

- Benchmark case -

Simply supported CLT plate (non-edge glued boards) with uniformly distributed loading

1 Objectives

This benchmark aims to offer reference values for the FE-modelling of simply supported CLT plates subjected to a uniformly distributed load regarding

- the maximal vertical displacement,
- the maximal bending stress at midspan taking into account the stress increase due to the lower shear stiffness perpendicular to the fibre (including rolling shear effect),
- the maximal shear stress (located in the centre layer) and
- the maximal (rolling-) shear stresses (located in the layers next to the centre layer 3).

This benchmark's data sheets and analytical reference results are based on the work of Bogensperger et al. [1].

2 Geometry definition, structural system and coordinate systems

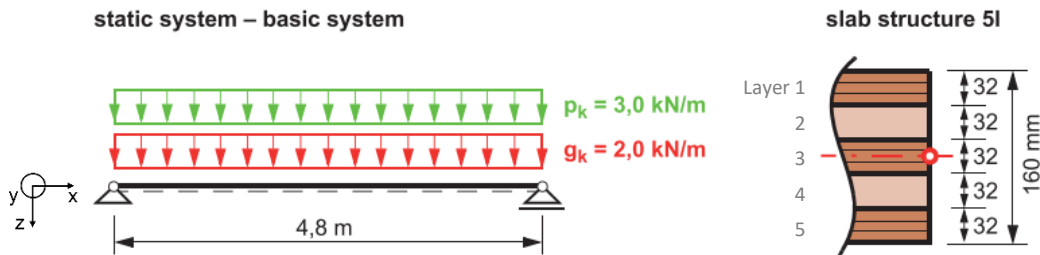


Fig. 1: Structural system and cross section definition [1]

In this benchmark the structural system of the CLT plate is modelled as a 1 m-wide-plate strip.

The global coordinate system is shown in Fig. 1. Additionally each layer has a local coordinate system to which the material orientations are referring. For the longitudinal layers (layer 1, 3 and 5) the local coordinate system corresponds to the global coordinate system whereas for the cross layers (layer 2 and 4) the local coordinate system is rotated by 90 degrees around the global z-axis. Thus y-axis of the local coordinate system of the cross layers runs in direction of the span.

3 Material properties

The wood stiffness properties are defined with indices referring to the wood fibres' directions [2]:

1 = longitudinal

2 = radial

3 = tangential

In this benchmark no differences are considered regarding the radial and tangential orientation of the wood fibres.

Tab. 1: Material properties (related to the layers and their local coordinate systems)

Modulus of elasticity [N/mm ²]	Shear Modulus [N/mm ²]	Poisson's ratio [-]
$E_{11} = 11600$	$G_{12} = 720$	$\nu_{12} = 0^{**}$
$E_{22} = 11600 / 30$ resp. 1 (0)*	$G_{13} = 720$	$\nu_{13} = 0^{**}$
$E_{33} = 11600 / 30 = 387$	$G_{23} = 720 / 10 = 72$	$\nu_{23} = 0^{**}$

* When modelling non edge-glued boards without modelling the single boards and the joints in between them E_{22} should be conservatively set to 0 which is here approximated with $E_{22} = 1$ N/mm² to obtain a meaningful stiffness matrix.

** Poisson's ratios' impact on the results of this benchmark is assumed to be negligible (all poisson's ratios are set to 0)

To take into account the effect of non-edge-glued boards without modelling single boards, E_{22} is conservatively set to zero in analytical calculations [1]. To obtain a meaningful stiffness matrix, E_{22} should be taken as $E_{22} = 1$ N/mm² in a FE-model. The reason why $E_{22} = 0$ is not adequate for a FE-model is shown in equation (1) to (5). A symmetrical stiffness matrix with a solution like shown in equation (1) is assumed [2].

