omega, p)\|_F^2 \,\mathrm{d}\omega}{\int_{-\omega_{\max}}^{\omega_{\max}} \|\mathbf{H}(\mathrm{i}\omega, p)\|_F^2 \,\mathrm{d}\omega}} \tag{35}$$

over the parameter space. Error measure Eq. (35) is motivated from the $\mathscr{H}_2$ system norm, but is only calculated up to a finite frequency $\omega_{\max} = 2\pi 1000\,\mathrm{rad/s}$. The mean of Eq. (35) is given as

$$\varepsilon_M = \frac{1}{p_{\max} - p_{\min}} \int_{p_{\min}}^{p_{\max}} \varepsilon(p) \,\mathrm{d}p. \tag{36}$$

The relative output error in time domain is measured with

$$\|\mathbf{y} - \bar{\mathbf{y}}\|_2 = \sqrt{\frac{|y_1(t) - \bar{y}_1(t)|^2 + \ldots + |y_m(t) - \bar{y}_m(t)|^2}{|y_1(t)|^2 + \ldots + |y_m(t)|^2}}. \tag{37}$$

The scalar measure for the error in time domain is

$$\|\mathbf{y} - \bar{\mathbf{y}}\|_{L_2[0,T]} = \sqrt{\int_0^T \|\mathbf{y}(t) - \bar{\mathbf{y}}(t)\|_2^2 \,\mathrm{d}t}. \tag{38}$$

4.3. Individual Model Order Reduction

In [12] and [13] it was shown that CMS-based reduction methods are suitable for the individual reduction in local parametric model order reduction. Therefore, the CMS-approach from Sec. 2.3 is used here for the local approaches from [4] and [5] as well. The parameter samples for the subsystems are selected as $p_i \in \{0.0, 0.25, 0.5, 0.75, 1.0\}$, where all local subsystems are reduced to $n = 16$. The local subsystem at $p = 0.5$ is used as reference system for the approach from [5]. Both local approaches from [4] and [5] use cubic spline interpolation to interpolate the reduced system matrices. Different reduction methods have been tested for the individual reduction for the global approach. Tangential interpolation as presented in Sec. 2.4 turned out to yield the most promising results and will therefore be used in the following. The subsystems are reduced to $n = 16$ as well. After concatenating the individual projection matrices and removing the rank deficient columns, the order of the globally reduced model is $n_{\mathrm{gl}} = 77$.

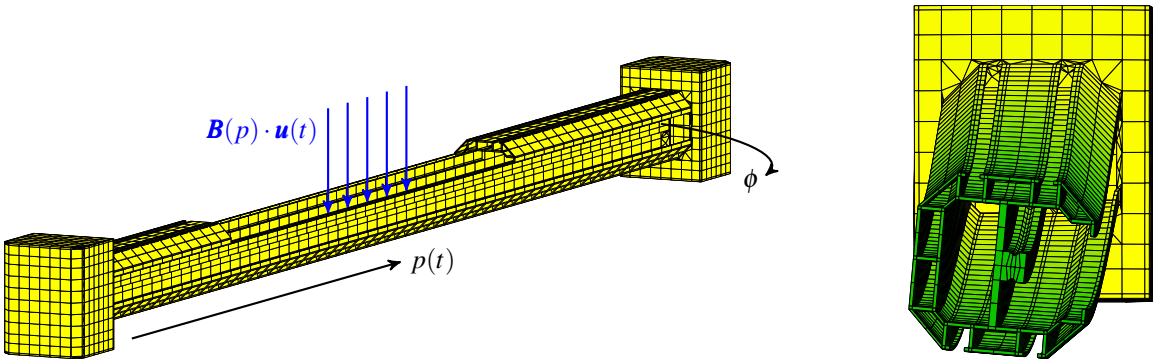

Figure 1: Parameter dependent load position of carrier and cross section of carrier under load, the slide is not shown.

