

# Dynamic Properties of Fluids from Molecular Simulations and Entropy Scaling

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*Sometimes science is a lot more art than science.  
A lot of people don't get that.*

RICK SANCHEZ  
Earth Dimension C-137



# Contents

|  |            |
|--|------------|
| <b>Abstract</b>  | <b>i</b>   |
| <b>Kurzfassung</b>   | <b>iii</b> |
| <b>1 Introduction</b>  | <b>1</b>   |
| 1.1 Molecular Simulations & Force Fields . . . . .                               | 3          |
| 1.1.1 Classical Force Fields . . . . .   | 4          |
| 1.1.2 Molecular Simulations . . . . .  | 7          |
| 1.2 Dynamic Properties from Molecular Dynamics . . . . .                         | 8          |
| 1.2.1 Non-Equilibrium Molecular Dynamics . . . . .                               | 9          |
| 1.2.2 Transport Coefficients from Equilibrium MD Simulations . . . . .           | 10         |
| 1.3 Entropy Scaling . . . . .  | 15         |
| 1.4 Outline of this Thesis . . . . .   | 19         |
| <b>Bibliography</b>  | <b>21</b>  |
| <b>2 Force Fields with Fixed Bond Lengths and with Flexible Bond Lengths</b>     | <b>31</b>  |
| 2.1 Introduction . . . . .   | 32         |
| 2.2 Methods . . . . .  | 34         |
| 2.2.1 Molecular Model . . . . .  | 34         |
| 2.2.2 Data Analysis . . . . .  | 35         |
| 2.2.3 Simulation Details . . . . .   | 40         |
| 2.3 Results and Discussion . . . . .   | 42         |
| 2.4 Conclusion . . . . .   | 50         |
| <b>Bibliography</b>  | <b>53</b>  |
| <b>3 TAMie Force Field: Transport Properties from Equilibrium MD Simulations</b> | <b>59</b>  |
| 3.1 Introduction . . . . .   | 60         |
| 3.2 Methods . . . . .  | 62         |
| 3.2.1 Molecular Model . . . . .  | 62         |
| 3.2.2 Simulation Setup . . . . .   | 65         |
| 3.2.3 Data Analysis . . . . .  | 68         |
| 3.3 Results and Discussion . . . . .   | 72         |
| 3.3.1 Viscosity . . . . .  | 72         |
| 3.3.2 Self-Diffusion . . . . .   | 78         |

|                     |  |            |
|---------------------|--|------------|
| 3.3.3               | Thermal Conductivity . . . . .   | 82         |
| 3.4                 | Conclusion . . . . .   | 85         |
| <b>Bibliography</b> |  | <b>87</b>  |
| <b>4</b>            | <b>Assessing Entropy Scaling for Mixture Viscosities</b>                           | <b>95</b>  |
| 4.1                 | Introduction . . . . .   | 95         |
| 4.2                 | Methods . . . . .  | 97         |
| 4.2.1               | Molecular Model . . . . .  | 97         |
| 4.2.2               | Entropy Scaling . . . . .  | 99         |
| 4.2.3               | Calculation of Residual Entropy . . . . .  | 100        |
| 4.2.4               | Calculation of Viscosity . . . . .   | 103        |
| 4.3                 | Results . . . . .  | 105        |
| 4.4                 | Conclusion . . . . .   | 113        |
| <b>Bibliography</b> |  | <b>115</b> |
| <b>5</b>            | <b>Conclusion</b>  | <b>123</b> |
| <b>Appendices</b>   |  | <b>125</b> |
| <b>A</b>            | <b>SI: Force fields with fixed bond lengths and with flexible bond lengths</b>     | <b>127</b> |
| A.1                 | Details of the calculation of the Thermal Conductivity . . . . .                   | 127        |
| A.2                 | Distribution of Bond Lengths . . . . .   | 128        |
| A.3                 | Diagram of pressure of propane for high values of period time T . . . . .          | 129        |
| A.4                 | Simulation results in tables . . . . .   | 129        |
| <b>B</b>            | <b>SI: TAMie Force Field: Transport Properties from Equilibrium MD Simulations</b> | <b>141</b> |
| B.1                 | SAFT Parameters used in this Study . . . . .                                       | 142        |
| B.2                 | Parameters of the TAMie force field . . . . .                                      | 143        |
| B.3                 | Correlation Parameters from Simulation . . . . .                                   | 145        |
| B.4                 | Simulation Results of each Substance . . . . .                                     | 147        |
| B.5                 | Verification of the Nosé-Hoover thermostat . . . . .                               | 153        |
|                     | Bibliography . . . . .   | 154        |
| <b>C</b>            | <b>SI: Assessing Entropy Scaling for Mixture Viscosities</b>                       | <b>156</b> |
| C.1                 | Tabulated Results of the MD Simulations . . . . .                                  | 156        |
| C.2                 | Tabulated Results of the isothermal GCMC Simulations . . . . .                     | 162        |
| C.3                 | Tabulated Results of the Vapor Liquid Equilibrium GCMC Simulations . . . . .       | 177        |

## Abstract

The design of most processes in chemical industry depends on reliable estimates of the transport properties of fluids. Various approaches exist for the prediction of these quantities, which can be used to compensate for insufficient experimental data. The present work deals with two of the approaches: *Molecular simulations* and *entropy scaling*. According to the latter approach, transport coefficients, such as shear viscosity, thermal conductivity or self diffusion coefficients, defined as dimensionless quantities using a suitable reference, are univariant functions of only the residual entropy of the fluid. The two methods, molecular simulations and entropy scaling are used jointly in order to achieve synergistic effects. A suitable mixture-model for entropy scaling models was investigated in molecular simulations as part of this work. Mixtures of simple model fluids, namely Lennard-Jones mixtures, are regarded and it is found that the principle of entropy scaling holds also for mixtures, to excellent approximation. Entropy scaling, in turn, is used to more efficiently design and evaluate molecular simulations. In this context, the *TAMie* force field developed in Stuttgart is assessed with respect to the accuracy of predicted transport coefficients. The *TAMie* model, like many other force fields developed for thermodynamic properties, uses rigid bond lengths between interaction sites within a molecule. In order to ensure a meaningful assessment of transport coefficients in Molecular Dynamic simulations, an analysis of bond-length models is conducted: what is the influence of the model for intramolecular atomic bonds on the predicted static and dynamic fluid properties? It is shown that it is possible to obtain the same results for transport coefficients with flexible atomic bonds, within statistical accuracy, as with the same force field but using a rigid description of the bonds. Within the context of the simulation studies carried out in this thesis, a workflow has been developed that enables efficient evaluation of simulations for determining transport properties. In combination with entropy scaling, this work presents a methodology that can be used to efficiently determine transport quantities from molecular simulations, thus enabling extensive simulation studies for either predicting fluid properties or to enable force field development where transport coefficients are considered in the objective function.





## Kurzfassung

Für die Entwicklung und Auslegung der meisten Prozesse der chemischen Industrie ist die verlässliche Vorhersage der Transporteigenschaften von Fluiden eine unverzichtbare Grundvoraussetzung. Dafür gibt es verschiedene Ansätze, mit denen fehlende experimentelle Daten kompensiert werden können. Diese Arbeit behandelt zwei dieser Ansätze: *Molekularsimulationen* und die *Entropieskalierung*. Transportkoeffizienten, wie Scherviskosität, Wärmeleitfähigkeit und Selbstdiffusionskoeffizienten, sind nach dem Entropieskalierungsansatz univariate Funktionen einzig der residuellen Entropie des Fluids, sofern sie mit einer geeigneten Referenz als dimensionslose Größen definiert werden. Die beiden Methoden, Molekularsimulationen und Entropieskalierung, werden kombiniert eingesetzt um Synergieeffekte zu erzielen. So wird in dieser Arbeit ein geeignetes Mischungsmodell für Entropieskalierungsmodelle untersucht. Dafür wurden Gemische einfacher Modellflüssigkeiten, insbesondere Lennard-Jones-Gemische, in Molekularsimulationen betrachtet. Dabei zeigt sich, dass das Prinzip der Entropieskalierung in hervorragender Näherung auch für Gemische gilt. Die Entropieskalierung wird wiederum dafür genutzt, Molekularsimulationen effizienter zu gestalten und zu bewerten. Dabei wird das in Stuttgart entwickelte *TAMie* Kraftfeld hinsichtlich der Güte seiner Vorhersage von Transportkoeffizienten untersucht. Das *TAMie*-Modell verwendet – wie viele andere für die Bestimmung thermodynamischer Eigenschaften entwickelte Kraftfelder auch – starre Bindungslängen zwischen Wechselwirkungszentren innerhalb eines Moleküls. Um eine aussagekräftige Bewertung der Transportkoeffizienten in molekulardynamischen Simulationen zu gewährleisten, wird eine Analyse der Bindungslängenmodelle durchgeführt: Welchen Einfluss hat die Modellierung der intramolekularen Atombindungen auf die Vorhersage von statischen und dynamischen Fluideigenschaften? Es wird gezeigt, dass es möglich ist, mit flexiblen Atombindungen innerhalb statistischer Genauigkeit die gleichen Ergebnisse für Transportkoeffizienten zu erzielen, wie mit einer starren Beschreibung der Bindungen. Im Rahmen der für diese Arbeit durchgeführten Simulationsstudien wurde ein Workflow entwickelt, der eine effiziente Auswertung der Simulationen für die Bestimmung der Transportgrößen ermöglicht. In Kombination mit der Entropieskalierung wird in dieser Arbeit damit eine Methodik vorgestellt, mit der sehr effizient Transportgrößen aus Molekularsimulationen bestimmt werden können. Dies ermöglicht umfangreiche Simulationsstudien zur Vorhersage von Fluideigenschaften oder zur Kraftfeldentwicklung, bei der Transportkoeffizienten in der Zielfunktion berücksichtigt werden.



## Journal publications

This thesis led to the following publications:

- Chapter 2: M. Fischer, G. Bauer, J. Gross: Force Fields with Fixed Bond Lengths and with Flexible Bond Lengths: Comparing Static and Dynamic Fluid Properties. *Journal of Chemical & Engineering Data*, **65**:1583-1593, 2020. doi:10.1021/acs.jced.9b01031
- Chapter 3: M. Fischer, G. Bauer, J. Gross: Transferable Anisotropic United-Atom Mie (TAMie) Force Field: Transport Properties from Equilibrium Molecular Dynamic Simulations. *Industrial & Engineering Chemistry Research*, **59**:8855-8869, 2020. doi:10.1021/acs.iecr.0c00848

The chapters 2 and 3 present literal quotes of the published work. Any addition with respect to the published work is marked. Any deletion is indicated with square brackets as “[...]”. Cross-references between chapters of this thesis are added to the published version of the text to increase readability. The Supporting Information to the published work is presented in the Appendix of this thesis.



# 1 Introduction

Dynamic properties of gases and liquids are decisive for processes from the nanoscale, such as the folding of proteins<sup>1,2</sup> or the transport of drugs in cells<sup>3,4</sup>, up to global scale phenomena, like for climate and weather.<sup>5-7</sup> Knowledge regarding dynamic properties is also crucial for the design and optimization of processes in engineering applications in chemical industry, like chemical reactions, distillation, and absorption. The present work focuses on this chemical engineering aspect, in particular on the dynamic properties of hydrocarbons as they are used in various chemical processes. Dynamic properties are needed for the optimization of already established processes in chemical industry.<sup>8</sup> Moreover, they are essential for the development of new technologies, such as the synthesis of climate-neutral fuels,<sup>9-11</sup> or the storage of the greenhouse gas CO<sub>2</sub>.<sup>12,13</sup> The accurate prediction of dynamic properties furthermore enables the integrated design and optimization of processes and working fluids. This allows for already known processes, such as the Organic Rankine Cycle, to be operated more effectively.<sup>14-16</sup>

In this work I refer to shear viscosity, thermal conductivity and self-diffusion coefficients as *dynamic properties*. Shear viscosity and thermal conductivity are transport coefficients, i.e. they appear in constitutive (force-flux) equations as linear coefficients.<sup>17</sup> Self-diffusion is not a transport phenomenon driven by a driving force. It describes the averaged translocation of molecules per unit time due to thermal motion of molecules for a system in equilibrium.<sup>18,19</sup>

Transport coefficients act as the proportionality factors relating the thermodynamic driving forces to the flux densities.<sup>17,20</sup> These coefficients thus describe how a system responds to a small external disturbance, namely a perturbation out of equilibrium.<sup>21</sup> For small disturbances, the relation is linear and the proportionality of a given flux  $J$  towards the conjugate driving force, expressed as (negative) gradient  $\nabla f$ , can be described in general terms by the equation<sup>18-21</sup>

$$J = -\gamma \nabla f. \tag{1.1}$$

Here,  $\gamma$  is the transport coefficient, which defines the proportional relation of flux and gradient. For some transport phenomena and their associated transport coefficient, eq. (1.1) reveals widely known relationships for mass, heat and momentum transport, summarized in table 1.1.<sup>20</sup> The flux equations listed here and discussed in this work are solely uncoupled

**Table 1.1:** Common constitutive (flux-force) equations

|  | Gradient                       | Flux                          | Transport Coefficient            |
|--|--------------------------------|-------------------------------|----------------------------------|
| Fourier's law<br>$J_q = -\lambda \frac{\partial T}{\partial x}$    | $T$ : temperature              | $J_q$ : heat flux             | $\lambda$ : thermal conductivity |
| Fick's law<br>$J_i = -D \frac{\partial \rho_i}{\partial x}$        | $\rho_i$ : mass density of $i$ | $J_i$ : mass flux of $i$      | $D$ : diffusion coefficient      |
| Newton's law<br>$J_p^{xy} = -\eta \frac{\partial v_x}{\partial y}$ | $v$ : velocity                 | $J_p$ : shear pressure tensor | $\eta$ : shear viscosity         |

terms. Coupled fluxes are also possible.<sup>20,22</sup> One example is the Soret coefficient, which describes the transport of mass due to the temperature gradient. Even though several relevant applications can be found for the coupled coefficients,<sup>20</sup> they will not be subject of this work.

The thermal conductivity  $\lambda$  and the shear viscosity  $\eta$  are characteristic of pure substances and mixtures and as such they are content of this work. For the case of diffusion, I do not consider the Fickian diffusion coefficient shown in table 1.1, but the self-diffusion coefficient  $D_{\text{self}}$ . Self-diffusion describes the movement of a labeled molecule in the pure substance, in other words, the diffusion of a substance within itself.<sup>18</sup> Self-diffusion of course also occurs in mixtures. The corresponding coefficient is then sometimes referred to as tracer diffusion coefficient  $D_A^T$  of component A.<sup>18</sup> With known tracer diffusion coefficients, Maxwell-Stefan diffusion coefficients  $D_{AB}$  can be approximated<sup>23</sup> by approaches from Darken and Vignes.<sup>18</sup>

Once the transport coefficients are available, they can be used in force-flux equations within the balance equations for mass of each species, linear momentum and energy.<sup>17</sup> Or the transport coefficients can be used to predict the entropy production of a process.<sup>20</sup> Reliable and accurate knowledge of transport properties is therefore indispensable for the accurate prediction of dynamic processes, not only in the chemical industry.

This thesis deals specifically with the prediction of dynamic fluid properties from *molecular simulations* and from *entropy scaling*. Both methods can be used to predict  $\eta$ ,  $\lambda$ , and  $D_{\text{self}}$  over a very wide range of temperatures and pressures (or densities) in both, the vapor and liquid phases, at state points for which no experimental data are available.<sup>24-32</sup> The challenges addressed in this thesis through means of molecular simulations and entropy scaling can be broadly condensed into three objectives:

1. Evaluation of the effects of specific force field properties on the results of molecular simulations of dynamic properties.

2. Development of methods for efficient calculation of transport coefficients from molecular simulations and the robust evaluation of results obtained with force fields adjusted exclusively to phase equilibria data.
3. Coupling of molecular simulations with the entropy scaling method to achieve synergy effects in order to minimize the simulation effort and to gain new insights for entropy scaling.

The following sections of chapter 1 are devoted to a brief introduction of the two central methods applied in this thesis: Section 1.1 gives an overview of the fundamentals of molecular simulations and classical force fields and presents the relevant methods and models used in this work. Section 1.2 describes how dynamic quantities can be determined from equilibrium molecular simulations. To conclude the introductory section, section 1.3 presents the basics of the entropy scaling correlation method.

## 1.1 Molecular Simulations & Force Fields

Statistical mechanics offers the possibility to determine macroscopic thermodynamic quantities (and the dynamic properties, as will be shown in the following sections) by averaging over microscopic states.<sup>33</sup> These molecular states are characterized by the positions of all  $N$  molecules ( $\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N \equiv \mathbf{r}^N$ ) and their momenta ( $\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_N \equiv \mathbf{p}^N$ ).<sup>21</sup> For molecular fluids,  $\mathbf{r}_1$  defines not only a (say) center of mass coordinate of molecule 1, but the entire configuration of that molecule. Analogously  $\mathbf{p}^N$  defines all momenta, including the intramolecular contributions.  $\mathbf{p}^N$  and  $\mathbf{r}^N$  span the *phase space* in which microscopic states are defined. The average  $A$  of a state variable  $a$  can be determined as integrals over all positions and momenta via<sup>34</sup>

$$A = \langle a \rangle = \int a(\mathbf{r}^N, \mathbf{p}^N) P(\mathbf{r}^N, \mathbf{p}^N) d\mathbf{p}^N d\mathbf{r}^N \quad (1.2)$$

with the phase space density distribution function  $P(\mathbf{r}^N, \mathbf{p}^N)$ , which is the probability density of encountering the system with its molecules positioned in the configuration  $\mathbf{r}^N$  with the molecules carrying momenta  $\mathbf{p}^N$ . The phase space density distribution function is related to the non-normalized phase space probability function  $f(\mathbf{r}^N, \mathbf{p}^N)$  by<sup>35</sup>

$$P(\mathbf{r}^N, \mathbf{p}^N) = \frac{f(\mathbf{r}^N, \mathbf{p}^N)}{\int f(\mathbf{r}^N, \mathbf{p}^N) d\mathbf{r}^N} \quad (1.3)$$

The integral in the denominator is the partition function, which is a measure of the total “number” of accessible microscopic states. It is a high-dimensional integral that cannot be solved directly for the applications considered in this work. Therefore, it is generally not possible to calculate the thermodynamic quantities analytically.

In the context of molecular simulations, eq. (1.2) thus rather describes the average value over occurring configurations in phase space. For this purpose, the integral in eq. (1.2) is approximated by a finite sum which allows us to determine averages of thermodynamic quantities without explicitly knowing the partition function. For generating the configurations to be averaged, molecular simulations provide two techniques to explore the phase space, both discussed in the following sections: *Monte Carlo* simulations (MC) and *Molecular Dynamics* simulations (MD). In both methods, macroscopic thermodynamic properties are determined by simulating a finite, manageable number of molecules in a simulation volume. Bulk systems are emulated by ensuring the simulation volume is surrounded by infinite copies of itself using periodic boundary conditions (PBC). The interactions between the observed molecules are defined by *force fields*, which are addressed in the following section.

### 1.1.1 Classical Force Fields

In classical molecular simulations, force fields are used to describe interactions between atoms of a configuration  $\mathbf{r}^N$ . A force field defines a set of equations and parameters that is used to calculate the distinct contributions to the potential energy  $E_{\text{pot}}$  of a system. The potential energy is used in MC simulations to evaluate the acceptance criterion of a new configuration, while in MD the force acting on molecules ( $\mathbf{F} = -\partial E_{\text{pot}}/\partial \mathbf{r}^N$ ) is used to determine the time evolution of the system. The quality of simulation results depends heavily on the quality and suitability of the force field used, and hence the development and improvement of force fields is the subject of a great number of research efforts.<sup>36–49</sup> The field of applications ranges from relatively general to very specific. On one side of the spectrum are force fields designed for good transferability of parameters to different substances. Examples for transferable force fields are the AUA<sup>36</sup> force field, TraPPE,<sup>37</sup> and GROMOS.<sup>38–41</sup> On the other hand, there are force fields optimized precisely for a specific substance and even for the description of some peculiar substance quantities, such as specific protein solvents.<sup>48</sup>

In the present work, the Transferable Anisotropic United-Atom Mie Potential force field (*TAMie*) developed by Hemmen *et al.*,<sup>42,43</sup> Weidler and Gross,<sup>44–46</sup> and Baz *et al.*<sup>47</sup> is used to evaluate the potential energy. The force field uses an united-atom (UA) approach, in which hydrogen atoms are grouped together with larger atoms and treated as a single interaction site. For example, methyl ( $\text{CH}_3$ ) and methylen ( $\text{CH}_2$ ) groups are considered as single interaction



sites, which significantly reduces the computational effort while yielding good predictions. TAMie is an anisotropic force field, because the bond length between the methylen and the methyl end groups of the hydrocarbon molecules is increased by  $\Delta l$ .

Like other common force fields, TAMie calculates  $E_{\text{pot}}$  as a sum of various contributions to the potential energy of a system as<sup>50</sup>

$$E_{\text{pot}}(\mathbf{r}^N) = E_{\text{VdW}}(\mathbf{r}^N) + E_{\text{charge}}(\mathbf{r}^N) + E_{\text{bond}}(\mathbf{r}^N) + E_{\text{bend}}(\mathbf{r}^N) + E_{\text{torsion}}(\mathbf{r}^N) \quad (1.4)$$

The contributions can be broken down into two categories. The Van-der-Waals interactions  $E_{\text{VdW}}$  and the Coulombic interactions  $E_{\text{charge}}$  characterize the *nonbonded* interactions between atoms of different molecules and between atoms of the same molecule that are sufficiently far apart. The *covalent* terms describe the intramolecular contributions due to stretching of bond lengths  $E_{\text{bond}}$ , bond angles  $E_{\text{bend}}$ , and torsion angles  $E_{\text{torsion}}$ . In the case of the TAMie force field, the individual energy contributions are calculated as<sup>44</sup>

$$\begin{aligned} E_{\text{pot}}(\mathbf{r}^N) = & \underbrace{\sum_{\text{pairs } i,j} c_{ij} \varepsilon_{ij} \left[ \left( \frac{\sigma_{ij}}{r_{ij}} \right)^{n_{ij}} - \left( \frac{\sigma_{ij}}{r_{ij}} \right)^6 \right]}_{E_{\text{VdW}}} + \underbrace{\sum_{\text{pairs } i,j} \frac{q_i q_j}{4\pi \epsilon_0 r_{ij}}}_{E_{\text{charge}}} \\ & + \underbrace{\sum_{\text{bonds}} k_{ij}^1 (r_{ij} - r_{ij}^0)^2}_{E_{\text{bond}}} + \underbrace{\sum_{\text{angles}} \frac{k_{ijk}^\theta}{2} (\theta_{ijk} - \theta_{ijk}^0)^2}_{E_{\text{bend}}} \\ & + \underbrace{\sum_{\text{prop. dih.}} \sum_{n=0}^3 c_{n,ijkl} [1 + d_{n,ijkl} \cos(n\phi_{ijkl})]}_{E_{\text{torsion}}} + \sum_{\text{imp. dih.}} \frac{d_{ijkl}^0}{2} (\phi_{ijkl} - \phi_{ijkl}^0)^2 \end{aligned} \quad (1.5)$$

where  $r_{ij} = |\mathbf{r}_j - \mathbf{r}_i|$  is the distance between two interaction sites  $i$  and  $j$ , located at position  $\mathbf{r}_i$  and  $\mathbf{r}_j$ , respectively.

The Mie- $n$ -6 potential given in eq. (1.5) that is used to describe  $E_{\text{VdW}}$  in the TAMie force field is defined by the energy parameter  $\varepsilon_{ij}$ , a size parameter  $\sigma_{ij}$ , and the exponent  $n_{ij}$ . The parameters of interactions between different types of UA centers are calculated via Lorentz-Berthelot rules.<sup>51,52</sup> Furthermore, an additional cross-interaction parameter  $k_{ij}$  can be introduced for mixtures.<sup>46</sup> A more detailed description of the potential and the calculated prefactor  $c_{ij}$  will be given in a later chapter. Electrostatic interactions are modelled as Coulomb interactions by placing point charges  $q_i, q_j$  on the centers of the VdW interactions or as additional off-center sites, where  $q_i$  is a fixed point charge (given as a factor of the electron charge) and  $\epsilon_0$  is the dielectric constant in vacuum. Applying multiple charges effectively leads

to multipole interactions. That way, dipole-dipole and quadrupole-quadrupole interactions can also be modeled using point charges.

The covalent terms in eq. (1.5) assign energy penalties for the displacements in distance ( $E_{\text{bond}}$ ) and angle ( $E_{\text{bend}}$ ) between bound interaction sites from a defined bond length  $r_{ij}^0$  by  $r_{ij}$ , or from a defined angle  $\theta_{ijk}^0$  by the angle  $\theta_{ijk}$ . The energy that results from torsion of dihedral angles  $\phi_{ijkl}$  between  $i$ ,  $j$ ,  $k$ , and  $l$  also adds to this contribution. The magnitude of the intramolecular contributions to the potential energy is defined by the respective force constants  $k_{ij}^1$ ,  $k_{ijk}^\theta$ ,  $c_{n,ijkl}$ , and  $d_{ijkl}^0$ .

The force field parameters of the intramolecular potentials, namely equilibrium values and force constants, are typically determined either from experimental data or from quantum mechanics calculations.<sup>53–56</sup> For a number of reasons, many force fields use certain length or angle constraints.<sup>57,58</sup> Also the original version of TAMie uses fixed bond lengths between neighboring interaction sites  $i-j$ , instead of a harmonic potential  $k_{ij}^1(r_{ij} - r_{ij}^0)^2$ , as in eq. (1.5). Fixed bond lengths can be justified in certain cases<sup>35</sup> and are convenient in Monte Carlo simulations, but also in molecular dynamics simulations, because the high-frequency vibrations of typical interatomic bond-lengths then do not have to be resolved numerically.<sup>59–61</sup> Consequently, for rigid bond lengths, the time step can be significantly increased, enabling longer simulation times. At the same time, the bond length vibrations do not significantly contribute to many physical properties.<sup>35</sup>

However, there are downsides when using constant bond lengths, that can be subtle. The averages obtained in a system with rigid bonds are different from those obtained in a system of infinitely strong but non-rigid bonds.<sup>21</sup> For the heat capacity  $c_v$ , as well as for the thermal conductivity  $\lambda$ , vibrational contributions have to be considered and hence correction terms have to be introduced to “correct” for rigid bonds.<sup>33,62</sup> Furthermore, modifying parts of the intramolecular interactions can significantly change the dynamic properties of a fluid.<sup>63–65</sup> It is difficult to predict how or to which degree individual parameters influence these properties. For some substances, such as water, the influence of, for example, bond flexibility on static and dynamic properties has been investigated.<sup>66,67</sup> Other studies show, that the parameterization of the torsional energy contribution  $E_{\text{torsion}}$  has a strong influence on the viscosity without affecting other properties, such as the VLE.<sup>68</sup>

Several questions addressed in this thesis concern force fields and the influence of their parameters on dynamic properties. Chapter 2 investigates the influence of bond strength (defined by  $k_{ij}^1$ ) on the transport quantities for various hydrocarbons and answers the question whether simulations with rigid and flexible bond lengths yield sufficiently similar results within typical statistical uncertainty. Chapter 3 considers the TAMie force field and addresses the

question: How well can a force field predict transport properties of fluids, whose parameters were fitted solely to VLE data (vapor pressure and liquid density), with no information on dynamic properties included in the optimization process? In chapter 4, simulations of simple model fluids are used to assess the concept of entropy scaling for mixtures. The force fields of the atomic Lennard-Jones fluids used in this study feature only a comparably small number of parameters, allowing the influence of individual parameters on the properties of the mixtures to be thoroughly investigated. In order to answer such questions, this work employs force fields in molecular simulations of both techniques, MC and MD, which are therefore briefly outlined in the following section.

### 1.1.2 Molecular Simulations

The historically earlier molecular simulation method is the **Monte Carlo** method, which was proposed by Metropolis *et al.*<sup>69</sup> The method uses *importance sampling*, to generate configurations according to the probability of their occurrence  $P(\mathbf{r}^N)$  by stepping through phase space in distinct moves.<sup>35</sup> This allows the ensemble averages  $A$  of a macroscopic quantity  $a$  of the system to be calculated simply by averaging over the sampled values of  $a$  for all moves.<sup>34</sup>

$$A = \langle a \rangle = \frac{1}{N_{\text{moves}}} \sum_{i=1}^{N_{\text{moves}}} a_i \quad (1.6)$$

In simulations in the canonical ensemble – the one natural to MC – each move is a displacement of one or more arbitrary atoms to a random new position according to the principle of importance sampling.<sup>34</sup> The MC simulations in this work are performed in the grand canonical ensemble. Such simulations, at constant  $\mu$ ,  $V$ , and  $T$ , are realized by introducing deletion and insertion of molecules as additional moves besides the displacement moves.<sup>35</sup> Grand canonical MC simulations (GCMC) provide efficient methods for determining vapor-liquid equilibria (VLE) via molecular simulations,<sup>42,70</sup> which are also used in this work and will be therefore introduced in chapter 4. Mainly, however, GCMC simulations are used in this work to determine entropy values along isotherms via the Gibbs equation, as described in section 4.2.3. Here, only a single GCMC simulation is needed at each temperature. The entropy at different pressures along the isotherm is then determined via histogram reweighting, which significantly reduces the computational effort.<sup>71</sup> As such, Monte Carlo simulations and histogram reweighting provide us with very efficient methods for the determination of static thermodynamic quantities.

However, if one is interested in dynamic properties, like viscosity, thermal conductivity, or

diffusion, one must rely on **Molecular Dynamics** simulations, in which a system is allowed to evolve in time. In this method, the motion of molecules in the system is estimated along a temporal trajectory by integrating Newton's equation of motion.<sup>35</sup> In a simulation at constant number of molecules, volume and energy (NVE), the trajectory thereby conserves the Hamiltonian  $\mathcal{H}$ , being the operator describing the total energy of the system (potential energy  $E_{\text{pot}}$  and kinetic energy  $E_{\text{kin}}$ ) as a function of the phase space variables  $\mathbf{r}^N$  and  $\mathbf{p}^N$ .<sup>34</sup>

$$\mathcal{H}(\mathbf{r}^N, \mathbf{p}^N) = E_{\text{kin}}(\mathbf{p}^N) + E_{\text{pot}}(\mathbf{r}^N) \quad (1.7)$$

For a Hamiltonian system in cartesian coordinates the following relations, known as Hamilton's equations of motion, apply.<sup>21</sup>

$$\dot{\mathbf{r}}^N = \frac{\partial \mathcal{H}}{\partial \mathbf{p}^N} = \frac{\mathbf{p}^N}{m} \quad (1.8)$$

$$\dot{\mathbf{p}}^N = -\frac{\partial \mathcal{H}}{\partial \mathbf{r}^N} = -\frac{\partial E_{\text{pot}}(\mathbf{r}^N)}{\partial \mathbf{r}^N} = \mathbf{F}(\mathbf{r}^N) \quad (1.9)$$

where  $\mathbf{F}$  is the vector of forces on each molecule of a system with the configuration  $\mathbf{r}^N$ . The dots above  $\dot{\mathbf{r}}^N$  and  $\dot{\mathbf{p}}^N$  symbolize the time derivative of the respective quantity.

Thermodynamic properties can be determined directly from the time integration of these equations of motion as an average over the time trajectories of the sampled quantities.<sup>35</sup> Yet there are several details that must be taken into account when simulating in order to ensure reliable results.<sup>31,72</sup> For instance, the algorithm used for time integration and the applied time step size may affect the result of a simulation.<sup>73</sup> In order to perform MD simulations at constant temperature or pressure, a thermostat or barostat, respectively, is required.<sup>34</sup> Selecting an unsuitable method for this purpose can lead to incorrect results, especially when investigating dynamic properties.<sup>74</sup> When determining dynamic properties in MD simulations, there are generally some additional caveats to consider compared to static quantities. For example, in order to determine transport coefficients, either the equations of motion have to be extended or additional quantities have to be sampled and processed.<sup>21,31</sup> Being a central subject of this thesis, the following chapter will introduce in more detail the specifics of determining dynamic coefficients via molecular simulations.

## 1.2 Dynamic Properties from Molecular Dynamics

Two general approaches are possible for determining transport coefficients using molecular simulations, both of which are MD methods. In addition to the equilibrium MD simulations

already introduced in section 1.1, an alternative technique are non-equilibrium MD simulations (NEMD), which will be regarded first.

### 1.2.1 Non-Equilibrium Molecular Dynamics

The NEMD approach appears to be the more intuitive one, since the transport properties are determined in a way comparable to related experiments.<sup>21</sup> A gradient is induced on the system and the corresponding transport quantity can be determined from the response of the system to this perturbation of the equilibrium. For this purpose, in the computer experiment, the equations of motion (eqs. (1.8) and (1.9)) are extended, as<sup>21</sup>

$$\dot{\mathbf{r}} = \frac{\partial \mathcal{H}}{\partial \mathbf{p}} + C(\mathbf{r}, \mathbf{p}) F_e(t) \quad (1.10)$$

$$\dot{\mathbf{p}} = -\frac{\partial \mathcal{H}}{\partial \mathbf{r}} + D(\mathbf{r}, \mathbf{p}) F_e(t) \quad (1.11)$$

$F_e(t)$  is a time-dependent driving force,  $C(\mathbf{r}, \mathbf{p})$  and  $D(\mathbf{r}, \mathbf{p})$  are functions of the phase space, which are specified by the respective perturbation.

Taking the example of the viscosity of a Couette flow in the  $x$  direction, this yields a set of equations of motion known as the SLLOD algorithm.<sup>75</sup>

$$\dot{\mathbf{r}} = \mathbf{p}/m + \gamma_{yx} (\mathbf{r} \cdot \bar{\mathbf{e}}_y) \bar{\mathbf{e}}_x \quad (1.12)$$

$$\dot{\mathbf{p}} = \mathbf{F} - \gamma_{yx} (\mathbf{p} \cdot \bar{\mathbf{e}}_y) \bar{\mathbf{e}}_x \quad (1.13)$$

Here,  $\bar{\mathbf{e}}_\alpha$  with  $\alpha = x, y, z$  is the unit vector in  $\alpha$  direction and  $\gamma_{yx}$  is the shear rate  $\frac{\partial v_x}{\partial y} = \gamma_{yx}$ , the change in the velocity in  $y$  direction (orthogonal to the direction of the flow). The systems response can be determined, manifesting as response of the non-diagonal entries of the pressure tensor  $J_p^{xy}$ . Using with Newton's shear law from table 1.1, the viscosity can be obtained as<sup>34</sup>

$$\eta = - \lim_{t \rightarrow \infty} \frac{\langle J_p^{xy} \rangle_t}{\gamma_{yx}} \quad (1.14)$$

The non-equilibrium average  $\langle J_p^{xy} \rangle_t$  can be determined as the time average of a trajectory following the equations of motion according to eqs. (1.12) and (1.13). It is necessary to apply certain specific periodic boundary conditions that take into account both the deformation of the simulation box due to shear, as well as the flow.<sup>76</sup> Moreover, high shear rates are required to ensure a low noise-to-signal ratio. To reach the viscosity at the shear rate of interest (often

the viscosity at  $\gamma_{yx} = 0$ ) it is necessary to extrapolate from several simulations at different  $\gamma_{yx}$ .<sup>77</sup> Furthermore, since the perturbation of the equations of motion introduces energy into the system from external sources, a thermostat must be used in this case for NEMD simulation.<sup>78</sup>

Some of the problems mentioned are overcome by a different non-equilibrium method, the Reverse NEMD (RNEMD).<sup>78,79</sup> In this approach, cause and effect are interchanged. A flux is artificially created in the system, which results in a gradient in the simulated system. In the example of viscosity, a momentum flux is applied by swapping the velocities of two particles by a Maxwell daemon. For this purpose, the simulation box is divided into slices in the  $y$  direction. The daemon now regularly exchanges the velocities of two particles with opposed velocities, which are located in two slices that are half a box length apart. This way, momentum is transferred artificially, which results in a physical momentum flux that compensates for this disturbance. A linear velocity profile evolves in the system in  $y$  direction, from which the viscosity can be determined. RNEMD generally converges comparatively well<sup>80</sup> and is considered to conserve energy, so no (or only a very weak) thermostat has to be used. Moreover, ordinary periodic boundary conditions can be applied. For high momentum fluxes, nonlinear velocity profiles are exhibited, leading to incorrect values for  $\eta$ .