$$\mathbf{C} = \begin{bmatrix} (1 - \nu_{23}\nu_{32})\Delta E_{11} & (\nu_{12} + \nu_{13}\nu_{32})\Delta E_{22} & (\nu_{13} + \nu_{12}\nu_{23})\Delta E_{33} & 0 & 0 & 0 \\ (\nu_{12} + \nu_{13}\nu_{32})\Delta E_{22} & (1 - \nu_{13}\nu_{31})\Delta E_{22} & (\nu_{23} + \nu_{21}\nu_{13})\Delta E_{33} & 0 & 0 & 0 \\ (\nu_{13} + \nu_{12}\nu_{23})\Delta E_{33} & (\nu_{23} + \nu_{21}\nu_{13})\Delta E_{33} & (1 - \nu_{12}\nu_{21})\Delta E_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{23} \end{bmatrix} \quad (1)$$

$$\text{with } \Delta = \frac{1}{(1 - \nu_{12}\nu_{21} - \nu_{13}\nu_{31} - \nu_{23}\nu_{32} - 2\nu_{13}\nu_{21}\nu_{32})}$$

$$\nu_{32} = \nu_{23} \cdot \frac{E_{33}}{E_{22}} = \zeta \quad \text{if } E_{22} = 0 \quad (2)$$

$$\Delta = \frac{1}{(1 - \nu_{12}\nu_{21} - \nu_{13}\nu_{31} - \nu_{23}\nu_{32} - 2\nu_{13}\nu_{21}\nu_{32})} = \zeta \quad \text{if } \nu_{32} = \zeta \quad (3)$$





As the first 3 x 3 entries in the stiffness matrix C are multiplied by (2), setting $E_{22} = 0$ is producing a stiffness matrix, which is leading to a numerical error (see equation (3)).

$$C = \begin{bmatrix} DIV/0 & DIV/0 & DIV/0 & 0 & 0 & 0 \\ DIV/0 & DIV/0 & DIV/0 & 0 & 0 & 0 \\ DIV/0 & DIV/0 & DIV/0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 720 & 0 & 0 \\ 0 & 0 & 0 & 0 & 720 & 0 \\ 0 & 0 & 0 & 0 & 0 & 72 \end{bmatrix} \text{ [N/mm}^2\text{]} \quad (4)$$

Whereas when setting $E_{22} = 1 \text{ N/mm}^2$ and $\nu_{12} = \nu_{13} = \nu_{23} = 0$, and using the material parameters from Tab. 1, following stiffness matrix can be derived.

$$C = \begin{bmatrix} 11600 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 387 & 0 & 0 & 0 \\ 0 & 0 & 0 & 720 & 0 & 0 \\ 0 & 0 & 0 & 0 & 720 & 0 \\ 0 & 0 & 0 & 0 & 0 & 72 \end{bmatrix} \text{ [N/mm}^2\text{]} \quad (5)$$

Note: The here shown material modelling should not be used for more sophisticated loading-scenarios where e.g. the CLT plate spans in two directions. In those cases also the low torsional stiffness of the non-edge-glued boards should be considered.

4 Loading

Uniformly distributed loading: $f_k = g_k + q_k = 5 \text{ kN/m}^2$

The dead weight of the members is neglected.

Abaqus Note: Modelled as surface traction in z-direction with the load not following the rotation.

5 Boundary conditions and constraints

Boundary conditions and constraints and their application in Abaqus are shown in Fig. 2.