4.4. Approximation Quality in Frequency Domain

The approximation error in frequency domain is shown in Fig. 2 for all approaches. The location of the subsystems is marked by dashed lines. The approach from [4] using direct interpolation shows the smallest error around the subsystems. This makes sense since at these parameter sample points no system matrices have to be interpolated but the reduced system matrices are simply the reduced system matrices from the individual reduction. Between the subsystems the error is increasing significantly. The mean error is $\varepsilon_M = 8.88 \cdot 10^{-2}$. At $p = 0.52$ the error shows a distinct peak. As mentioned in Sec. 3.2 the approach from [4] does not guarantee that the interpolated, reduced system matrices have full rank. In this example, the elastic mass matrix $\tilde{\mathbf{M}}(p)$ of the reduced body becomes singular around $p = 0.52$ causing the error peak. The condition number of the elastic mass matrix is shown in Fig. 4. This rank deficiency has no physical meaning but is only a result of the interpolation. This is clearly a disadvantage of the approach from [4] since the ordinary differential equation from Eq. (2) requires an elastic mass matrix with full rank for time integration. To the authors' knowledge there is no method to predict singular system matrices except for the special case of linear interpolation, where full rank system matrices are guaranteed. However, the approximation quality becomes bad when using linear interpolation which will be shown later. The local approach from [5] using interpolation on matrix manifolds shows a similar error pattern. However, the error is with $\varepsilon_M = 1.19 \cdot 10^{-1}$ slightly larger compared to the error using direct interpolation. An advantage is that there are no error peaks since the interpolation on matrix manifolds guarantees interpolated system matrices with full rank. The global approach shows the smallest approximation errors around the subsystems as well. The error is increasing between the subsystems, but the growth of the error is smaller compared to the local approaches. There are no error peaks and the mean error is with $\varepsilon_M = 9.72 \cdot 10^{-4}$ about hundred times smaller compared to the local approaches.

A convergence study for an increasing number of subsystems is shown in Fig. 3. The left diagram shows the relative mean error ε_M for an increasing number of subsystems. The global approach shows a logarithmic decrease of the mean error. Both local approaches show an decreasing error as well, but the decrease is less distinct. The right diagram of Fig. 3 shows the mean error for an increasing order of the reduced subsystems. The global approach shows a

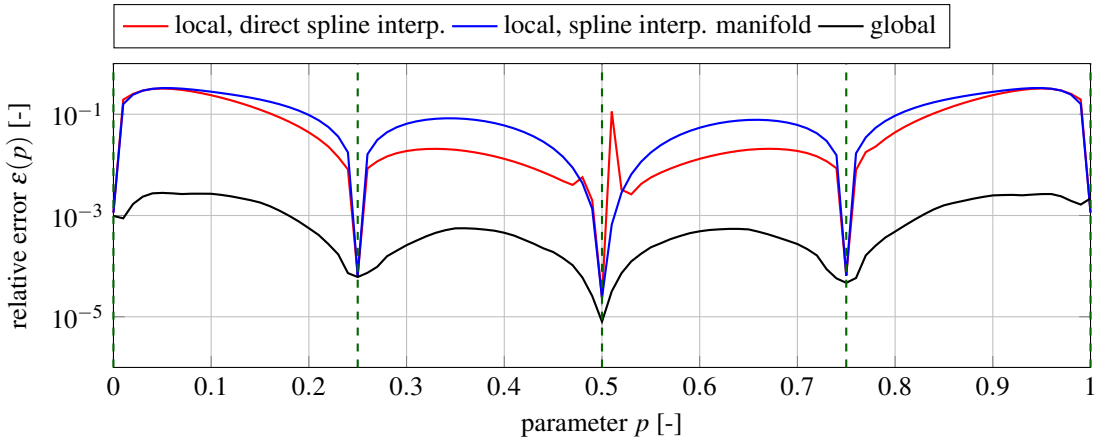

Figure 2: Relative error ε_p in frequency domain for the local approaches from [4] with direct interpolation, from [5] with interpolation on matrix manifolds, and for the global approach.