A drawback – which RNEMD has in common with other NEMD methods – is that a dedicated flux (or driving force) has to be applied for any transport coefficient. Besides the viscosity shown above, thermal conductivity and diffusion can also be calculated via NEMD using the appropriate  $C(\mathbf{r}, \mathbf{p})$ ,  $D(\mathbf{r}, \mathbf{p})$  and  $F_e(t)$  terms in eqs. (1.10) and (1.11). This is where the equilibrium approach for determining the transport properties in MD has a strong advantage: The method determines the transport coefficients from equilibrium simulations and can thus calculate all coefficients simultaneously.

### 1.2.2 Transport Coefficients from Equilibrium MD Simulations

Along with the NEMD methods shown, dynamic quantities such as the transport coefficients can also be determined in simulations in the equilibrium ensemble in which no perturbations are imposed. The method is based on the fact that all the information about the response of a system to a perturbation is contained in the *equilibrium time correlation function* (TCF), which can be determined in equilibrium simulations. The origin of the TCF, as well as its relation to the dynamic properties, will be briefly presented in this section. The argumentation follows a chapter of the text book by Tuckerman,<sup>21</sup> where the interested reader can find a more detailed derivation.

The initial point for the derivation is – as for NEMD – the perturbed equations of motion, shown in eqs. (1.10) and (1.11). At this point, we assume just a small perturbation from equilibrium, so that the driving forces lead only to a small perturbation of Hamilton's equations. For the time-dependent phase space density distribution function of the non-equilibrium ensemble a perturbation approach can then be applied as

$$P(\chi, t) = P_0(\mathcal{H}(\chi)) + \Delta P(\chi, t) \quad (1.15)$$

where  $\chi$  abbreviates the phase space vector  $\chi = (\mathbf{r}, \mathbf{p})$  for a more compact notation.  $P_0$  describes the equilibrium phase space density distribution function as a function of the Hamiltonian  $\mathcal{H}(\chi)$  of the undisturbed system;  $\Delta P(\chi, t)$  denotes the time-dependent perturbation. For the average of an arbitrary phase space function  $a(\chi)$  this perturbation approach yields

$$\langle a \rangle_t = \int a(\chi) P(\chi, t) d\chi \quad (1.16)$$

$$= \int a(\chi) P_0(\mathcal{H}(\chi)) d\chi + \int a(\chi) \Delta P(\chi, t) d\chi \quad (1.17)$$

$$= \langle a \rangle + \int a(\chi) \Delta P(\chi, t) d\chi \quad (1.18)$$

$\langle a \rangle_t$  is the average in the non-equilibrium ensemble and corresponds to the time-dependent macroscopic quantity  $A(t)$ , whereas  $\langle a \rangle$  is the average of  $a(\chi)$  in equilibrium (see eq. (1.2)). Since the disturbance of the system due to the driving force is assumed to be small, the perturbation term can be linearized. This approach is the basis of the linear response theory, which allows to solve the Liouville equation of the system. As result, the following equation is obtained for  $\langle a \rangle_t$ .

$$\langle a \rangle_t = \langle a \rangle - \beta \int_0^t F_e(s) ds \int P_0(\mathcal{H}(\chi)) a(\chi_{t-s}) j(\chi) d\chi \quad (1.19)$$

The integration over the time variable  $s$  in eq. (1.19) originates from the solution of the Liouville equation.  $F_e(s)$  is the driving force from eqs. (1.10) and (1.11) with respect to the time  $s$ . Equation (1.19) further introduces the *dissipative flux*  $j(\chi)$ , which is defined by the equation

$$j(\chi) = - \sum_{i=1}^{3N} \left[ D_i(\chi) \frac{\partial \mathcal{H}}{\partial \mathbf{p}_i} + C_i(\chi) \frac{\partial \mathcal{H}}{\partial \mathbf{r}_i} \right] \quad (1.20)$$

Also note, that for eq. (1.19) the system is assumed to be in equilibrium by time  $t \leq 0$ . So the

driving forces are zero for  $t < 0$ , giving also  $\Delta P(\chi, t = 0) = 0$  and  $\langle a \rangle_{t=0} = \langle a \rangle$ .

For the second integral in eq. (1.19), Tuckerman gives a quite accessible interpretation:<sup>21</sup> Each point  $\chi$  in the phase space is considered as a starting point. The system is then allowed to evolve from this point for a time  $t - s$  according to Hamilton's equations of motion. After this time a new point in the phase space is reached:  $\chi_{t-s}$ . As a solution of Hamilton's equations of motion,  $\chi_{t-s}$  depends solely on the respective starting point  $\chi$ , which is why it is denoted as  $\chi_{t-s}(\chi)$  in eq. (1.19). The integral has the form of an average, similar to that in eq. (1.2). It corresponds to the average of  $a(\chi_{t-s}(\chi))j(\chi)$  over all possible initial conditions, with respect to the phase space density distribution function of the unperturbed ensemble  $P_0(\mathcal{H}(\chi))$ . The resulting average is multiplied with  $F_e(s)$ .  $s$  is then integrated from 0 to  $t$  to obtain the time-dependent non-equilibrium average of  $a$  at time  $t$ :  $\langle a \rangle_t = A(t)$ .

It is noteworthy at this point that, with eq. (1.19), it is possible to determine the non-equilibrium-average, exclusively via averages of the equilibrium ensemble, characterized by  $P_0(\mathcal{H}(\chi))$ . The average value calculated in the second part of eq. (1.19) is the aforementioned *equilibrium time correlation function* (TACF). By expressing the equilibrium TACF as an average, eq. (1.19) can be rewritten.

$$A(t) = \langle a \rangle_t = \langle a \rangle - \beta \int_0^t F_e(s) \langle a(t-s)j(0) \rangle ds \quad (1.21)$$

All quantities in this equation – the equilibrium average  $\langle a \rangle$ , just as the quantities in the TACF – can be sampled in an equilibrium MD simulation. In order to be able to calculate the transport quantities via TACFs, the respective perturbation terms  $F_e$ ,  $C(\chi)$  and  $D(\chi)$  still have to be identified. They can be expressed in quantities that are captured during the simulation. Using the example of viscosity, this step will be illustrated in the following.

The perturbed equations of motion of a Couette flow in the  $x$  direction (eqs. (1.12) and (1.13)) again serve as a basis. By comparison with the unperturbed equations of motion (eqs. (1.8) and (1.9)) and the extended form of the equations of motion from eqs. (1.10) and (1.11),  $C$  and  $D$  can be identified as

$$C(\mathbf{r}, \mathbf{p}) = \gamma_{yx} (\mathbf{r} \cdot \bar{\mathbf{e}}_y) \bar{\mathbf{e}}_x \quad (1.22)$$

$$D(\mathbf{r}, \mathbf{p}) = \gamma_{yx} (\mathbf{p} \cdot \bar{\mathbf{e}}_y) \bar{\mathbf{e}}_x \quad (1.23)$$



with a resulting value of  $F_e(t) = 1$ . As a matter of that, the dissipative flux is calculated as

$$\begin{aligned}
 j(\chi) = j(\mathbf{r}, \mathbf{p}) &= \sum_{i=1}^{3N} \left[ C_i(\mathbf{r}, \mathbf{p}) \mathbf{F}_i - D_i(\mathbf{r}, \mathbf{p}) \frac{\mathbf{p}_i}{m_i} \right] \\
 &= \gamma_{yx} \sum_{i=1}^{3N} \left[ (\mathbf{r} \cdot \bar{\mathbf{e}}_y) (\bar{\mathbf{e}}_x \cdot \mathbf{F}_i) + (\mathbf{p} \cdot \bar{\mathbf{e}}_y) \left( \bar{\mathbf{e}}_x \cdot \frac{\mathbf{p}_i}{m_i} \right) \right] \\
 &= \gamma_{yx} V J_p^{xy}
 \end{aligned} \tag{1.24}$$

with  $J_p^{xy}$  as a non-diagonal entry of the instantaneous pressure tensor

$$J_p(\mathbf{r}, \mathbf{p}) = \frac{1}{3V} \sum_{i=1}^N \left[ \frac{\mathbf{p}_i^2}{m_i} + \mathbf{r}_i \cdot \mathbf{F}_i \right] \tag{1.25}$$

As a result, the non-equilibrium-average of the non-diagonal entry of the pressure tensor  $\langle J_p^{xy} \rangle_t$  can be calculated according to eq. (1.21) as

$$\langle J_p^{xy} \rangle_t = \langle J_p^{xy} \rangle - \beta \gamma_{yx} V \int_0^t \langle J_p^{xy}(0) J_p^{xy}(t-s) \rangle ds \tag{1.26}$$

With the definition of viscosity from eq. (1.14) as  $\eta = -\lim_{t \rightarrow \infty} \langle J_p^{xy} \rangle_t / \gamma_{yx}$  and the fact that  $\langle J_p^{xy} \rangle$  equals zero according to the virial theorem, one obtains the famous Green-Kubo<sup>81,82</sup> equation for viscosity

$$\eta = \beta V \int_0^{\infty} \langle J_p^{xy}(0) J_p^{xy}(\tau) \rangle d\tau \tag{1.27}$$

introducing a change of variables  $\tau = t - s$  into the integral. The Green-Kubo equations can also be derived for thermal conductivity and self-diffusion. The corresponding quantities of the TACF are the heat flux  $J_q$  for  $\lambda$  and the velocities of the particles  $\mathbf{v}_i$  for  $D_{\text{self}, i}$ .

According to Onsager's regression hypothesis, any TACF decays to zero after a sufficiently long time.<sup>83</sup> Thus, essentially constant values for the transport coefficients of a fluid with low molecular mass can be determined from finite integration times in eq. (1.21). How long the decay time (correlation time) actually is, in the specific case, depends strongly on the nature of the system under study. In most cases considered in this work, the TACF decayed to zero after only a few simulation time steps. When using the Green-Kubo equations, it is crucial to record the required values for the TACF (i.e.,  $J_p^{xy}$ ,  $J_q$ , and  $\mathbf{v}_i$ ) at a high frequency, because

the values at very small times  $\tau$  substantially contribute to the value of the integral of the autocorrelation function, eq. (1.27). For higher values of  $\tau$ , the noise-to-signal ratio of the TACF becomes significantly worse. Accordingly, for the evaluation of the Green-Kubo integrals, it is necessary to balance between selecting an integration time  $\tau_{\max}$  that is long enough to ensure that the TACF has decayed while still providing good statistics for the integral.

Associated with each Green-Kubo equation is a corresponding Einstein relation, into which the long-time behavior of the trajectories enters, in contrast to the short correlation times which are relevant for the Green-Kubo equations. Green-Kubo and Einstein relation can be straightforwardly transformed into each other.<sup>34</sup> In the present work, the diffusion coefficient is determined from an Einstein relation that calculates  $D_{\text{self}}$  from the Mean Squared Displacement (MSD) of the molecule center of mass, according to<sup>35</sup>

$$D_{\text{self}} = \frac{1}{6N} \lim_{\tau \rightarrow \infty} \frac{d}{d\tau} \sum_{i=1}^N \langle |\mathbf{r}_i(\tau) - \mathbf{r}_i(0)|^2 \rangle \quad (1.28)$$

We use the Einstein equation instead of the velocity autocorrelation function in this case as it only requires storing the positions of the molecules at comparatively large time intervals and does not require additional sampling of the molecule velocities at high frequency.

In general, the Green-Kubo and Einstein relations shown here can be used to determine dynamic properties from quantities that can be sampled in ordinary equilibrium MD simulations. This is where the advantages of this method over NEMD methods become apparent. The equations of motion do not need to be modified, no special periodic boundary conditions need to be applied, and no (additional) thermostating is required. And most significantly, all three transport coefficients can be determined from one and the same simulation, which reduces the simulation effort. The evaluation of the dynamic properties can be done in post-processing; in fact, the simulation is independent of the evaluation of the coefficients – as long as the required quantities are recorded at a sufficiently high frequency during the simulation. A particular challenge in post-processing lies in a statistically optimized analysis of the data in order to improve the noise-to-signal ratio, which has long been considered the major weakness in the determination of transport properties from EMD simulations.<sup>84</sup> There have been advancements in this field in recent years, and new algorithms have been developed to efficiently compute the TACF and to improve and accurately quantify statistical uncertainties. Some examples of such techniques, such as the moving window algorithm,<sup>85</sup> the time decomposition method,<sup>86</sup> and bootstrapping methods for determining error bars are collected in the Best Practice Paper for Computing  $\eta$  and  $D_{\text{self}}$  with EMD by Maginn *et al.*,<sup>31</sup> which gives a highly recommended overview of the subject. The best statistics at the highest efficiency can be achieved by computing the TACF using the Fast Fourier Transform

(FFT) algorithm.<sup>21</sup> Here, the TACF are computed as convolutions in Fourier space and are subsequently transformed back. No data is omitted in the process and the calculation is performed in a fraction of the time required by conventional algorithms, while at the same time, requiring comparatively little memory resources.

One goal of the present work is to build an efficient framework for robust determination of transport coefficients from EMD simulations. It addresses both, optimal post-processing and the rigorous setup of simulation sets. By combining Fast Fourier Transform algorithms with best-practice methods, I have developed a workflow that allows – especially in combination with the entropy scaling methods that will be introduced in section 1.3 – to determine dynamic properties over a wide range of states easily and in a comparably short time. A detailed description of the workflow and techniques used can be found in chapters 2 and 3 of this work.

### 1.3 Entropy Scaling

In determining transport properties, the present thesis is not limited to molecular simulations alone. The goal of this work is to combine simulations with the entropy scaling method in various ways in order to achieve synergy effects on multiple levels.

Entropy scaling is a semi-empirical correlation approach by which transport properties can be determined based on the residual entropy  $s_{\text{res}}$  of a given state defined by  $\rho$  and  $T$ . The residual entropy is the entropy minus the entropy of an ideal gas at the same temperature  $T$  and density  $\rho$ , as

$$s_{\text{res}}(T, \rho) = s(T, \rho) - s_{\text{id. gas}}(T, \rho) \quad (1.29)$$

The connection between  $s_{\text{res}}$  and dynamic properties was discovered and first researched by Yaakov Rosenfeld already in the 1970s.<sup>87</sup> However, the method received little attention until it was brought back into spotlight by a study by Dzugutov<sup>88</sup> published in Nature in 1996. Without providing a complete physical derivation, Rosenfeld’s original article was the first to describe a monovariate relationship between excess entropy and a transport property  $\zeta = (\eta, \lambda, D_{\text{self}})$  in reduced form  $\zeta^{\#}$ , i.e. the property divided by a ‘reference’ property. Rosenfeld initially demonstrated the principle only for viscosity and diffusion; it was later applied to thermal conductivity as well.<sup>89</sup> Rosenfeld’s investigations included model fluids based on Lennard-Jones and hard sphere potentials, whose dynamic properties were determined in molecular simulations. Subsequent work on entropy scaling also continued using molecular simulations

to demonstrate the applicability of the principle.<sup>25,90–92</sup> Thus, entropy scaling has historically been closely related to molecular simulation.

The rather involved property behavior of transport coefficients for varying state variables, such as temperature and pressure, makes the development of engineering correlations difficult – especially across phase boundaries. Using Rosenfeld’s approach, transport coefficients can be described as a function of a single state variable as

$$\zeta^\#(T, \rho) = \frac{\zeta(T, \rho)}{\zeta^{\text{ref}}(T, \rho)} = \zeta^\#(s_{\text{res}}(T, \rho)) \quad (1.30)$$

where  $\zeta^{\text{ref}}(T, \rho)$  stands for a suitable reference for each specific transport coefficient, which is used to obtain a dimensionless property. Several approaches using different references can be found in literature, which yield more or less satisfactory results. Rosenfeld’s original approach was to define  $\zeta^{\text{ref}}$  in such a way that  $\zeta^\#$  becomes dimensionless, using combinations of thermodynamic quantities and constants, such as  $\rho, T, k_B$ . Towards the low-density limit (for which eq. (1.29) gives  $s_{\text{res}} \rightarrow 0$ ), this approach, however, leads to a diverging increase of  $\zeta^\#$  for gases.<sup>24</sup> This behavior can be avoided for  $\eta$  and  $D_{\text{self}}$  by using the Chapman-Enskog approximation as a reference – a first-order approximation of the Boltzmann-equation, as proposed by Novak<sup>93</sup> and Lötgering-Lin and Gross,<sup>24</sup> as well as Hopp *et al.*<sup>26</sup> for self-diffusion. The new reference leads to finite values at the low density limit and a simple parameterization of correlation functions.

In recent years, entropy scaling has received increased attention, partly in connection with the *isomorph theory*, giving a lead for the physical derivation of the entropy scaling approach. The theory, developed by the *Glass & Time* group from Roskilde, describes the occurrence of special lines in the thermodynamic state space, so-called *isomorphs*. Isomorphs occur in fluids whose fluctuations in energy  $U$  and virial  $W$  (from the virial equation  $pV = \frac{N}{\beta} + W$ ) are strongly correlated. Fluids with a correlation factor of  $R = \frac{\langle \Delta W \Delta U \rangle}{\sqrt{\langle \Delta W^2 \rangle \langle \Delta U^2 \rangle}} > 0.9$  are called *Roskilde fluids*<sup>94</sup> and can be characterized by some interesting behaviors and invariants of structural, thermodynamic and dynamic properties, which hold along an isomorph  $I$ .<sup>95</sup> For example, the radial pair distribution function in reduced coordinates  $\tilde{g}(\tilde{r})$  does not change along isomorph  $I$ .<sup>94</sup> The isomorph can thereby cover states of different pressures, temperatures and densities. Another fascinating phenomenon is the aging behavior of a system along an isomorph  $I$ . After a jump from an equilibrated state point on the isomorph to another state on the same isomorph (for example in a molecular simulation), the system is *instantaneously* back in equilibrium. Isomorphs thus can be thought of as “*wormholes in phase space*”.<sup>95</sup>

In the context of entropy scaling, though, the following two invariants are most interesting:

Both, the residual entropy and the TACF are considered constant in reduced coordinates along isomorph  $I$ .<sup>94,96</sup> From the immediate relationship between transport coefficients and the TACF as used in the Green-Kubo relations (see, e.g., eq. (1.27) for viscosity), it can be deduced that the reduced transport coefficients are also constant along isomorph  $I$ .<sup>97</sup> That means, that entropy scaling holds along isomorphs. Consequently, a number of studies of entropy scaling have already been carried out from the perspective of isomorph theory.<sup>98</sup> For the isomorph theory, model fluids have been extensively studied by means of molecular simulations.<sup>97–100</sup> Instead of a reference, dimensionless transport coefficients are usually considered directly in this context, similar to those also used in this work in chapter 4 (see table 4.1).

However, entropy scaling is not only applicable to Roskilde fluids. Monovariation of transport coefficients and  $s_{\text{res}}$  also occurs for fluids and state points where  $\Delta U$  and  $\Delta W$  are not correlated. Therefore the isomorph theory does not give a conclusive derivation for entropy scaling.

Although the fundamental physical derivation for entropy scaling has not yet been uncovered – what speaks for the approach is that it works very convincingly in application and provides a powerful engineering tool. Beyond model fluids and proofs of concepts, entropy scaling can be used for real fluids and applications in industrial practice.<sup>14–16</sup> With a suitable choice of reference fluid, the monovariation profile of a reduced transport coefficient over residual entropy  $\ln(\zeta^\#(s_{\text{res}}))$  is so benign that a simple correlation function can be adjusted to match it. In the simplest case, for shear viscosity, even a linear Ansatz function can give good results. Once the correlation function and its parameters are determined, the corresponding transport coefficient is accessible for any state point for which the residual entropy is known.

In this work, the entropy scaling variant developed in Stuttgart by members of the Thermodynamics group is used. Approaches for viscosity,<sup>24,25</sup> self-diffusion,<sup>26</sup> and thermal conductivity<sup>27–29</sup> have already been published and further research is ongoing. In the mentioned studies, an ideal gas approximation of the respective transport quantity (e.g., the mentioned CE viscosity<sup>19</sup> for  $\eta^\#$ <sup>25</sup>) is chosen as reference, and entropies are determined from the PC-SAFT equation of state.<sup>101</sup> This choice ensures a great flexibility in terms of the range of substances and accessible state points. For all transport coefficients, correlations were adjusted to a rather extensive set of experiments, e.g. from the Dortmund database.<sup>102</sup> Using a PC-SAFT group contribution method (the GC-PC-SAFT<sup>103</sup>), it was possible to develop a group contribution method for entropy scaling of viscosity<sup>24</sup> and thermal conductivity,<sup>27</sup> which provides remarkable results, even for substances to which the method was not adapted in the first place. The excellent results of the method are not limited to pure substances. Using mixing rules for the parameters of the  $\eta$  correlations of the pure substances, the viscosity of their mixture can also be predicted in very good agreement with experiment, without adjusting

further parameters.<sup>25</sup> The Stuttgart entropy scaling variant reaches its limits, among others, for the viscosity of associating substances<sup>25</sup> and the diffusion coefficients in the gas phase<sup>26</sup>, which in the latter case is due to the sparse data available. Apart from the fundamental question of an underlying physical theory, there is thus a need for further research in the field of applied entropy scaling.

In the present work, entropy scaling is combined with molecular simulations in several ways:

1. *The use of entropy scaling for selecting meaningful state points at which transport coefficients of real fluids are calculated from MD simulations.* For this purpose, the state points are chosen to be equidistant in residual entropy  $s_{\text{res}}$  based on the entropy scaling. Simulations at a few state points are then sufficient to allow the adjustment of a correlation function for the respective transport coefficient of the considered force field. The correlation function can then be used to estimate the transport property of the force field for any other state point, provided  $s_{\text{res}}$  is known.
2. *The efficient assessment of the performance of a force field in the determination of transport coefficients.* By estimating the transport properties using a correlation from entropy scaling, associated to the force field (as described in point 1), the results of the simulations can be compared against all experimental data available. To do this, the corresponding transport coefficient of the force field's correlation is simply estimated at each state point where experiments are available. This avoids having to simulate at all experimental state points, and the comparison with a large number of data provides an assessment of the performance of the force field.
3. *The assessment and further development of entropy scaling for mixtures using results from molecular simulations for mixtures of simple model fluids as a reference.* For this purpose, mixtures of Lennard-Jones fluids are simulated, which differ only in one parameter of eq. (1.5). In this particular case, both, the transport coefficient and the entropy of the mixtures are determined from molecular simulations. Thus, GCMC simulations are used in addition to MD simulations.

On the one hand, molecular simulations are thus used to further explore and evaluate the entropy scaling method, as it has been done in studies since the beginning of the theory. On the other hand, entropy scaling is used vice versa to make molecular simulation of transport properties simpler and more efficient.

## 1.4 Outline of this Thesis

**Chapter 2** investigates whether classical force fields with *rigid bond lengths* yield equivalent results (within statistical uncertainty) to models with *harmonic bond length* potentials that are identical in all other respects. Static and dynamic physical properties, namely pressure, viscosity, self-diffusion, and thermal conductivity of homogeneous phases, are analyzed under varying force constants of the harmonic bond length potentials for ethane, propane, and dimethyl ether described with the TAMie potential. The anticipated offset in the thermal conductivity of the force field with rigid bond lengths compared to the harmonic bond length models, is approximated by an analytical correction term. The study examines different time steps for solving the equations of motion, and investigates the question of whether an integrator that allows a separate, shorter time step for rather high frequency bond vibrations offers advantages over a simple velocity Verlet integrator.

**Chapter 3** gives an assessment of the Transferable Anisotropic united-atom Mie (TAMie) force field for predicting dynamic properties. Equilibrium molecular dynamics simulations are used to calculate three dynamic properties ( $\eta$ ,  $D_{\text{self}}$ ,  $\lambda$ ) in one MD simulation using the corresponding Green-Kubo methods. The simulated state points are distributed in temperature and pressure based on an entropy scaling approach, using the PC-SAFT equation of state to calculate the residual entropy. As a result, few simulations are sufficient to parameterize a correlation function that allows comparison with experimental data over a wide range of  $T$  and  $p$ . The combination of a few simulations with the entropy scaling method therefore allows a very efficient estimation of the transport properties for a large temperature and pressure range, while at the same time providing a consistent evaluation of the results by comparing them with experiments.

The study in **chapter 4** uses molecular simulations of model-mixtures to systematically validate the applicability of entropy scaling for viscosities of mixtures. The considered binary mixtures are composed of two LJ-fluids only differing in  $\epsilon$  or  $\sigma$ , respectively. We further focus on non-ideal mixtures of two identical LJ-fluids with  $\epsilon_{11} = \epsilon_{22} \neq \epsilon_{12}$  which are known to show a strongly non-ideal phase behavior. We determine the viscosity  $\eta$  of several binary mixtures of Lennard-Jones fluids in equilibrium molecular dynamic simulations at different state points. Expanding a study<sup>25</sup> published together with Lötgering-Lin (first author), I calculate the dimensionless viscosity  $\eta^\#$  for mixtures. Using the residual entropy of the mixtures  $s_{\text{res}}$ , which I determined in grand canonical MC simulations, an entropy-scaled representation shows a monovariate dependence of the reduced transport coefficient with  $s_{\text{res}}$ , so that all results of the study indeed collapse onto a single line.





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## 2 Force Fields with Fixed Bond Lengths and with Flexible Bond Lengths: Comparing Static and Dynamic Fluid Properties

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### Abstract

This study investigates the equivalence or differences between classical force fields with rigid bond lengths and the same models but with (harmonic) bond length potentials. For ethane, propane and dimethyl ether described with the Transferable Anisotropic Mie potential we vary the force constant of the harmonic bond length potentials and analyze static and dynamic physical properties, namely pressure, viscosity, self-diffusion, and thermal conductivity of homogeneous phases. We find a range of values for the force constant of the bond length potentials (expressed in terms of the period-length of bond-oscillations) where force fields with harmonic bond lengths give equivalent results as the model with rigid bond lengths for static properties, for viscosity and for self-diffusion coefficients. The thermal conductivity of the force field with rigid bond lengths has an offset compared to the harmonic bond length models, which can be approximated through an analytic correction term. After adding the correction term results of the rigid model and the flexible models are in rather close agreement. Our study varies time-steps for solving the equations of motion and investigates whether the rRESPA integrator with a small time step associated to the (rather high frequency) bond length potentials has advantages compared to a simple velocity Verlet integrator. Furthermore, this work proposes a fast and memory efficient prescription to calculate autocorrelation functions for the calculation of Green-Kubo integrals. We then estimate average values and meaningful error bars for dynamic physical properties based on the time-decomposition approach [Zhang, Y.; Otani, A.; Maginn, E.J.: Reliable Viscosity Calculation from Equilibrium Molecular Dynamics Simulations: A Time Decomposition Method, *J. Chem. Theory Comput.* **2015**, *11*, 3537–3546].

### 2.1 Introduction

Reliable prediction of transport coefficients for fluids, such as viscosity, thermal conductivity and diffusion is an important prerequisite for process design. Next to experimental measurements and semi-empirical models, molecular simulations are an attractive alternative for the determination of dynamic properties of fluids. Simulations are used where experiments are expensive, time-consuming and particularly challenging, for example at extreme temperatures or pressures. Semi-empirical models, such as the entropy scaling method<sup>1-3</sup>, require a set of experimental data as basis for predicting transport properties, which may not be available for many substances. Also in this case, molecular simulations can be used to provide data where no experiments are available for which the models can be parameterized. The quality of results from molecular simulations strongly depends on the used force field, as corroborated in recent studies.<sup>4</sup> Force fields are composed of potentials that describe the interaction between molecules and the interaction between atoms within these molecules, such as binding forces, binding angles and torsion angles. Some parameters of force fields are adjusted to experimental data, especially those corresponding to Van der Waals interactions, because they are usually defined to effectively capture multi-body corrections. Numerous force fields can be found in literature for a wide variety of applications; for condensed liquids some of the most commonly used are TraPPE<sup>5</sup>, GAFF<sup>6</sup>, OPLS<sup>7</sup>, CHARMM<sup>8,9</sup> and GROMOS<sup>10-13</sup>.

In this work we study the influence of the way bonds are modeled with an emphasis on the influence on transport properties. In most force fields, harmonic potentials are used to model the bond energy between two neighboring atoms or atom groups. The two required parameters, the reference length  $r_0$  and force constant  $k_l$ , are determined *a priori* for many force fields, for example, from crystallographic experiments, microwave data, and vibration frequencies<sup>8</sup>. Alternatively, *ab initio* quantum mechanics calculations can be used to parameterize the potential<sup>6,14,15</sup>. In either case, these parameters are not part of the optimization used to determine intermolecular parameters of a force field. The high frequency of the bond vibrations limit the time step that can be used for the integration of the equations of motion in the simulation. This also affects the calculation of transport properties. A study by Gordon<sup>16</sup> showed an influence of the time step and the chosen integration method on the simulated viscosity  $\eta$  of fully flexible *n*-octane and *n*-dodecane. Using multi-time step integration schemes allows the simulation of high-frequency bond vibrations with larger time steps<sup>17,18</sup>. In this context, the influence of the frequency of the bonds of a Lennard-Jones dimer on the energy conservation of the integration approach<sup>18,19</sup> as well as dynamic properties such as the velocity autocorrelation function and the friction kernel of the model fluid were investigated<sup>20,21</sup>.

In practice, bonds are often constrained to a constant bond length in order to increase the time step<sup>22-24</sup>. A further simplification are so-called united atom force fields, in which bonds including hydrogen atoms are implicitly considered: the hydrogen atoms are combined with neighboring atoms to pseudo atoms, which act as single centres of interaction. These simplifications are justified in many cases, because the bond vibrations are not excited quantum mechanically at room temperature<sup>25,26</sup>. Importantly, these constraints also show little influence on the dynamics in molecular simulations<sup>27,28</sup>. The comparability of the two approaches in describing bonds has earlier been investigated in literature. Considering viscosity, Bird *et al.*<sup>29</sup> showed the equivalence of bead-spring molecules and of bead-rod molecules whose atoms are connected by infinitely stiff springs. Chandler and Berne<sup>30</sup> found differences in the conformational structure of rigid and flexible models of *n*-butane. Tironi *et al.*<sup>31</sup> discussed the relative merits of either rigid or flexible models of water. The authors concluded that introducing flexibility is possible but not recommended. The quality of the results could not be improved, whereas flexible bonds increase the danger of introducing further artifacts. The velocity autocorrelation function of flexible and rigid water models was also compared by Hess *et al.*<sup>32</sup> who proposed flexible constraints as an alternative approach for the calculation of intramolecular bonds.

The difference in transport properties between fixed and flexible bonds were compared for N<sub>2</sub> and H<sub>2</sub>O by Bordat and Müller-Plathe<sup>33</sup>. Differences for the viscosity were found to be within statistical uncertainties. Other studies for water show more accurate results for flexible molecules in diffusion and viscosity<sup>34-36</sup> compared to a rigid model. However, in these cases, results were compared to fully rigid water models. The effects of flexible bond lengths and flexible bond angles were not considered separately in these studies. Aimoli *et al.*<sup>37</sup> found, that a flexible TraPPE description of CO<sub>2</sub> is able to estimate transport properties with accuracies comparable to that of rigid models. However, the authors report higher deviations to experimental data for the thermal conductivity calculated with the flexible TraPPE force field. Considerable deviations from the respective force field with rigid bonds were also found for the thermal conductivity in flexible models of *n*-hexane<sup>38</sup> and polyamide-6,6<sup>39</sup>.

MD simulations with bond length constraints require iterative adjustment of molecules' space coordinates for every time step which is particularly undesired for large simulations. That is why force fields with fixed bond lengths were in several studies converted to a "flexible" equivalent force field. A corresponding "flexible" model is usually defined by assuming harmonic bond length potentials with force constant  $k_l$  and zero-force length  $r_0$  defined equal to the fixed bond length. For example, studies in the literature show flexible TraPPE variants for CO<sub>2</sub><sup>40,41</sup>, for *n*-alkanes<sup>16,42</sup>, for propane<sup>43</sup> and for *n*-octane, cyclohexane, squalane, and other hydrocarbons<sup>44</sup>. The origin of the  $k_l$  values in the *TraPPE-flex* force fields is often not

quite clear. Usually, the force constants are taken over from other force fields, like AMBER<sup>6</sup>, as in the force field of Rane *et al.*<sup>44</sup> Depending on the originating force field, the  $k_l$  value for the same bond type may differ significantly between the two force fields.

In this work we systematically investigate how the force constant  $k_l$  of bond length potentials influences the static and transport properties of short-chain hydrocarbons. We thereby regard the pressure  $p$ , the shear viscosity  $\eta$ , the self-diffusion coefficient  $D_{\text{self}}$  and the thermal conductivity  $\lambda$  of ethane, propane and dimethyl ether as determined from equilibrium molecular dynamics (MD) simulations. We introduce a harmonic bond potential in the Transferable Anisotropic Mie (TAMie) force field, which was originally parameterized for fixed bond lengths<sup>45,46</sup>. The (inverse) bond length constants are thereby scaled with the mass of the two corresponding interaction sites, leading to the period time of single bond length vibrations. Expressing bond length constants in this way makes the results transferable to other force fields and allows for a meaningful comparison of different substances. The “flexible force field” is studied for a range of  $k_l$  values. For varying force constants of the harmonic bond length potentials we investigate different time steps and MD-integration schemes and observe how the results for static and dynamic properties change for increasing large time steps.

## 2.2 Methods

### 2.2.1 Molecular Model

The united atom TAMie force field<sup>45–48</sup> is considered in this study. TAMie originally utilizes fixed bond lengths whereas angles and torsions are modeled as flexible potentials. Intermolecular interactions are described as Mie (n–6) potentials (vdW), and fixed point charge (Coulombic) potentials, as

$$u_{ij}(r) = c_{ij}\varepsilon_{ij} \left[ \left( \frac{\sigma_{ij}}{r} \right)^{n_{ij}} - \left( \frac{\sigma_{ij}}{r} \right)^6 \right] + \frac{q_i q_j}{4\pi\epsilon_0 r} \quad (2.1)$$

with the prefactor

$$c_{ij} = \left( \frac{n_{ij}}{n_{ij} - 6} \right) \left( \frac{n_{ij}}{6} \right)^{6/(n_{ij}-6)} \quad (2.2)$$

where  $r_{ij}$  denotes the distance between two interaction sites  $i$  and  $j$ . The size parameter  $\sigma_{ij}$ , the well depth  $\varepsilon_{ij}$  and the repulsive exponent  $n_{ij}$  characterize the vdW interactions. These parameters were adjusted to experimental data of vapor-liquid equilibria (VLE). Lorentz-

Berthelot combining rules are applied<sup>49,50</sup>, where repulsive exponents are combined arithmetically as  $n_{ij} = (n_{ii} + n_{jj})/2$ <sup>51</sup>. Coulombic interactions are calculated with the partial charge  $q_i$ , given as a factor of an electron charge  $|e|$ , and the dielectric constant  $\epsilon_0$ .

Molecules regarded in this study consist of three or less interaction sites. The intramolecular potential of the molecules is composed of the bending angle potential of the original force field and the bond length potential investigated in this study. Bending angles are generated according to a harmonic potential

$$u_{\text{bend}}(\theta) = k_{\theta}/2(\theta - \theta_0)^2 \quad (2.3)$$

where  $k_{\theta}$  is the force constant,  $\theta$  is the bending angle, and  $\theta_0$  is the zero-force angle. TAMie was originally parameterized for fixed bond lengths. In order to study the influence of the bond vibrations on static and dynamic fluid properties, we introduce flexible bond lengths according to a harmonic bond potential

$$u_{\text{bond}} = k_l(r - r_0)^2 \quad (2.4)$$

The zero-force distance  $r_0$  is adopted from the original TAMie force field.  $k_l$  denotes a force constant with unit  $\text{kcal mol}^{-1} \text{\AA}^{-2}$ . Note, that we use  $k_l$  in eq. (2.4) instead of  $k_l/2$ , in agreement to the notation in LAMMPS<sup>52</sup>. We investigate how the stiffness of the harmonic potential influences different static and harmonic properties of fluids.

An overview of all TAMie parameters used in this study is given in table 2.1 and table 2.2.