Left support:

- Displacement boundary condition in x-direction (modelled as a line support over the plate's width)
- Displacement boundary condition in y-direction (modelled as a line support over the plate's height)
- Spring displacement boundary condition in z-direction with an overall stiffness of $K_z = 10^8 \text{ N/mm}^*$

Right support:

- Displacement boundary condition in y-direction (modelled as a line support over the plate's height)



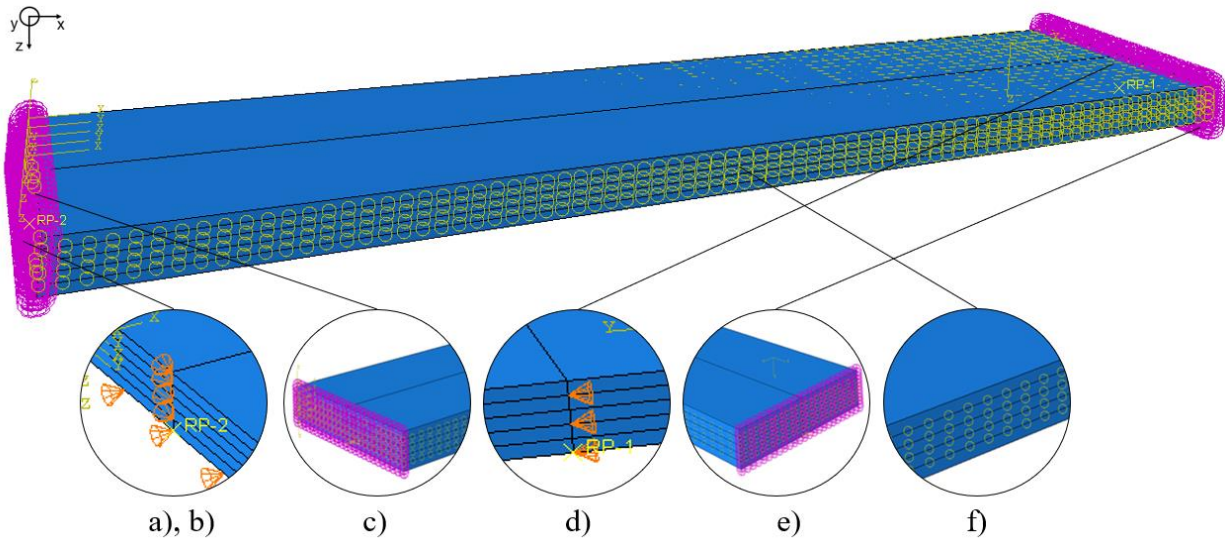


Fig. 2: Abaqus model: Boundary conditions and constraints

- e) Spring displacement boundary condition in z -direction with an overall stiffness of $K_z = 10^8 \text{ N/mm}^*$

* *Note: The spring boundary condition on either side is used to connect all points in z -direction to the ground. With an overall spring stiffness of $K_z = 10^8 \text{ N/mm}$ a stiffness converging to infinity is approximated. This is done to obtain a model that is comparable to the analytical one [1]. Coupling all nodes to a support node which receives a fixed displacement boundary condition in z -direction doesn't yield the desired results especially with regard to the shear stresses.*

Coupling of CLT layers:

- f) Tie constraints are used to couple the respective nodes of the surfaces of the different CLT layers. The surfaces of layer 1, 3 and 5 which are orientated in x -direction are chosen to be the master surfaces whereas the surfaces of layer 2 and 4 are selected to be the slave surfaces. Considerations regarding a modelling of an adhesive film connecting the single layers (gluing) are not part of this benchmark.

6 Modelling

In this benchmark different so called “parts” in Abaqus are used to model the single layers, which are assembled to one cross section, with local material coordinate systems depending on the single layers orientations in each part. A detailed modelling of the single boards and the joints in between the boards next to each other is not part of this benchmark.

Abaqus Note: Alternatively the cross section can be modelled as one part, which is partitioned into different partition cells to model the CLT's subdivision into different layers, with local material coordinate systems in all partition cells depending on the single layers' orientations. Both modelling techniques generate a similar model and thus yield equal results.



7 Discretization

20 node quadratic hexahedral volume elements with reduced integration (Abaqus: C3D20R) are used in this benchmark. The CLT plate's mesh contains 40 elements in span width in x-direction, 12 elements over the plate's width in y-direction and 5 elements, which is one element per layer, over the plate's height in z-direction (see Fig. 3). Within a verification process according to [3] it could be shown that there are no significant differences using a mesh with one element per layer or six elements per layer in z-direction. Thus to reduce the computational effort in larger models one element per layer is assumed to be sufficient. Further refinement of the meshing does not lead to a notable increase in calculation accuracy.

