Nature-inspired generation scheme for shell structures

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Summary

Although less researched and put into practice in the building environment, pure plate structures are to be observed frequently in biological structures. The 3-plate principle which is common in the morphology and growth pattern of natural systems is also found to be of a structurally optimum content when considered from a plate point of view. This is for instance the case of the sea urchin's plate skeleton morphology, which served as biological inspiration for the recently built ICD/ITKE Research Pavilion 2011 at the University of Stuttgart. The current paper will focus on the 3-plate principle and its mechanical features, also presenting study models to analyse the structural characteristics and advantages of the principle. Along with the theoretical background, the paper will introduce the structural concept of the pavilion, as well as the analysis methods used for its design and engineering.

Keywords: *3-plate principle; bio-inspiration; complex geometry; shell structure; structural design; structural morphology.*

1. Introduction

The stability of polyhedral structures is uniquely determined by the valency of their vertices, where the valency indicates the number of structural elements converging at one node. According to the type of elements that make up the structure we may distinguish between lattice and plate systems, meaning that the first ones are materialised by the edges whilst the latter by the faces of the polyhedron. Systems with triangular faces which only rely on bars and joints make up pure lattice structures, whereas systems composed of just Y vertices (only three edges meeting at one point) which depend on plates and hinges along the edges are called pure plate structures. Such structures are said to comply with the 3-plate principle.

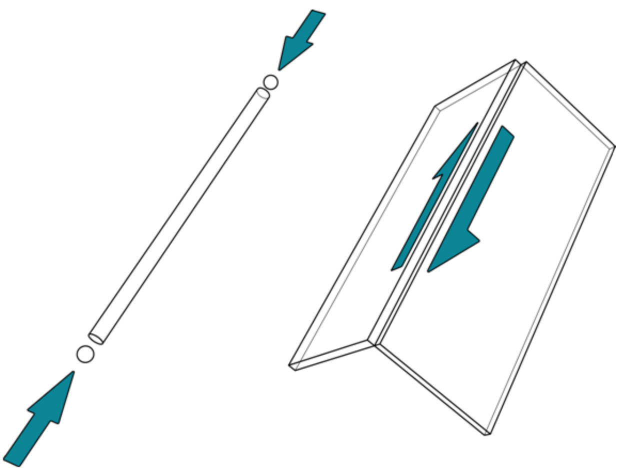

Figure 1: Axial force action in lattice systems and shear force action in plate systems

In triangulated structures, where the loads are carried by concentrated tension and compression forces in the edges and vertices (Fig. 1), a minimum of three bars is needed to fix a node in space, i.e. three mutually independent possibilities of translations for an unconnected node. For this reason, any triangulated lattice system will be inherently stable. In plate structures, the two basic elements are plates and edges (Fig. 1), and a minimum of three lines of support are needed to stabilize a plate plane in space, for instance two translations along perpendicular directions plus one rotation in the plane of the plate. By eliminating the nodal force concentration of lattice structures, the plate is stabilized by resisting internal forces which lie in the plane of the plate itself. Each face in a pure plate structure may be reduced to carry plate forces only, i.e. plates must not rely on carrying bending moments and torsion for their own stability. On the other hand, the lines of support enable the transmission of normal and shear forces but no bending moments between the joints. Three plates with three edges meeting at just one point are necessarily to stabilize themselves, thus resulting in a bending bearing but yet deformable structure.

2. Dual principle

In geometry, each polyhedron is related to its dual [1], where the vertices of one correspond to the faces of the other. The transformation process to produce a polyhedron's dual is known as polar reciprocation: in the case of a plate system, chosen a reference point (pole), the vertex of the lattice structure is located on the line from the pole perpendicular to the plane. Finally, if the distance of the vertex from the origin is chosen to follow the polarity relation [2]:

$$|\mathbf{p}| \cdot |\mathbf{P}| = 1 \tag{1}$$

where **p** represents the distance of the plate from the origin and **P** the distance of the vertex from the origin (Fig. 2), then the two systems will be one the dual of the other. The same relation of polarity as described above for the transformation of a plate structure into its lattice dual is also valid for the opposite transformation of a lattice structure into its plate dual. Geometrical dualism has the property of transforming a fully triangulated lattice polyhedron into a pure plate system, and vice versa. This implies that a lattice structure in equilibrium will still be granted the minimum stability conditions once it has been reciprocated into its dual. The duality as described so far acts upon the topology and rigidity of systems [3]. However, dualism is not only limited to morphological features, but it may also be applied to the mechanical properties of the structure.