### 2.2.2 Data Analysis

Dynamic properties in this study were determined from equilibrium molecular dynamics simulations. The three investigated dynamic properties (shear viscosity  $\eta$ , thermal conductiv-

| bond type   |                       | $r_0/\text{\AA}$                  |
|---|-----------------------|-----------------------------------|
| CH <sub>3</sub> —CH <sub>3</sub>                  |                       | 1.94                              |
| CH <sub>3</sub> —CH <sub>2</sub>                  |                       | 1.74                              |
| CH <sub>3</sub> —O                                |                       | 1.61                              |
| angle type  | $\theta_0/\text{deg}$ | $k_{\theta}/\text{kcal mol}^{-1}$ |
| CH <sub>3</sub> —CH <sub>2</sub> —CH <sub>3</sub> | 114.0                 | 124.20                            |
| CH <sub>3</sub> —O—CH <sub>3</sub>                | 112.0                 | 120.03                            |

**Table 2.1:** Intramolecular Parameters

| united atom $i$             | $\epsilon_{ii}/\text{kcal mol}^{-1}$ | $\sigma_{ii}/\text{\AA}$ | $n_{ii}$ | $q_i/e$                 |
|-----------------------------|--------------------------------------|--------------------------|----------|-------------------------|
| —CH <sub>3</sub> —          | 0.270 89                             | 3.6034                   | 14       | 0.175 (next to ether O) |
| —CH <sub>3</sub> — (ethane) | 0.259 89                             | 3.6463                   | 14       | —                       |
| —CH <sub>2</sub> —          | 0.105 15                             | 4.0400                   | 14       | —                       |
| —O— (DME)                   | 0.108 02                             | 3.213                    | 12       | −0.35                   |

**Table 2.2:** Intermolecular Parameters

ity  $\lambda$ , and self-diffusion coefficient  $D_{\text{self}}$ ) for a given state point can be obtained in a single simulation run.

Shear viscosity and thermal conductivity are determined through Green-Kubo relations<sup>53,54</sup>. Green-Kubo integrals associate dynamic properties with autocorrelation functions (ACF) of quantities sampled during simulation. The ACF of the sampled time series,  $R_{JJ}$ , of a general quantity  $J$  is calculated as

$$R_{JJ}(\tau) = \lim_{\Theta \rightarrow \infty} \frac{1}{2\Theta} \int_{-\Theta}^{\Theta} J(t + \tau)J(t)dt \quad (2.5)$$

where  $\tau$  is the time lag between two evaluations of  $J$  and  $t$  is the instantaneous time. Integration of  $R_{JJ}$  (with consideration of a corresponding prefactor  $C$ ) yields the cumulative transport property

$$\zeta_{\text{run}}(\tau) = C \int_0^{\tau} R_{JJ}(\tau')d\tau' \quad (2.6)$$

as function of the lag time. The ACF's decay to zero in the limit of infinite lag times  $\tau$  and the respective transport coefficient  $\zeta$  is therefore derived from the limit of infinite integration times of the running integral as

$$\zeta = \zeta_{\text{run}}(\tau \rightarrow \infty) \quad (2.7)$$

In the specific case of the the shear viscosity, the Green-Kubo relation uses the ACF of off-diagonal elements of the shear-stress tensor in the simulation box,  $J_p^{\alpha\beta}$ , (with  $\alpha, \beta \in (x, y, z)$  and  $\alpha \neq \beta$ ), calculated as

$$R_{J_p J_p}(\tau) = \lim_{\Theta \rightarrow \infty} \frac{1}{2\Theta} \int_{-\Theta}^{\Theta} J_p^{\alpha\beta}(t + \tau)J_p^{\alpha\beta}(t)dt \quad (2.8)$$



The shear viscosity as function of  $\tau$  is then

$$\eta_{\text{run}}(\tau) = \frac{1}{Vk_{\text{B}}T} \int_0^{\tau} R_{J_p J_p}(\tau') d\tau' \quad (2.9)$$

where the shear viscosity is formally obtained as the limit  $\eta = \eta_{\text{run}}(\tau \rightarrow \infty)$ . The stress tensor is calculated during the simulation using the molecule velocities  $v_i$ ,  $i \in x, y, z$  and the virial  $\mathbf{r}_{ij}(\partial u(r_{ij})/\partial \mathbf{r}_{ij})$ , according to

$$J_p^{\alpha\beta} = \sum_{i=1}^N m_i v_i^{\alpha} v_i^{\beta} - \sum_{i=1}^N \sum_{j>i}^N r_{ij}^{\alpha} \frac{\partial u(r_{ij})}{\partial r_{ij}^{\beta}} \quad (2.10)$$

In order to improve the statistic of each simulation, we use the six independent shear components<sup>55,56</sup> of the stress tensor.

The correlated quantities needed for the calculation of the thermal conductivity are the three spatial entries of the heat flux  $\mathbf{J}_q = (J_q^x, J_q^y, J_q^z)$ . The thermal conductivity is determined via the ACF  $R_{J_q J_q}$ , analogous to the viscosity. Details of the calculation are shown in the Supporting Information.

For computing the self-diffusion coefficient  $D_{\text{self}}$  we use the Einstein relation, that calculates  $D_{\text{self, run}}$  from the mean-squared displacement (MSD) of the centre-of-mass (COM) positions  $W_i(t')$  of each molecule  $i$ , according to

$$W_i(\tau) = \lim_{\Theta \rightarrow \infty} \frac{1}{\Theta} \int_0^{\Theta} [\mathbf{r}_i^{\text{COM}}(\tau + t) - \mathbf{r}_i^{\text{COM}}(t)]^2 dt \quad (2.11)$$

The transport coefficient is defined as the limit of the running value ( $D_{\text{self}} = D_{\text{self, run}}(\tau \rightarrow \infty)$ ), which is calculated as the slope of the average of the MSD over all molecules, according to

$$D_{\text{self, run}}(\tau) = \frac{1}{6N} \frac{d}{d\tau} \sum_{i=1}^N W_i(\tau) \quad (2.12)$$

During molecular simulations, the time series of  $J_p^{\alpha\beta}(t)$ ,  $\mathbf{J}_q(t)$  and  $\mathbf{r}_i^{\text{COM}}(t)$  are sampled as described in the Simulation Details section. The autocorrelation of  $J_p^{\alpha\beta}$  and  $\mathbf{J}_q$ , as well as the mean-squared displacement are calculated in a post-processing step.

The autocorrelation integral is defined as

$$(f * f)(\tau) = \int_{-\infty}^{\infty} f(t + \tau)f(t)dt \quad (2.13)$$

and can be calculated using efficient Fourier transform algorithms. This allows us to calculate the ACF in three steps<sup>57</sup>.

1. Calculate the Fourier transform of the time series.

$$\hat{J}(f) = \mathcal{F}\{J(t)\}$$

2. Multiply the transform with its complex conjugate  $\hat{J}^*(f)$ .

$$\hat{R}_{JJ}(f) = \hat{J}(f)\hat{J}^*(f)$$

3. Calculate the ACF as inverse Fourier transform of the result.

$$R_{JJ}(\tau) = \mathcal{F}^{-1}\{\hat{R}_{JJ}(f)\}$$

Using the fast Fourier transform algorithm (FFT), the computational complexity depending on the number of observations  $N_t$  is reduced from  $\mathcal{O}(N_t^2)$ , for the direct calculation, to  $\mathcal{O}(N_t \log_2(N_t))$ <sup>58</sup>. FFT algorithms assume periodicity of the transformed functions, and the sequence has to be zero-padded to twice the length before the calculation of the autocorrelation. The first half of the resulting sequence is the desired ACF. This approach allows us to calculate the ACF very efficiently without having to apply a moving window scheme as typically used<sup>59</sup>, which, in practice, would reduce the number of considered data points. Our approach rigorously correlates all observations with all other observations, whereby every observation is also a starting point for a new time series. We therefore do not have to consider any parameters, such as window width, lag time, or the complexity due to the use of multiple time origins.

Although the mean-squared displacement is not an autocorrelation function in the first place, it can be determined more efficiently using FFT algorithms as well. For this purpose eq. (2.11) is decomposed into two parts, where one part is solved by FFT algorithms and the second part is calculated by a simple recursion. Details on efficient computation of mean-squared displacements are reported by Calandrini *et al.*<sup>58</sup>.

The Green-Kubo method suffers from increasingly high statistical uncertainties and numerical noise for increasing lag times  $\tau$ . Therefore, one typically truncates data after a certain correlation time  $\tau_{\max}$ . The time decomposition method proposed by Zhang *et al.*<sup>60</sup> helps us identify a suitable trade-off between too low values for  $\tau_{\max}$ , where the ACF has not

sufficiently dropped to zero and too high  $\tau_{\max}$  values, where statistical uncertainty degrades the calculation results. We generate several independent replicate trajectories for each state point and calculate a running standard deviation  $\sigma(\tau)$  and a running average  $A(\tau)$  of  $\eta_{\text{run}}$ ,  $\lambda_{\text{run}}$  and  $D_{\text{self, run}}$  of all replicates. An on-the-fly algorithm is used for this purpose in order to save memory<sup>61</sup>. The cut-off time  $\tau_{\max}$  is set to the time when the ratio of standard deviation and running average,  $\sigma(\tau_{\max})/A(\tau_{\max})$ , reaches a heuristic value of  $\sigma_{\max}^* = 0.4$ <sup>60</sup>. The data with  $\tau > \tau_{\max}$  is ignored for the further procedure. An exponential function for the estimated standard deviation

$$\sigma_{\text{fit}}(\tau) = A\tau^b \quad (2.14)$$

is then fitted to  $\sigma(\tau)$ . In the next step, a double exponential function<sup>62,63</sup> is adjusted to the running average  $A(\tau)$ . We rearranged it to the following form<sup>64</sup>

$$\eta_{\text{fit}}(\tau) = \eta \frac{\alpha\beta_1(1 - e^{-\tau/\beta_1}) + (1 - \alpha)\beta_2(1 - e^{-\tau/\beta_2})}{\alpha\beta_1 + (1 - \alpha)\beta_2} \quad (2.15)$$

with the adjustable parameters  $\eta$ ,  $\alpha$ , and the decay times  $\beta_1$  and  $\beta_2$ . These parameters are identified by minimizing the objective function

$$f = \sum_{i=N_{\tau}^{\min}}^{N_{\tau}^{\max}} ((\eta_{\text{fit}}(\tau_i) - \eta_{\text{run}}(\tau_i))w(\tau_i))^2 \quad (2.16)$$

where index  $i$  runs over observations of  $\eta_{\text{run}}(\tau)$ . Adjusting the double exponential ansatz function to the sampled data is done with weight function  $w(\tau) = 1/\tau^b$  in order to account for the increasing statistical uncertainty with increasing decay time where parameter  $b$  is from eq. (2.14). At very short decay times, large fluctuations make the fitting difficult due to the heavy weight in this  $\tau$  range, which is why we adjust the function for decay times greater than  $\tau_{i=N_{\tau}^{\min}} = 2$  ps determining the lower bound index  $i = N_{\tau}^{\min}$ .<sup>60</sup> The upper bound index is given as  $\tau_{i=N_{\tau}^{\max}} = \tau_{\max}$ , with the cut-off time  $\tau_{\max}$  as defined above. To prevent nonphysical decay times longer than  $\tau_{\max}$ , the parameters  $\beta_1$  and  $\beta_2$  are limited to  $\beta_1, \beta_2 < 0.01\tau_{\max}$  during the fitting procedure. The desired transport coefficient is determined by parameter  $\eta$  in eq. (2.15). Thermal conductivity  $\lambda$  is evaluated analogously.

As the value of  $D_{\text{self, run}}$  is an average over all  $N$  molecules, the statistical uncertainty is much lower than for  $\eta_{\text{run}}$  and  $\lambda_{\text{run}}$ . We do not apply an elaborate adjustment procedure for determining  $D_{\text{self}}$ . The adjustment of one constant parameter to  $D_{\text{self, run}}(\tau)$  with a weight of  $1/\sigma(\tau)$  is sufficient. The data is truncated at  $\tau_{\max, D} = \max(\tau_{\max, \eta}, \tau_{\max, \lambda})$ . The value of  $D_{\text{self}}$  is corrected for finite size effects applying the analytic correction factor proposed by Yeh and

Hummer<sup>65</sup>.

Because the running standard deviation based on replicate trajectories is used for estimating the transport quantities in terms of the weight function  $w(\tau)$  and for choosing of the cut-off time  $\tau_{\max}$  of the fitting procedure, the method yields an estimate for the transport property, and it is not straight forward in determining statistical uncertainties. We propose an approach to estimate statistical uncertainties of the calculated transport properties by using a bootstrapping method<sup>4,66</sup>. One bootstrap sample is a random set of the replicate simulations, generated by randomly selecting (with replacement) 30 time series of the complete set of replicates. Transport properties are calculated for each bootstrap sample using the methods described above. In order to account for the uncertainty of the heuristic value of  $\sigma_{\max}^* = 0.4$ , a new value for  $\sigma_{\max}^*$  is randomly selected between 0.4 and 0.6 for each subset. The procedure is repeated 500 times to generate a distribution of  $\eta$ ,  $\lambda$  and  $D_{\text{self}}$  values, differing in the time series used as well as in  $\sigma_{\max}^*$ . We then construct the 95 % confidence interval of the distribution using the percentile method<sup>67</sup>. The results given below are mean values of the bootstrap distributions and error bars are defined as 95 % confidence interval.

The reported errors for the pressure are calculated as the standard error of the mean of  $p$  of  $n = 10$  independent replicate simulations as  $s(\bar{p}) = s(p)/\sqrt{n}$ , where  $s(p)$  is the unbiased estimate of the standard deviation as obtained from the independent replicates. The reported 95 % confidence interval is then calculated as  $\bar{p} \pm 2.262s(\bar{p})$ , where the factor 2.262 comes from the t-distribution using  $(n - 1)$  degrees of freedom.<sup>68</sup>

### 2.2.3 Simulation Details

Dynamic as well as equilibrium properties are obtained from equilibrium molecular dynamics simulations using the MD code LAMMPS. General simulation specifications are given in table 2.3.

**Table 2.3:** General Simulation Specifications.

|   |                     |
|---|---------------------|
| equilibration time/ns                                     | 20                  |
| production time/ns  | 10                  |
| cut-off length/Å  | 14                  |
| number of molecules                                       | 1000                |
| tail-corrections  | U and P             |
| constrained bonds   | SHAKE <sup>22</sup> |
| long range solver   | PPPM <sup>69</sup>  |
| with $1.0 \cdot 10^{-5}$ desired relative error in forces |                     |

We regard two substances (ethane and propane) where partial charges  $q_i$  are zero and dimethyl ether as a polar species. Besides the original TAMie force field with constant bond lengths, we simulated “flexible” versions with varying constants  $k_l$  of the harmonic bond length potential as given in eq. (2.4). We investigated 10 values for  $k_l$  varying from loose bonds ( $k_l = 50 \text{ kcal } \text{\AA}^{-2}$ ) to very stiff bonds ( $k_l = 10\,000 \text{ kcal } \text{\AA}^{-2}$ ).

Two liquid state points were simulated for each investigated force field variation. Properties are calculated at fixed temperatures  $T$  and densities  $\rho$ .  $T$  and  $\rho$  were taken from VLE calculations with the original TAMie force field as obtained from the literature<sup>45,46</sup>. An overview of the simulations is given in table 2.4.

The same simulation stages were applied to all state points. 1000 molecules are placed on a lattice as the initial configuration. After conducting an energy minimization, all systems run through a 10 ns *NVT* equilibration, proceeded by another 10 ns *NVE* equilibration. We added this step to make sure that energy is sufficiently conserved for the selected integration algorithm and time step. The production step of the simulation is carried out in the *NVT* ensemble. Temperature is controlled using a Nosé-Hoover thermostat, with a coupling time of 2000 fs. In order to improve statistical precision and provide estimates of uncertainties, we utilize 10 to 20 replicate simulations for the production stage. A single equilibrated configuration is used as the initial configuration for each replicate where different randomized initial velocities are taken from a Boltzmann distribution for each replicate simulation. Every second time step during the production stage, the pressure tensor entries and the entries of the heat flux are written to a file. The centre-of-mass positions of each molecule, used in the calculation of mean-squared displacement in eq. (2.11), are sampled every 100 steps.

Two types of numeric methods were used in this study to integrate the equations of motion. For the analysis of the fixed bonds, we use a velocity Verlet (VV) algorithm<sup>70</sup>. Bonds are constrained using the SHAKE algorithm<sup>22</sup>. For the simulation of the molecules with flexible bonds, we used the rRESPA multi-time step integrator<sup>18</sup>. rRESPA allows using different time

**Table 2.4:** Overview over Simulated States for Different Substances. For all Simulations,  $N = 1000$  Molecules were Used

| Substance      | $T/\text{K}$ | $\rho/\text{kg m}^{-3}$ |
|----------------|--------------|-------------------------|
| ethane         | 205.0        | 512.83                  |
|                | 300.0        | 315.44                  |
| propane        | 295.0        | 497.24                  |
|                | 360.0        | 361.27                  |
| dimethyl ether | 300.0        | 648.93                  |
|                | 380.0        | 482.04                  |

steps for the calculation of different contributions of the potential-energy function. If not stated otherwise, we use a maximum time step of  $1 \times \Delta t = 1$  fs for all simulations using rRESPA. Intermolecular interactions and angle potentials are evaluated at this outer level. Forces due to bond-stretching are calculated with an “inner” time step of  $\frac{1}{n} \times \Delta t$  with  $n = 2$  and  $n = 10$ . This is meant to allow for sufficiently small time steps for the calculation of (high frequency) bond vibrations without exerting high computational costs in calculating expensive vdW interactions or Coulombic interactions. For  $n = 1$  the rRESPA time integrator is equivalent to the VV integrator. For propane, we performed additional simulations of the flexible molecules with the VV algorithm, using time steps  $\Delta t = 0.5$  fs and  $\Delta t = 2.0$  fs for comparison.

### 2.3 Results and Discussion

The properties of three substances were investigated in this study: ethane, propane and dimethyl ether (DME). Ethane has a simple geometry with only one bond and no bonding angle. For ethane, it is easy to single out the influence of the bond interactions on simulation results. The number of rigid bonds that can be simulated with LAMMPS using the SHAKE algorithm is limited to two adjacent bonds. Propane with two bonds is the longest alkane for which we are able to determine a reference calculation with rigid bond lengths. The third substance, dimethyl ether, has a similar topology as propane but carries partial charges on each of the three united atom interaction sites.

We emphasize that this study is not committed to assess how well the chosen force field reproduces experimental data. We are focused on the equivalence or difference between force fields with rigid bond lengths and models with (harmonic) bond length potentials. Two saturated liquid state points were investigated for all substances. One state is at a temperature near the critical point and one condition is at a lower temperature, as summarized in table 2.4. The near-critical temperature was chosen because changes in the force field often lead to strong responses in observed quantities near the critical point. If not mentioned otherwise, all results presented below were calculated at a near critical temperature ( $360 \text{ K}/T^{\text{crit}} = 0.97$  for propane,  $300 \text{ K}/T^{\text{crit}} = 0.98$  for ethane, and  $380 \text{ K}/T^{\text{crit}} = 0.95$  for dimethyl ether). Results for the lower temperature are given in the Supporting Information.

One static property (pressure  $p$ ) and three transport properties (shear viscosity  $\eta$ , self-diffusion coefficient  $D_{\text{self}}$ , and thermal conductivity  $\lambda$ ) are analyzed in this study. Figure 2.1 illustrates results for calculated pressures of propane plotted over the period of the bond

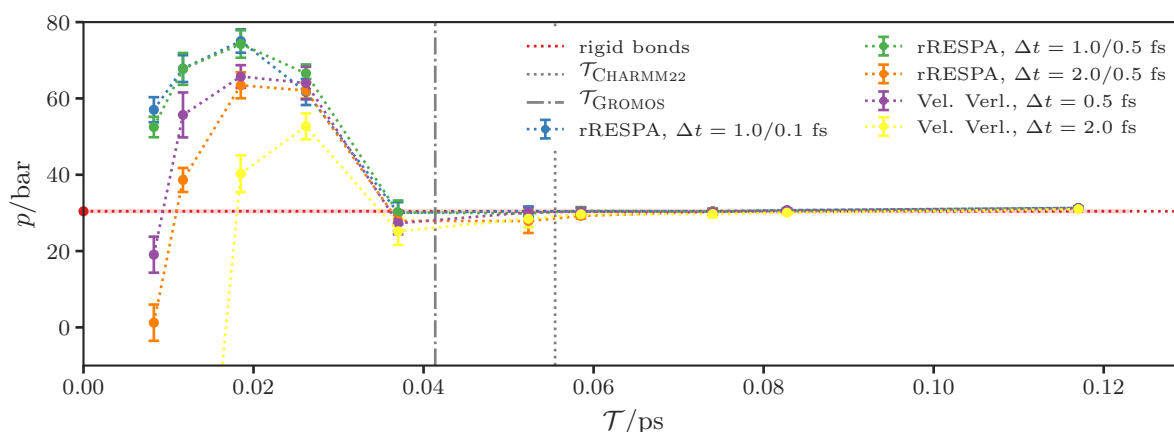
oscillations

$$\mathcal{T} = 2\pi\sqrt{\bar{M}/k_l} \quad (2.17)$$

with reduced molar mass  $\bar{M} = (M_i M_j)/(M_i + M_j)$  of the two bonded united atom groups  $i$  and  $j$ . Presenting results in terms of  $\mathcal{T}$  allows for comparison of different molecular bonds where low  $\mathcal{T}$  values correspond to high values for  $k_l$ , indicating stiff bonds. The limit  $\mathcal{T} = 0$  represents rigid bonds. It is noteworthy that for the limit  $k_l \rightarrow \infty$ , flexible bonds differ energetically and entropically from fixed bonds.<sup>28</sup> However, the thermal equation of state as well as calculated dynamic properties may well be practically equivalent.

Each point in fig. 2.1 is the result of one set of replicate simulations. Lines between points are merely visual aids. The red dashed horizontal line represents the result of a set of simulations with rigid bonds, as calculated using the VV algorithm with a time step of 1 fs and the SHAKE algorithm to ensure the bond length constraints. Other colors are results for propane with flexible bonds, obtained with different integration schemes and different time steps.

The first observation in fig. 2.1 is the high spread as well as statistical uncertainty for short  $\mathcal{T}$  values, that is, for high values of  $k_l$ . For  $\mathcal{T} < 0.035$  ps, we expect that the pressure from the flexible model should approach the pressure from the rigid bond force field. However, for periods below a  $\mathcal{T}_{\min}$  of 0.035 ps, the large frequency of the bonds' oscillations,  $1/\mathcal{T}$ , cannot be sufficiently solved by the integration algorithms. As a result, the pressure cannot be properly estimated and tends to be overestimated or underestimated compared to fixed bond length simulations. It is remarkable that the two-stage integration scheme rRESPA is also suffering



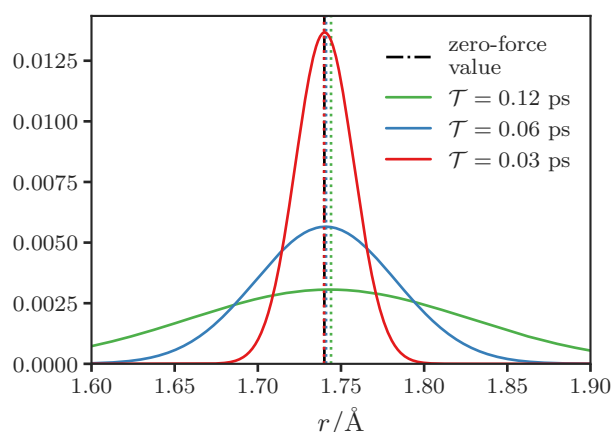
**Figure 2.1:** Pressure  $p$  of propane at  $T = 360$  K over the period of the bond vibrations  $\mathcal{T}$ . Colors represent different integration schemes with different time steps  $\Delta t$ . The red horizontal line shows result of rigid bonds. Vertical lines highlight  $\mathcal{T}$  values of the CHARMM (dotted) and the GROMOS force field (dashed).

from this issue at low values of  $\mathcal{T}$ . For  $\mathcal{T} > 0.1$  ps, the  $k_l$  values are small and therefore the flexible bonds are so loose that results of the flexible bond model deviate from results of a rigid bond model. The results at  $\mathcal{T} = 0.12$  ps are no longer equivalent to the results of the stiff bonds within statistical uncertainty. The scale of fig. 2.1 is too coarse to clearly confirm this statement, which is why the Supporting Information gives the results of fig. 2.1, but shown on a finer scale.

For a wide range of moderate values of  $\mathcal{T}$  (0.035 ps to 0.085 ps), the pressures as determined from the force field with flexible bonds agree within statistical uncertainties with the pressure  $p_{\text{rigid}}$  as determined for the rigid bonded molecule. The exact choice of  $k_l$  is not important, the  $k_l$  values in this plateau region result in statistically equivalent pressures, independent of time steps and integration schemes used.

Grey vertical lines in fig. 2.1 highlight  $\mathcal{T}$  values for C–C bonds of the CHARMM22<sup>8</sup> (dotted line) and GROMOS<sup>11</sup> (dashed line) force field, determined from experimental crystallographic and microwave data. The  $\mathcal{T}$  values of these two models are located in the plateau region for all considered time steps. We conclude, that for liquid-phase pressure as a rather sensitive static property, a time resolution of 1.0 fs or even 2.0 fs appears to be sufficient for the simulation of bond potentials with force constants in the range  $100 \text{ kcal mol}^{-1} \text{ \AA}^{-2} < k_l < 500 \text{ kcal mol}^{-1} \text{ \AA}^{-2}$ . For this range we find equivalent results of the flexible force field to the model with rigid bond lengths.

In the preceding discussion we argued that the rigid bond end of the plateau of constant  $p$  values at  $p = p_{\text{rigid}}$ , that is, for high values of  $\mathcal{T}$  or low values of  $k_l$ , is reached when the flexible bonds are so loose that the results for the flexible model deviate from results of the rigid

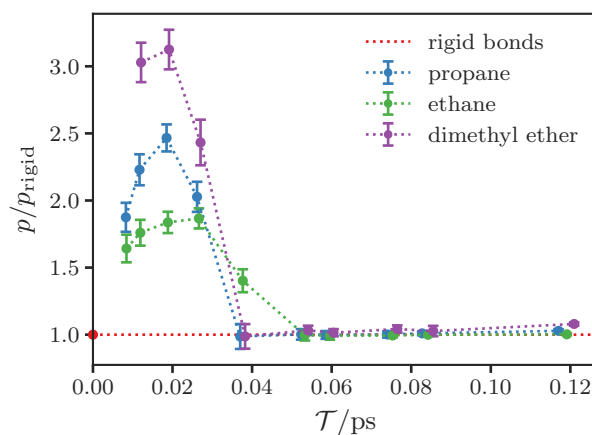


**Figure 2.2:** Distribution of bond lengths of liquid propane at  $T = 360$  K and  $\rho = 361.27 \text{ kg m}^{-3}$  (rRESPA with inner time step  $\Delta t = 0.1$  fs) for different values of bonding force constant  $k_l$ . Vertical colored lines represent the respective maximum value of the distribution. The black vertical line shows the reference length  $r_0$ .



force field. This explanation is supported by fig. 2.2 which shows the distribution of the bond lengths of propane for three simulations with different  $k_l$  values. The higher the  $k_l$  values get, the more stiff are the bonds and the more narrow are the bond length distributions. The maximum of the distributions are shown as vertical dashed lines. For loose flexible bonds with  $\mathcal{T} = 0.12$  ps and  $k_l = 50 \text{ kcal mol}^{-1} \text{ \AA}^{-2}$ , which is only one-eighth of the  $k_l$  value of GROMOS, one sees deviations of the maximum from the zero-force value (black line). For too low  $k_l$  values one can not expect to yield the equivalent results of a rigid force field with bond length defined as the vertical red line.

Figure 2.3 compares results for pressure obtained for propane with the results for the other two substances, ethane and dimethyl ether. The previous results for propane showed that the chosen time step (in the range of 0.1 fs to 2 fs) and the integration scheme have no significant impact on the results. For the other substances, these observations are confirmed. In Figure 2.3 we show results using the rRESPA integrator with an outer time step  $\Delta t = 1.0$  fs and an inner time step of 0.1 fs for ethane and propane and of 0.5 fs for dimethyl ether, respectively. To be able to compare different substances, in fig. 2.3 we scale the pressure by the pressure obtained for the model with rigid bond lengths,  $p/p_{\text{rigid}}$ . The figure shows that for all three substances, the flexible bond model and the rigid bond model are statistically equivalent for sufficiently high values of  $\mathcal{T}$ , while deviations appear for low values of  $\mathcal{T}$ . The lowest period of the bond vibrations  $\mathcal{T}_{\text{min}}$  for which the flexible bond model can be regarded equivalent to a rigid bond force field is higher for ethane ( $\approx 0.052$  ps) than for propane or dimethyl ether ( $\approx 0.035$  ps). The higher number of bonds in a molecule appears to be more well-behaved in the time-integration scheme. We observe no influence of the presence of partial charges in dimethyl ether on the numerical stability of the problem. It is noteworthy that the period of the bond vibrations  $\mathcal{T}_{\text{GROMOS}}$  calculated for the bond-energy constant  $k_l$  of the GROMOS force

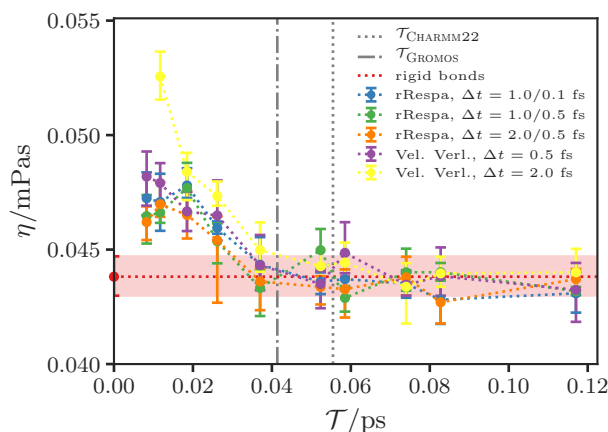


**Figure 2.3:** Reduced pressure  $p/p_{\text{rigid}}$  of ethane, propane and dimethyl ether over the period of the bond vibrations  $\mathcal{T}$ . The red horizontal line shows the result of rigid bonds as reference.

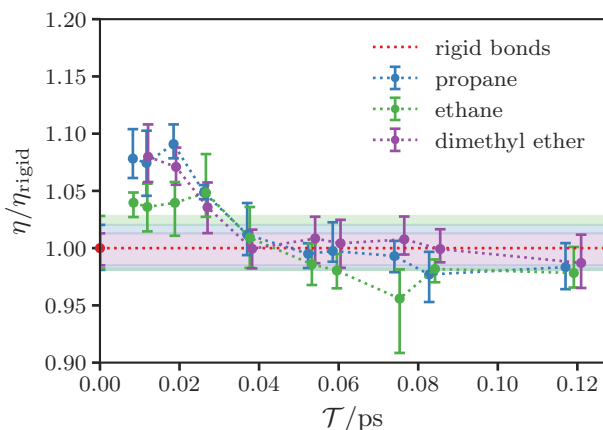
field is below the left end of the plateau  $\mathcal{T}_{\min}$ , where time steps below  $\Delta t = 0.5$  fs are needed. The main focus of this work is on transport properties. To investigate dynamic properties we regard viscosity, self-diffusion, and thermal conductivity. Figure 2.4 depicts the shear viscosity of propane for varying  $\mathcal{T}$  values. The dashed red horizontal line is for the result of the force field with rigid bond lengths, and the 95 % confidence interval is indicated by the red-shaded region. For models with flexible bond lengths, we regard different time steps and different integration methods. As before, we observe a plateau with a viscosity  $\eta$  that is constant within error bars where all computed viscosities are statistically equivalent. We see high deviations for short periods of the bond vibrations ( $\mathcal{T} < 0.035$  ps) where the integrators (with time steps considered in this work) are not any more appropriate for the high frequent bond vibrations. The rRESPA integrator with a small time step allocated to the harmonic bond length potential does not improve the numerical stability at low  $\mathcal{T}$  values. The viscosity of propane from models with flexible bond lengths (for  $0.035 > \mathcal{T}$ ) is equal to the viscosity of the model with rigid bonds within statistical uncertainty.

Figure 2.5 confirms that results for viscosity of propane are transferable to ethane and dimethyl ether. All viscosities of molecules with flexible bonds of  $\mathcal{T} > 0.35$  within the error bars agree with the results of molecules with rigid bonds. The normalized 95 % confidence interval around the red dotted line representing the rigid molecules is indicated for each substance by the region shaded in the respective color.

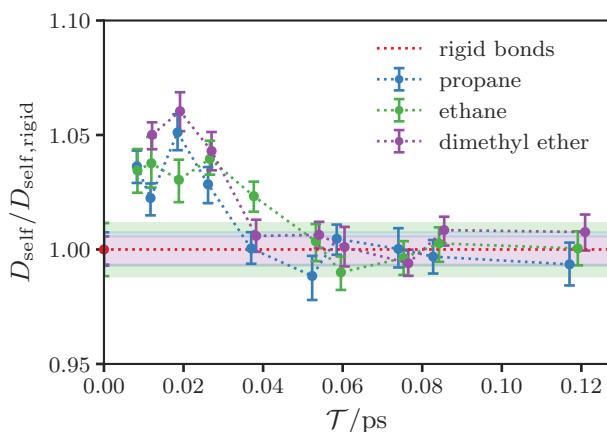
An analysis of the diffusion coefficient  $D_{\text{self}}$  for the three substances is given in fig. 2.6. The range in  $\mathcal{T}$  for which a plateau in self-diffusion coefficient occurs is comparable to the range found for pressure and viscosity above. The lowest period  $\mathcal{T}_{\min}$ , which yields stable results,



**Figure 2.4:** Viscosity  $\eta$  of propane at  $T = 360$  K over the period of the bond vibrations  $\mathcal{T}$ . Different colors represent different integration schemes with different time steps  $\Delta t$ . The red horizontal line shows result of rigid bonds. Vertical lines show  $\mathcal{T}$  values of common force fields.



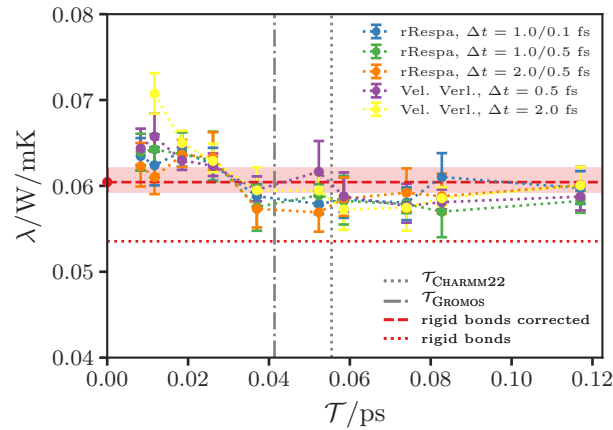
**Figure 2.5:** Reduced viscosity  $\eta/\eta_{\text{rigid}}$  of ethane, propane and dimethyl ether over the period of the bond vibrations  $\mathcal{T}$ . The red horizontal line shows the result of rigid bonds as reference.



**Figure 2.6:** Reduced self-diffusion coefficient  $D_{\text{self}}/D_{\text{self,rigid}}$  of ethane, propane and dimethyl ether over the period of the bond vibrations  $\mathcal{T}$ . The red horizontal line shows the result of rigid bonds as reference.

can apparently be observed in several quantities. For the diffusion coefficient, we do not observe higher deviations of flexible bond models from the fixed-bond length models for molecules with two bonds as compared to molecules with only a single bond. For all considered substances, the diffusion coefficient of the original rigid bond force field is reproduced by the flexible bond models within statistical errors, provided that the force constant is appropriately chosen.

Now, we consider thermal conductivity, which is different from the other transport properties, in that the rigid bond length model suppresses intramolecular vibrations. However, these degrees of freedom must be taken into account in order to properly compare thermal conductivity. We approximately correct the fixed-bond model for intramolecular bond-vibrational degrees of freedom, by adding an analytic vibrational thermal conductivity  $\lambda^{\text{vib}}$ , to the thermal



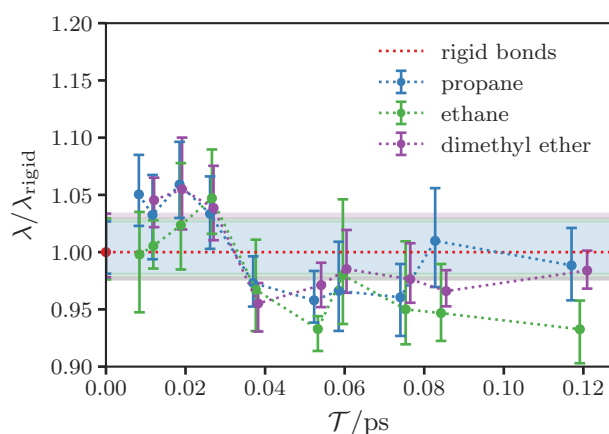
**Figure 2.7:** Thermal conductivity  $\lambda$  of propane at  $T = 360\text{K}$  over the period of the bond vibrations  $\mathcal{T}$ . Different colors represent different integration schemes with different time steps  $\Delta t$ . The red dashed horizontal lines show uncorrected (thin line) and corrected (thick line) result of rigid bonds. Vertical lines show  $\mathcal{T}$  values of common force fields.

conductivity determined from molecular simulations, with

$$\lambda^{\text{vib}} = c_v^{\text{vib}} \rho D_{\text{self}} \quad (2.18)$$

as proposed by Liang and Tsai<sup>71</sup>. The expression accounts for the conductivity of thermal energy stored in intramolecular vibrational modes of the considered molecule in the form of the heat capacity  $c_v^{\text{vib}}$  whereby the transport mechanism is self-diffusion. The density  $\rho$  and the self-diffusion coefficient  $D_{\text{self}}$  are determined from the simulations. The vibrational contribution of the heat capacity is approximated as  $c_v^{\text{vib}} = k_B N_{\text{bonds}}$ , which means, we assume independent harmonic oscillators for all rigid bonds  $N_{\text{bonds}}$ <sup>72</sup>. Especially for molecules with several rigid bond length potentials, this is a somewhat coarse approximation. The statistical error of the two quantities,  $D_{\text{self}}$  and  $c_v^{\text{vib}}$ , is negligible, as compared to that of the thermal conductivity.