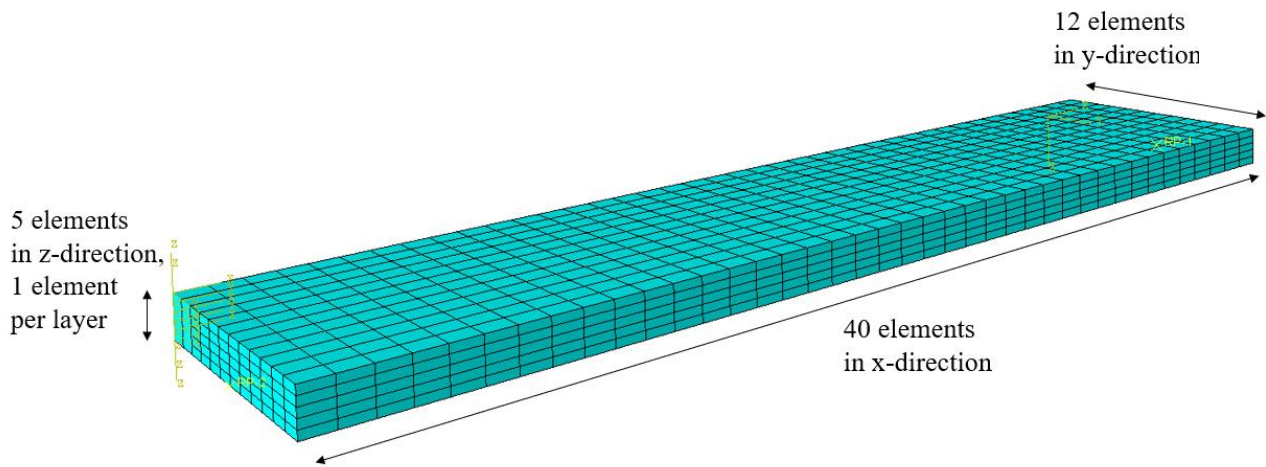


Fig. 3: Abaqus model: Discretization

8 Analytical target results

Tab. 2: Analytical reference values [1]

Reference values [1]	$w_{z,max}$ [mm]	$\sigma_{xz,max}$ [N/mm ²]	$\tau_{xz,max}$ [N/mm ²]	$\tau_{xz,max,r}$ [N/mm ²]
Gamma method	11,77	4,313	0,097	0,091
Shear analogy method	11,83	4,301	0,099	0,088

Note: $\tau_{xz,max}$ refers to the maximum shear stress in the middle longitudinal layer 3.

$\tau_{xz,max,r}$ refers to the maximum (rolling-) shear stress in the two cross layers 2 and 4 next to the middle longitudinal layer. If different local coordinate systems are used for the different layers, global $\tau_{xz,max,r}$ can be shown by choosing $\tau_{yz,max}$.

9 Finite element analysis results

With the structural system, boundary conditions, material parameters and discretization described in the preceding chapters the results displayed in Tab. 3 can be obtained within finite element calculations. Fig. 4 to Fig. 9 show graphically the results of the deformations and stresses of the FE-calculations.