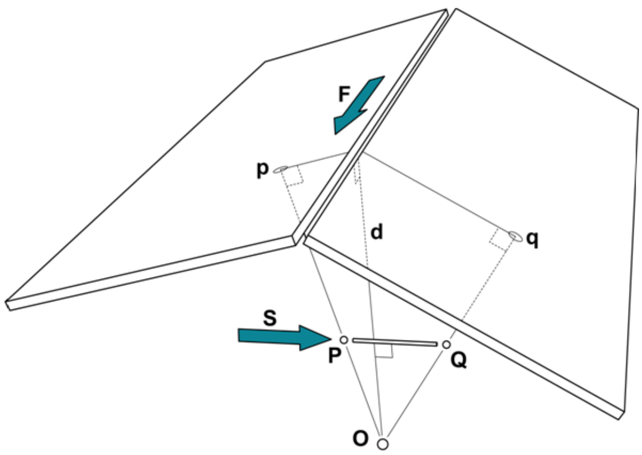

Figure 2: Relationship between geometry and forces in a dual system [1]

If we consider all the forces $\mathbf{F}_{p,n}$ acting along the n edges of a plate P (Fig.2), the equilibrium of the plate imposes that the sum of the vectorial product between the force vector and the position vector with respect to the chosen pole is zero [2]:

$$\sum^{n} \mathbf{F}_{p,n} \times \mathbf{d}_{p,n} = \mathbf{0} \tag{2}$$

which must obviously result in a closed polygon of forces. Analogously, the axial forces $\mathbf{S}_{p,n}$ in all the n bars that converge to the vertex P dual to the plate p (Fig. 2), have to be in mechanical equilibrium [2]:

$$\sum^{n} \mathbf{S}_{p,n} = \mathbf{0} \tag{3}$$

also indicating a spatial string polygon in equilibrium. Following the right-hand rule to define the direction of the vectorial product, it is easy to see that the two vectors **F**x**d** and **S** are always parallel. which means that the two polygons of forces are necessarily identical [2]:

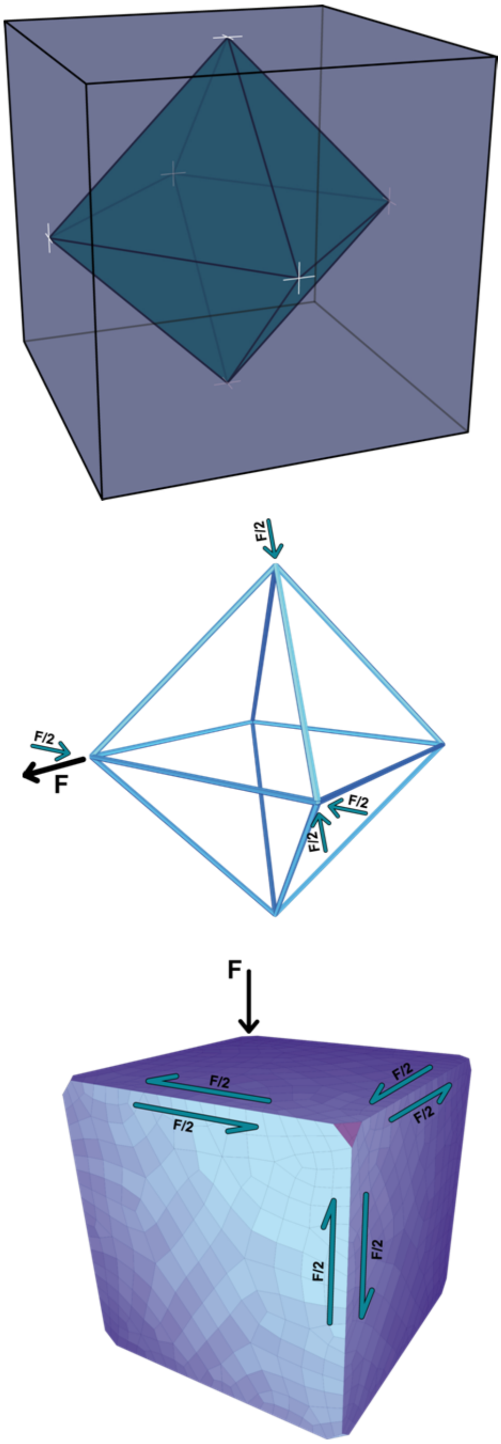

Figure 3: Force distribution in plate structure and its lattice dual

$$\mathbf{F}_{p,n} \times \mathbf{d}_{p,n} = \mathbf{S}_{p,n} \tag{4}$$

i.e. d = S/F for all corresponding parts of the structure. Equations (1) and (4) state the correlated geometrical and statical formulas for transformations between dual plate and lattice structures. It has been shown that the plate-lattice dualism is not limited to the topology and the simple geometrical features of the system, but it may also be extended to the mechanical characteristics of the structure. An interesting consequence of this last statement is that a simple way to calculate the forces and reactions in a spatial plate structure is through its lattice dual. This is done by translating all the data for the plate structure (including geometry, loads and boundary conditions) into data for the dual lattice structure which is successively solved with any truss calculation software. Finally, by reversing the dual transformation process, all the information concerning the lattice structure can be brought back to the original plate structure, where the axial forces of the lattice's bars will now represent the corresponding shear forces and reactions along the line of support of the plate system.

The concept is illustrated in Fig. 3 where the FE analysis of a simple plate structure, a cube in this case, is compared with its lattice dual which corresponds to the regular octahedron, having chosen the centroid of the cube as pole for the reciprocal transformation. A test point load **F** has been applied to one of the cube's vertices having the direction of the edge, which is transformed into a point load of the same magnitude applied to the reciprocal node of the truss structure and having the same direction of the concurrent bar. To avoid any nodal effect which would have induced stress concentrations, the vertices of the cube have been trimmed off without affecting the general stability of the structure. The connecting edges of the plates have been simulated by a set of axial springs of infinite stiffness to easily access the values of the shear forces in the lines of support. Apart slight discrepancies due to numerical approximation, the axial and shear forces values of the two systems result equal, thus confirming the dual principle as exposed so far.

3. Research project

In the summer 2011 the Institute for Computational Design (ICD) and the Institute of Building Structures and Structural Design (ITKE), together with the students at the University of Stuttgart have realized a temporary, bionic research pavilion made of wood (Fig. 4). The project explored the transfer to architecture of biological principles of the sea urchin's plate skeleton morphology. This was achieved by means of novel computer-based design and simulation methods, along with computer-controlled manufacturing methods for the building implementation. A particular innovation consisted in the possibility of extending, through computer-based applications, the recognized bionic principles to different geometries achieving high-performing lightweight structures, which was demonstrated by the fact that the complex morphology of the pavilion could be built exclusively with extremely thin sheets of plywood (6.5 mm).

Figure 4: External view of the pavilion

3.1 Biological model

The project aimed at integrating the performative qualities of biological structures into architectural design and at testing these results on a spatial and structural material-system. The focus was set on the development of a modular system which allowed a high degree of adaptability and performance due to the differentiation of its geometric components. During the analysis of biological structures, the morphology of the sea urchin (Echinoidea) was of particular interest, which provided the basic principles of the bionic structure that was realized. The shell of the sea urchin (Fig. 5) consists of a modular system of polygonal plates, which are linked together at the edges by finger-like calcite protrusions [4]. Shell action is very close to plate action because a finely faceted plate polyhedron is nothing but a slightly discontinuous shell, stabilized only by shear forces, acting across the edges. High load bearing capacity is thus achieved by the particular geometric arrangement of the plates and their joining system. Therefore, the sea urchin serves as a perfect model for shells made of prefabricated elements. Similarly, the traditional finger-joints typically used in carpentry as connection elements, may be seen as the equivalent of the sea urchin's calcite protrusions.