Figure 2.7 presents results for the thermal conductivity. The lower dashed horizontal line represents the results from molecular simulations determined for the rigid bond model. The upper dashed horizontal line is shifted by  $\lambda^{\text{vib}}$  corresponding to the results of the corrected rigid bond model. Comparable to other transport coefficients, the thermal conductivity as calculated for the flexible bond force field exhibits a plateau within  $0.035\text{ ps} < \mathcal{T} < 0.1\text{ ps}$ . For bond-energy constants  $k_l$  corresponding to  $\mathcal{T} < 0.035\text{ ps}$ , we observe a systematic deviation from the values for  $\mathcal{T} > 0.035\text{ ps}$ , as we have noticed for the other transport properties. The investigated time steps of 0.5 and 2 fs and both types of integration schemes only act weakly on the results. The plateau value for the thermal conductivity of the flexible bond model agrees within the statistical uncertainty to the corrected rigid bond force field. However, the



**Figure 2.8:** Reduced thermal conductivity  $\lambda/\lambda_{\text{rigid}}$  of ethane, propane and dimethyl ether over the period of the bond vibrations  $\mathcal{T}$ . The red dashed horizontal line shows the result of rigid bonds corrected by  $\lambda^{\text{vib}}$ , eq. (2.18), as a reference.

error bars of thermal conductivity are significantly wider than those of the other properties because, as described above, the ACF  $R_{J_q J_q}$  can only be averaged over fewer samples as opposed to viscosity or self-diffusion.

Figure 2.8 compares thermal conductivities for three different components, relative to the results of their fixed-bond counterpart. Flexible bond force fields of ethane and propane lead to lower values of  $\lambda$ , compared to the fixed-bond models, if we ignore the region of insufficient time step resolution at  $\mathcal{T} < 0.035$ . Because the results of the rigid bond models seem to systematically underestimate results from models with flexible bonds for all substances and all time steps, it seems that the rigid bond value is overcorrected by the consideration of  $\lambda^{\text{vib}}$ . Nevertheless, it is evident in fig. 2.7 that the correction leads to a better agreement of the  $\lambda$  results of molecules with rigid and flexible bonds.

The results shown for liquids at near critical temperature could also be confirmed for the lower temperatures for each substance. Tables with all results of these simulations can be found in the Supporting Information. Also, for exemplary simulations of the gas phase of propane at the VLE at 360 K, the exact same statements can be made as for the results of the liquids: molecules with harmonic binding potentials provide statistically equivalent results as models with rigid bonds. The corresponding simulation results are also summarized in tables in the Supporting Information.

### 2.4 Conclusion

This study investigates whether a force field with rigid bond lengths can be applied with harmonic bond length potentials and how static and transport properties are thereby affected. Introducing harmonic bond length potentials leads to results for thermodynamic properties and transport properties that are in good agreement to results obtained for the model with rigid bond lengths, provided that the bond-energy constants  $k_l$  are appropriately chosen. Suitable values for  $k_l$  are in the range of those from fully flexible united atom force fields. We consider ethane and propane as substances with one and two bond length potentials as well as dimethyl ether as a species with (partial) point charges. These substances are described with the TAMie force field, which was developed with rigid bond lengths. The same force field but with flexible bond lengths reproduces results of the rigid bond counterpart to within the overlap of the corresponding 95 % confident intervals for (liquid phase) pressure, shear viscosity, and self-diffusion coefficient. For thermal conductivity a force field with rigid bond lengths and a model with flexible bond lengths are conceptually different. For a flexible bond length model, a certain part of the thermal conductivity is due to the energy stored in intramolecular vibrational bond length modes of the considered molecule, which is transported through self-diffusion of the considered species. We use an approximation for this contribution of the thermal conductivity to correct the rigid bond length model for missing degrees of freedom, which makes the “corrected rigid bond” model and the model with flexible bond lengths comparable. Both models lead to satisfyingly similar values in thermal conductivity.

With increasingly large  $k_l$  values, an integration scheme, such as the velocity Verlet integration with a time step of 2 fs, leads at some point to deviations from the expected results, because the highly frequent bond length vibrations can no longer be resolved. The rRESPA integrator offers the possibility of assigning a shorter (inner) time step to the bond length potential and a longer (outer) time step for the other contributions of the force field. The rRESPA integrator with this setting, however, does not improve on the velocity Verlet integrator with the same outer time step. Smaller steps for the integration of the binding potentials did not lead to improvements.

## **Acknowledgement**

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## **Supporting Information**

Tables with a summary of temperatures, pressures, viscosities, diffusion coefficients and thermal conductivities for all integration methods and time steps; tables with results for a lower temperature for each substance and the results of gas phase simulations of propane at 360K calculated with rRESPA; more detailed description of the thermal conductivity calculation.





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### 3 Transferable Anisotropic United-Atom Mie (TAMie) Force Field: Transport Properties from Equilibrium Molecular Dynamic Simulations

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#### Abstract

Reliable prediction of transport coefficients for fluids, such as the viscosity, thermal conductivity and diffusion coefficients, is an important prerequisite for process design. Besides experimental measurements and semiempirical correlations, molecular simulations are a promising method to estimate transport properties of fluids over wide ranges of temperatures and pressures. Transport properties are sensitive to the underlying intermolecular potentials. In this work we assess the Transferable Anisotropic united-atom Mie (TAMie) force field regarding the calculation of transport properties. The force field was parametrized for thermodynamic properties with emphasis on vapor–liquid coexistence properties. Equilibrium molecular dynamic simulations are used to calculate all transport properties in a single simulation, using the corresponding Green–Kubo methods. The simulated state points were distributed in the temperature and pressure based on an entropy scaling approach, where the PC-SAFT equation of state is used to calculate residual entropy. Utilizing the favorable behavior of dynamic properties when plotted over the residual entropy, only few simulations are needed to parametrize a correlation function that furthermore enables comparison with experimental data over a wide range of temperatures and pressures. TAMie yields good results for all transport properties and substances investigated (with average absolute deviations of 13 % for viscosity, 18 % for diffusion, and 10 % for thermal conductivity), given that only static properties were considered in the parametrization of the force field. Combining few simulations with the entropy scaling method enables very efficient prediction of transport properties for a large temperature and pressure region.

### 3.1 Introduction

Transport coefficients such as viscosity, thermal conductivity, or diffusion coefficients are essential for process and apparatus design in chemical engineering. Data for these transport coefficients are less abundant as compared to many static properties, say density to prescribed temperature and pressure. Molecular simulations offer the possibility of predicting these quantities for pure substances and for mixtures.

Transport coefficients can be determined from various molecular dynamics (MD) techniques. In nonequilibrium MD approaches one maintains or creates driving forces and samples the resulting fluxes<sup>1-3</sup>, or in reverse nonequilibrium MD one imposes fluxes and samples the resulting gradients of conjugate field variables<sup>4,5</sup>. For viscosity, Maginn and co-workers suggested a method where the relaxation of an initially imposed momentum impulse is observed within a system, and they showed good statistical results with a modest computational (wall-clock time) demand<sup>6</sup>. Alternatively, transport coefficients can be determined from systems in equilibrium, given a suitable ensemble, as a time-dependent response to spontaneous fluctuations. All transport coefficients can thereby be obtained from a single simulation or, for practical reasons, more likely from a set of statistically independent simulations of the same state condition. Our work is concerned with MD simulations of systems in equilibrium, by determining autocorrelation functions according to the Green–Kubo formalism<sup>7-9</sup>.

It was observed for Lennard-Jones fluids, that equilibrium MD simulations at fluid conditions give thermal conductivities<sup>10</sup> and viscosities<sup>11</sup> with a small dependence on system size. A comprehensive analysis for shear viscosity was recently given by Kim *et al.* who confirmed results for viscosity are weakly dependent on the size of the simulation box unless rather small simulation boxes are considered, where shear viscosity exhibits complex, oscillatory size-dependent behavior<sup>12</sup>. Self-diffusion coefficients as determined from equilibrium MD simulations, in contrast, show a rather pronounced dependence on the choice of the system size. Dünweg and Kremer found a linear dependence of the self-diffusion coefficient with inverse length of the simulation box<sup>13</sup>. Yeh and Hummer proposed an analytic expression for the size dependence that can conveniently be applied to estimate self-diffusion coefficients for the limit of infinite system size<sup>14</sup>. Their development is based on a hydrodynamic model of particles diffusing in a continuum with defined shear viscosity with periodic boundary conditions. The shear viscosity thus needs to be sampled and known. The correction of Yeh and Hummer is sufficiently reliable, so that Jamali *et al.* proposed applying the correction to determine the viscosity of a fluid by studying the system size dependence of the self-diffusion coefficients<sup>15</sup>. This approach is motivated from the fact that shear viscosities (like thermal conductivities) are sampled with higher statistical uncertainty than self-diffusion coefficients,



because the autocorrelation function of self-diffusion coefficients can be sampled for every molecule of a considered simulation, whereas shear viscosity only has six independent entries, namely the six independent cross-coefficients of the pressure tensor.

Transport coefficients are an important test for force fields and they can serve as training data for optimizing force fields. Our group has proposed the Transferable Anisotropic Mie (TAMie) force field optimized toward phase equilibrium properties, namely vapor pressure and liquid densities<sup>16–20</sup>. The force field has been shown to have considerable predictive capabilities for phase equilibria of mixtures<sup>20,21</sup>. It is a united-atom force field, where van der Waals interactions of hydrogen atoms are grouped together with larger neighboring atoms and the united-atom group is described as a single effective van der Waals interaction site<sup>22–28</sup>. A methylene group ( $-\text{CH}_2$ ), for example, is treated as a single effective van der Waals interaction site. In order to better account for the presence of hydrogen atoms, the methylene groups in the TAMie model are moved outward from the position of the carbon atom, as proposed by Toxvaerd<sup>29</sup> and adopted in other force fields<sup>30–32</sup>. The Mie potential is considered for van der Waals interactions with a prescribed attractive exponent of  $n = 6$ , whereas the repulsive exponent is an adjustable parameter. The additional degree of freedom, compared to a Lennard-Jones potential, with predefined repulsive exponent of  $m = 12$ , leads to substantially better agreement of simulated properties with experimental properties<sup>16–18,33–35</sup>.

It is well recognized that phase equilibrium properties are a meaningful test for force fields, or in turn, phase equilibrium properties are valuable ingredients of a training set for optimizing force fields. With this study, we wish to assess how a force field parametrized solely to (static) phase equilibrium properties predicts transport coefficients of pure fluids. Fernández *et al.* already found good agreement with experiments for transport properties of simple real fluids<sup>36,37</sup>, quadrupolar fluids<sup>38</sup> and dipolar fluids<sup>39</sup>, using force fields that were developed with no information on transport properties included in the optimization procedures. Guevara-Carrion *et al.* extended the studies on transport properties of force fields adjusted to static properties to methanol, ethanol<sup>40</sup> and water and their mixtures<sup>41</sup>. First assessments of the TAMie force fields were reported by Messerly *et al.*<sup>42</sup> with deviations of approximately 10% for viscosities of linear alkanes and roughly 20% for branched alkanes. Our work on 1-alcohols<sup>21,43</sup> showed reasonably good agreement of predicted transport coefficients to experimental data. The deviations from experimental data, however were systematic, i.e. too high diffusion coefficients and too low shear viscosities are found, as expected from a united-atom force field. Messerly *et al.*<sup>44</sup> investigated the force field proposed by Potoff and co-workers, which is also based on a Mie ( $m$ -6) potential for the van der Waals interactions<sup>33,35</sup>. They observe overestimated viscosities at rather high pressures, above 600 MPa, attributed to a too repulsive 16-6 Mie force field (with repulsive exponent of  $m = 16$ ).

Our study is facilitated by recent developments in correlating transport coefficients using entropy scaling, as proposed by Rosenfeld<sup>45,46</sup>. Whereas Rosenfeld initially limited entropy scaling to simple, spherically symmetric atomic fluids, it was later observed that entropy scaling also holds for molecular species with complex intermolecular interactions<sup>47-57</sup>, namely for substances with strongly nonspherical interactions or directional polar or hydrogen-bonding interactions and for mixtures of such substances<sup>58</sup>. Previous work from our group showed that entropy scaling can be used to develop models for transport properties. Those models allowed experimental viscosities of pure components<sup>57</sup> and of mixtures<sup>58</sup> to be correlated and predicted. Also thermal conductivities<sup>59</sup> and self-diffusion coefficients<sup>60</sup> of pure substances were regarded. A strength of entropy scaling is that a model parametrized to a few data points (well-distributed in state conditions, as we will show below) can be used for predicting transport properties well outside the range of temperatures and pressures considered for training the model.

In this work, we assess the TAMie force field (initially developed for static thermodynamic properties and phase equilibria) for predicting transport coefficients. We regard self-diffusion coefficients, shear viscosity, and thermal conductivities of pure substances. For a meaningful comparison of the TAMie force field within the entire fluid region (with pressures below  $\sim 100$  MPa), we select state points about equally distributed in residual entropy. Entropy scaling can then be used to estimate transport coefficients in wide ranges of gaseous and liquid states.

## 3.2 Methods

### 3.2.1 Molecular Model

The TAMie force field uses a united atom (UA) description of molecules. Carbon atoms and their adjacent hydrogen atoms are combined to single interaction sites, whereas other atoms are usually considered as individual interaction sites. The intermolecular interactions are described as sum of interactions between pairs of sites. The (nonbonded) interactions between two sites  $i$  and  $j$  are described by a Mie potential and by fixed point charges

$$u_{ij}(r_{ij}) = c_{ij}\epsilon_{ij} \left[ \left( \frac{\sigma_{ij}}{r_{ij}} \right)^{n_{ij}} - \left( \frac{\sigma_{ij}}{r_{ij}} \right)^6 \right] + \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}} \quad (3.1)$$

where  $r_{ij}$  denotes the distance between two interaction sites  $i$  and  $j$  and  $\sigma_{ij}$  and  $\epsilon_{ij}$  are size and energy parameters of the Mie potential, respectively. Point charge  $q_i$ , defined as a unitless

negative factor of the electron charge, and vacuum permittivity  $\epsilon_0$  characterize the Coulombic interactions. The attractive exponent was set to  $m_{ij} = 6$  for all sites, whereas the repulsive exponent  $n_{ij}$  was part of the parameter adjustment. The constant

$$c_{ij} = \left( \frac{n_{ij}}{n_{ij} - 6} \right) \left( \frac{n_{ij}}{6} \right)^{6/(n_{ij}-6)} \quad (3.2)$$

ensures a minimum of the Mie potential at a value of  $-\epsilon_{ij}$ . Two interaction sites located on the same molecule but separated by more than three bonds also interact through the potential, eq. (3.1). The parameters for nonbonded interactions of like ( $i$ - $i$ ) pairs were extracted from previous publications<sup>16-18,20</sup> and are given in the supporting information. For the parameters of unlike site pairs, Lorentz–Berthelot combining rules are applied as

$$\sigma_{ij} = (\sigma_i + \sigma_j) / 2 \quad (3.3)$$

$$\epsilon_{ij} = \sqrt{\epsilon_i \epsilon_j} \quad (3.4)$$

and the arithmetic mean

$$n_{ij} = (n_i + n_j) / 2 \quad (3.5)$$

is used for the repulsive exponent. This combination rule may lead to fractional exponents if even and odd exponents are combined. There are alternative combination rules for the exponents discussed in literature that circumvent this issue.<sup>61</sup> The TAMie force field was constructed to only apply even exponents, precisely to circumvent fractional exponents, so that the arithmetic mean can be used in this work. It should be emphasized that parameters of the TAMie force field describing the intermolecular interactions were adjusted to experimental liquid–vapor equilibrium data (saturation pressure  $p_{\text{sat}}$  and liquid density  $\rho_L$ ). The calculated dynamic quantities are predictions, since no transport properties were included in the parametrization of the TAMie force field.

The intramolecular (bonded) interactions are defined in the TAMie force field through potential functions for bond lengths, angle bending, and torsions. The angle bending potential is described by a harmonic potential as

$$u_{\text{bend}}(\theta) = \frac{k_\theta}{2} (\theta - \theta_0)^2 \quad (3.6)$$

where  $\theta_0$  is the zero-force angle and  $k_\theta$  is the force constant. For all substances considered in this study, the torsional potential is defined as

$$\begin{aligned}
 u_{\text{torsion}} = & c_0 + c_1 [1 + \cos(\phi)] \\
 & + c_2 [1 - \cos(2\phi)] \\
 & + c_3 [1 + \cos(3\phi)]
 \end{aligned}
 \tag{3.7}$$

While the TAMie force field was originally proposed with fixed bond lengths, we here adopt harmonic potentials for the bond lengths. Constant bond lengths require additional, often iterative algorithms to enforce constraints. These constraints suppress vibrational degrees of freedom and consequently have to be accounted for when evaluating e.g. the thermal conductivity as discussed in our previous work.<sup>62</sup> When bonds are modeled with a harmonic potential, typically smaller time steps have to be used to properly sample the fast bond vibrations – a disadvantage we alleviate by using appropriate time steps.

The bond potentials are modeled as

$$u_{\text{bond}} = k_l (r - r_0)^2,
 \tag{3.8}$$

whereas the original force field uses fixed bond lengths between interaction sites<sup>16–18</sup>. The zero-force bond-length  $r_0$  is set equal to the rigid bond-length of the TAMie model. Our previous study<sup>62</sup> regarded short-chain alkanes and ethers and showed that results of the TAMie force field with properly chosen force constants are equivalent to results of the TAMie model with fixed bond lengths. Equivalent results within reasonably small statistical uncertainties were found for the thermal equation of state, for self-diffusion coefficient and for viscosity. Thermal conductivity, as calculated from the original TAMie model with fixed bond lengths and from the TAMie force field with flexible bond lengths cannot be expected to be equivalent. But both models can be brought to close agreement with one another when correcting for the thermal conductivity contribution from bond-vibrations by assuming independent harmonic oscillators<sup>62</sup> for every fixed bond-length. Entropic properties of a model with fixed bond lengths and the same model with flexible bond lengths, of course, cannot be equivalent. The force constant  $k_l$  is, in agreement to our earlier study<sup>62</sup>, defined as  $100644 \text{K}\text{\AA}^{-2}$  for all binding types. All bonded parameters of the substances used in this study are summarized in the supporting information.

### 3.2.2 Simulation Setup

We find it appealing to select state points for conducting simulations based on the principle of entropy scaling, which is briefly introduced here. Rosenfeld<sup>45,46</sup> first proposed that there exists, to excellent approximation, a univariate relation between dimensionless transport coefficients and the residual entropy  $s_{\text{res}} = s - s_{\text{id. gas}}(T, \rho)$ . The coefficients he originally regarded are the self-diffusion coefficient, shear viscosity, and thermal conductivity. The dimensionless transport coefficients are the simulated or experimental values of the transport coefficients divided by an adequate reference. Entropy scaling was initially proposed by Rosenfeld for simple fluids only, but in the meantime several studies showed that the principle also holds for strongly nonspherical, polar and hydrogen-bonding fluids and models were developed for viscosity  $\eta$ <sup>57,58</sup>, thermal conductivity  $\lambda$ <sup>59</sup> and self-diffusion coefficient  $D_{\text{self}}$ <sup>60</sup>. With such a model, say for shear viscosity  $\eta^*(s_{\text{res}})$ , one calculates  $s_{\text{res}}(T, \rho, x)$  for temperature  $T$ , composition  $x$  and density  $\rho(p)$  (for given pressure  $p$ ) from an equation of state and it is then possible to calculate the corresponding transport property  $\eta$  from  $\eta^* = \eta/\eta_{\text{ref}}$ .

In this study we use entropy scaling in two ways. First, we select state points approximately equally spaced in residual entropy space. Doing so, we cover the entire fluid region of a considered substance with a defined number of simulations (here 5 simulations for each pure substance). Second, we conduct simulations and use the results to parametrize simple (univariate) models for each transport property for the TAMie force field. These models can subsequently be used for predicting transport properties for the considered substance at any fluid state point (with pressures below  $\sim 100$  MPa), when combined with  $s^{\text{res}}$  from an equation of state.

The five state points simulated for each substance in this study are selected using the PC-SAFT<sup>63</sup> equation of state. For this we select pressures (in most cases  $p_0 = 1$  MPa). The lower limit of the considered  $s_{\text{res}}$  range is determined by selecting a temperature close to the triple point temperature as  $T_0 = T^{\text{TP}} \cdot 1.1$  to  $1.5$ . We use index 0 to indicate the state with the lowest value of  $s_{\text{res}}$ , corresponding to the highest density and to low temperatures of a liquid phase. With PC-SAFT we calculate the dimensionless entropy  $s_0^* = s_{\text{res}}(T_0, p_0)/(N_A k_B m)$ , where  $m$  is the segment number parameter of PC-SAFT for the substance. We then divide the range of  $s^*$ , from  $s_0^*$  to  $s_{\text{id. gas}}^* = 0$ , into four equal parts and determine the entropy values of the next three liquid state points,  $s_1^*$ ,  $s_2^*$  and  $s_3^*$ . We use PC-SAFT to iterate the temperatures to  $T_1$ ,  $T_2$  and  $T_3$  in such a way that the values  $s^*(p_1, T_1)$ ,  $s^*(p_2, T_2)$  and  $s^*(p_3, T_3)$  are evenly distributed. We thereby ensure a liquid state by defining  $p_i = \min(p_0, p^{\text{sat}}(T_i) \cdot 1.25)$ ,  $i \in \{1, 2, 3\}$ . The fifth state point is chosen in the gas phase and its entropy is determined as  $s_4^* = s^*(T_3, 0.8 \cdot p^{\text{sat}}(T_3))$ .  $p^{\text{sat}}$  values in this procedure are also calculated from PC-SAFT.

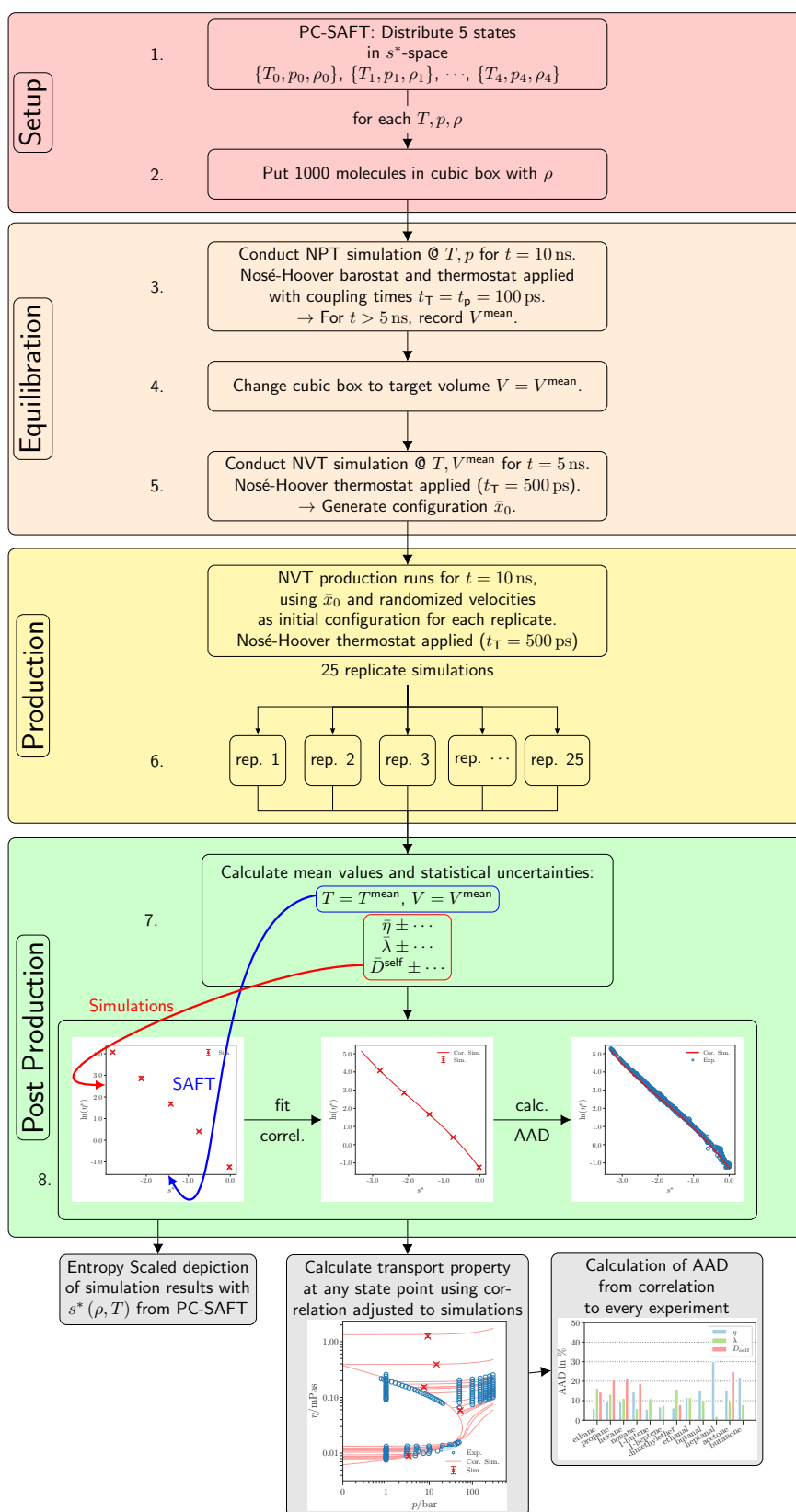


Figure 3.1: Visualization of the workflow used to calculate transport properties in equilibrium molecular dynamics simulations.

This approach yields five state conditions  $(T, p)$  that are used to conduct equilibrium MD simulations. All states used in this study are listed in the Supporting Information. We emphasize that calculations with the PC-SAFT equation of state are only used to deliver estimates for the initial conditions of simulations with the TAMie force field. Properties such as equilibrium pressures and densities that are used for analysis in this study are the results of the MD simulations described below and the values from simulations only approximately agree with the initial PC-SAFT state points. The properties as obtained from the simulations are used as input to PC-SAFT to recalculate  $s_{0-4}^*$  afterward.

After defining initial conditions  $T, p, \rho$  using PC-SAFT, the simulation setup is completed by placing 1000 molecules on a lattice as initial configuration in a box at the density  $\rho$ . For each state point found in the way described above, the same simulation procedure is now conducted as shown in the flowchart in fig. 3.1. Simulation details, such as simulation times  $t$  and choices for barostat and thermostat, including the coupling times  $t_p$  and  $t_T$ , are summarized in the flowchart of fig. 3.1. All simulations are performed using the simulation package LAMMPS<sup>64</sup>.

The equilibration procedure comprises steps 3 to 5 of fig. 3.1. After conducting an energy minimization, the density of the system is equilibrated in an  $NpT$  simulation (step 3). The box volume  $V$  is sampled for the second half of the  $NpT$  equilibration time and a average equilibrium volume  $V^{mean} = \langle V \rangle$  is calculated. After that, the size of cubic box is dynamically adjusted to  $V^{mean}$  (step 4). In step 5, the system undergoes another  $NVT$  equilibration, yielding a representative configuration which is used to initiate 25 replicate production simulations in  $NVT$ , each with different randomized initial velocities, taken from the Boltzmann distribution. In order to ensure a constant temperature over the whole simulation time we apply a Nosé-Hoover thermostat ( $t_T = 500$  ps) with weak coupling. This thermostat does not influence the kinetics of MD simulations nor the calculation of transport properties.<sup>65</sup> To confirm this fact, we repeated the simulations for hexane without a thermostat in the production phase under otherwise identical simulation conditions. We obtain equivalent results within the statistical accuracy. The results of the NVE simulations are shown in the supporting information. During production (step 6), the pressure tensor entries and the entries of the heat flux, needed for the calculation of shear viscosity  $\eta$  and thermal conductivity  $\lambda$  are written to a file every second time step. The center of mass position of each molecule, used in the calculation of  $D^{self}$ , is sampled every 500 steps. For all simulations, the multiple-time-scale integrator rRESPA<sup>66</sup> is used to integrate the equations of motion. We evaluate the bond vibrations with an inner time step of 0.5 fs. All other interactions are calculated with an outer time step of 1.0 fs. An overview of general simulation specifications is given in table 3.1.

|   |                    |
|---|--------------------|
| Time-step / fs (outer/inner)                              | 1/0.5              |
| Cut-off length / Å  | 14                 |
| Number of molecules                                       | 1000               |
| Tail-corrections  | $U$ and $p$        |
| Long range solver   | PPPM <sup>67</sup> |
| with $1.0 \cdot 10^{-5}$ desired relative error in forces |                    |

**Table 3.1:** General Simulation Specifications.

### 3.2.3 Data Analysis

We compute transport properties in this study in equilibrium MD simulations using Green–Kubo methods. Using these methods all transport coefficients,  $\eta$ ,  $\lambda$ , and  $D_{\text{self}}$ , can be calculated from data that originates from the same simulation. The Green–Kubo equations relate transport coefficients to an integral of an autocorrelation function (ACF) of time series that are sampled during simulations. The viscosity  $\eta$  is given by the Green–Kubo equation

$$\eta = \frac{1}{Vk_{\text{B}}T} \int_0^{\infty} R_{J_p J_p}(\tau) d\tau, \quad (3.9)$$

as the integral of the ACF  $R_{J_p J_p}$  of the stress-tensor  $J_p$ , according to

$$R_{J_p J_p}(\tau) = \lim_{\Theta \rightarrow \infty} \frac{1}{2\Theta} \int_{-\Theta}^{\Theta} J_p^{\alpha\beta}(t + \tau) J_p^{\alpha\beta}(t) dt \quad (3.10)$$

as a function of the lag time  $\tau$ .  $J_p^{\alpha\beta}$  is any off-diagonal element of the stress-tensor (with  $\alpha, \beta \in x, y, z$  and  $\alpha \neq \beta$ ). In order to improve the statistics of each simulation, we use the six independent shear components of the stress-tensor  $((J_p^{xy} + J_p^{yx})/2, (J_p^{yz} + J_p^{zy})/2, (J_p^{zx} + J_p^{xz})/2, (J_p^{xx} - J_p^{yy})/2, (J_p^{yy} - J_p^{zz})/2$  and  $(J_p^{xx} - J_p^{zz})/2$ ) to calculate an averaged  $R_{J_p J_p}(\tau)$ <sup>68</sup>.  $J_p^{\alpha\beta}$  is calculated from the molecule’s center of mass (COM) velocity  $v_i$  and the virial  $\mathbf{r}_{ij}(\partial u_{ij}(r_{ij})/\partial \mathbf{r}_{ij})$  as

$$J_p^{\alpha\beta} = \sum_{i=1}^{N^{\text{mol.}}} m_i v_i^{\alpha} v_i^{\beta} - \sum_{i=1}^{N^{\text{mol.}}} \sum_{j>i}^{N^{\text{mol.}}} r_{ij}^{\alpha} \frac{\partial u_{ij}(r_{ij})}{\partial r_{ij}^{\beta}}. \quad (3.11)$$



In the same manner, we can calculate the thermal conductivity as

$$\lambda = \frac{V}{3k_B T^2} \int_0^{\infty} R_{J_q J_q}(\tau) d\tau \quad (3.12)$$

from the ACF

$$R_{J_q J_q}(\tau) = \lim_{\Theta \rightarrow \infty} \frac{1}{2\Theta} \int_{-\Theta}^{\Theta} \mathbf{J}_q(t + \tau) \cdot \mathbf{J}_q(t) dt \quad (3.13)$$

of the heat flux  $\mathbf{J}_q$ , which is determined through the per-atom energy  $e_i$  and the force  $\mathbf{f}_{ij} = -\frac{\partial u_i}{\partial \mathbf{r}_j}$  due to the per-atom potential energy  $u_i$  as

$$\mathbf{J}_q = \frac{1}{V} \left[ \sum_{i=1}^{N^{\text{at.}}} \mathbf{v}_i e_i + \sum_{i=1}^{N^{\text{at.}}} \sum_{j>i}^{N^{\text{at.}}} \mathbf{r}_{ij} \left( \mathbf{v}_j \cdot \frac{\partial u_i}{\partial \mathbf{r}_j} \right) \right] \quad (3.14)$$

The self-diffusion coefficient was calculated using an Einstein relation (that can also be converted into a Green–Kubo equation). The Einstein relation formulates  $D_{\text{self}}$  in terms of the slope of the mean squared displacement as

$$D_{\text{self}} = \frac{1}{6N} \lim_{\tau \rightarrow \infty} \frac{d}{d\tau} \sum_{i=1}^{N^{\text{mol.}}} W_i(\tau), \quad (3.15)$$

where  $W_i$  denotes the mean squared displacement of the center of mass position  $\mathbf{r}_i^{\text{COM}}$  of molecule  $i$  at time  $\tau$  relative to the initial time  $\tau_0 = 0$ , as

$$W_i(\tau) = \lim_{\Theta \rightarrow \infty} \int_0^{\Theta} \left[ \mathbf{r}_i^{\text{COM}}(t + \tau) - \mathbf{r}_i^{\text{COM}}(t) \right]^2 dt. \quad (3.16)$$

The integration in eq. (3.16) averages over different initial times. Moreover,  $D_{\text{self}}$  is averaged over all  $N$  molecules and therefore has the best statistics of all transport coefficients. After evaluating eq. (3.15), the finite size correction of Yeh and Hummer<sup>14</sup> is applied to the values of  $D_{\text{self}}$ .

Calculation of the transport coefficients is done in post processing. We thereby follow the suggestions of Maginn *et al.*, who provided a very helpful and comprehensive summary of best practices in calculating transport properties from equilibrium MD simulations including estimates for statistical uncertainties using a bootstrapping procedure<sup>69</sup>. We use the *fast*

*fourier transform* algorithm to calculate the ACFs as a convolution in Fourier space.<sup>70,71</sup> The Fourier Transform is convenient because one can effortlessly correlate all samples to one another; i.e., every sample is also the starting point of another time series. This approach replaces elaborate methods to reduce memory demand and computation time in evaluating the autocorrelation function, like the staged time-interval sampling proposed by Dubbeldam *et al.*<sup>72</sup>. The evaluation of the Green–Kubo integrals is done using the *time decomposition method*, proposed by Zhang *et al.*, where the maximum time for the autocorrelation integral is determined by approximating the increasing uncertainty for increasingly long upper bounds of the autocorrelation integral<sup>73</sup>. Statistical errors are given as 95 % confidence interval as obtained from a bootstrapping procedure.<sup>69,74</sup> We generate bootstrap samples by randomly selecting (with replacement) 25 time series of the complete set of 25 replicate simulations. For each bootstrap sample we calculate the transport properties as described above. The procedure is repeated 500 times to generate a distribution of  $\eta$ ,  $\lambda$  and  $D_{\text{self}}$  values, each based on a different subset of all simulated time series. The distributions are used to construct the 95 % confidence interval using the percentile method<sup>74</sup>. For a detailed description of the post processing procedure we refer the reader to our previous work.<sup>62</sup>

The transport coefficients resulting from the MD simulations are analyzed using the entropy scaling method. To do this, we first calculate the entropy for each of the five state points we have simulated per substance. We calculate  $s^*(T^{\text{sim}}, \rho^{\text{sim}})$  with PC-SAFT and use the mean value of the temperature from the time series of the production simulation runs as  $T^{\text{sim}}$  and the constant density of these  $NVT$  simulations as  $\rho^{\text{sim}}$ . The dimensionless quantities  $\lambda^* = \lambda/\lambda^{\text{ref}}$ ,  $D_{\text{self}}^* = D_{\text{self}}/D_{\text{self}}^{\text{ref}}$  and  $\eta^* = \eta/\eta^{\text{ref}}$ , needed for the reference values for entropy scaling, are determined by the equations in table 3.2. The collision integrals  $\Omega^{(1,1)*}$  and  $\Omega^{(2,2)*}$  are calculated after Neufeld *et al.*<sup>75</sup>; the PC-SAFT parameters  $m_{ii}$ ,  $\sigma_{ii}$  and  $\varepsilon_{ii}$ , needed for calculating the references are summarized in the supporting information. The critical reduced entropy  $s_c^*$  in the calculation of  $\alpha$  is also calculated with PC-SAFT.