Tab. 3: Finite element results

$w_{z,max}$ [N/mm ²]	$\sigma_{xz,max}$ [N/mm ²]	$\tau_{xz,max}$ [N/mm ²]	$\tau_{xz,max,r}$ [N/mm ²]
11,82	4,304	0,097	0,089

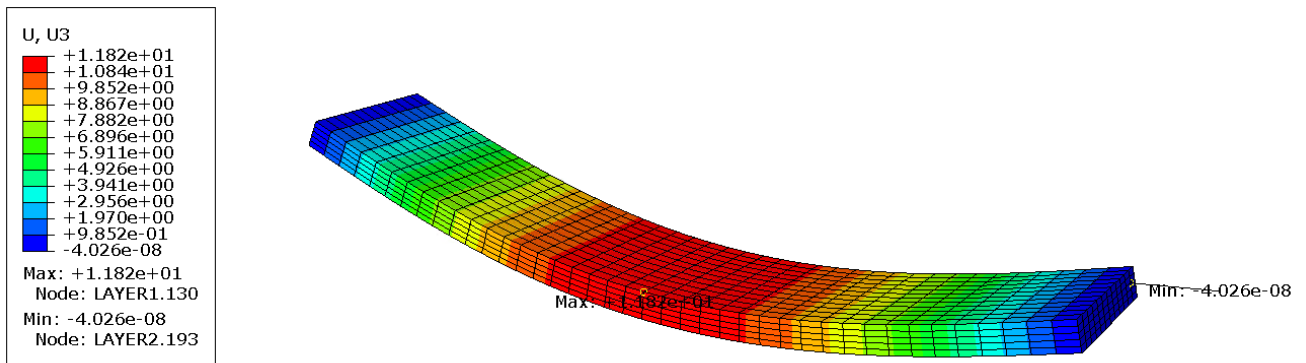


Fig. 4: Abaqus results: Vertical deformations w_z in z-direction in [mm]

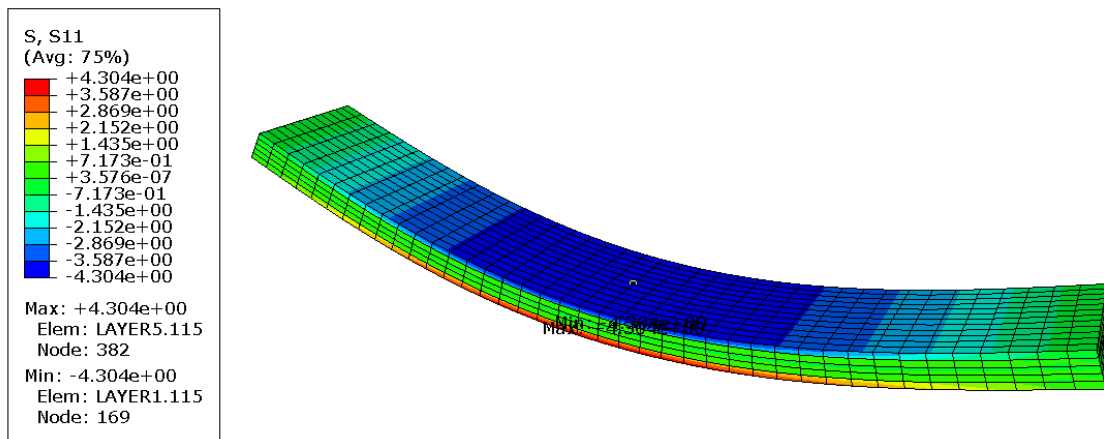


Fig. 5: Abaqus results: Bending stresses σ_x in [N/mm²]