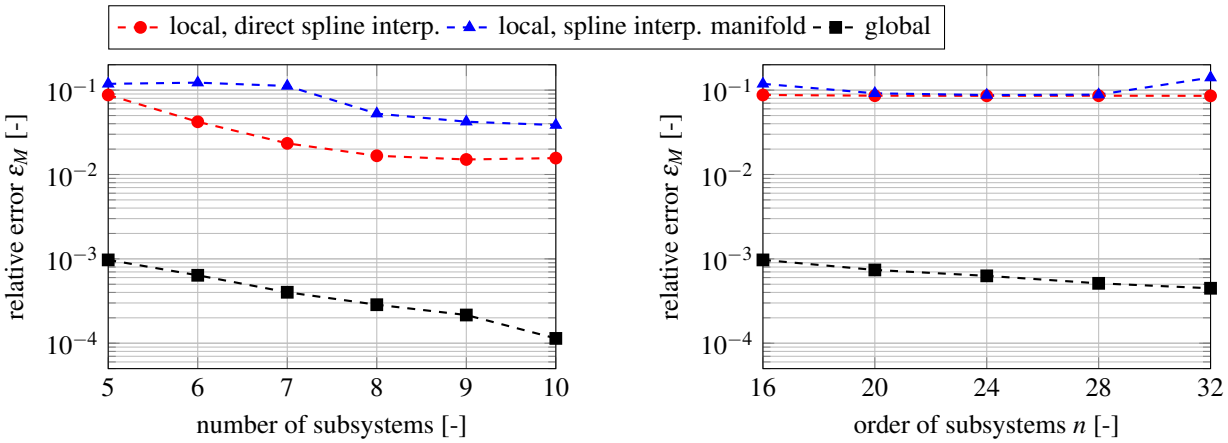

Figure 3: Relative mean error ε_M in frequency domain for increasing number of subsystems and increasing reduced order n.

logarithmic decrease as well, but it is less distinct compared to increasing the number of subsystems. The mean error for the local approach from [4] using direct interpolation does not decrease. The mean error for the approach from [5] using interpolation on matrix manifolds decreases first, but is increasing for a reduced order of $n = 32$. Increasing the order of the reduced models improves the approximation quality at the subsystems. However, the main error of the local approaches is caused by the interpolation which is not positively influenced by a smaller individual reduction error. In the case of the approach from [5] a higher order of the reduced subsystems even delivers reduced system matrices which are less appropriate to interpolate. Summarized, the approximation quality can be improved more efficiently by increasing the number of subsystems rather than increasing the order of the local subsystems both for the local approaches and the global approach. In this example, the global approach shows a better convergence behavior, as well.

4.5. Approximation Quality in Time Domain

To rate the approximation quality in time domain a typical load case is simulated. The trajectory $p(t)$ describes a step-by-step movement of the slide position from the left end to the right end and is shown in Fig. 4. The approximation errors in time domain are shown in Fig. 5 up to $t = 0.8\,\mathrm{s}$.

As mentioned before, cubic spline interpolation cannot be used for the local approach from [4] due to the singular elastic mass matrix. Therefore, linear interpolation has to be used in this transient error analysis. The relative error is around $\|\mathbf{y} - \bar{\mathbf{y}}\|_2 \approx 1.1 \cdot 10^{-1}$. Only as $p(t)$ is close to the location of a subsystem the error decreases significantly to about $\|\mathbf{y} - \bar{\mathbf{y}}\|_2 \approx 6.1 \cdot 10^{-3}$. The mean error is $\|\mathbf{y} - \bar{\mathbf{y}}\|_{L_2[0,T]} = 1.4 \cdot 10^{-1}$, which is not satisfactory. The error from the approach from [5] with the interpolation on matrix manifolds is about three times smaller, since here cubic spline interpolation can be applied. The smallest errors occur around the subsystems as well and the mean error is