Having determined the reduced transport properties  $\eta^*$ ,  $D_{\text{self}}^*$  and  $\lambda^*$  as well as the reduced entropy  $s^*$  at each simulated state point, we can adjust correlation functions known from the literature to our simulation results. The functions used for this purpose are listed in table 3.3. Three ( $\eta, \lambda$ ) and four ( $D_{\text{self}}$ ) parameters are adjusted with a least squares fit. Using the equations for the references from table 3.2 for each transport property, it is now straight forward to calculate an estimate for the viscosity  $\eta^{\text{cor}}(s^*) = \eta^*(s^*) \cdot \eta^{\text{ref}}$ , the thermal conductivity  $\lambda^{\text{cor}}(s^*) = \lambda^*(s^*) \cdot \lambda^{\text{ref}}$  and the diffusion coefficient  $D_{\text{self}}^{\text{cor}}(s^*) = D_{\text{self}}^*(s^*) \cdot D_{\text{self}}^{\text{ref}}$  corresponding to the TAMie force field, at state points that were not simulated explicitly.

We use this correlation in the residual entropy to compare the simulation results with a wide range of experimental data. Using PC-SAFT we determine  $s^*(T, p)$  of a state point at which an

| Transport Coefficient              | Reference Value   |
|------------------------------------|---|
| Viscosity <sup>58</sup>            | $\eta^{\text{ref}} = \frac{5}{16} \frac{\sqrt{M_{ii} k_b T / (N_A \pi)}}{\sigma_{ii}^2 \Omega_{ii}^{(2,2)*}}$   |
| Self-diffusion <sup>60</sup>       | $D_{\text{self}}^{\text{ref}} = \frac{3}{8\sigma^2 \rho N_A} \frac{1}{\Omega^{(1,1)*}} \sqrt{RT / (\pi M_{ii} m_{ii})}$   |
| Thermal Conductivity <sup>59</sup> | $\lambda^{\text{ref}} = \lambda_{\text{CE}}^{\text{ref}} + \alpha(s^*) \lambda^*$ $\text{with } \lambda_{\text{CE}}^{\text{ref}} = \frac{83.235}{10^3} \frac{\sqrt{T / (M_{ii} m_{ii})}}{\sigma_{ii}^2 \Omega_{ii}^{(2,2)*} m_{ii}}$ $\text{and } \alpha(s^*) = \exp(-s^*/s_c^*)$ |

**Table 3.2:** Reference Transport Coefficients  $\eta^{\text{ref}}$ ,  $D_{\text{self}}^{\text{ref}}$  and  $\lambda^{\text{ref}}$  for the Calculation of Reduced Transport Properties.

| Transport Coefficient              | Entropy Correlation  |
|------------------------------------|--|
| Viscosity <sup>58</sup>            | $\ln(\eta^*) = A_\eta + B_\eta s^* + C_\eta s^{*2} + D_\eta s^{*3}$  |
| Self-diffusion <sup>60</sup>       | $\ln(D_{\text{self}}^*) = A_{D_{\text{self}}} - B_{D_{\text{self}}} (1.0 - \exp(s^*)) s^{*2} + C_{D_{\text{self}}} s^{*3}$ $\text{with } C_{D_{\text{self}}} \geq 0$ |
| Thermal Conductivity <sup>59</sup> | $\ln(\lambda^*) = A_\lambda + B_\lambda s^* + C_\lambda (1.0 - \exp(s^*)) + D_\lambda s^{*2}$  |

**Table 3.3:** Entropy correlations of reduced transport properties.

experimental value for a transport property is available. The transport property corresponding to the  $s^*(T, p)$  condition can then be calculated. This allows us to estimate transport properties the force field would deliver at  $(T, p)$ , where experiments are available, but which are not covered from the actually simulated state points. We illustrate this procedure for the viscosity of dimethyl ether (DME). The viscosity of dimethyl ether determined experimentally at pressure  $p^{\text{exp}} = 100$  bar and at temperature  $T^{\text{exp}} = 293.14$  K is  $\eta^{\text{exp}} = 0.145$  mPa.s. Although neither pressure nor temperature of the simulated state points is close to this experimental point, we can estimate the deviation of the simulation from this experiment. We determine the residual entropy of the experimental state as  $s^{*,\text{exp}} = s^*(T^{\text{exp}}, \rho(p^{\text{exp}})) = -1.7481$  with PC-SAFT. Then, we evaluate the correlation function for  $\ln(\eta^{*,\text{cor}})(s^{*,\text{exp}})$  from table 3.3, where parameters  $A_{\eta, \text{DME}}$ ,  $B_{\eta, \text{DME}}$ ,  $C_{\eta, \text{DME}}$ , and  $D_{\eta, \text{DME}}$  were adjusted to the five simulated viscosities of dimethyl ether. Using the reference  $\eta^{\text{ref}}$  from table 3.2 we determine  $\eta^{\text{cor}}(s^{*,\text{exp}}) = 0.157$  mPa.s and calculate its deviation from the experiment as  $|\eta^{\text{exp}} - \eta^{\text{cor}}| / \eta^{\text{exp}} = 7.97\%$ . It is important to point out that the analytic PC-SAFT model is used to calculate the residual entropy for the simulated conditions and for the experimental conditions. Errors of the PC-SAFT model therefore cancel to some extent.

Following this procedure, we estimate absolute average deviations (AAD) of the correlations (adjusted to the simulations) and all experimental data, for which  $s^{*,\text{exp}}$  can be determined with PC-SAFT. For example, the AAD for viscosity is calculated for each substance as

$$AAD_{\eta} = \frac{1}{N_{\text{exp}}} \sum_{i=1}^{N_{\text{exp}}} \frac{|\eta_i^{\text{exp}} - \eta_i^{\text{cor}}|}{\eta_i^{\text{exp}}}, \quad (3.17)$$

where  $N_{\text{exp}}$  is the number of experimental data points considered. The AADs of the other transport coefficients are calculated analogously.

### 3.3 Results and Discussion

In this study we simulated the transport properties of 13 different substances. The substances were selected based on availability of experimental data and existing TAMie parameters in the literature, and we can group them into five classes: alkanes, alkenes as nonpolar species, ketones, aldehydes, and ethers as polar substances. For each class, 1 to 5 substances were selected for which most experimental data is available in the Dortmund Data Bank<sup>76</sup> for  $\eta$  and  $\lambda$  and in several other sources<sup>77-91</sup> for  $D_{\text{self}}$  in order to compare the simulations with the largest possible data set using entropy scaling. Despite our selection criteria, for some substances experimental data for certain transport properties are not reported and in these cases it is not possible to quantitatively evaluate the simulation results through AAD values. In total 19334 experimental data points were considered in the calculation of the AADs shown here, up to 3154 per substance.

The results of this study are organized per transport property and presented in the same way in each case. The structure will be explained in the following subsection.

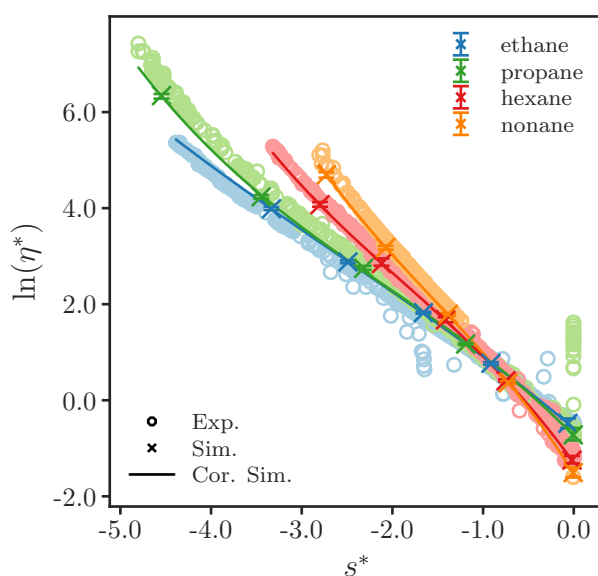
#### 3.3.1 Viscosity

Figures 3.2–3.6 show the entropy scaled representation of the simulation results, whereby the logarithm of the reduced viscosity is plotted over  $s^*$ , as calculated from PC-SAFT. Results of the simulations are represented as crosses with error bars, the correlations (table 3.3) adjusted to the simulation points are shown as solid lines and the experimental data – if available – is represented as circles. Each figure summarizes results of a chemical class, whereby different colors represent the individual substances.

Most substances were studied in the class of alkanes shown in fig. 3.2, namely ethane,

propane, hexane, and nonane. The figure illustrates that the qualitative behavior of the relation  $\eta^*(s^*)$  from experimental points is well captured by both, the individual simulation results at five state conditions, and by the correlations (solid lines) adjusted to them. The rightmost simulated point corresponds to a gas phase, noting that ideal gas conditions are characterized by  $s^* = 0$ . Strongly negative values of  $s^*$  represent dense liquid states. In the range  $s^* \leq -2$ , we observe deviations of simulated results from experimental data exceeding the range, where experimental data scatters. Although state points at low values of residual entropy  $s^* \leq -2$  are rather challenging to simulate, because they correspond to high densities and low temperatures, we are convinced the deviations are not due to deficiency in the simulation technique. Rather, deficiencies of the force field in predicting the dynamics of a fluid will be most prominently seen at state points at low temperatures and high densities. Results for ethane shows a typical behavior, already reported in an earlier study of Messerly *et al.*<sup>42</sup>. While the experimental viscosities are underestimated for the alkanes with longer chains, for ethane, in contrast, the predicted viscosities are overestimated.

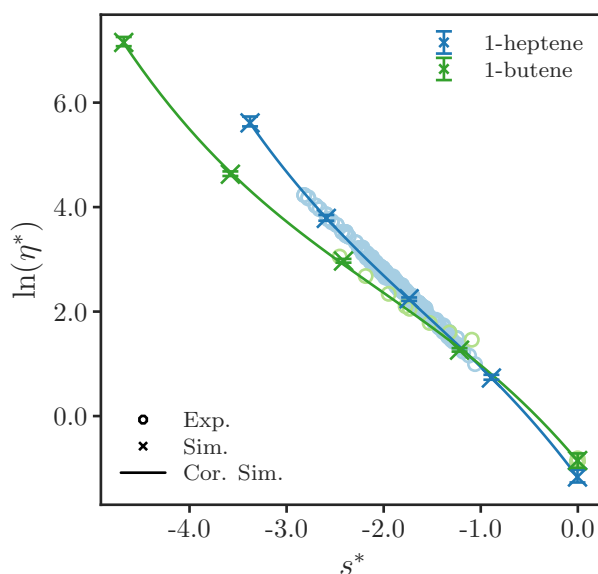
Figure 3.2 illustrates the large number of experimental data with which we compare our simulation results. For alkanes in fig. 3.2 there are between 382 and 2606 points per substance for a temperature range of 88.1 K (for propane) to 222.15 K (for nonane) and a pressure range of 2.6 kPa (ethane) up to 400 MPa (propane). The experimental data originates from different sources and the entropy scaled representation reveals data that does not follow the univariate behavior and has to be considered as outlier. Entropy scaling can provide strong indications



**Figure 3.2:** Logarithmic depiction of reduced viscosity  $\eta^*$  over the residual entropy  $s^*$  for different alkanes. Circles show experimental data from the DDB<sup>76</sup>, and crosses represent simulation results. Lines are correlations adjusted to the simulation data.

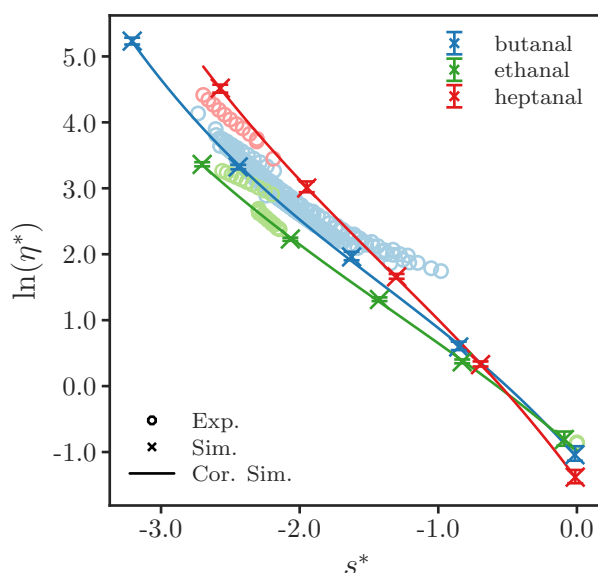
of non-plausible or inconsistent experimental data<sup>59</sup>, as can be seen for example for ethane in the range of  $-3 \leq s^* \leq -1$ . Nevertheless, to be conservative, all experimental data shown here are included in the calculation of the AAD according to eq. (3.17). The AADs for viscosity summarized in fig. 3.7 confirm rather good agreement with the experimental data for alkanes with AADs below 15 % for the entire fluid region, from gaseous state to liquid states at low temperature and high pressures. A systematic tendency towards higher deviations for longer alkane chains can be observed. There are two possible explanations for the weaker description of long alkane chains. Either the intramolecular force field (i.e. angle and torsion potentials as well as intramolecular van der Waals interactions) does not sufficiently reflect the actual behavior of alkanes. Or, increasing deviations are a result of a coarse-grained (united-atom) force field, where hydrogen interaction sites are not explicitly modeled. The effect of this coarse-graining may act more substantially on longer chains than on molecules with small aspect ratio. The results in fig. 3.2 support this hypothesis for increasingly long chains, because coarse-graining artificially increases the dynamic of the system, leading to too low viscosities and too high self-diffusion coefficients. In order to better assess the quality of results from the TAMie force field, fig. 3.7 also gives the AADs for the entropy scaling method from the literature<sup>58</sup>, which resulted from adjusting correlations directly to experimental data. It is remarkable that the correlation from five simulation points for ethane delivers the same accuracy of the correlation as obtained from multiple hundred experimental data points. Also for other n-alkanes the accuracy of the model trained by the TAMie force field is in the same order of magnitude as results obtained by adjusting the model directly to the experimental data.

For the group of alkenes, fig. 3.3, there are noticeably fewer experimental data points available in literature. The simulated state points extend beyond the range of residual entropies  $s^*$  covered by the experimental data. In the  $s^*$ -ranges, where both, simulations and experiments are available, the simulated viscosities lie within the range of the experimental data. Correlations adjusted to all simulations represent the experimental data very satisfyingly, as also reflected by the AAD-value of less than 7 %, which for 1-butene is in the range of what can best be achieved with the entropy scaling method without filtering experimental data (as seen in fig. 3.7). In the case of 1-heptene, a potential application for the determination of transport properties by means of simulations can be identified. The entropy scaling method requires a broad set of data to reasonably adjust the correlations. For 1-heptene experimental data is scarce and limited to a small range in residual entropy, so that no viscosity model could be parametrized in our earlier work<sup>58</sup>. Results from the TAMie force field or from other force fields might be used to supplement experimental data in order to propose a model for the viscosity of such substances.



**Figure 3.3:** Logarithmic depiction of reduced viscosity  $\eta^*$  over the residual entropy  $s^*$  for different alkenes. Circles show experimental data from the DDB<sup>76</sup>, and crosses represent simulation results. Lines are correlations adjusted to the simulation data.

Larger deviations from experimental data can be seen in fig. 3.4 for the viscosity of aldehydes, where also the qualitative behavior of the experiments is partly not represented by the simulations. Taking a closer look at experimental data for ethanal reveals two ‘sets’ of points with different courses of  $\eta^*(s^*)$ , in the range  $s^* < -2.0$ . One set is accurately traced by the

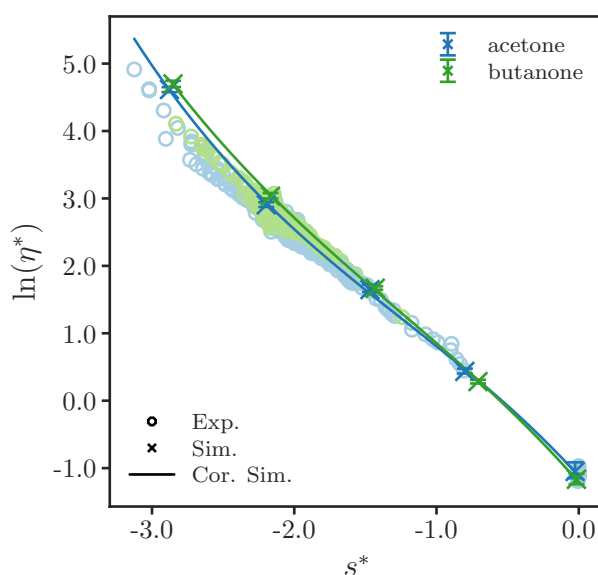


**Figure 3.4:** Logarithmic depiction of reduced viscosity  $\eta^*$  over the residual entropy  $s^*$  for different aldehydes. Circles show experimental data from the DDB<sup>76</sup>, and crosses represent simulation results. Lines are correlations adjusted to the simulation data.

correlation adjusted to the simulations. The experimental data of this set comes from four different references. The data set with higher viscosities, that clearly contradicts predictions from the TAMie force field, originates from a single reference and was measured at high pressures of 137 bar to 1378.9 bar. If one chooses to exclude this data set, say by limiting the considered pressures to values  $p \leq 100$  bar, the deviation of the TAMie-correlation to the experimental values drops to 4%. Also 80% of the experimental data shown in fig. 3.4 for heptanal, the substance with the highest AAD for the viscosity, were measured above 100 bar. For butanal, with an AAD of 15%, a clear analysis of the TAMie results and of experimental data is difficult. The data shows rather pronounced spread in the entropy scaling diagram, where experimental data is partly contradicting with other experimental data, which also translates to relatively high AAD of 9% of the original entropy scaling correlation in the literature, that was adjusted to the experiments<sup>58</sup>.

The results for the next chemical group, ketones, are summarized in fig. 3.5. Qualitatively the simulations follow the  $\eta^*(s^*)$  behavior of the experiments. The four rightmost state points of both substances, located in the range  $s^* > -2.5$ , show simulated viscosities in satisfying quantitative agreement with experimental values. The point furthest to the left, with the lowest entropy, however, overestimates the viscosity for both ketones. Correspondingly, the correlation runs above the experimental points, leading to comparatively high AADs of 15% and 21%, respectively.

As a representative for ethers, we here regard the viscosity of dimethyl ether, shown in fig. 3.6.

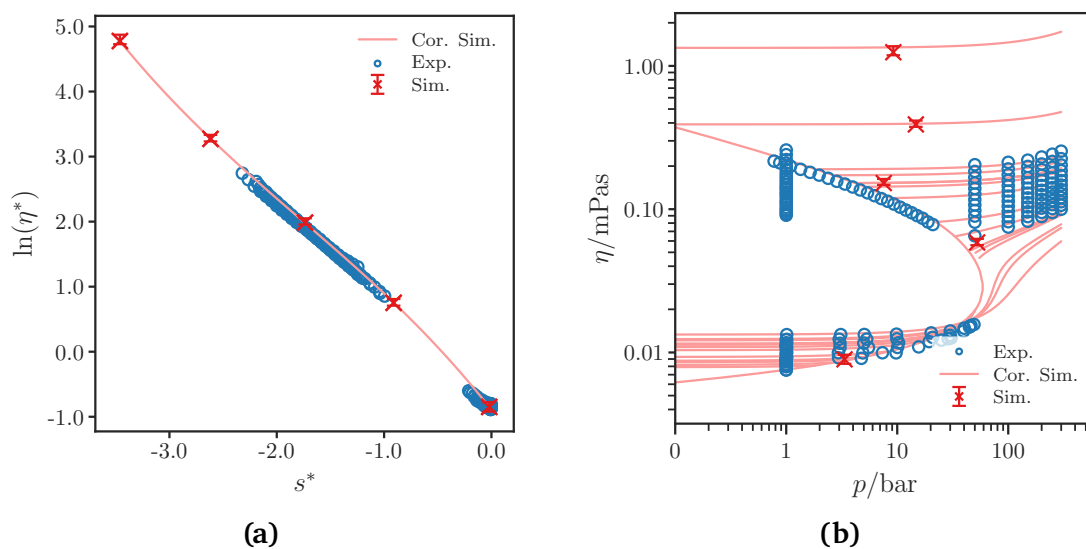


**Figure 3.5:** Logarithmic depiction of reduced viscosity  $\eta^*$  over the residual entropy  $s^*$  for acetone and butanone. Circles show experimental data from the DDB<sup>76</sup>, and crosses represent simulation results. Lines are correlations adjusted to the simulation data.

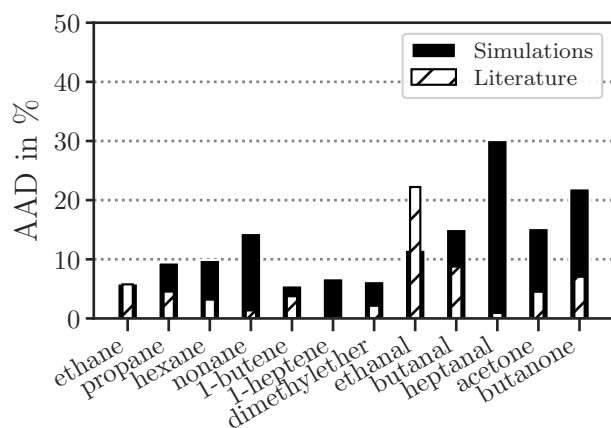


The entropy scaled representation in fig. 3.6a illustrates that the experimental viscosities in both, the gas phase ( $-0.1 < s^*$ ) and the liquid phase ( $-2.5 < s^* < -1$ ) can be predicted very well with the TAMie force field. Figure 3.6b shows a double logarithmic representation of the unscaled viscosity versus pressure. Open circles symbolize experiments, red crosses the results of our simulations. The red lines in fig. 3.6b represent viscosities along isotherms as calculated from the TAMie-correlation. Although the isotherms are selected rather arbitrarily for the purpose of illustrating the course of isotherms in the fluid region, the figure reveals the appeal of the method used: from simulation results at five states (that do not even have to be close to the experimental data in the state space), the transport coefficients can be predicted for any  $T$  and  $p$  in the fluid region. From the diagram we see that the TAMie-correlation predicts experimental viscosities both at vapor liquid equilibrium (VLE) states – gas-side and liquid-side –, as well as liquid viscosities at high pressures up to 400 bar rather well. For dimethyl ether this visual assessment is also reflected in a low AAD value of 6%.

The deviations of the viscosity correlations determined with TAMie are summarized for all substances in fig. 3.7. The mean of all AADs is 13%. This value is surprising, (1) in view of the large number of experimental data that entered the AAD values, (2) considering the fact that we did not filter (non-plausible) experimental data, and (3) noticing that we conducted simulations at merely five state points.



**Figure 3.6:** Two logarithmic depictions of the results for the viscosity  $\eta$  of dimethyl ether. In both figures, circles show experimental data from the DDB<sup>76</sup>, and crosses represent simulation results. (a) shows the reduced viscosity  $\eta^*$  over the residual entropy  $s^*$ . Lines in (a) are correlations adjusted to the simulation data. (b) shows viscosity  $\eta$  over pressure  $p$ . Lines in (b) are isotherms calculated with a correlation adjusted to the simulation data.



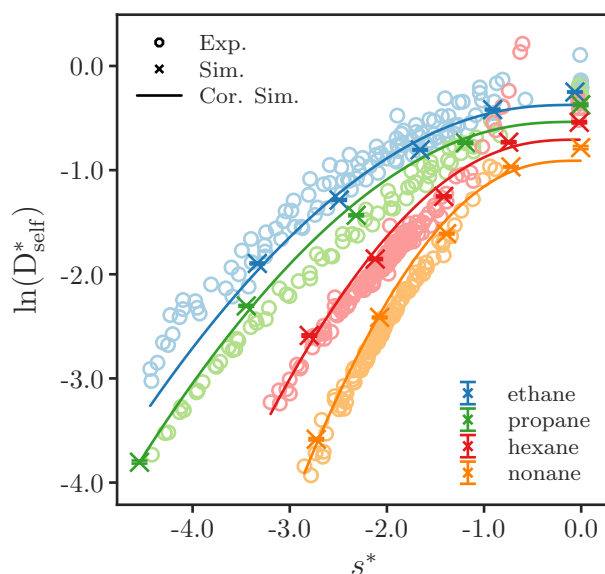
**Figure 3.7:** AAD in percent of the correlation adjusted to simulation compared to all experimental viscosities available. Additionally, AAD of correlations from the literature<sup>58</sup>, adjusted directly to experiments are given.

### 3.3.2 Self-Diffusion

Results for the self-diffusion coefficients  $D_{\text{self}}$  are shown in figs. 3.8–3.13. For diffusion there are considerably fewer experimental data available than for the other transport properties. Data with which we can assess the TAMie force field are only available for seven of the substances investigated. Gas diffusion experiments are particularly scarce.

For alkanes, however, data for the gas phase is available, with the exception of nonane, as shown in fig. 3.8. The results of alkanes are therefore important for assessing the predictive quality of the simulations, especially for the gas phase. The qualitative behavior of  $D_{\text{self}}^*(s^*)$  is reproduced by the simulated results. The scatter in experimental data is more pronounced for self-diffusion coefficients than for shear viscosity. In our earlier work on self-diffusion<sup>60</sup> we have come to believe the experimental scatter in the entropy scaling diagram is mainly rooted in contradicting data and does not necessarily reveal deficiencies of the entropy scaling principle. The simulated values  $D_{\text{self}}$  are in reasonable agreement to the experimental data. The simulations are for longer n-alkanes systematically overestimating experimental values of  $D_{\text{self}}$ , and for ethane they are underestimating  $D_{\text{self}}$ -values. This may hint either at a non-optimal choice for the anisotropic shift of the  $\text{CH}_3$ -group, or at the need for partial charges to better represent the quadrupolar charge distribution of ethane. Diffusion coefficients in the gas phase are determined within the experimental range, with a slight tendency to underestimate them.

From our simulation results for self-diffusion coefficients (fig. 3.8) it becomes apparent the ansatz function in table 3.3 for  $D_{\text{self}}$  is not well suitable for correlating the function  $D_{\text{self}}(s^*)$  at low values of  $s^*$ , i.e. for the gas phase. The ansatz function enforces a slope of zero for



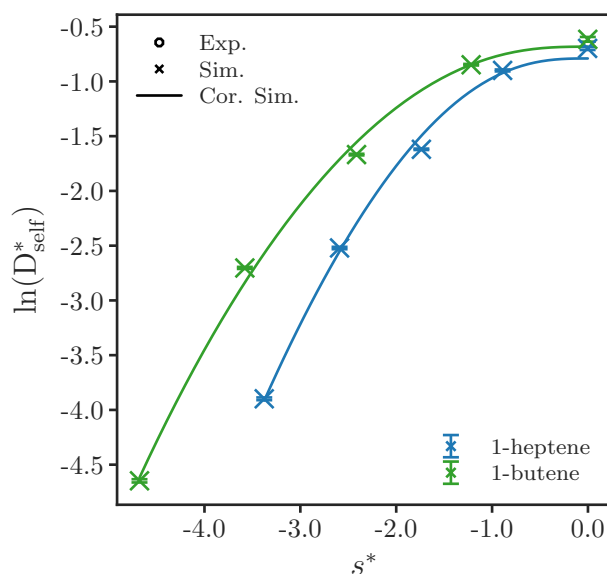
**Figure 3.8:** Logarithmic depiction of reduced self-diffusion coefficient  $D_{\text{self}}^*$  over the residual entropy  $s^*$  for different alkanes. Circles show experimental data from the literature<sup>77–83,85,87–91</sup>, and crosses represent simulation results. Lines are correlations adjusted to the simulation data.

$s^* = 0$  which does not model the experiments nor the simulations adequately. That is an important finding. When we proposed the ansatz function, ref.<sup>60</sup>, we had collected fewer experimental data for the vapor phase and had no results from molecular simulations. The few substances, where we had vapor phase data (xenon, argon, carbon dioxide, water) are spherical substances or only mildly elongated. For these species the slope of  $D_{\text{self}}(s^*)$  at  $s^* = 0$  is indeed close to zero. Now, with an extended database and with the results from the molecular simulations from the TAMie force field, we need to revise the ansatz function. For the same reason, namely the deficient ansatz function for  $D_{\text{self}}$ , the calculated AADs are worse than the simulation results would suggest.

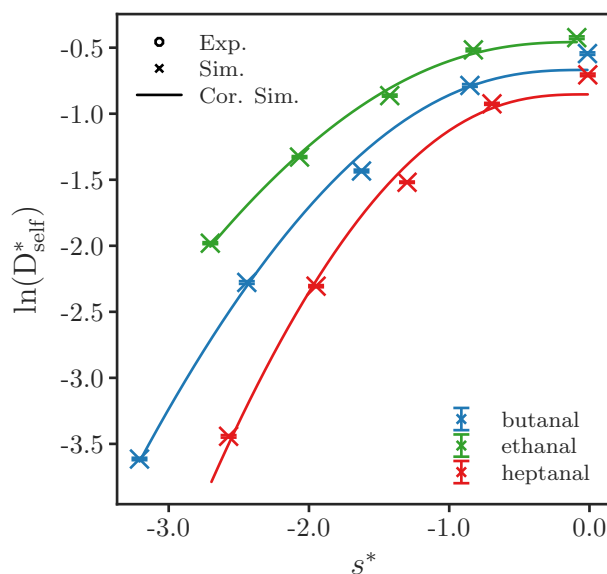
The results for alkenes and aldehydes given in figs. 3.9 and 3.10 demonstrate the scarcity of experimental values for self-diffusion coefficients.

Figure 3.11 gives results of the TAMie force field for ketones, i.e. for acetone and for butanone. The experimental self-diffusion coefficients available for acetone are clearly underestimated by the force field.

Simulation results for dimethyl ether are shown in fig. 3.12. Predictions of TAMie are in good agreement to the experimental data in the liquid phase; experimental values for the gas phase are not available. The strength of expressing results of simulations in an entropy scaled form is evident from fig. 3.12b. The experiments all correspond to comparatively high pressures, whereas the simulations were performed at moderate 30 bar to 50 bar. The predictive power



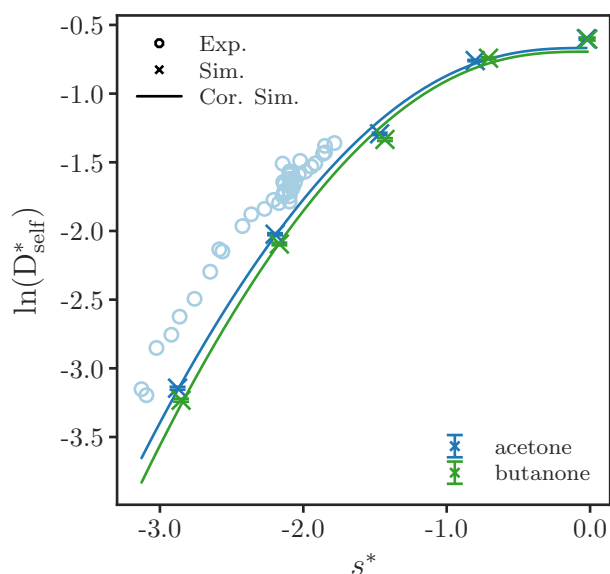
**Figure 3.9:** Logarithmic depiction of reduced self-diffusion coefficient  $D_{\text{self}}^*$  over the residual entropy  $s^*$  for different alkenes. Lines are correlations adjusted to the simulation data.



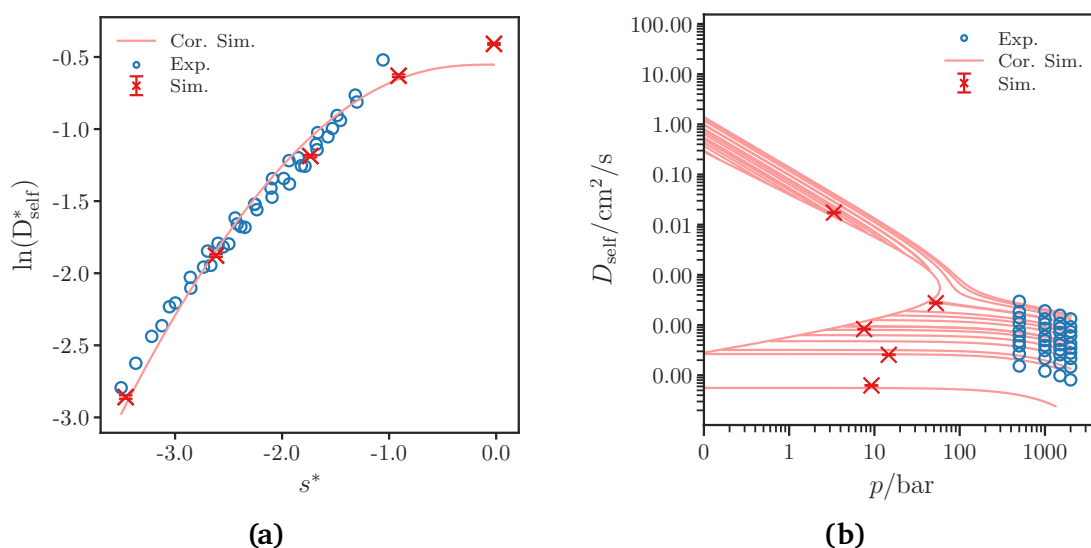
**Figure 3.10:** Logarithmic depiction of reduced self-diffusion coefficient  $D_{\text{self}}^*$  over the residual entropy  $s^*$  for different aldehydes. Lines are correlations adjusted to the simulation data.

of entropy scaling is remarkably good as seen from the isotherms in fig. 3.12b, reproducing the experimental isothermal data series.

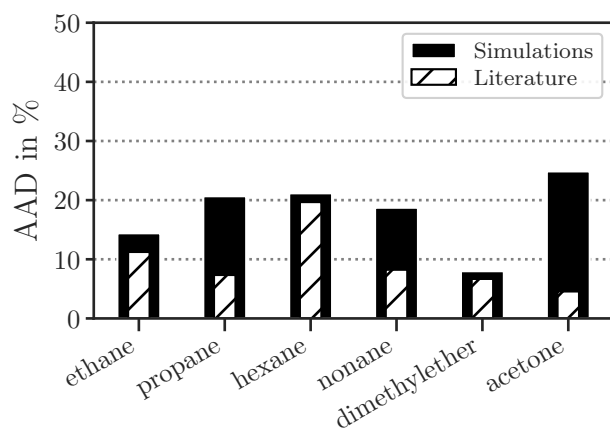
The deviations of the TAMie-correlations from experimental data are summarized in fig. 3.13. We find AADs between 8 % and 25 %, a result that can be appreciated when compared to the native entropy scaling model based on PC-SAFT, also presented in fig. 3.13.



**Figure 3.11:** Logarithmic depiction of reduced self-diffusion coefficient  $D_{\text{self}}^*$  over the residual entropy  $s^*$  for acetone and for butanone. Circles show experimental data from the literature<sup>91</sup>, and crosses represent simulation results. Lines are correlations adjusted to the simulation data.



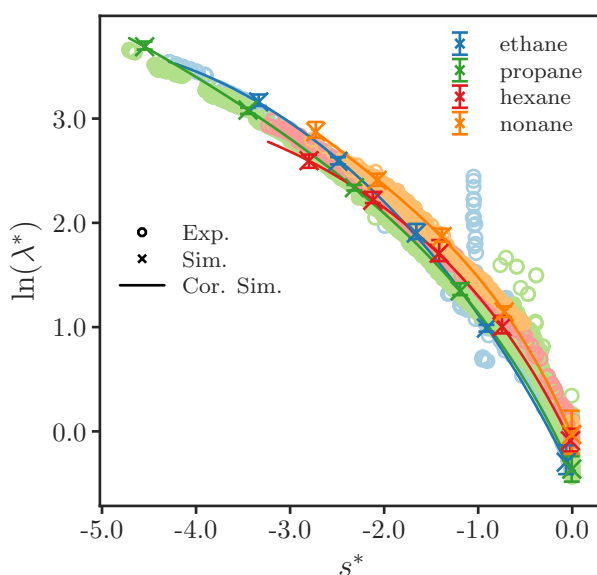
**Figure 3.12:** Two logarithmic depictions of the results for the self-diffusion coefficient  $D_{\text{self}}$  of dimethyl ether. In both figures, circles show experimental data from the literature<sup>91</sup>, and crosses represent simulation results. (a) shows the reduced self-diffusion coefficient  $D_{\text{self}}^*$  over the residual entropy  $s^*$ . Lines in (a) are correlations adjusted to the simulation data. (b) shows  $D_{\text{self}}$  over pressure  $p$ . Lines in (b) are isotherms calculated with a correlation adjusted to the simulation data.