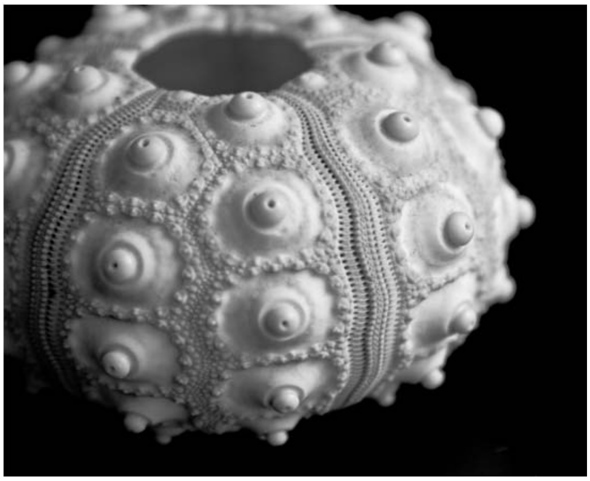

Figure 5: Detail of sea urchin's plate structure

3.2 Morphology transfer

The most important feature of the sea urchin's plate morphology is its compliance to the 3-plate principle, meaning that it is fully trivalent and can thus be regarded as a pure plate structure [5]. This property allows the sand dollar to grow without interfering with the transfer of stabilizing forces, as the direction of growth will be perpendicular to the shear lines, hence perpendicular to the direction of the stabilizing shear forces. Following the analysis of the sand dollar, the morphology of its plate structure was translated into the design of a pavilion. Three plate edges always meet together at just one point, enabling the transmission of normal and shear forces but no bending moments between the joints, thus resulting in a bending bearing but yet deformable structure. As the plates are mainly subjected to in-plane forces, the degree of utilization of the material's strength is maximized, guaranteeing an optimal structural configuration. The high lightweight potential of this approach is evident as the pavilion, despite its considerable size, could be built out of 6.5 mm thin sheets of birch plywood and therefore primarily needed anchoring to the ground to resist wind suction loads.

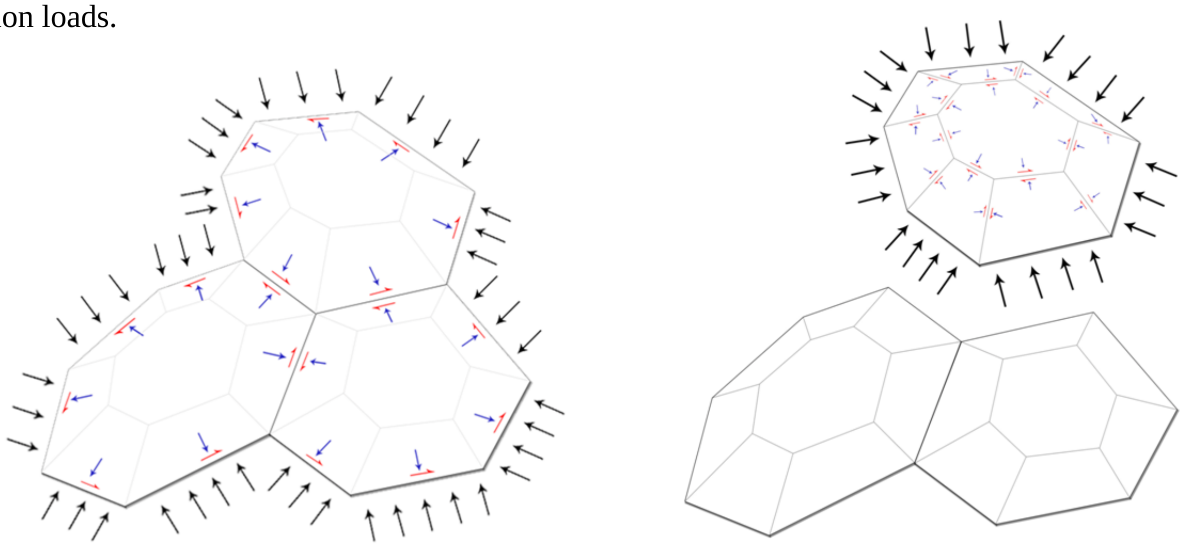

Figure 6: Cell hierarchy and hierarchical distribution of the 3-plate principle

Besides these constructional and organizational principles, hierarchy is another fundamental property of biological structures which has been applied in the project. The pavilion is organized on a two-level hierarchical structure (Fig. 6). On the first level, the finger joints of the plywood sheets are glued together to form a cell. On the second hierarchical level, a simple screw connection joins the cells together, allowing the assembling and disassembling of the pavilion. Within each hierarchical level only three plates - respectively three edges – meet exclusively at one point, therefore assuring bendable edges for both levels.