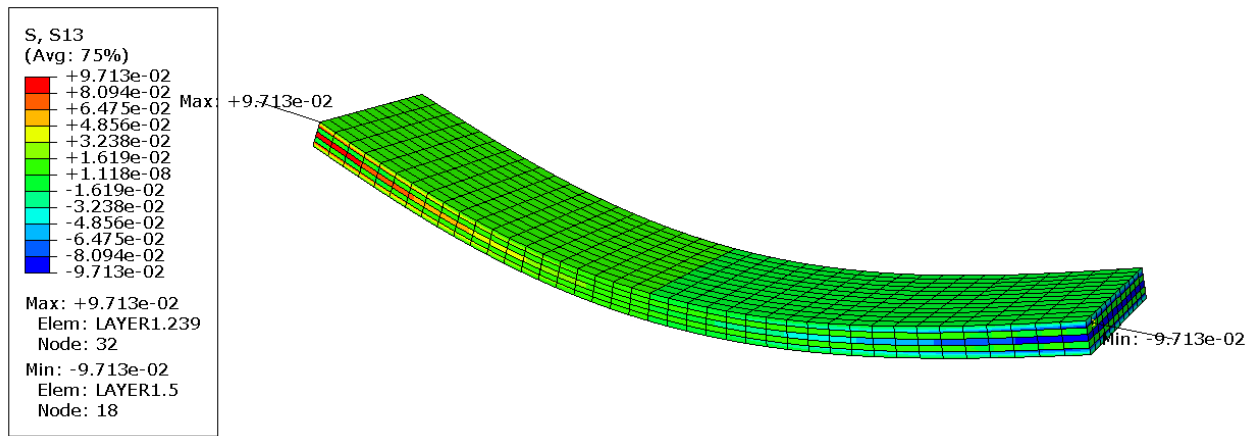


Fig. 6: Abaqus results: Shear stresses τ_{xz} of longitudinal layers in [N/mm²]

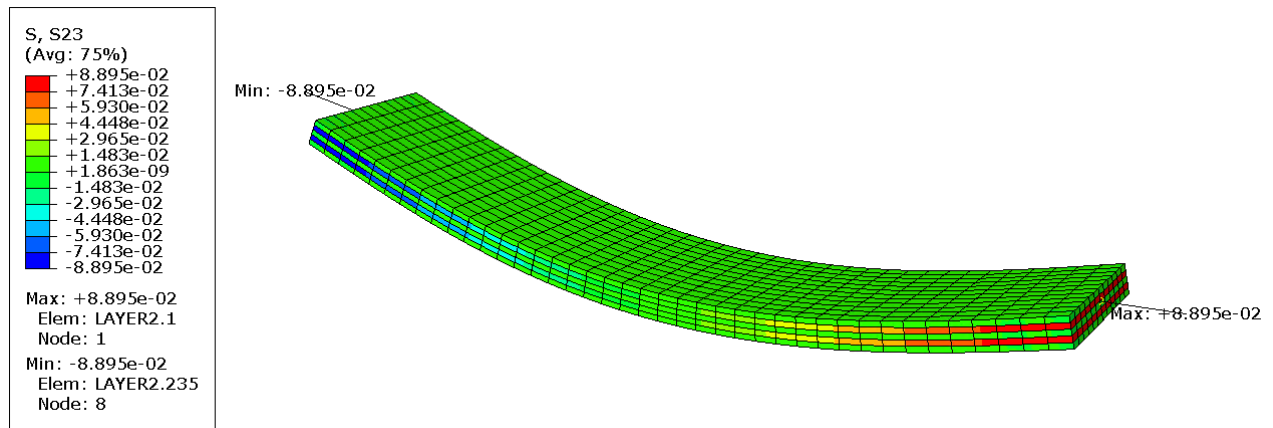


Fig. 7: Abaqus results: (Rolling) shear stresses $\tau_{xz,r}$ of cross layers 2 and 4 in [N/mm²]

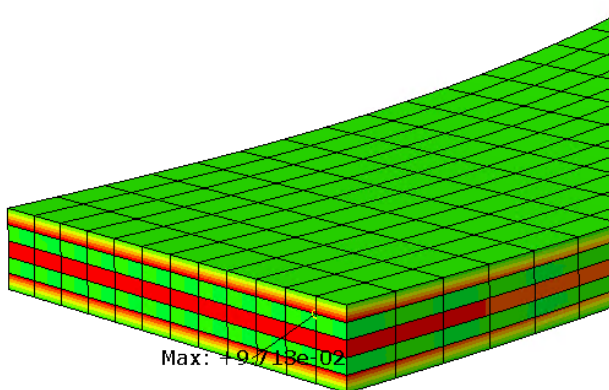


Fig. 8: Abaqus results, close-up: Shear stresses τ_{xz} of longitudinal layers 1,3 and 5 in [mm]