**Figure 3.13:** AAD in percent of the correlation adjusted to simulation compared to all experimental  $D_{\text{self}}$  values available. Additionally, AAD of correlations from the literature<sup>60</sup>, adjusted directly to experiments are given.

### 3.3.3 Thermal Conductivity

Figures 3.14–3.19 show the results for the thermal conductivity, starting with the alkanes in fig. 3.14. Results from the TAMie force field reproduce the qualitative behavior of  $\ln(\lambda^*)(s^*)$  of the experiments accurately. The simulation results lie within the distribution of the experiments in the entropy scaled diagram, both for the liquid state points as well as for the gas phase. For alkanes we find smaller deviations of the TAMie force field from experiments for longer chain molecules, with the lowest deviations of 6% for nonane.



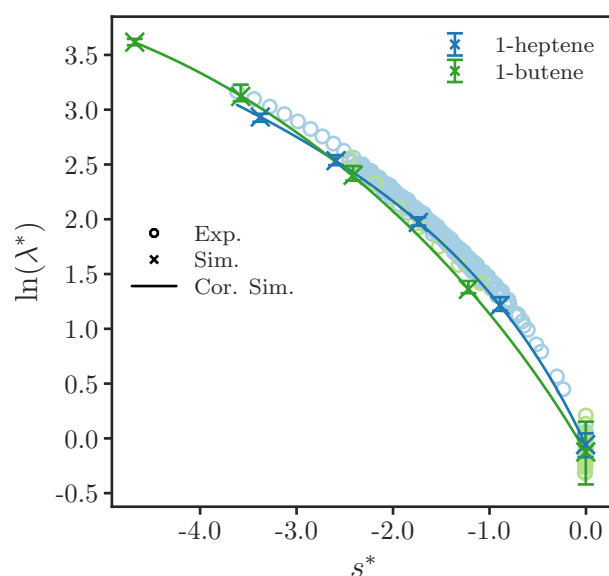
**Figure 3.14:** Logarithmic depiction of the reduced thermal conductivity  $\lambda^*$  over the residual entropy  $s^*$  for different alkanes. Circles show experimental data from the DDB<sup>76</sup>, and crosses represent simulation results. Lines are correlations adjusted to the simulation data.

The results for the alkenes in fig. 3.15 are satisfying. In the liquid phase, the thermal conductivities tend to be underestimated, with a total AAD of 11 % for 1-butene. For 1-heptene the simulations underestimate the experimental values for  $\lambda$ , especially in the range  $s^* < -2.5$ , leading to a total deviation of below 8 %.

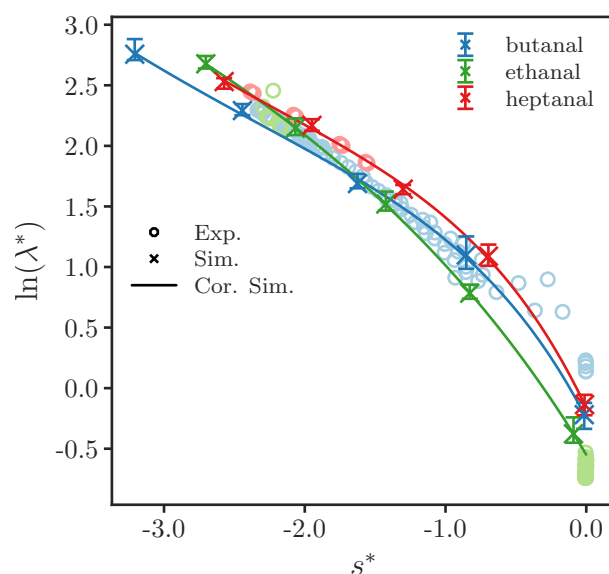
Experimental values of thermal conductivity for aldehydes are scarce, see fig. 3.16. The TAMie model is in rather good agreement with the available experimental data for the liquid phase, whereas some deviations are seen for the gas phase. The AAD of the TAMie-correlation is calculated to about 10 % for the aldehydes.

For the ketones, fig. 3.17 shows deviations from the experiments in the vicinity to the critical point ( $s^* \approx 0.4$  for both ketones). Thermal conductivity is known significantly increase close to the critical point<sup>92</sup> and it was observed that the critical enhancement is not captured by entropy scaling<sup>59</sup>. However, the critical enhancement is seen for conditions rather close to the critical point and cannot explain the deviations of simulated thermal conductivities and experimental data for ketones. Good overall agreement of the gas phase and liquid phase simulations with experiments results in averaged deviations of less than 10 % for both ketones.

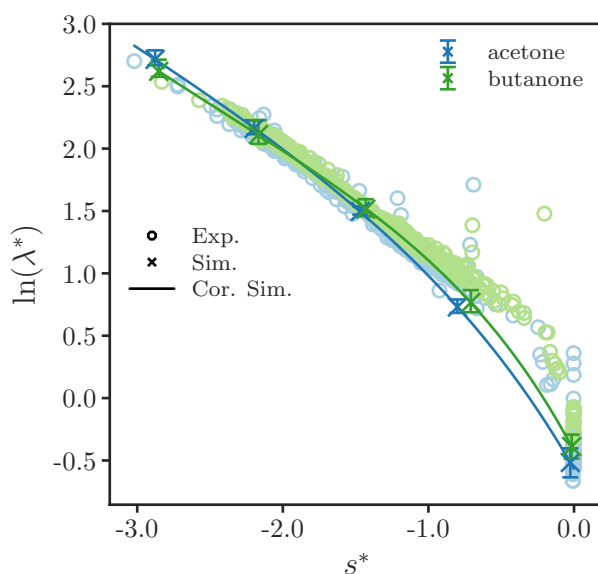
Very satisfying results for thermal conductivities are found for dimethyl ether, as seen in Figure 3.18a. The TAMie force field tends to underestimate the experimental data in the gas phase and overestimates thermal conductivities in the dense liquid phase. fig. 3.18b shows thermal conductivity of dimethyl ether versus pressure, where lines represent isotherms as



**Figure 3.15:** Logarithmic depiction of the reduced thermal conductivity  $\lambda^*$  over the residual entropy  $s^*$  for different alkenes. Circles show experimental data from the DDB<sup>76</sup>, and crosses represent simulation results. Lines are correlations adjusted to the simulation data.



**Figure 3.16:** Logarithmic depiction of the reduced thermal conductivity  $\lambda^*$  over the residual entropy  $s^*$  for different aldehydes. Circles show experimental data from the DDB<sup>76</sup>, and crosses represent simulation results. Lines are correlations adjusted to the simulation data.

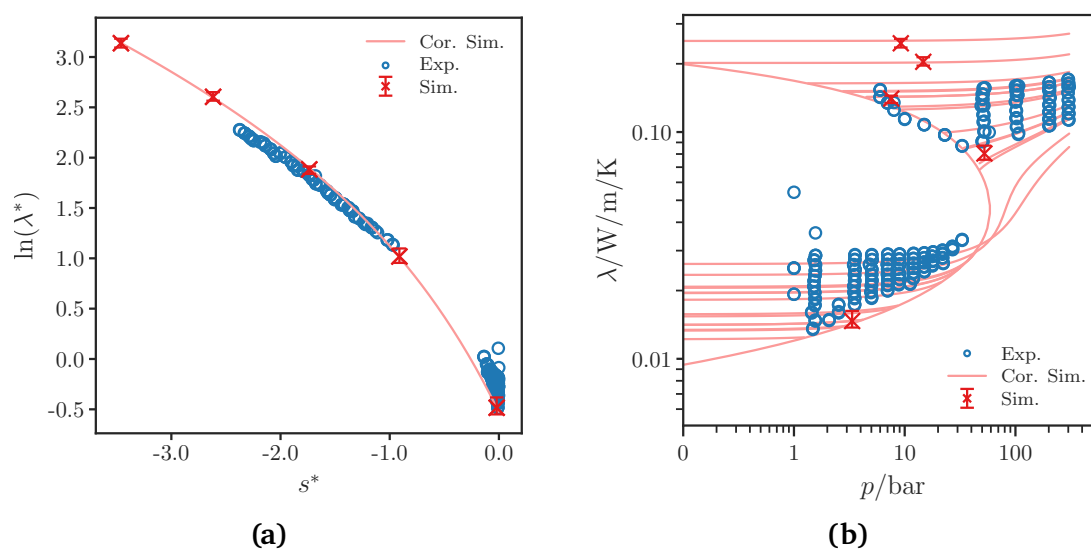


**Figure 3.17:** Logarithmic depiction of the reduced thermal conductivity  $\lambda^*$  over the residual entropy  $s^*$  for acetone and butanone. Circles show experimental data from the DDB<sup>76</sup>, and crosses represent simulation results. Lines are correlations adjusted to the simulation data.

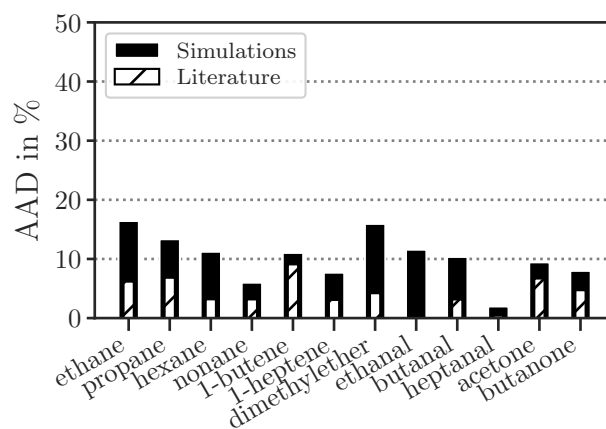
determined from TAMie-correlations.

Figure 3.19 summarizes the deviations of predicted thermal conductivities from TAMie-correlations to experimental data for all substances. The overall average AAD are 10 % for all substances, which we consider a very satisfactory result.





**Figure 3.18:** Two logarithmic depictions of the results for the thermal conductivity  $\lambda$  of dimethyl ether. In both figures, circles show experimental data from the DDB<sup>76</sup>, and crosses represent simulation results. (a) shows the reduced thermal conductivity  $\lambda^*$  over the residual entropy  $s^*$ . Lines in (a) are correlations adjusted to the simulation data. (b) shows  $\lambda$  over pressure  $p$ . Lines in (b) are isotherms calculated with a correlation adjusted to the simulation data.



**Figure 3.19:** AAD in percent of the correlation adjusted to simulation compared to all experimental  $\lambda$  available. Additionally, AAD of correlations from the literature<sup>59</sup>, adjusted directly to experiments are given.

### 3.4 Conclusion

The TAMie force field was originally parametrized using VLE data without considering any dynamic properties. In this study we investigate the predictive power of the TAMie force field for the transport properties, namely shear viscosity, self-diffusion coefficient and thermal conductivity. We conclude that the TAMie force field is rather well suited for the prediction of

transport coefficients.

Simulation results for all transport properties and their statistical uncertainties are obtained simultaneously from a series of equilibrium MD simulations. We propose a workflow, where the state points at which simulations are conducted are selected based on their residual entropy (which we estimate a priori using the PC-SAFT equation of state). We define five state points for each substance, approximately equally distributed in residual entropy. According to entropy scaling that ensures a meaningful distribution of states for conducting simulations covering the entire fluid region. The simulation results are used to adjust a simple correlation function for each transport property, based on the entropy scaling method. We assess predictions of the TAMie force field by comparing the correlation parametrized using simulations with all experimental data available in literature – even for thermodynamic states far from those we have simulated. We observe good agreement of the model, based on the TAMie force field with literature data for all transport coefficients, with AAD-values of 13 % for viscosity, 18 % for diffusion and 10 % for thermal conductivity. The deviations are only mildly higher as compared to an entropy scaling model that directly uses experimental data<sup>57–60</sup>.

Generally, molecular simulations are seen to be promising for species with scarce or nonexistent experimental data for one of the transport coefficients. For self-diffusion it is particularly important to provide estimates for self-diffusion coefficients in the gaseous phase. In fact, with this study it became apparent that our ansatz function for the entropy-scaling correlation proposed in earlier work<sup>60</sup> (originally developed with rather limited experimental data for gas phases) does not adequately model the course of the simulation results.

#### **Acknowledgement**

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#### **Supporting Information**

PC-SAFT parameters, as well as the parameters of the TAMie force field used in this study, are provided along with tables summarizing temperatures, pressures, viscosities, diffusion coefficients, and thermal conductivities for all simulated state points of all investigated substances; tables with the resulting parameters of the adjusted correlations; and the results of an NVE study of hexane.

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## 4 Assessing Entropy Scaling for Mixture Viscosities using Molecular Simulations

### Abstract

The present work uses molecular simulations of model-mixtures to systematically validate the applicability of entropy scaling for mixtures. I determine the viscosity  $\eta$  of several binary mixtures of simple Lennard-Jones fluids in equilibrium molecular dynamic simulations at different state points. The extended Chapman-Enskog viscosity for mixtures is considered as reference viscosity  $\eta^{CE}$  in order to define the dimensionless viscosity  $\eta^\#$ . Grand-Canonical Monte Carlo simulations are used to determine residual entropy  $s_{res}$  in order to avoid assumptions inherent to any analytic EoS, but remain consistent within molecular simulations. The considered binary mixtures are composed of two LJ-fluids differing in their  $\epsilon$  or  $\sigma$  parameters. The study further focuses on non-ideal mixtures of two identical LJ-fluids with  $\epsilon_{11} = \epsilon_{22} \neq \epsilon_{12}$  which can be parameterized to show a strongly non-ideal phase behavior. In an entropy scaled depiction, the viscosities of all considered mixtures collapse well onto a universal curve and show a monovariabile dependence of  $\eta^\#$  on residual entropy. The results of the non-ideal mixtures of two identical LJ-fluids enabled us to formulate a general mixing rule for the correlation parameters of pure components.

### 4.1 Introduction

Approaches for correlating or predicting transport coefficients of mixtures proposed in literature are based on the principle of corresponding states<sup>1-5</sup>, the free volume theory<sup>6-12</sup>, friction theory<sup>13-15</sup> and thermodynamic scaling<sup>16-21</sup>. A broad overview of the existing correlation methods for viscosity is given in the recent review of Baled et al.<sup>22</sup>. The authors compare established methods and find unsatisfactorily high deviations between predicted viscosities and experiments, especially in temperature and pressure ranges with scarce experimental data. The most precise and promising predictive results were provided by a group contribution entropy scaling method of Lötgering-Lin and Gross<sup>23</sup>. This method will be assessed in this work with regard to the entropy scaling behavior of the viscosity of mixtures. For this purpose

simple model mixtures are investigated using molecular simulations. Simulation results allow the distinct analysis of the influence of individual model parameters on fluid properties.

This benefit of molecular simulation of model fluids has been used in the literature for a long time to develop and assess different approaches for the calculation of dynamic properties of mixtures. Several studies considered the influence of different parameters defining Lennard-Jones fluids (LJ) and Mie fluid mixtures on the viscosity. Investigated were mixtures, where two Mie fluids have different mass<sup>24</sup>, or energy parameters, or size parameters of the intermolecular potentials,<sup>25</sup> or different repulsive exponents of the Mie potential<sup>26</sup>. The model mixtures helped to develop correlation functions for transport coefficients of mixtures, based on various approaches, such as thermodynamic scaling<sup>26</sup> or the corresponding-state principle<sup>24,25,27-29</sup>. Another interesting approach that allows to predict dynamic properties is the isomorph theory, introduced in chapter 1. At this point it is worth recalling two specifics of isomorphs which are of particular interest in the context of this work: Both, the residual entropy  $s_{\text{res}}$  and the reduced dynamic quantities  $\tilde{D}_{\text{self}}$ ,  $\tilde{\lambda}$ , and  $\tilde{\eta}$  are invariant along isomorphs. These two observations support the basic idea of entropy scaling: The reduced dynamic properties are a monovariate function of residual entropy. Numerous simple model fluids, such as the LJ fluid and the dumbbell fluid, have been identified as *Roskilde fluids* and exhibit isomorph properties.<sup>30</sup> Also mixtures of model fluids, like the Kob-Andersen binary LJ fluid, behave like *Roskilde fluids*.<sup>31</sup> Although isomorph theory seeks approaches to explain entropy scaling, a full understanding can not be provided.<sup>32</sup> It has been shown that the monovariate relationship between entropy and dynamic properties is also valid for fluids and regions in phase space to which  $R < 0.9$  applies and where therefore no isomorphs occur<sup>33</sup>. The relation between entropy and dynamic properties therefore exceeds the isomorph theory and has been discovered long before the isomorphs.

In later studies, dynamic properties of more complex model fluids, like LJ chains<sup>34,35</sup> and Dumbbell particles<sup>36</sup>, were investigated using entropy scaling. The findings regarding model fluids were applied to molecular simulation studies of real fluids, for example water<sup>37</sup>, n-alkanes<sup>38</sup> and polyethylene<sup>39</sup>. One advantage of the investigation of simple model fluids is that  $s_{\text{res}}$  is accessible through well established and highly accurate equations of state<sup>34,35,38-41</sup>, or even directly through molecular simulations<sup>36,37</sup>. The approach was also used to investigate mixtures. Results based on Dzugotov's scaling approach implied a general scaling law for LJ-mixtures.<sup>42,43</sup> Recently, I used molecular simulations of LJ model mixtures to find a mixing rule that takes the correlation parameters of the pure fluids as input, as published jointly in Lötgering-Lin *et al.*<sup>44</sup>. The mixing rule leads to excellent predictions for viscosities of mixtures containing non-polar and polar fluids, without any further adjustment of parameters.

In this chapter I extend on our previous work<sup>44</sup> in analyzing the scaling behavior of mixture

viscosities. For a consistent assessment, I calculate  $\eta^\# = \eta/\eta_{CE,mix}$  as well as  $s_{res}$  using molecular simulations. This approach circumvents any inaccuracy underlying analytical equation-of-state models. Additionally, also the reference quantity  $\eta_{CE,mix}$  can be calculated using the molecular parameters of the considered force fields. The mixtures investigated here consist of two LJ fluids, that differ in one model parameter, which allows the dedicated investigation of the influence of individual parameters. In addition, a mixture of two identical LJ fluids with a non-Berthelot cross-interaction is studied to assess the influence of non-idealities.

## 4.2 Methods

This section discusses the methodical background of this work. First, the molecular model and the model mixtures are described and a brief overview of entropy scaling and the required reference quantities is given. Subsequently, it is explained how entropy and viscosity are calculated from molecular simulations.

### 4.2.1 Molecular Model

The mixtures investigated in this work are composed of spherical mono-atomic model fluids. The interactions two interaction sites are modeled by a Lennard–Jones potential,

$$\phi^{LJ}(r_{ij}) = 4\varepsilon_{ij} \left[ \left( \frac{\sigma_{ij}}{r_{ij}} \right)^{12} - \left( \frac{\sigma_{ij}}{r_{ij}} \right)^6 \right], \quad (4.1)$$

with  $r_{ij}$  as the distance of two centers of interaction  $i$  and  $j$ . Here, the size parameter  $\sigma_{ii}$  and the energy parameter  $\varepsilon_{ii}$  are the parameters that distinguish the pure components of the mixtures. Unless otherwise specified, the parameters of the unlike site pairs of the mixture are calculated using a Lorentz combining rule and an non–Berthelot cross-energy parameter  $\varepsilon_{ij}$ , as

$$\sigma_{ij} = \frac{\sigma_{ii} + \sigma_{jj}}{2} \quad (4.2)$$

$$\varepsilon_{ij} = \sqrt{\varepsilon_{ii}\varepsilon_{jj}}(1 - k_{ij}), \quad (4.3)$$

where the binary interaction parameter  $k_{ij}$  (with  $k_{ii} = 0$ ) acts as a deviation from the geometric Berthelot combining rule. The parameters  $\sigma_{11}$  and  $\varepsilon_{11}$  of the first component of each mixture are used to define reduced (dimensionless) quantities. All results are given in these reduced

LJ units that are summarized in table 4.1.

A well studied example for a binary mixture of LJ fluids is the Kob-Andersen model<sup>45,46</sup>, that has also been investigated with respect to entropy scaling and is known as a *Roskilde fluid* in isomorph theory.<sup>31</sup> The two LJ components of the Kob-Andersen fluid differ in  $\sigma_{22}/\sigma_{11}$  as well as in  $\varepsilon_{22}/\varepsilon_{11}$  and a non-zero binary interaction parameter  $k_{ij}$ .<sup>47</sup> Since the aim of this study is to systematically analyze the impact of the parameters of the LJ potential on viscosity and entropy, two LJ fluids are mixed, differing only in their  $\varepsilon$  or  $\sigma$ -values, respectively. Mixture I is therefore designed as a mixture of two LJ fluids with a ratio of the energy parameters of  $\varepsilon_{22} = \varepsilon_{11} \cdot 0.7$ . The other parameters are identical for both components of the mixture ( $\sigma_{11} = \sigma_{22}$ ,  $m_1 = m_2$ ). In a second mixture II, only the size parameters differ ( $\varepsilon_{11} = \varepsilon_{22}$ ,  $m_1 = m_2$ , but  $\sigma_{22} = \sigma_{11} \cdot 1.5$ ). For both mixtures, the Lorentz-Berthelot combining rules eq. (4.2) and eq. (4.3) with  $k_{ij} = 0$  are applied. The viscosity of mixtures comparable to I and II have been investigated using a one-fluid model, which defines one pseudo-component based on the parameters of the two involved LJ fluids<sup>25</sup>. The study showed the limitations of the one-fluid model regarding size asymmetric mixtures. It is thus of interest how well mixture II can be described with entropy scaling.

The third mixture III is a mixture of two LJ-fluids with an identical set of parameters ( $\varepsilon_{11} = \varepsilon_{22}$ ,  $m_1 = m_2$ ,  $\sigma_{11} = \sigma_{22}$ ), but with a non-ideal cross interaction term for the energy parameter of  $k_{12} = 0.25$ . This mixture is of special interest, as both boundaries of the composition,  $x_1 = 0$  and  $x_1 = 1$ , are well defined as the regular LJ fluid, whereas the non-pure compositions will show a different behavior. All parameters of defining the three investigated model mixtures are summarized in table 4.2

**Table 4.1:** Reduced Lennard-Jones units

| Quantity         | Symbol | Reduced units   |
|------------------|--------|---|
| Time             | $t$    | $t^* = t \sqrt{\varepsilon_{11} / (M_1 \sigma_{11}^2)}$     |
| Distance         | $r$    | $r^* = r / \sigma_{11}$                                     |
| Energy           | $u$    | $u^* = u / \varepsilon_{11}$                                |
| Temperature      | $T$    | $T^* = T / (\varepsilon_{11} / k_b)$                        |
| Volume           | $V$    | $V^* = V / \sigma_{11}^3$                                   |
| Particle Density | $\rho$ | $\rho^* = \rho \sigma_{11}^3$                               |
| Pressure         | $p$    | $p^* = p \sigma_{11}^3 / \varepsilon_{11}$                  |
| Viscosity        | $\eta$ | $\eta^* = \eta \sigma_{11}^2 / \sqrt{\varepsilon_{11} M_1}$ |

**Table 4.2:** Parameters of the LJ potential of the investigated model mixtures

| Mixture | $\sigma_{22}/\sigma_{11}$ | $\varepsilon_{22}/\varepsilon_{11}$ | $k_{12}$ |
|---------|---------------------------|-------------------------------------|----------|
| I       | 1                         | 0.7                                 | 0        |
| II      | 1.5                       | 1                                   | 0        |
| III     | 1                         | 1                                   | 0.25     |

### 4.2.2 Entropy Scaling

Rosenfeld's Ansatz allows for calculating transport properties of a substance at any fluid state point, using a simple correlation function. For a wide range of substances, a third order polynomial in terms of reduced viscosity  $\eta^\# = \eta/\eta_{ref}$  is sufficient to calculate the viscosity  $\eta$  for a wide range of thermodynamic conditions. For spherical fluids, even a linear correlation gives good results<sup>40,41,48</sup>. The monovariate dependence of  $\eta^\#$  on  $s_{res}$ , and thus the applicability of entropy scaling, critically depends on the choice of the reference viscosity  $\eta^{ref}$ . For pure fluids, Novak<sup>48</sup> proposed the Chapman-Enskog viscosity as a reference, which was shown to lead to accurate predictions of viscosities of various real substances<sup>23,44,49</sup>. The CE-viscosity for a pure substance of index  $i$  is defined as<sup>50</sup>

$$\eta_{CE,i} = \frac{5}{16} \frac{\sqrt{M_i k_b T / (N_A \pi)}}{\sigma_{ii}^2 \Omega_i^{(2,2)*}}, \quad (4.4)$$

with the molecular mass  $M_i$ , Boltzmann's constant  $k_b$ , temperature  $T$ , Avogadro's constant  $N_A$  and the molecular diameter  $\sigma_{ii}$ . The collision integral  $\Omega_i^{(2,2)*}$  is calculated using the empirical correlation of Neufeld et al<sup>51</sup>. In this study the approximation of Wilke<sup>52,53</sup> is used to define the Chapman-Enskog viscosity for mixtures,  $\eta_{CE,mix}$ .

$$\eta_{CE,mix} = \frac{\sum_{i=1}^N x_i \eta_{CE,i}}{\sum_{j=1}^N x_j \phi_{ij}} \quad (4.5)$$

where

$$\phi_{ij} = \frac{\left(1 + (\eta_{CE,i}/\eta_{CE,j})^{1/2} (M_j/M_i)^{1/4}\right)^2}{\left(8(1 + M_i/M_j)\right)^{1/2}}. \quad (4.6)$$

Since this study is concerned with LJ mixtures only, the reference values can also be given in

reduced units (cf. table 4.1). The CE viscosity in eq. (4.4) is then reduced to

$$\eta_{CE,i}^* = \frac{5}{16} \frac{\sqrt{T^*/(\pi)}}{\Omega_i^{(2,2)*}}. \quad (4.7)$$

For the case  $M_i = M_j$ , eq. (4.6) simplifies to

$$\phi_{ij} = \frac{\left(1 + (\eta_{CE,i}/\eta_{CE,j})^{1/2}\right)^2}{4}. \quad (4.8)$$

A second prerequisite for the precise prediction of  $\eta$  via entropy scaling is an accurate estimate of the molar residual entropy  $s_{\text{res}}$ , which is defined as the molar entropy minus the molar ideal gas entropy, according to  $s_{\text{res}}(T, \rho) = s(T, \rho) - s_{\text{id. gas}}(T, \rho)$ . For the application to real substances, the calculation of  $s_{\text{res}}$  is relatively straightforward, and can be done based on a suitable equations of state (EoS)<sup>23,44,48,49</sup>, such as PC-SAFT<sup>54</sup>. In principle, such an approach is also applicable to mixtures, provided that the used EoS is applicable to mixtures; however, the use of an EoS leads to approximate, i.e. non-exact mixture properties. All properties are therefore calculated based on molecular simulations.

### 4.2.3 Calculation of Residual Entropy

I calculate the residual entropy of the investigated model mixtures using Grand Canonical Monte Carlo (GCMC) simulations applying transition matrix sampling, following the scheme proposed by Errington and Shen<sup>55</sup>. The GCMC simulations are performed at constant temperature  $T$ , volume  $V$  and  $\mu_i$  for each component  $i$ , whereas the particle numbers  $N_1$  and  $N_2$  are fluctuating. Temperature and volume are defined a priori for every simulation, estimates for excess chemical potentials  $\mu_1$  and  $\mu_2$  suiting the simulated temperature are taken from the PC-SAFT EoS<sup>54</sup>. Estimates of  $\boldsymbol{\mu} = (\mu_1, \mu_2)$  are thereby sufficient, as earlier studies have shown<sup>56</sup>. The thermodynamic quantities, including residual entropy  $s_{\text{res}}$ , of a mixture at a certain pressure  $p^*$  and composition  $\mathbf{x}$  are calculated in post processing using histogram reweighting<sup>57</sup>. For a more detailed description of the implementation I refer to the work of Hemmen *et al.*<sup>56</sup>.

The simulations are carried out in a triangle in the  $(N_1, N_2)$  plane with  $(N_1 = N_2 = 0)$ ,  $N_1^{\text{max}}$  and  $N_2^{\text{max}}$  as vertices. As this study focuses on state points in a homogeneous liquid, the maximum numbers of  $N_1$  and  $N_2$  are chosen in a way that the region of liquid densities is included in the sampling area. The triangle also includes the gas phase and the two phase region of the mixtures. The vapor liquid equilibrium (VLE) of each mixture can be calculated in post



processing using the simulation results. In order to allow parallel computation, the sampled  $(N_1, N_2)$  space is divided in squared windows of a fixed size  $\Delta N_1 \times \Delta N_2$  with an overlap of one particle to each neighboring window<sup>58</sup>.

In each window, every  $\mathbf{N} = (N_1, N_2)$  point is sampled for a fixed number of trial moves. The following types of trial moves are thereby considered: particle displacements, particle insertion, particle deletion and particle identity swaps. All trial moves that might change the particle numbers are never accepted until the desired number of samples is reached in this particular  $(N_1, N_2)$  point. Only then will the simulation proceed on to an adjacent point in the  $\mathbf{N}$  plane (e.g.  $(N_1 + 1, N_2)$ ). In this way, the simulation meanders over the  $\mathbf{N}$  plane and it is ensured that each point is sampled equally. All transition probabilities, including those of the trivially rejected moves, are recorded in a transition matrix (TM). From the TM the probability distribution of a particle composition  $P(N_1, N_2; \mu_1, \mu_2, V, T)$  in this window for a given  $(\mu, V, T)$  can be calculated<sup>55,59</sup>. Combining the results of all windows a histogram of the normalized probability density  $P(N_1, N_2; \mu, V, T)$  is obtained over the complete predefined  $(N_1, N_2)$  triangle. Besides the probability  $P$ , the histogram of the average energy of a state  $\langle U \rangle(N_1, N_2; \mu, V, T)$  is sampled throughout the simulation.

The resulting histogram is then post processed using histogram reweighting<sup>57</sup>, which is used to determine VLE data from GCMC simulations. In this procedure,  $P(N_1, N_2; \mu, V, T)$ , simulated at  $T$  and the before estimated  $\mu$  is reweighted to a probability distribution at a desired  $\mu^{\text{target}}$  not too far away from the sampled  $\mu$ , via the relation

$$P(\mathbf{N}, T, \mu^{\text{target}}) = P(T, \mathbf{N}, \mu) \exp[\beta \mathbf{N} \cdot (\mu^{\text{target}} - \mu)] \quad (4.9)$$

with  $\beta = \frac{1}{k_B T}$ . An iterative approach is used. The  $\mathbf{N}$ -domain needs to be divided into a vapor and a liquid domain. The equilibrium requires equal pressure  $p$  for both phases. This condition is fulfilled if the sum under the peak in the vapor domain of the distribution  $I^{\text{vap}} = \sum_{N_1, N_2}^{\text{vap}} P(\mathbf{N}, T, \mu^{\text{coex}})$  is equal to the corresponding sum under the liquid peak  $I^{\text{liq}}$ . The two chemical potentials  $\mu_1^{\text{coex}}$  and  $\mu_2^{\text{coex}}$  are determined iteratively by two requirements,  $I^{\text{vap}} = I^{\text{liq}}$  and the composition of species 1 is equal to a target value,  $x_1 = x_1^{\text{target}}$ .

For determining the thermodynamic properties along an isobar for various compositions,  $P$  is iteratively reweighted to a  $\mu^{\text{isobar}}$  belonging to a state point at the desired pressure  $p^{\text{target}}$  and the required composition  $\mathbf{x}^{\text{target}}$ .  $p^{\text{target}}$  is selected in such a way that a state in the liquid-like phase is established. In each iteration step pressure  $p$  is calculated via the ideal gas limit as<sup>60</sup>

$$p = -\frac{1}{\beta V} \ln(P(N_1 = 0, N_2 = 0; \mu, V, T)) \quad (4.10)$$

and the number of particles of species  $i$ , corresponding to the target composition is

$$\langle N_i \rangle = \sum_{N_1=0, N_2=0}^{N_{1,\max}, N_{2,\max}} N_i \cdot P(N_1, N_2; \boldsymbol{\mu}, V, T) \quad (4.11)$$

$\boldsymbol{\mu}^{\text{isobar}}$  is thereby iterated in a way that the target composition

$$x_i^{\text{target}} = \frac{\langle N_i^{\text{target}} \rangle}{(\langle N_1^{\text{target}} \rangle + \langle N_2^{\text{target}} \rangle)} \quad (4.12)$$

and the pressure condition  $p^{\text{target}}$  are fulfilled simultaneously. This procedure is repeated for several  $\mathbf{x}^{\text{target}}$  along an isobar. At each state point, it is now also possible to determine the average intermolecular energy of the system from the sampled energy histogram, as

$$\langle U_{\text{res}} \rangle = \sum_{N_1=0, N_2=0}^{N_{1,\max}, N_{2,\max}} U(N_1, N_2; \boldsymbol{\mu}, V, T) \cdot P(N_1, N_2; \boldsymbol{\mu}, V, T) \quad (4.13)$$

The internal energy  $U$  coming from a GCMC simulation does not include the kinetic contribution and is therefore a residual quantity. The specific residual entropy  $s^*$  can now be calculated starting from the residual Gibbs Free Energy

$$G_{\text{res}} = \langle U \rangle + p_{\text{res}} V - T S_{\text{res}} \quad (4.14)$$

The residual pressure has to be calculated by subtracting the ideal gas contribution. For pressure  $p$  as determined from eq. (4.10), one gets the residual pressure from

$$p_{\text{res}} V = \left( p V - \frac{\langle N \rangle}{\beta} \right) \quad (4.15)$$

The residual Gibbs energy can be expressed by the residual chemical potential of the components, as

$$G_{\text{res}} = \sum_{i=1}^{N_{\text{comp}}} N_i \mu_{\text{res},i} \quad (4.16)$$

A chemical potential of species  $i$  contains the de Broglie wavelength  $\Lambda_i$  with  $\beta \mu_i = \ln(\rho_i \Lambda_i^3) + \mu_{\text{res}}(T, \boldsymbol{\rho})$ . Without loss of generality,  $\Lambda_i$  can be set to  $\sigma_i$  in this chapter, so that

$$\beta \mu_{\text{res},i}(\mathbf{N}, V, T) = \beta \mu_i - \ln \left( \frac{\langle N_i \rangle}{V^*} \right) \quad (4.17)$$

Combining these equations finally gives the residual entropy.

$$S_{\text{res}}(\mathbf{N}, V, T) = \frac{1}{T} \langle U_{\text{res}} \rangle + (\beta p V - \langle N \rangle) - \sum_{i=1}^{N_{\text{comp}}} N_i (\beta \mu_{\text{res},i}) \quad (4.18)$$

The dimensionless and molar specific form  $s^*$  is used in the following for the entropy scaling of the viscosity results.

$$s^* = \frac{S_{\text{res}}}{\langle N \rangle k_{\text{b}}} = \beta \frac{\langle U_{\text{res}} \rangle}{\langle N \rangle} + \left( \frac{\beta p V}{\langle N \rangle} - 1 \right) - \sum_{i=1}^{N_{\text{comp}}} x_i (\beta \mu_{\text{res},i}). \quad (4.19)$$

Note that although non-reduced units are used in the equations of this chapter for the sake of convenience, the equations are also valid when using dimensionless quantities.

#### 4.2.4 Calculation of Viscosity

The viscosity of the mixtures is calculated in equilibrium Molecular Dynamics (MD) simulations using the MD package DL\_POLY\_4<sup>61</sup>. NVT simulations are performed at constant particle number  $N$ , Volume  $V^*$  and temperature  $T^*$ . The simulations were carried out at densities in the liquid region along several isotherms at one specific pressure  $p^*$  for each mixture and varying compositions  $\mathbf{x}$ . The density  $\rho^*$  corresponding to each simulated state point at  $(p^*, T^*, \mathbf{x})$  are taken from the results of the GCMC simulations at the respective isobaric line. MC and MD, provide the same pressures for a simulation at the same density. The initial configurations for the NVT MD simulations are generated each in a short MC NVT simulation. This MC configuration is used as starting point for multiple MD NVT simulations at each considered state point using a velocity Verlet integrator with a time step of  $\Delta t^* = 4.6374e - 04$ . Initial particle velocities are drawn from the Boltzmann distribution for the considered temperature, with a different random seed for each replicate simulation. Each MD replicate is equilibrated for  $10^6$  time steps. The shear components of the stress tensor are then sampled for another  $10^6$  time steps.  $T^*$  is held constant using a weakly coupled Berendsen thermostat, which has been shown to preserve realistic dynamics<sup>62</sup>. An overview of general simulation specifications of the MD simulations is given in table 4.3.