3.3 Digital design and robotic production

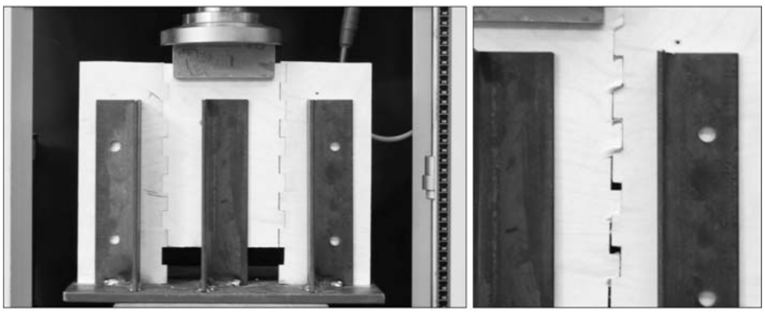

Figure 7: Shear testing of the finger joints

A requirement for the design, development and realization of the complex morphology of the pavilion was a closed, digital information chain linking the project's model, finite element simulations and machine control. Form finding and structural design were closely intertwined. An optimized data exchange scheme made it possible to repeatedly read the complex geometry into a Finite Element program to analyze and modify the critical points of the model. Shear load tests of the joints (Fig. 7) were performed in order to obtain experimental data for the Finite Element analysis of the overall structure. The FE setup (Fig. 8) was modeled with plane surfaces and lines of axial springs acting in the direction of the edges of connection between the plates. The stiffness values of the linear springs were obtained from the laboratory tests on the shear strength of the finger joints.

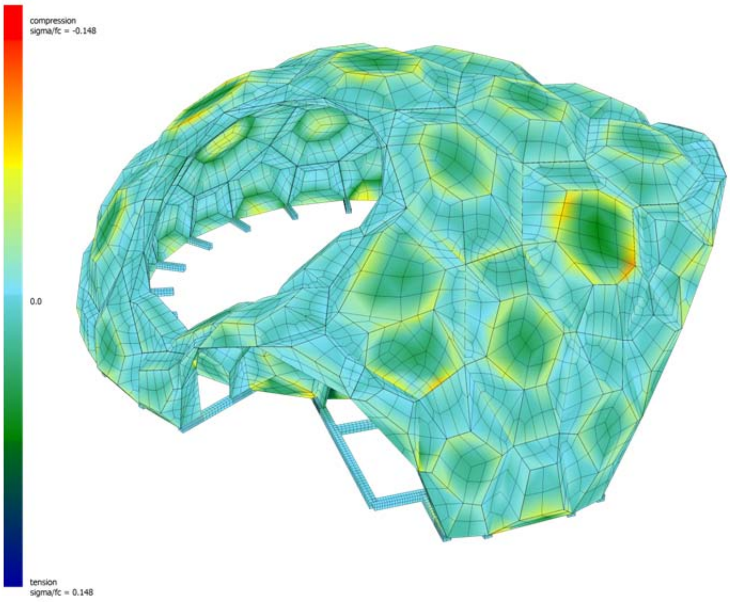

Figure 8: Finite Element model of the pavilion

The plates of each cell were finally produced with the university's robotic fabrication system. Using custom scripting routines the digital building model provided the basis for the automatic generation of the machine code (NC-Code) for the control of the industrial robot. This enabled the economical production of more than 850 geometrically different components, as well as more than 100,000 finger joints freely arranged in space. Following the robotic production, the plywood panels were joined together to form the cells, which were successively primed and stained. The prefabricated modules were finally joined with a bolted connection and assembled at the downtown campus of the University of Stuttgart.

4. Conclusions

Figure 9: Night view of the pavilion

Plate action in today’s buildings is more or less limited to the resisting of horizontal forces. The research pavilion showed how this principle can be successfully implemented in a complex and expressive spatial system and how it is possible to create an extremely light but at the same time very stiff supporting structure only by the optimal arrangement of its constitutive elements. Through the plate-lattice dualism it has been shown that any structure which complies with the 3-plate principle is inherently stable. This fundamental principle, along with the reciprocal transformation process, offers a powerful tool to investigate a wide range of structural morphologies by simple equilibrium assumptions. Finally, the research pavilion offered the opportunity to investigate methods of modular bionic construction using freeform surfaces with different geometric characteristics.

5. Acknowledgments

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