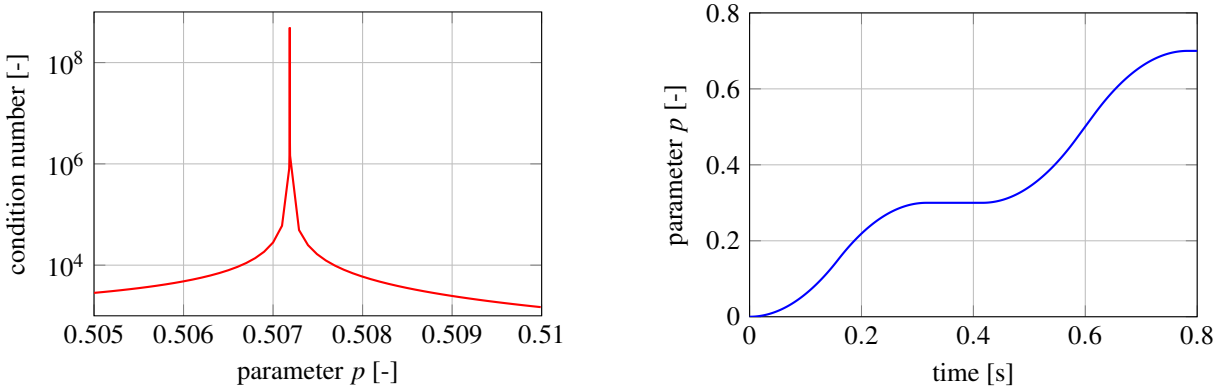

Figure 4: Condition number of elastic mass matrix $\tilde{\mathbf{M}}(p)$ for direct cubic spline interpolation and parameter trajectory $p(t)$.

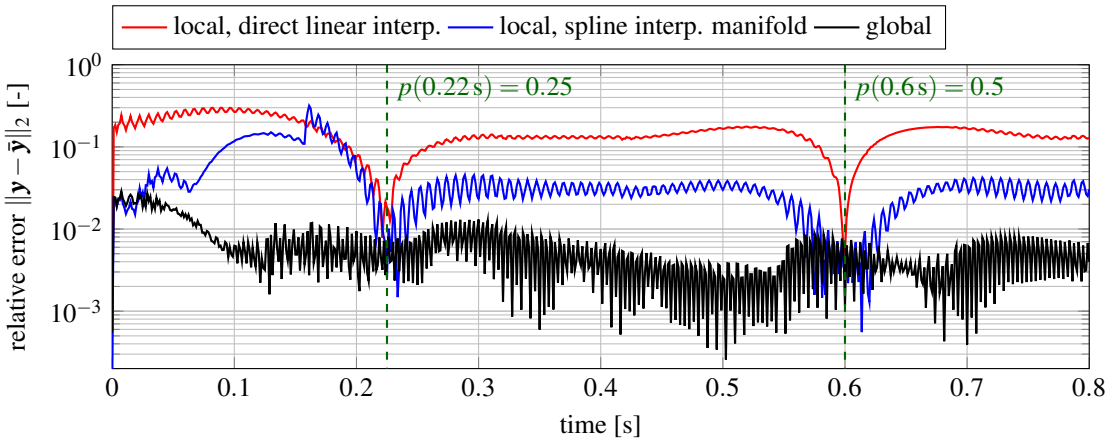

Figure 5: Relative error $\|\mathbf{y} - \bar{\mathbf{y}}\|_2$ in time domain for the local approaches from [4] with direct linear interpolation and [5] with cubic spline interpolation on matrix manifolds and for the global approach.

$\|\mathbf{y} - \bar{\mathbf{y}}\|_{L_2[0,T]} = 3.5 \cdot 10^{-2}$, which is still a rather large approximation error. Only the global approach shows satisfying results. It shows the smallest relative errors with $\|\mathbf{y} - \bar{\mathbf{y}}\|_{L_2[0,T]} = 4.9 \cdot 10^{-3}$. An interesting fact is, that the error is not decreasing at the subsystem but is rather equally distributed over the simulation time.