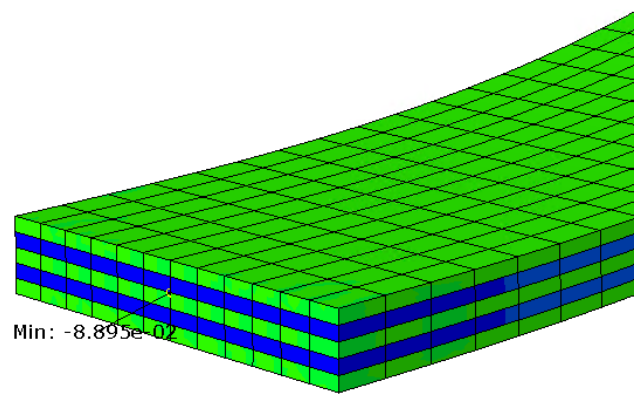


Fig. 9: Abaqus results, close-up: (Rolling) shear stresses $\tau_{xz,r}$ of cross layers 2 and 4 in [N/mm²]



10 Comments upon difficulties that might be encountered during FE-modelling of CLT

The following section is giving hints on different kinds of general and modelling issues which might occur depending on the numerical software that is used.

- Modelling targeted support conditions can be challenging. In the frame of this benchmark it was observed that depending type of modelling of the supports parasitic / not expected stresses might appear. Thus considerations of the support modelling have a huge impact especially on shear- and rolling shear stresses.
- All nodes at the cross section where the supporting conditions are applied should be coupled in z-direction and receive the same supporting conditions in z-direction. Therefore a constraint was intended to be used that should have coupled all nodes in z-direction to one support node where the boundary conditions were applied. Each of this nodes at the cross section needs so two coupling conditions. One to model the described support and another one to tie the single CLT layers.*
- The applied model is a simplification that renounces a detailed modelling of the single boards per layer. In order to nevertheless take into account that non-edge-glued boards are used, E_{22} is analytically set to zero. In numerical computations E_{22} has to be set to a value unequal zero, so $E_{22} = 1 \text{ N/mm}^2$ can be chosen.
- If all single boards are modelled, the joints in between the single boards and any resulting stress peaks have to be dealt with. In general such sophisticated models are needed as soon as two-axial-spanning systems are investigated where E_{22} cannot be set to 0 or 1 N/mm².

**Abaqus Notes:*

- The here used software is in some cases not accepting double coupling conditions per node. This leads to the fact that in the tie constraint that is coupling the single CLT layers, the end nodes have to be excluded. This in turn showed a high impact on the shear stresses at the supports and thus this modelling idea was rejected. Using a nodes-to-ground spring and coupling the layers to each other with a tie constraint is in contrary to the double node coupling with support coupling in z-direction and coupling the layers to each other with a tie constraint possible with this software.
- In all cases it turned out that it is all about carefully and specifically selected node set definitions. Choosing surfaces instead of previously defined node sets often leads to errors.





11 References

- [1] Bogensperger, T.; Silly, G.; Schickhofer, G.: *Comparison of Methods of Approximate Verification Procedures for Cross Laminated timber*. Graz University of Technology, Institute of Timber Engineering and Wood Technology; holz.bau forschungs gmbh, 2012.
- [2] Sandhaas, C.: *Mechanical behaviour of timber joints with slotted-in steel plates*. Delft University of Technology, Department of Structural Engineering, Dissertation, 2012.
- [3] Kuhlmann, U.; Töpler, J.; Buchholz, L.; Machanek, S.: *Guidelines for a Finite Element Based Design of Timber Structures*. Institute of Structural Design, University of Stuttgart, 2022.