The Green-Kubo approach<sup>63,64</sup> is used to calculate viscosities. The method determines  $\eta^*$  by integrating the auto correlation function (ACF) of the stress tensor, according to

$$\eta = \frac{1}{V k_{\text{B}} T} \int_0^{\infty} R_{J_p J_p}(\tau) d\tau, \quad (4.20)$$

|                                     |                                      |
|-------------------------------------|--------------------------------------|
| integration method                  | Velocity Verlet                      |
| time-step $\Delta t^*$              | $4.6374 \cdot 10^{-4}$               |
| equilibration steps                 | $1 \cdot 10^6$                       |
| production steps                    | $1 \cdot 10^6$                       |
| number of molecules                 | 1000                                 |
| thermostat (coupling time $t_T^*$ ) | Berendsen ( $4.6374 \cdot 10^{-6}$ ) |
| cut-off length / $\sigma$           | 4.112                                |
| tail-corrections                    | applied on $U$ and $p$               |

**Table 4.3:** General Specifications of the MD Simulations.

with the ACF  $R_{J_p J_p}$  of the stress tensor  $J_p$ , according to

$$R_{J_p J_p}(\tau) = \lim_{\Theta \rightarrow \infty} \frac{1}{2\Theta} \int_{-\Theta}^{\Theta} J_p^{\alpha\beta}(t + \tau) J_p^{\alpha\beta}(t) dt \quad (4.21)$$

as a function of the lag time  $\tau$ .  $J_p^{\alpha\beta}$  (with  $\alpha, \beta = x, y, z$ ) are entries of the shear-stress tensor of the simulation box. The stress tensor is calculated using the particle velocities  $v_i$  and the virial  $\mathbf{r}_{ij}(\partial \phi(r_{ij})/\partial \mathbf{r}_{ij})$ .

$$J_p^{\alpha\beta} = \sum_{i=1}^N m_i v_i^\alpha v_i^\beta - \sum_{i=1}^N \sum_{j>i}^N r_{ij}^\alpha \frac{\partial \phi(r_{ij})}{\partial r_{ij}^\beta} \quad (4.22)$$

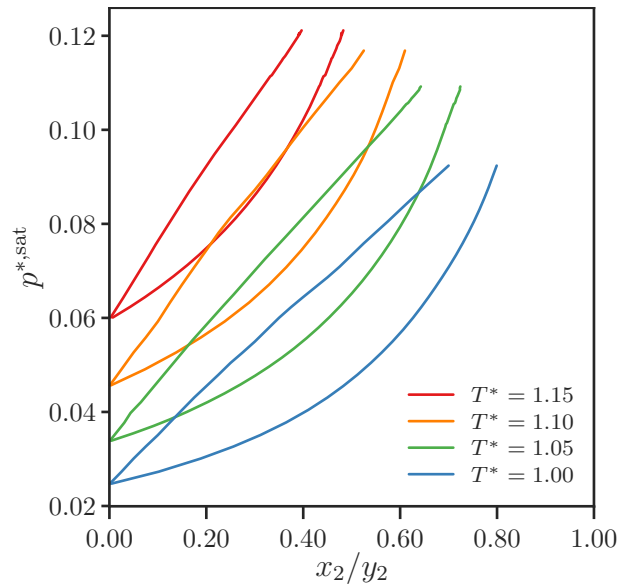
In order to improve the statistic of each simulation,  $J_p^{\alpha\beta}$  is calculated as the average of the six independent shear components  $1/2(J_p^{xy} + J_p^{yx})$ ,  $1/2(J_p^{yz} + J_p^{zy})$ ,  $1/2(J_p^{xz} + J_p^{zx})$ ,  $1/2(J_p^{xx} - J_p^{yy})$ ,  $1/2(J_p^{yy} - J_p^{zz})$  and  $1/2(J_p^{xx} - J_p^{zz})$ . The last three terms are determined from  $45^\circ$  rotations of the stress tensor about all axes<sup>65,66</sup>.

As the the Green-Kubo method suffers from a high level of noise, statistics are improved by generating 100 replicate trajectories at each state point. The autocorrelation function (ACF) of each time series is computed in post processing. According to the Wiener-Khinchin Theorem<sup>67,68</sup>, the correlation integrals can be calculated very efficiently in Fourier-space.<sup>69,70</sup> Using the *Fast Fourier Transform* algorithm, the computational complexity depending on the number of samples  $N_t$  is reduced from  $\propto N_t^2$  to  $\propto N_t \log_2(N_t)$  compared to the direct calculation<sup>71</sup>. This procedure allows us to evaluate ACFs very efficiently and without sacrificing any data points. Therefore, the autocorrelations of every sampled time step with all other time steps of the simulation is considered. The Green-Kubo integrals are evaluated with the Time Decomposition Method of Zhang *et al.*<sup>72</sup>, a fitting method where the statistical uncertainty is part of the evaluation. The stated statistical uncertainties of the results are

95 % confidence intervals, obtained from a bootstrapping technique.<sup>73–75</sup> For this purpose 500 bootstrap samples are evaluated, generated by randomly drawing (with replacement) 50 times from the complete set of replicate simulations. For a more detailed description of the analysis of the viscosity data I refer to our recent work<sup>75,76</sup>.

### 4.3 Results

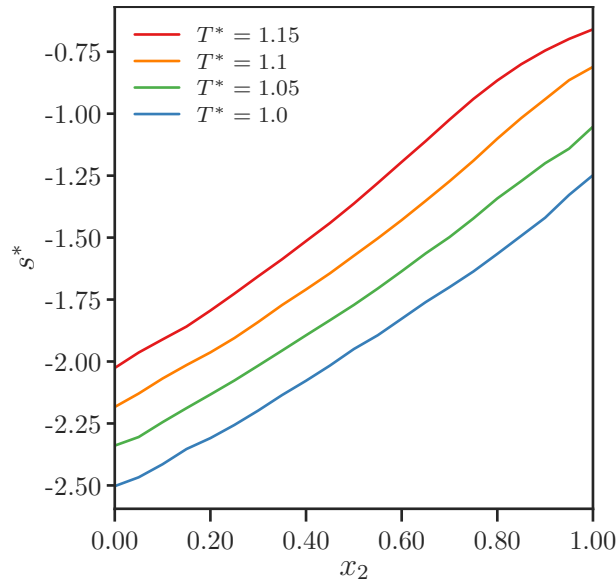
The central result of the study of the model mixtures is the analysis of entropy scaling of mixture-viscosities. In order to lead to this result, this section first separately considers the calculated properties needed for entropy scaling – entropy  $s^*$  and viscosity  $\eta^*$ . The results are presented successively for the different mixtures. For a more comprehensive appraisal of the individual mixtures, I also show phase equilibrium properties of the mixtures. The vapour-liquid equilibria (VLE) can be computed from the data generated for the calculation of entropy. The VLEs are determined via histogram reweighting, as described in section 4.2.3, at the temperatures at which the GCMC simulations for the entropy calculation were performed. Therefore no additional simulations are required to determine the VLEs.



**Figure 4.1:** Vapor-liquid equilibrium of the energy asymmetric mixture I ( $\sigma_{22} = \sigma_{11}$ ,  $\varepsilon_{22} = 0.7 \cdot \varepsilon_{11}$ , and  $\varepsilon_{12} = \sqrt{\varepsilon_{11}\varepsilon_{22}}$ ) at four temperatures. For reduced units refer to table 4.1.

Figure 4.1 shows the VLE of the energy asymmetric mixture I. As the second component of the mixture is supercritical at all considered temperatures, a closed-loop phase behavior is observed for all shown VLEs. Component 2 is the low boiling component, due to its less attractive interactions ( $\varepsilon_{22} = 0.7 \cdot \varepsilon_{11}$ ). Thus, a liquid phase, rich in component 1, and a gas phase, enriched with component 2, are obtained in the phase equilibrium. The phase

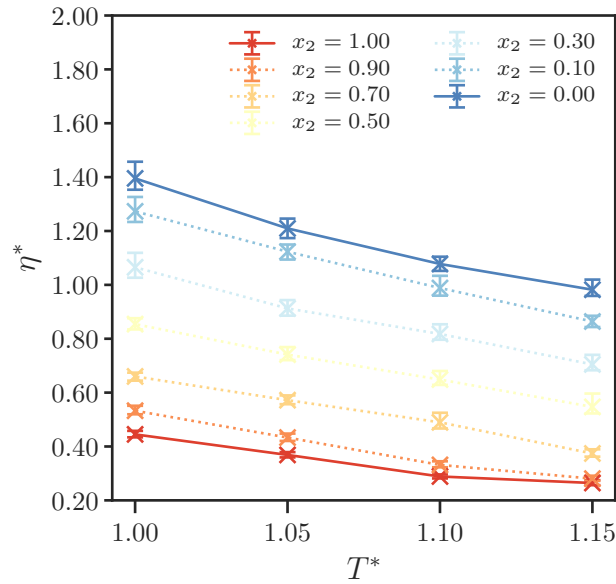
envelopes in fig. 4.1 do not close towards the critical point, because in regions near the critical pressure, phase equilibria can no longer be determined using simple histogram rescaling. The probability surfaces of the two phases increasingly overlap towards the critical point.<sup>77</sup> Although the critical point can be estimated using other methods<sup>56,78</sup>, the required finite-size analysis was not performed in this work, as it is not a focus of this study. Since the two-phase region should be avoided for the investigation of entropy scaling of the mixture, the VLE in fig. 4.1 was also used to determine a pressure at which a homogeneous phase is ensured for the determination of  $s^*$  and  $\eta^*$ . In order to ensure a homogeneous phase for a wide range of  $T^*$  and  $\mathbf{x}$ , the pressure for the further studies was chosen supercritical for both components, but moderate with  $p^* = 0.2$ . This pressure results in densities in the liquid regime at the regarded temperatures.



**Figure 4.2:** GCMC results for the energy asymmetric mixture I ( $\sigma_{22} = \sigma_{11}$ ,  $\epsilon_{22} = 0.7 \cdot \epsilon_{11}$ , and  $\epsilon_{12} = \sqrt{\epsilon_{11}\epsilon_{22}}$ ): Reduced entropy  $s^*$  as a function of composition  $x_2$  at  $p^* = 0.20$  and four temperatures.

Figure 4.2 shows the composition dependence of  $s^*$  of mixture I at different  $T^*$ . For each composition, entropy increases with rising temperature, as suggested by the partial derivative  $(\partial s^{\text{res}} / \partial T)_{p,\mathbf{x}} = c_p^{\text{res}} / T$ . With increasing composition of component 2 (with  $\epsilon_{22} = 0.7 \cdot \epsilon_{11}$ ), the entropy  $s^*$  of the mixture tends to higher values. Component 1, the high boiler, has significantly more negative entropies ( $s^* \sim -2$  to  $-2.5$ ), which are typical values of liquids. The values of the pure component 2 ( $s^* \sim -0.7$  to  $-1.3$ ) are in a range close to the critical point of pure Lennard-Jones fluids.

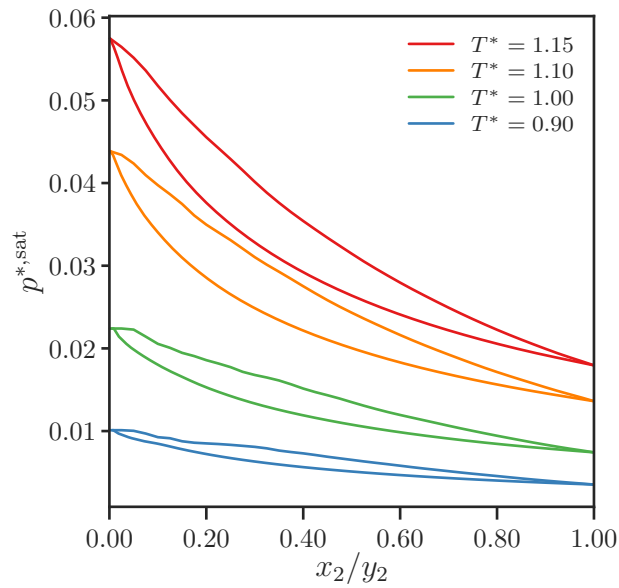
Figure 4.3 depicts the temperature dependence of the viscosity of mixture I for several lines of constant compositions (isopleths). The viscosity in this figure is the dimensionless quantity  $\eta^* = \eta \sigma_{11}^2 / \sqrt{\epsilon_{11} M_1}$ , not to be mistaken with the reduced viscosity  $\eta^\#$  in entropy scaling. While



**Figure 4.3:** MD simulation results for the viscosities  $\eta^*$  of mixtures I ( $\sigma_{22} = \sigma_{11}$ ,  $\varepsilon_{22} = 0.7 \cdot \varepsilon_{11}$ , and  $\varepsilon_{12} = \sqrt{\varepsilon_{11}\varepsilon_{22}}$ ) at a pressure of  $p^* = 0.2$  and several temperatures. Different colors represent different compositions, solid lines representing the pure fluids. Lines are a guide to the eyes.

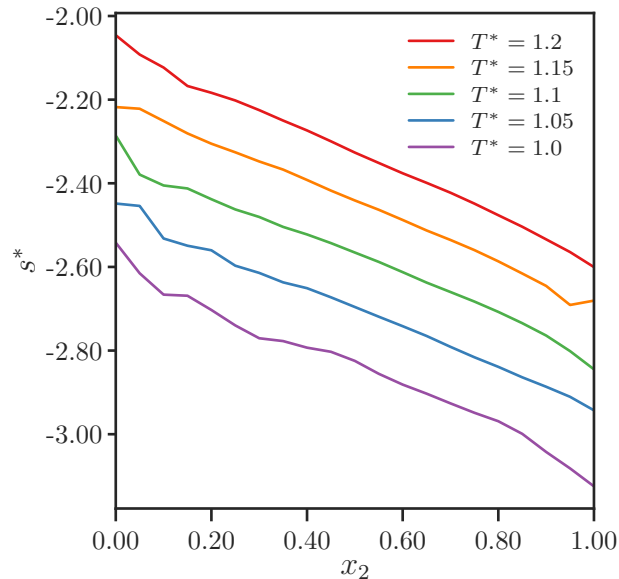
the results of entropy are derived from histogram reweighting and can therefore be determined for any composition, the viscosity points are derived from individual MD simulations at discrete state points. The conditions ( $T^*$ ,  $v^*$  and  $\mathbf{x}$ ) at which the MD simulations are conducted were chosen in such a way that GCMC results are available for each viscosity. The isopleths of mixtures of varying composition lay between the lines of the pure components. The viscosities decrease for higher concentrations of the low boiling component 2 with  $\varepsilon_{22} = 0.7\varepsilon_{11}$ . This behavior seems plausible as the second component has a lower energy parameter  $\varepsilon$ , which defines the attractive interactions. With a higher concentration of this component, the interaction between the particles of the mixture is less attractive and therefore the viscosity decreases.

Figures 4.4–4.6 analyze the results of the size-asymmetric mixture II. This kind of mixture is known to show a thin VLE envelope in contrast to mixture I<sup>60</sup>. The small difference in vapor pressure between the two components impedes GCMC simulations of this mixture and necessitates a high number of samples. This has also been reported in the literature for Gibbs-ensemble calculations of similar mixtures dominated by size-asymmetric effects.<sup>79</sup> As can be seen in the VLEs at several temperatures shown in fig. 4.4, both pure components remain subcritical for temperatures below  $T^* = 1.2$ . At a concentration of  $x_2 = 0$  the same pure LJ fluid is present as in mixture I in fig. 4.1. So at the same temperature the same vapor pressures should be obtained for the left boundary of both mixtures. However, the vapor



**Figure 4.4:** Vapor-liquid equilibrium of the size asymmetric mixture II ( $\sigma_{22} = 1.5 \cdot \sigma_{11}$ ,  $\epsilon_{22} = \epsilon_{11}$ , and  $\epsilon_{12} = \sqrt{\epsilon_{11}\epsilon_{22}}$ ) at four temperatures.

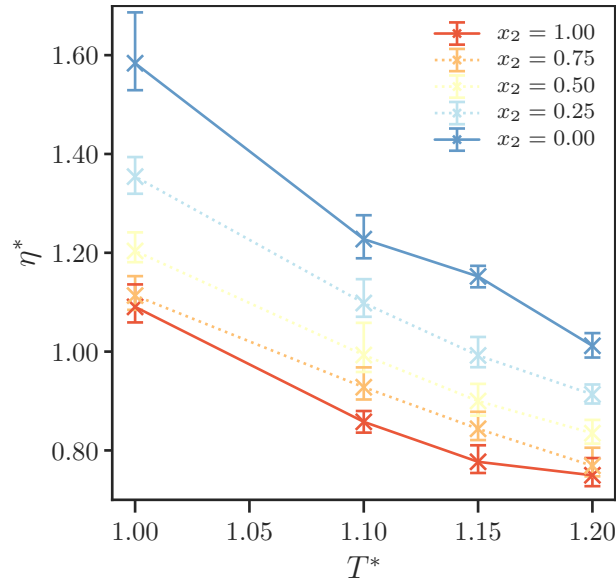
pressures of the pure LJ fluid in mixture II are predicted slightly too low, which indicates the difficulties in sampling the size-asymmetric mixture. The low boiler in this mixture is component 1. The larger particles of component 2 tend rather to remain in the liquid phase. Based on fig. 4.4, a pressure is selected for the entropy calculation which is again far from the two-phase region, in the case of mixture II it is set to  $p^* = 0.4$ .



**Figure 4.5:** GCMC results for the size asymmetric mixture II ( $\sigma_{22} = 1.5 \cdot \sigma_{11}$ ,  $\epsilon_{22} = \epsilon_{11}$ , and  $\epsilon_{12} = \sqrt{\epsilon_{11}\epsilon_{22}}$ ): Reduced entropy  $s^*$  as a function of concentration  $x_2$  at  $p^* = 0.40$  and five temperatures. For reduced units refer to table 4.1.



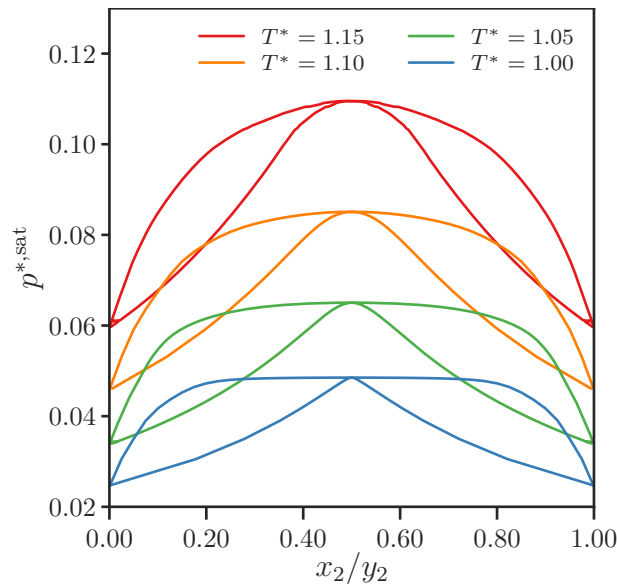
Figure 4.5 exhibits a similar concentration dependence for entropy as for mixture I, if one considers that component 1 is the low-boiling component of the mixture in this case. If the fraction of high boiler increases with  $x_2$ , the entropy  $s^*$  of the mixtures becomes more negative. Also for entropy, near the pure substances and for low temperatures, deviations from the nearly linear course are observed, which was already seen for mixture I.



**Figure 4.6:** MD simulation results for the viscosities  $\eta^*$  of the size asymmetric mixtures II ( $\sigma_{22} = 1.5 \cdot \sigma_{11}$ ,  $\varepsilon_{22} = \varepsilon_{11}$ , and  $\varepsilon_{12} = \sqrt{\varepsilon_{11}\varepsilon_{22}}$ ) at a pressure of  $p^* = 0.4$  and several temperatures. Different colors represent different compositions, solid lines representing the pure fluids. Lines are a guide to the eyes.

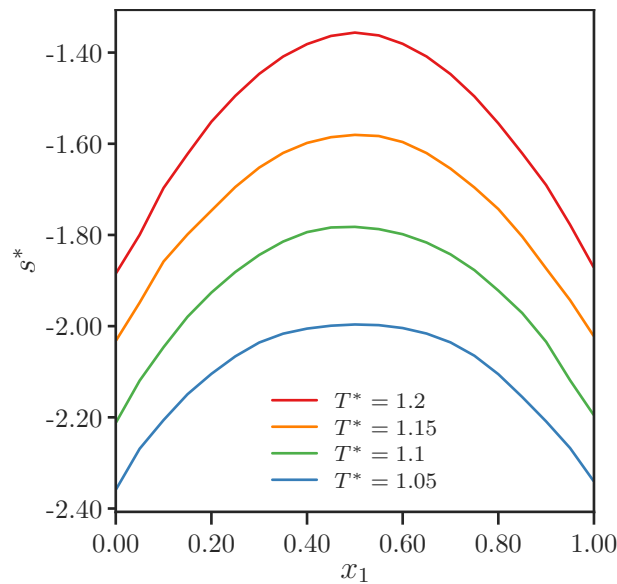
The viscosity in fig. 4.6 decreases for each temperature as the proportion of component 2 in the mixture increases. So I obviously find a clear dependence of  $\eta^*$  on the composition at the moderate pressure  $p^* = 0.4$ , unlike earlier studies of the viscosity of size asymmetric LJ-mixtures<sup>29</sup> at a high pressure of  $p^* = 4.18$ .

Mixture III is a mixture of two LJ fluids with identical  $\sigma$  and  $\varepsilon$  parameters, but with a cross-energy parameter  $\varepsilon_{12} = 0.75\sqrt{\varepsilon_{11}\varepsilon_{22}}$ . This mixture is known for a symmetric phase behavior with an VLE azeotrope at equimolar composition and a liquid-liquid equilibrium<sup>60</sup> at temperatures  $T^* < 1.0$ . Figure 4.7 shows the calculated vapor liquid equilibrium of the mixture at various temperatures. According to the character of the mixture a symmetrical behavior of the VLEs can be seen over the composition with the azeotropic point in the centre at  $x_2 = 0.5$ . The left ( $x_2 = 0$ ), as well as the right boundary ( $x_2 = 1$ ) of mixture III corresponds to the LJ fluid. The vapour pressures for the pure species boundaries ( $x_1 = 1$  and  $x_2 = 1$ ) are in good agreement with those of mixture I at  $x_2 = 0$  in fig. 4.1. As before, using the VLE from fig. 4.7, a pressure is selected, that ensures conditions well away from the two-phase region



**Figure 4.7:** Vapor-liquid equilibrium of mixture III with non-Berthelot cross-energy parameters ( $\sigma_{22} = \sigma_{11}$ ,  $\epsilon_{22} = \epsilon_{11}$ , and  $\epsilon_{12} = 0.75 \cdot \sqrt{\epsilon_{11}\epsilon_{22}}$ ) at four temperatures.

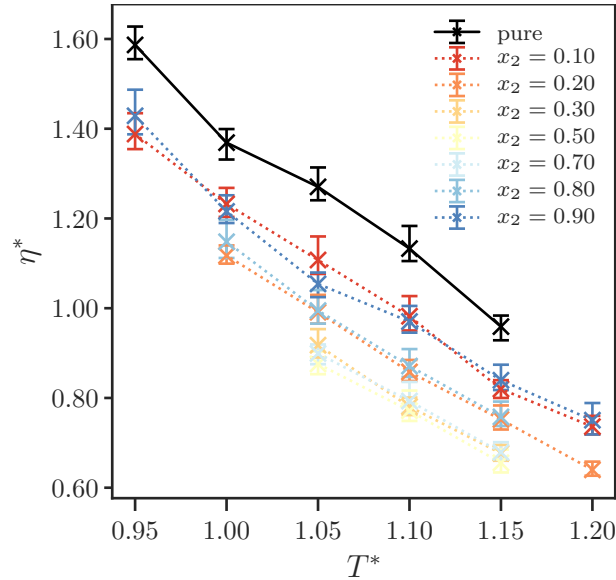
at which the analysis is carried out. For mixture III the pressure is set to  $p^* = 0.15$ . The value was chosen moderately enough to avoid high densities at which the pure components tend to form solids at lower temperatures.



**Figure 4.8:** GCMC results for mixture III with non-Berthelot cross-energy parameters ( $\sigma_{22} = \sigma_{11}$ ,  $\epsilon_{22} = \epsilon_{11}$ , and  $\epsilon_{12} = 0.75 \cdot \sqrt{\epsilon_{11}\epsilon_{22}}$ ): Reduced entropy  $s^*$  as a function of concentration  $x_2$  at  $p^* = 0.20$  and four temperatures.

In fig. 4.8 one can see the entropy of mixture III, which is also symmetric with the composition. As can be seen in fig. 4.8, the highest value for  $s^*$  at each temperature is reached at equimolar

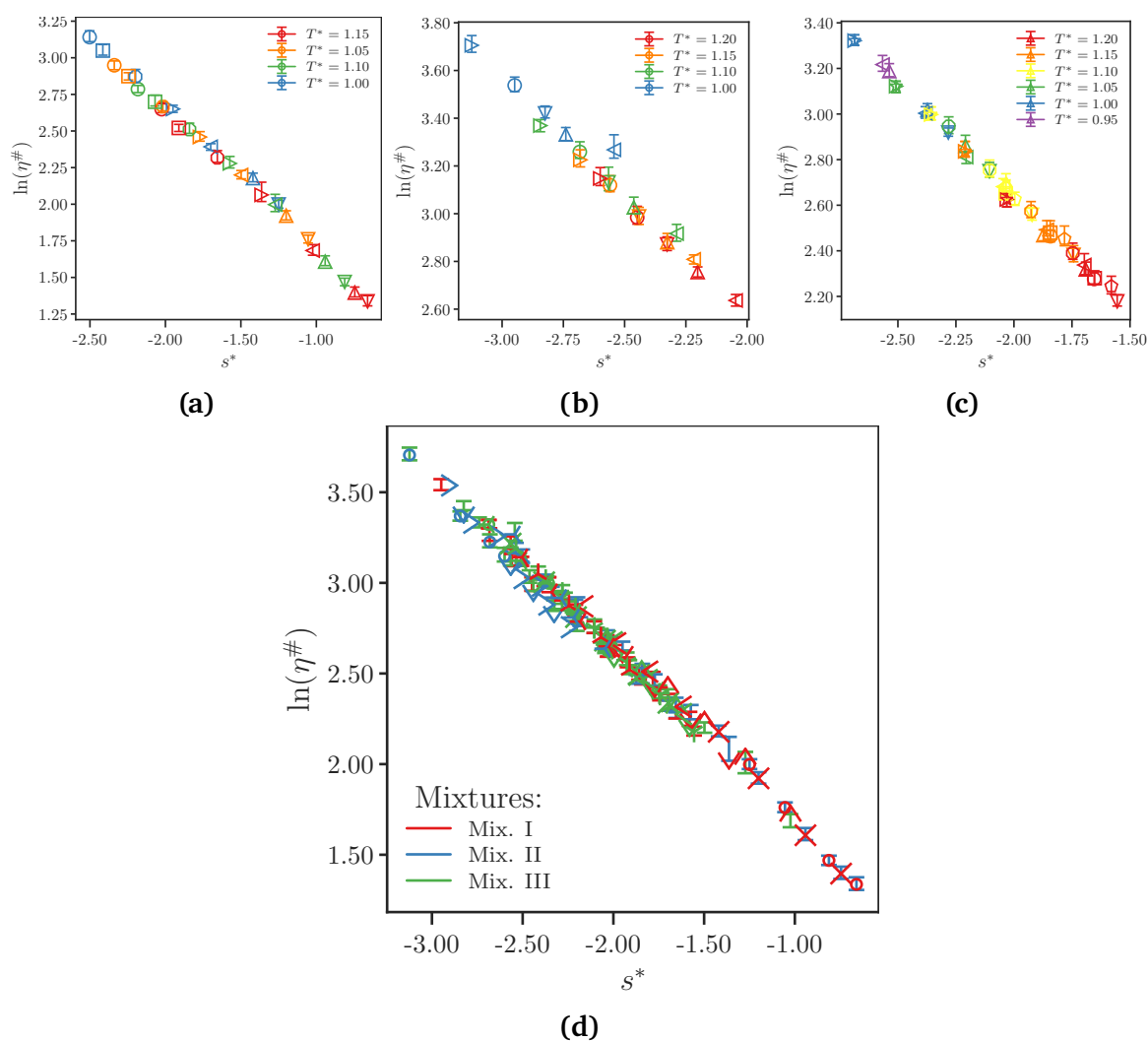
composition. This reflects the behavior of the other model mixtures being considered. For the “low boiler” – here the azeotropic mixture at  $x_2 = 0.5$  – the entropy is less negative, towards the high boilers – here the two pure substances – the entropy decreases.



**Figure 4.9:** MD simulation results for the viscosities  $\eta^*$  of mixtures III ( $\sigma_{22} = \sigma_{11}$ ,  $\varepsilon_{22} = \varepsilon_{11}$ , and  $\varepsilon_{12} = 0.75 \cdot \sqrt{\varepsilon_{11}\varepsilon_{22}}$ ) at a pressure of  $p^* = 0.15$  and several temperatures. Different colors represent different compositions, solid lines representing the pure fluids. Lines are a guide to the eyes.

The fact that the low boiler in mixture III is at equimolar composition leads to an interesting symmetric behavior also in viscosity, which can be seen in fig. 4.9 and was already noticed in our previous work.<sup>44</sup> The viscosity decreases with increasing amount of component 2 until it reaches its lowest value at the azeotropic composition. With further addition of component 2,  $\eta^*$  rises again. In the depiction of the isopleths versus temperature in fig. 4.9, this symmetry can be seen in the superimposed isopleths of complementary composition. Thus, for example, the viscosities at  $x_2 = 0.1$  and  $x_2 = 0.9$  ( $x_2 = 0.2$  and  $x_2 = 0.8$ , and so on) are the same within the error bars for each temperature. The isopleths of the azeotropic mixture and those of the pure LJ fluids limit the viscosity at the bottom and top, respectively.

Figure 4.10 summarizes and visualizes the central result of the study on the model mixtures. The diagram combines the results presented in the figures above and shows the viscosity of all three mixtures in an entropy scaled analysis. On the y-axis of each figure, the logarithm of the dimensionless viscosity  $\eta^\# = \eta^*/\eta_{CE,mix}$  is shown, using the Chapman-Enskog viscosity of the mixture (eq. (4.5)) as reference. On the x-axis is the related  $s^*$  value of each state point, respectively. Figures 4.10a–4.10c show the results of the individual mixtures, (a) gives the results of mixture I, (b) the results of mixture II, and (c) the results of of mixture III. The different colors in each figure indicate different temperatures. At each temperature several



**Figure 4.10:** Dimensionless mixture viscosities  $\ln(\eta^\#)$  as a function of the residual entropy  $s^*$  of all non-associating mixtures I-III. Simulated temperatures and pressures of each mixture refer to figs. 4.3, 4.6 and 4.9. In (a), (b), and (c) different colors represent different temperatures; different symbols represent different compositions. (a) shows the results of mixture I, (b) the results of mixture II, and (c) the results of of mixture III. (d) shows the results of all three mixtures in one diagram. Here, different colors represent the three different types of mixtures. Different symbols show different compositions.

compositions are shown, represented by different symbols.

All mixtures follow the monovariate (almost linear) relationship of  $\eta^\#$  and  $s^*$ . For mixture II, which has proven to be difficult to sample, an outlier from this linear course can be seen in (b). The point (at  $s^* \sim -2.5, T^* = 1.0$  and  $x_2 = 1$ ) is one at which both the GCMC simulations of entropy and the MD simulations of viscosity showed results deviating from the overall trend and I consider this point as an “outlier”. It is worth noting that the results in (a) - (c) extend over various  $s^*$  ranges that only partially overlap. All results combined in one figure

cover a very wide range of liquid entropies.

Figure 4.10d contains all datapoints calculated in this study. It contains state points at various compositions and temperatures for three different mixtures. The remarkable result is that the dimensionless viscosity of all data points collapse on one mastercurve. A universal behavior can be seen for *all* LJ mixtures, not only for each mixture individually. The mastercurve can be accurately represented by a linear equation. The parameters defining the linear mastercurve can conveniently be obtained by adjustment to the viscosity results of pure LJ fluids. As a conclusion of these findings, it can be stated, that the viscosity of any mixture of LJ fluid equals the viscosity of a pseudo pure fluid at the same residual entropy as the mixture. The pseudo pure components can be described by general parameters adjusted to the pure LJ fluid. The only quantity needed to predict the mixture viscosity of any combination of LJ fluids differing in  $\sigma_{ii}$ ,  $\epsilon_{ii}$  or  $\epsilon_{ij}$  is the residual entropy  $s^*$  of the mixture.

#### 4.4 Conclusion

In this study I investigated the entropy scaling behavior of LJ model mixtures consistently using molecular simulations. By considering three model mixtures at various temperatures and compositions, the influence of individual LJ parameters on VLE, entropy  $s^*$  and viscosity ( $\eta^\#$ ) has been assessed. Using a Chapman-Enskog equation for defining a dimensionless viscosity  $\eta^\#$  as the only model equation, I related  $\eta^\#$  with the residual entropy  $s^*$  and found a general, nearly linear master curve that is valid for all state points of all mixtures. In the entropy scaling approach all LJ mixtures can thus be considered as a pseudo pure LJ substance.

This result confirms what has already been postulated by isomorph theory, declaring LJ mixtures as *Roskilde fluid* and thus suitable for entropy scaling. In the present study, scalability was shown even for states far outside the range in which the LJ fluid behaves like a *Roskilde fluid*.<sup>33</sup> The results also show that  $\ln(\eta^\#)$  is approximately linear with  $s^*$  for all LJ mixtures using the Chapman-Enskog viscosity as reference for making  $\eta^\#$  dimensionless. The CE reference is therefore very suited as a reference for viscosity of LJ fluids, contrary to what was claimed in a previous publication.<sup>33</sup>

Furthermore, GCMC simulations have been validated as a suitable method for determining the entropy of mixtures in the context of entropy scaling. In a previous work<sup>44</sup> it has been found that the combination of the PC-SAFT equation of state for the determination of  $s^*$  and the CE viscosity as a reference leads to well-defined courses in entropy-scaled depiction, which allows the adjustment of simple correlation functions for real mixtures. The present work

shows for LJ-mixtures that this also applies when residual entropy is determined consistently from molecular simulations.

The results also open exciting new applications for entropy scaling. If one knows the entropy scaling parameters (see equation for viscosity in table 3.3 in chapter 3) related to a force field (in our case this would correspond to the straight line equation for LJ fluids), the transport properties can be estimated from GCMC simulations and entropy scaling. Thus, efficient methods like histogram reweighting can be used to determine dynamic properties, which otherwise requires numerous individual MD simulations. This allows the dynamic properties to be taken into account in force field optimization, for example. A prerequisite for such a procedure is that entropy scaling parameters remain almost constant for mild changes in the force field parameters. First studies in this regard have shown that this requirement is fulfilled for simple hydrocarbons<sup>80</sup>.

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## 5 Conclusion

In the present work, entropy scaling and molecular simulations, two methods of predicting transport coefficients of fluids, are jointly employed in several ways, to achieve synergistic effects. Molecular simulations are used, on the one hand, to gain new findings and deeper insights concerning the entropy scaling method. On the other hand, approaches from entropy scaling are used to better define and assess molecular simulations for transport properties. This work establishes a framework to both robustly and efficiently determine transport coefficients of technically relevant fluids over a wide range of pressures and temperatures, and to perform larger-scale simulation studies to be used in force field development.

The first part of this thesis investigates whether a force field with rigid bond lengths can be applied with harmonic bond length potentials and how static and transport properties are thereby affected. The substances studied in this study are described using the *TAMie* force field, which was originally developed with fixed bond lengths, and to which harmonic bonds are introduced in this study. The new “flexible” version of *TAMie* manages to reproduce the results for liquid phase pressure, shear viscosity, thermal conductivity, and self-diffusion coefficients of the original force field within the 95 % confidence interval, provided that the binding energy constant of the harmonic bond  $k_l$  is appropriately chosen. In the study, a wide range of values for  $k_l$  were examined at different time step sizes of the integration scheme to evaluate whether the high-frequency oscillation of the bond can still be resolved correctly. It has been shown that with a Velocity Verlet integrator and a common time step of 2 fs, suitable values for  $k_l$  are in the range of fully flexible united atom force fields reported in the literature. Bonds with excessively high bond constants can no longer be resolved, which results in deviations. As a practical result of this study, it was shown that “flexible” versions can be introduced for force fields that were originally parametrized with fixed bonds (including *TAMie*), making it more straightforward to compute transport properties in molecular simulations.