In order to make a fair comparison between the different approaches it is also necessary to consider the calculating times. The calculating times are shown in Fig. 6 for all three approaches. They are subdivided in several parts. The 'assembling' part considers the calculating time for assembling the global mass matrix and the right-hand side of the equation of motion. The 'inverse of mass matrix' part considers the time for solving Eq. (2) for the accelerations. The local approaches have an additional part considering the time to interpolate the reduced system matrices and the approach from [5] considers also the time for the exponential matrix mapping. The left diagram shows the calculating time for one evaluation of the equation of motion. The 'assembling' part and the 'inverse of mass matrix' part are the fastest for the local approaches. This makes sense since the order of the reduced models is $n = 16$ for the local approaches while it is $n_{\mathrm{gl}} = 77$ for the global approach. However, the local approaches also have to interpolate the reduced system matrices in every time step. Considering this additional shows that the local approaches need more time for one evaluation of the equation of motion although the reduced order is smaller. The additional cost for the exponential matrix mapping makes an evaluation of the model from the approach from [5] almost twice as expensive as the model from the global approach.

The total calculating time is depicted in the right diagram of Fig. 6. It is influenced both by the cost and the number of evaluations of the equation of motion. The model using direct interpolation needed $3.40 \cdot 10^4$ function calls, the approach using interpolation on matrix manifolds needed $2.46 \cdot 10^4$ function calls and the model from the global approach $1.72 \cdot 10^4$ function calls. This makes the shortest calculating time for the global approach with 24 s. The local approach from [4] is two times slower and the approach from [5] is almost four times slower. Summarized, the model derived from the global approach is the most efficient model for transient simulations although its reduced dimension is the largest.

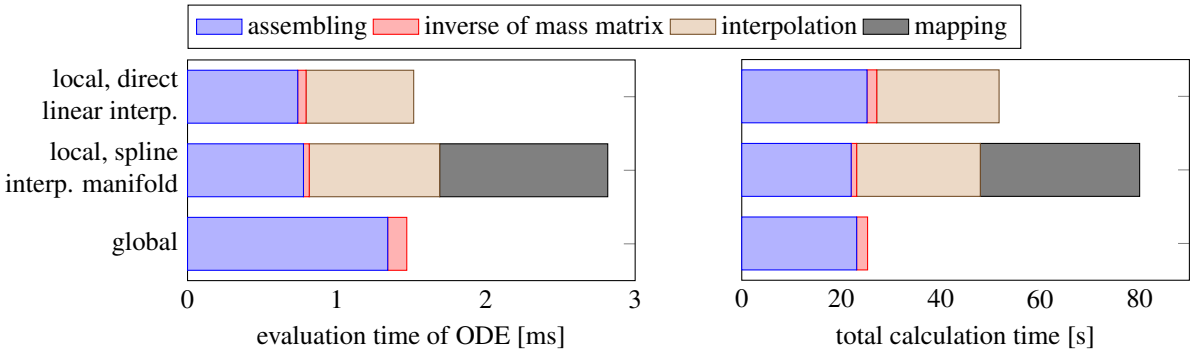

Figure 6: Calculating time for one evaluation of the equation of motion and total calculating time for transient simulation.

5. Conclusion

In this contribution, two local approaches and a global approach from parametric model order reduction for EMBS with moving loads are compared, where a linear drive with a spatially distributed load is investigated as a numerical example. In frequency domain the local approaches show rather large approximation errors, where the approximation quality of the global approach is satisfying. The global approach shows the most promising results in time domain as well. The local approaches only show satisfying results for parameter values close to a subsystem. Additionally, it turned out that the local approach with direct interpolation tends to deliver singular system matrices if higher order interpolation methods are used. This makes this approach rather inappropriate for transient simulations. The numerical cost for the local approaches is higher, since an additional interpolation and matrix mapping has to be performed in every time step. Therefore, the global approach was also the most efficient approach in a numerical sense although its reduced order is larger compared to the reduced models from the local approaches.

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