This insight is applied in the second part of the thesis, where the predictive power of the (“flexible”) *TAMie* force field for transport properties, namely shear viscosity, self-diffusion coefficient and thermal conductivity is assessed in a broad molecular simulation study. *TAMie* was initially developed for static thermodynamic properties and phase equilibria. This work proposes a workflow, where the state points at which simulations are conducted are selected based on their residual entropy in order to ensure a meaningful distribution of states for

conducting simulations covering the entire fluid region. The three transport coefficients and their statistical uncertainties are then determined simultaneously in a series of equilibrium molecular dynamics simulations at each state point, respectively. A simple correlation function is then fitted to the simulation results for each transport property, based on the entropy scaling method. By the careful choice of the simulated state points, four points corresponding to liquid states and one point to a gas state are sufficient to parameterize robust correlations. Using these correlations, transport coefficients in the entire fluid region can be predicted. The *TAMie* force field could then be assessed by comparing them with all the data available in literature – even for thermodynamic states far from those that were simulated. The model based on the *TAMie* force field achieved good agreement with literature data, with AAD values of 13 % for shear viscosity, 18 % for self diffusion, and 10 % for thermal conductivity.

Further, this thesis investigates the entropy scaling behavior of LJ model mixtures using molecular simulations. By considering three model mixtures at various temperatures and compositions, the influence of individual LJ parameters on VLE, entropy  $s^*$  and viscosity ( $\eta^\#$ ) has been assessed. All quantities are determined in molecular simulations; besides MD simulations for the determination of viscosity, GCMC simulations are conducted for the calculation of the mixture entropies and the VLE data. In the context of entropy scaling, GCMC simulations are a suitable choice for determining mixture entropies. Relating the dimensionless viscosity to the residual entropy from the GCMC simulations, a general, nearly linear master curve was found that is valid for all state points of all mixtures. The Chapman-Enskog viscosity is used as the only model equation in the study to define the dimensionless viscosity. The results of the study indicate that within the entropy scaling approach, all LJ mixtures can be considered as pseudo pure LJ substance.

The results obtained in this thesis open an exciting potential for future applications for the combination of molecular simulations and entropy scaling. The *TAMie* force field, is subject to further development and parameters for new substance classes have recently been published. The workflow proposed in this work can be used to efficiently evaluate the predictive power of *TAMie* for the transport coefficients of these compounds. The results obtained from the simulation of the model mixtures suggest that it can also be applied to real mixtures. Following the approach, it may even be possible to avoid the comparatively computationally intensive MD simulations for an initial estimate of transport properties: Knowing the entropy scaling parameters for a force field (as determined for *TAMie* in chapter 3) the transport properties can be estimated via entropy scaling and the results of GCMC simulations and histogram reweighting. My preliminary work on this subject indicated that the entropy scaling parameters remain nearly constant under slight changes in the force field parameters, suggesting the dynamical properties can be considered in force field optimization.



# Appendices



## A Supporting Information for: Force fields with fixed bond lengths and with flexible bond lengths: Comparing static and dynamic fluid properties

The content of this appendix is a literal quote of the supporting information:

M. Fischer, G. Bauer, J. Gross: Supporting Information for: Force Fields with Fixed Bond Lengths and with Flexible Bond lengths: Comparing Static and Dynamic Fluid Properties. *Journal of Chemical & Engineering Data*, **65**:1583-1593, 2020. doi:10.1021/acs.jced.9b01031

### A.1 Details of the calculation of the Thermal Conductivity

The correlated quantities needed for the calculation of the thermal conductivity are the three spatial entries of the heat flux  $\mathbf{J}_q = (J_q^x, J_q^y, J_q^z)$ .  $R_{J_q J_q}$  is the mean of the autocorrelation functions (ACF) of the three entries and is calculated via the dot product as

$$R_{J_q J_q}(\tau) = \lim_{\Theta \rightarrow \infty} \frac{1}{2\Theta} \int_{-\Theta}^{\Theta} \mathbf{J}_q(t + \tau) \cdot \mathbf{J}_q(t) dt \quad (\text{S1})$$

where  $\mathbf{J}_q$  is defined as

$$\mathbf{J}_q = \frac{1}{V} \left[ \sum_{i=1}^N \mathbf{v}_i e_i + \sum_{i=1}^N \sum_{j>i}^N \mathbf{r}_{ij} \left( \mathbf{v}_j \cdot \frac{\partial u_i}{\partial \mathbf{r}_j} \right) \right] \quad (\text{S2})$$

which contains the per-atom (potential and kinetic) energy  $e_i$  and the force  $\mathbf{f}_{ij} = -\frac{\partial u_i}{\partial \mathbf{r}_j}$  due to the per-atom potential energy  $u_i$ . Analogous to the viscosity, the running value of  $\lambda$  is calculated as integral of an ACF, according to

$$\lambda_{\text{run}}(\tau) = \frac{V}{3k_B T^2} \int_0^{\tau} R_{J_q J_q}(\tau') d\tau' \quad (\text{S3})$$

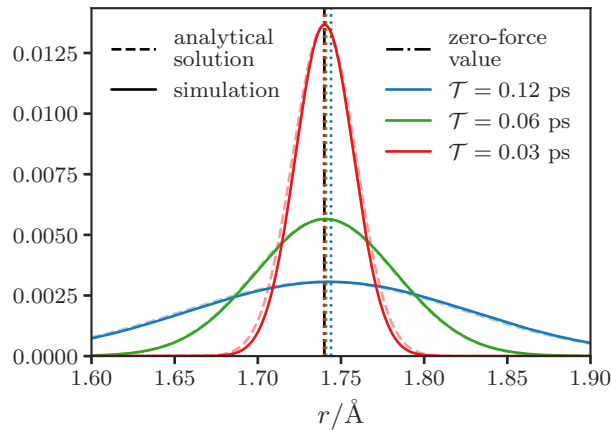
and the thermal conductivity is calculated as the limit  $\lambda = \lambda_{\text{run}}(\tau \rightarrow \infty)$ .

## A.2 Distribution of Bond Lengths

Figure A.1 shows the distribution of the bond lengths of propane, which can also be seen in fig. 2.2 of the main text. In fig. A.1 the simulation results from fig. 2.2 of the main text are compared with an analytical distribution, which calculates as

$$P(r) = \sqrt{\frac{k_l \beta}{\pi}} \exp(-\beta k_l (r - r_0)^2) \quad (\text{S4})$$

Each maximum value of  $P(r)$  is adjusted to the maximum value of the respective distribution of the simulations. The considered molecule, propane, comprises two harmonic bond length

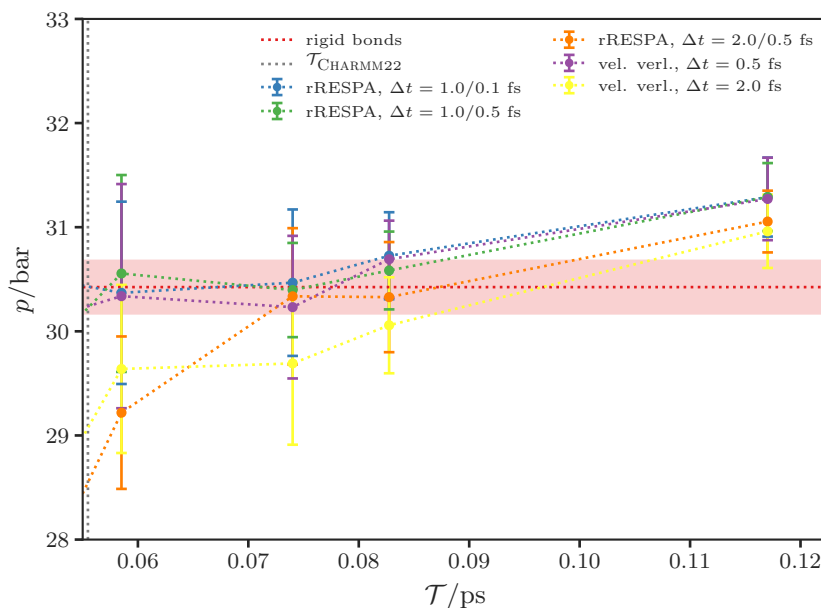


**Figure A.1:** Comparison of distributions of bond lengths from simulation (solid) and analytical solution (dashed). Results are shown for liquid propane at  $T = 360\text{K}$  and  $\rho = 361.27\text{kg m}^{-3}$  (rRESPA with inner time step  $\Delta t = 0.1\text{fs}$ ) for different values of bonding force constant  $k_l$ . Vertical colored lines represent the respective maximum value of the distribution. The black vertical line shows the reference length  $r_0$ .

potentials that are not independent from one another. The simple solution of a harmonic oscillator given in eq. (S4) can only be defended for uncorrelated harmonic oscillators of both bonds and no impact of surrounding molecules. These two assumptions are rather well justified, as we show in fig. A.1. For  $k_l = 50 / T = 0.12\text{ps}$  the total distribution shifts to higher distances compared to the analytical solution, which confirms the offset maximum value. For  $k_l = 1000 / T = 0.03\text{ps}$  the simulated distribution is slightly narrower than the analytical distribution.

### A.3 Diagram of pressure of propane for high values of period time $T$

In the main text we explain that for  $\mathcal{T} > 0.1$  ps, the  $k_l$  values are small and therefore the flexible bonds are so loose that results of the flexible bond model deviate from results of a rigid bond model. Figure A.2 illustrates this point by showing the section of high  $\mathcal{T}$  values in Figure 1 from the main text in a finer scale.



**Figure A.2:** Zoom into Figure 1 of the main text for high values of  $\mathcal{T}$ . Shown is the pressure  $p$  of propane at  $T = 360$  K over period of the bond vibrations  $\mathcal{T}$ . Colors represent different integration schemes with different time steps  $\Delta t$ . The red horizontal line shows result of rigid bonds. Vertical lines highlight  $\mathcal{T}$  values of the CHARMM (dotted) and the GROMOS force field (dashed).

### A.4 Simulation results in tables

This supporting information contains tables summarizing temperatures, pressures, viscosities, diffusion coefficients and thermal conductivities for the molecules with rigid bonds (table A.1) as well as for the models with flexible bonds, including all integration schemes and time steps (tables A.2–A.5). Additionally tables with results for one lower temperature for each substance (tables A.6–A.9) and the results of gas phase simulations of propane at 360 K, calculated with rRESPA (table A.10) are presented. The stated statistical uncertainties are the 95 % confidence interval, calculated as described in the main text.

**Table A.1:** Results of the rigid bonds, calculated with Velocity Verlet and SHAKE ( $\Delta t = 1.0$  fs)

| Substance       | $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$   | $\eta/\text{mPa s}$       | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{cm}^2 \text{s}^{-1}$ |
|-----------------|----------------------|--------------|------------------|---------------------------|---|---|
| ethane          | 315.43               | 299.990(51)  | 38.677(240)      | $3.75(9) \cdot 10^{-2}$   | $6.86(18) \cdot 10^{-2}$                | $3.38(4) \cdot 10^{-4}$                     |
| ethane          | 512.82               | 205.000(53)  | -0.659 48(68000) | $1.35(3) \cdot 10^{-1}$   | $1.58(5) \cdot 10^{-1}$                 | $7.49(5) \cdot 10^{-5}$                     |
| dimethylether   | 482.12               | 379.970(59)  | 31.868(290)      | $5.91(8) \cdot 10^{-2}$   | $8.11(22) \cdot 10^{-2}$                | $2.53(2) \cdot 10^{-4}$                     |
| dimethylether   | 649.04               | 299.980(64)  | 1.6168(6000)     | $1.34(2) \cdot 10^{-1}$   | $1.34(4) \cdot 10^{-1}$                 | $9.86(7) \cdot 10^{-5}$                     |
| propane         | 361.24               | 360.010(56)  | 30.425(250)      | $4.38(9) \cdot 10^{-2}$   | $6.04(14) \cdot 10^{-2}$                | $3.04(2) \cdot 10^{-4}$                     |
| propane         | 497.2                | 294.980(48)  | 7.4815(4900)     | $9.53(28) \cdot 10^{-2}$  | $9.35(32) \cdot 10^{-2}$                | $1.26(1) \cdot 10^{-4}$                     |
| ethane          | 315.43               | 299.990(51)  | 38.677(240)      | $3.75(9) \cdot 10^{-2}$   | $6.86(18) \cdot 10^{-2}$                | $3.38(4) \cdot 10^{-4}$                     |
| propane (vapor) | 18.481               | 295.000(24)  | 8.6049(74)       | $8.08(120) \cdot 10^{-3}$ | $1.68(19) \cdot 10^{-2}$                | $6.19(4) \cdot 10^{-3}$                     |
| propane (vapor) | 94.323               | 359.990(51)  | 35.201(37)       | $1.17(8) \cdot 10^{-2}$   | $2.75(19) \cdot 10^{-2}$                | $1.50(1) \cdot 10^{-3}$                     |

**Table A.2:** Results of ethane at 300 K, calculated with rRESPA

| $\Delta t/\text{fs}$ | $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $\mathcal{T}/\text{fs}$ | $\eta/\text{mPa s}$      | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{cm}^2 \text{s}^{-1}$ |
|----------------------|----------------------|--------------|----------------|-------------------------|--------------------------|---|---|
| 1.0 / 0.1            | 315.43               | 299.990(51)  | 63.518(4000)   | 0.008 422 1             | $3.90(4) \cdot 10^{-2}$  | $6.85(30) \cdot 10^{-2}$                | $3.50(3) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 315.43               | 300.000(64)  | 68.064(3700)   | 0.011 911               | $3.89(8) \cdot 10^{-2}$  | $6.90(14) \cdot 10^{-2}$                | $3.51(4) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 315.43               | 300.000(52)  | 71.039(3100)   | 0.018 832               | $3.90(9) \cdot 10^{-2}$  | $7.03(32) \cdot 10^{-2}$                | $3.48(3) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 315.43               | 300.000(47)  | 72.191(2900)   | 0.026 633               | $3.93(10) \cdot 10^{-2}$ | $7.19(25) \cdot 10^{-2}$                | $3.51(3) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 315.43               | 300.000(41)  | 54.226(3300)   | 0.037 665               | $3.79(10) \cdot 10^{-2}$ | $6.64(27) \cdot 10^{-2}$                | $3.46(2) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 315.43               | 300.000(58)  | 38.52(150)     | 0.053 266               | $3.70(7) \cdot 10^{-2}$  | $6.40(10) \cdot 10^{-2}$                | $3.39(3) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 315.43               | 299.990(75)  | 38.388(1200)   | 0.059 553               | $3.68(6) \cdot 10^{-2}$  | $6.73(37) \cdot 10^{-2}$                | $3.35(2) \cdot 10^{-4}$                     |

Continued on next page

**Continued:** Results of ethane at 300 K, calculated with rRESPA

| $\Delta t/\text{fs}$ | $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $\mathcal{T}/\text{fs}$ | $\eta/\text{mPa s}$      | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{cm}^2 \text{s}^{-1}$ |
|----------------------|----------------------|--------------|----------------|-------------------------|--------------------------|---|---|
| 1.0 / 0.1            | 315.43               | 299.980(62)  | 38.443(540)    | 0.075 33                | $3.59(14) \cdot 10^{-2}$ | $6.52(31) \cdot 10^{-2}$                | $3.37(2) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 315.43               | 300.000(51)  | 38.655(510)    | 0.084 221               | $3.68(4) \cdot 10^{-2}$  | $6.50(23) \cdot 10^{-2}$                | $3.39(3) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 315.43               | 299.990(53)  | 38.785(240)    | 0.119 11                | $3.67(7) \cdot 10^{-2}$  | $6.40(19) \cdot 10^{-2}$                | $3.38(3) \cdot 10^{-4}$                     |

**Table A.3:** Results of propane at 360 K, calculated with rRESPA

| $\Delta t/\text{fs}$ | $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $\mathcal{T}/\text{fs}$ | $\eta/\text{mPa s}$      | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{cm}^2 \text{s}^{-1}$ |
|----------------------|----------------------|--------------|----------------|-------------------------|--------------------------|---|---|
| 2.0 / 0.5            | 361.24               | 360.01(7)    | 1.2363(47000)  | 0.008 274 8             | $4.62(7) \cdot 10^{-2}$  | $6.23(25) \cdot 10^{-2}$                | $3.19(3) \cdot 10^{-4}$                     |
| 2.0 / 0.5            | 361.24               | 359.990(51)  | 38.625(3200)   | 0.011 702               | $4.70(8) \cdot 10^{-2}$  | $6.10(18) \cdot 10^{-2}$                | $3.16(2) \cdot 10^{-4}$                     |
| 2.0 / 0.5            | 361.24               | 359.980(55)  | 63.451(3400)   | 0.018 503               | $4.65(10) \cdot 10^{-2}$ | $6.36(16) \cdot 10^{-2}$                | $3.20(3) \cdot 10^{-4}$                     |
| 2.0 / 0.5            | 361.24               | 359.97(6)    | 62.065(2100)   | 0.026 167               | $4.54(19) \cdot 10^{-2}$ | $6.33(22) \cdot 10^{-2}$                | $3.16(1) \cdot 10^{-4}$                     |
| 2.0 / 0.5            | 361.24               | 360.030(55)  | 27.924(2600)   | 0.037 006               | $4.36(9) \cdot 10^{-2}$  | $5.73(18) \cdot 10^{-2}$                | $3.01(2) \cdot 10^{-4}$                     |
| 2.0 / 0.5            | 361.24               | 359.970(62)  | 27.853(3100)   | 0.052 334               | $4.34(6) \cdot 10^{-2}$  | $5.69(21) \cdot 10^{-2}$                | $3.01(2) \cdot 10^{-4}$                     |
| 2.0 / 0.5            | 361.24               | 360.000(62)  | 29.219(730)    | 0.058 511               | $4.33(11) \cdot 10^{-2}$ | $5.85(22) \cdot 10^{-2}$                | $3.04(2) \cdot 10^{-4}$                     |
| 2.0 / 0.5            | 361.24               | 360.000(52)  | 30.336(650)    | 0.074 012               | $4.38(8) \cdot 10^{-2}$  | $5.92(25) \cdot 10^{-2}$                | $3.03(2) \cdot 10^{-4}$                     |
| 2.0 / 0.5            | 361.24               | 359.980(55)  | 30.328(530)    | 0.082 748               | $4.27(10) \cdot 10^{-2}$ | $5.88(23) \cdot 10^{-2}$                | $3.05(2) \cdot 10^{-4}$                     |
| 2.0 / 0.5            | 361.24               | 359.990(78)  | 31.054(300)    | 0.117 02                | $4.37(5) \cdot 10^{-2}$  | $6.01(18) \cdot 10^{-2}$                | $3.05(2) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 361.24               | 359.990(41)  | 57.036(3300)   | 0.008 274 8             | $4.72(9) \cdot 10^{-2}$  | $6.35(19) \cdot 10^{-2}$                | $3.15(2) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 361.24               | 360.010(66)  | 67.816(3500)   | 0.011 702               | $4.71(12) \cdot 10^{-2}$ | $6.24(22) \cdot 10^{-2}$                | $3.11(2) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 361.24               | 360.000(53)  | 75.041(3100)   | 0.018 503               | $4.78(6) \cdot 10^{-2}$  | $6.40(20) \cdot 10^{-2}$                | $3.20(2) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 361.24               | 360.020(64)  | 61.681(3400)   | 0.026 167               | $4.59(3) \cdot 10^{-2}$  | $6.25(19) \cdot 10^{-2}$                | $3.13(2) \cdot 10^{-4}$                     |

Continued on next page

**Continued:** Results of propane at 360 K, calculated with rRESPA

| $\Delta t/\text{fs}$ | $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $\mathcal{T}/\text{fs}$ | $\eta/\text{mPa s}$      | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{cm}^2 \text{s}^{-1}$ |
|----------------------|----------------------|--------------|----------------|-------------------------|--------------------------|---|---|
| 1.0 / 0.1            | 361.24               | 359.990(65)  | 30.012(2800)   | 0.037006                | $4.43(10) \cdot 10^{-2}$ | $5.88(13) \cdot 10^{-2}$                | $3.04(2) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 361.24               | 359.970(48)  | 30.492(1200)   | 0.052334                | $4.36(5) \cdot 10^{-2}$  | $5.79(14) \cdot 10^{-2}$                | $3.01(3) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 361.24               | 359.990(66)  | 30.37(88)      | 0.058511                | $4.37(8) \cdot 10^{-2}$  | $5.84(24) \cdot 10^{-2}$                | $3.06(2) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 361.24               | 359.990(55)  | 30.467(700)    | 0.074012                | $4.35(6) \cdot 10^{-2}$  | $5.81(19) \cdot 10^{-2}$                | $3.04(3) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 361.24               | 360.00(7)    | 30.726(420)    | 0.082748                | $4.28(10) \cdot 10^{-2}$ | $6.10(26) \cdot 10^{-2}$                | $3.03(2) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 361.24               | 360.030(53)  | 31.289(380)    | 0.11702                 | $4.31(9) \cdot 10^{-2}$  | $5.97(19) \cdot 10^{-2}$                | $3.02(3) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 361.24               | 359.970(51)  | 52.506(2700)   | 0.0082748               | $4.65(11) \cdot 10^{-2}$ | $6.40(22) \cdot 10^{-2}$                | $3.19(1) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 361.24               | 359.980(58)  | 67.735(4200)   | 0.011702                | $4.66(4) \cdot 10^{-2}$  | $6.43(25) \cdot 10^{-2}$                | $3.21(3) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 361.24               | 360.000(35)  | 74.26(360)     | 0.018503                | $4.77(11) \cdot 10^{-2}$ | $6.37(17) \cdot 10^{-2}$                | $3.17(1) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 361.24               | 360.02(4)    | 66.556(2300)   | 0.026167                | $4.54(9) \cdot 10^{-2}$  | $6.30(29) \cdot 10^{-2}$                | $3.16(3) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 361.24               | 359.970(76)  | 30.101(3200)   | 0.037006                | $4.33(10) \cdot 10^{-2}$ | $5.75(27) \cdot 10^{-2}$                | $3.03(2) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 361.24               | 360.010(63)  | 29.882(1200)   | 0.052334                | $4.50(9) \cdot 10^{-2}$  | $5.89(13) \cdot 10^{-2}$                | $3.05(1) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 361.24               | 360.000(57)  | 30.556(950)    | 0.058511                | $4.29(6) \cdot 10^{-2}$  | $5.82(28) \cdot 10^{-2}$                | $3.00(2) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 361.24               | 360.000(53)  | 30.396(450)    | 0.074012                | $4.40(9) \cdot 10^{-2}$  | $5.79(13) \cdot 10^{-2}$                | $3.06(3) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 361.24               | 360.01(6)    | 30.584(370)    | 0.082748                | $4.40(4) \cdot 10^{-2}$  | $5.70(22) \cdot 10^{-2}$                | $3.04(1) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 361.24               | 360.000(79)  | 31.289(330)    | 0.11702                 | $4.32(2) \cdot 10^{-2}$  | $5.82(13) \cdot 10^{-2}$                | $3.07(2) \cdot 10^{-4}$                     |



**Table A.4:** Results of propane at 360 K, calculated with Velocity Verlet

| $\Delta t/\text{fs}$ | $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $\mathcal{T}/\text{fs}$ | $\eta/\text{mPa s}$      | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{cm}^2 \text{s}^{-1}$ |
|----------------------|----------------------|--------------|----------------|-------------------------|--------------------------|---|---|
| 2.0                  | 361.24               | 360.000(59)  | -116.75(820)   | 0.011 702               | $5.26(10) \cdot 10^{-2}$ | $7.07(24) \cdot 10^{-2}$                | $3.16(3) \cdot 10^{-4}$                     |
| 2.0                  | 361.24               | 360.010(49)  | 40.274(4800)   | 0.018 503               | $4.84(10) \cdot 10^{-2}$ | $6.50(18) \cdot 10^{-2}$                | $3.19(3) \cdot 10^{-4}$                     |
| 2.0                  | 361.24               | 360.000(55)  | 52.683(3400)   | 0.026 167               | $4.73(5) \cdot 10^{-2}$  | $6.29(17) \cdot 10^{-2}$                | $3.16(2) \cdot 10^{-4}$                     |
| 2.0                  | 361.24               | 360.020(66)  | 25.175(3600)   | 0.037 006               | $4.50(11) \cdot 10^{-2}$ | $5.95(25) \cdot 10^{-2}$                | $3.01(3) \cdot 10^{-4}$                     |
| 2.0                  | 361.24               | 359.990(42)  | 28.487(2300)   | 0.052 334               | $4.43(7) \cdot 10^{-2}$  | $5.95(16) \cdot 10^{-2}$                | $3.07(3) \cdot 10^{-4}$                     |
| 2.0                  | 361.24               | 359.990(64)  | 29.639(810)    | 0.058 511               | $4.44(8) \cdot 10^{-2}$  | $5.73(20) \cdot 10^{-2}$                | $3.03(2) \cdot 10^{-4}$                     |
| 2.0                  | 361.24               | 359.990(51)  | 29.692(780)    | 0.074 012               | $4.34(13) \cdot 10^{-2}$ | $5.75(22) \cdot 10^{-2}$                | $3.03(3) \cdot 10^{-4}$                     |
| 2.0                  | 361.24               | 359.990(74)  | 30.058(460)    | 0.082 748               | $4.40(7) \cdot 10^{-2}$  | $5.86(11) \cdot 10^{-2}$                | $3.05(1) \cdot 10^{-4}$                     |
| 2.0                  | 361.24               | 360.000(53)  | 30.961(350)    | 0.117 02                | $4.40(9) \cdot 10^{-2}$  | $6.01(16) \cdot 10^{-2}$                | $3.03(2) \cdot 10^{-4}$                     |
| 0.5                  | 361.24               | 360.000(65)  | 19.058(4700)   | 0.008 274 8             | $4.82(12) \cdot 10^{-2}$ | $6.44(21) \cdot 10^{-2}$                | $3.17(2) \cdot 10^{-4}$                     |
| 0.5                  | 361.24               | 360.000(59)  | 55.675(5900)   | 0.011 702               | $4.79(9) \cdot 10^{-2}$  | $6.57(22) \cdot 10^{-2}$                | $3.16(2) \cdot 10^{-4}$                     |
| 0.5                  | 361.24               | 360.030(53)  | 65.735(3000)   | 0.018 503               | $4.67(9) \cdot 10^{-2}$  | $6.30(13) \cdot 10^{-2}$                | $3.11(3) \cdot 10^{-4}$                     |
| 0.5                  | 361.24               | 360.010(61)  | 64.017(4300)   | 0.026 167               | $4.65(13) \cdot 10^{-2}$ | $6.23(13) \cdot 10^{-2}$                | $3.14(3) \cdot 10^{-4}$                     |
| 0.5                  | 361.24               | 359.990(83)  | 27.291(2900)   | 0.037 006               | $4.43(12) \cdot 10^{-2}$ | $5.95(15) \cdot 10^{-2}$                | $3.02(2) \cdot 10^{-4}$                     |
| 0.5                  | 361.24               | 359.990(67)  | 30.141(1300)   | 0.052 334               | $4.35(8) \cdot 10^{-2}$  | $6.16(32) \cdot 10^{-2}$                | $3.03(3) \cdot 10^{-4}$                     |
| 0.5                  | 361.24               | 359.98(7)    | 30.338(1100)   | 0.058 511               | $4.49(11) \cdot 10^{-2}$ | $5.88(23) \cdot 10^{-2}$                | $3.03(2) \cdot 10^{-4}$                     |
| 0.5                  | 361.24               | 360.010(53)  | 30.232(680)    | 0.074 012               | $4.35(5) \cdot 10^{-2}$  | $5.77(23) \cdot 10^{-2}$                | $3.03(2) \cdot 10^{-4}$                     |
| 0.5                  | 361.24               | 360.000(59)  | 30.692(370)    | 0.082 748               | $4.38(11) \cdot 10^{-2}$ | $5.81(11) \cdot 10^{-2}$                | $3.07(2) \cdot 10^{-4}$                     |
| 0.5                  | 361.24               | 359.980(79)  | 31.271(400)    | 0.117 02                | $4.32(13) \cdot 10^{-2}$ | $5.88(15) \cdot 10^{-2}$                | $3.06(2) \cdot 10^{-4}$                     |

**Table A.5:** Results of dimethylether at 380 K, calculated with rRESPA

| $\Delta t/\text{fs}$ | $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $\mathcal{T}/\text{fs}$ | $\eta/\text{mPa s}$      | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{cm}^2 \text{s}^{-1}$ |
|----------------------|----------------------|--------------|----------------|-------------------------|--------------------------|---|---|
| 1.0 / 0.5            | 482.12               | 380.03(8)    | 96.53(470)     | 0.012094                | $6.38(15) \cdot 10^{-2}$ | $8.48(18) \cdot 10^{-2}$                | $2.65(1) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 482.12               | 380.00(8)    | 99.587(4700)   | 0.019123                | $6.33(10) \cdot 10^{-2}$ | $8.55(33) \cdot 10^{-2}$                | $2.68(2) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 482.12               | 380.010(68)  | 77.516(5400)   | 0.027044                | $6.12(13) \cdot 10^{-2}$ | $8.42(26) \cdot 10^{-2}$                | $2.64(2) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 482.12               | 380.010(79)  | 31.465(2900)   | 0.038245                | $5.91(10) \cdot 10^{-2}$ | $7.74(17) \cdot 10^{-2}$                | $2.54(2) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 482.12               | 380.020(68)  | 32.886(1100)   | 0.054087                | $5.96(12) \cdot 10^{-2}$ | $7.87(16) \cdot 10^{-2}$                | $2.54(1) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 482.12               | 380.00(7)    | 32.34(85)      | 0.060471                | $5.94(12) \cdot 10^{-2}$ | $7.99(22) \cdot 10^{-2}$                | $2.53(2) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 482.12               | 380.000(66)  | 33.208(860)    | 0.076491                | $5.96(10) \cdot 10^{-2}$ | $7.92(21) \cdot 10^{-2}$                | $2.51(1) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 482.12               | 379.970(67)  | 32.733(1200)   | 0.085519                | $5.91(9) \cdot 10^{-2}$  | $7.83(13) \cdot 10^{-2}$                | $2.55(2) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 482.12               | 380.000(58)  | 34.411(380)    | 0.12094                 | $5.84(14) \cdot 10^{-2}$ | $7.98(13) \cdot 10^{-2}$                | $2.55(2) \cdot 10^{-4}$                     |

**Table A.6:** Results of ethane at 205 K, calculated with rRESPA

| $\Delta t/\text{fs}$ | $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $\mathcal{T}/\text{fs}$ | $\eta/\text{mPa s}$     | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{cm}^2 \text{s}^{-1}$ |
|----------------------|----------------------|--------------|----------------|-------------------------|-------------------------|---|---|
| 1.0 / 0.1            | 512.82               | 205.010(34)  | 81.712(7300)   | 0.0084221               | $1.44(3) \cdot 10^{-1}$ | $1.71(4) \cdot 10^{-1}$                 | $7.77(5) \cdot 10^{-5}$                     |
| 1.0 / 0.1            | 512.82               | 204.980(37)  | 84.29(660)     | 0.011911                | $1.40(3) \cdot 10^{-1}$ | $1.65(8) \cdot 10^{-1}$                 | $7.79(7) \cdot 10^{-5}$                     |
| 1.0 / 0.1            | 512.82               | 205.010(33)  | 85.827(7400)   | 0.018832                | $1.39(3) \cdot 10^{-1}$ | $1.71(6) \cdot 10^{-1}$                 | $7.73(5) \cdot 10^{-5}$                     |
| 1.0 / 0.1            | 512.82               | 205.010(43)  | 91.801(11000)  | 0.026633                | $1.39(1) \cdot 10^{-1}$ | $1.71(6) \cdot 10^{-1}$                 | $7.78(6) \cdot 10^{-5}$                     |
| 1.0 / 0.1            | 512.82               | 205.000(38)  | 73.402(13000)  | 0.037665                | $1.37(2) \cdot 10^{-1}$ | $1.62(6) \cdot 10^{-1}$                 | $7.68(4) \cdot 10^{-5}$                     |
| 1.0 / 0.1            | 512.82               | 205.000(37)  | -1.9202(78000) | 0.053266                | $1.33(2) \cdot 10^{-1}$ | $1.51(4) \cdot 10^{-1}$                 | $7.44(7) \cdot 10^{-5}$                     |
| 1.0 / 0.1            | 512.82               | 205.000(37)  | -5.2385(43000) | 0.059553                | $1.35(3) \cdot 10^{-1}$ | $1.48(7) \cdot 10^{-1}$                 | $7.45(4) \cdot 10^{-5}$                     |

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Continued: Results of ethane at 205 K, calculated with rRESPA

| $\Delta t/\text{fs}$ | $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $\mathcal{T}/\text{fs}$ | $\eta/\text{mPa s}$     | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{cm}^2 \text{s}^{-1}$ |
|----------------------|----------------------|--------------|----------------|-------------------------|-------------------------|---|---|
| 1.0 / 0.1            | 512.82               | 205.000(19)  | -4.2234(23000) | 0.07533                 | $1.31(2) \cdot 10^{-1}$ | $1.55(6) \cdot 10^{-1}$                 | $7.41(6) \cdot 10^{-5}$                     |
| 1.0 / 0.1            | 512.82               | 205.010(44)  | -4.8826(13000) | 0.084221                | $1.34(2) \cdot 10^{-1}$ | $1.54(6) \cdot 10^{-1}$                 | $7.49(6) \cdot 10^{-5}$                     |
| 1.0 / 0.1            | 512.82               | 204.990(36)  | -8.3183(6000)  | 0.11911                 | $1.30(2) \cdot 10^{-1}$ | $1.48(7) \cdot 10^{-1}$                 | $7.52(4) \cdot 10^{-5}$                     |

Table A.7: Results of propane at 295 K, calculated with rRESPA

| $\Delta t/\text{fs}$ | $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $\mathcal{T}/\text{fs}$ | $\eta/\text{mPa s}$      | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{cm}^2 \text{s}^{-1}$ |
|----------------------|----------------------|--------------|----------------|-------------------------|--------------------------|---|---|
| 2.0 / 0.5            | 497.2                | 295.000(51)  | -49.237(8500)  | 0.0082748               | $1.06(2) \cdot 10^{-1}$  | $1.03(5) \cdot 10^{-1}$                 | $1.33(1) \cdot 10^{-4}$                     |
| 2.0 / 0.5            | 497.2                | 295.010(53)  | 29.661(7500)   | 0.011702                | $1.01(2) \cdot 10^{-1}$  | $1.03(2) \cdot 10^{-1}$                 | $1.32(1) \cdot 10^{-4}$                     |
| 2.0 / 0.5            | 497.2                | 295.000(54)  | 66.258(4900)   | 0.018503                | $1.02(2) \cdot 10^{-1}$  | $1.04(1) \cdot 10^{-1}$                 | $1.32(1) \cdot 10^{-4}$                     |
| 2.0 / 0.5            | 497.2                | 295.01(4)    | 61.853(6600)   | 0.026167                | $9.99(13) \cdot 10^{-2}$ | $1.03(3) \cdot 10^{-1}$                 | $1.29(1) \cdot 10^{-4}$                     |
| 2.0 / 0.5            | 497.2                | 294.990(46)  | 4.0931(45000)  | 0.037006                | $9.68(16) \cdot 10^{-2}$ | $9.44(37) \cdot 10^{-2}$                | $1.25(1) \cdot 10^{-4}$                     |
| 2.0 / 0.5            | 497.2                | 295.010(55)  | 4.1473(100000) | 0.052334                | $9.57(20) \cdot 10^{-2}$ | $9.65(44) \cdot 10^{-2}$                | $1.26(1) \cdot 10^{-4}$                     |
| 2.0 / 0.5            | 497.2                | 295.000(57)  | 4.9439(18000)  | 0.058511                | $9.51(14) \cdot 10^{-2}$ | $9.80(61) \cdot 10^{-2}$                | $1.26(1) \cdot 10^{-4}$                     |
| 2.0 / 0.5            | 497.2                | 295.010(54)  | 5.7005(15000)  | 0.074012                | $9.46(36) \cdot 10^{-2}$ | $9.59(20) \cdot 10^{-2}$                | $1.26(1) \cdot 10^{-4}$                     |
| 2.0 / 0.5            | 497.2                | 295.010(61)  | 5.2461(8500)   | 0.082748                | $9.44(11) \cdot 10^{-2}$ | $9.45(19) \cdot 10^{-2}$                | $1.26(1) \cdot 10^{-4}$                     |
| 2.0 / 0.5            | 497.2                | 295.000(37)  | 5.2348(6000)   | 0.11702                 | $9.33(21) \cdot 10^{-2}$ | $9.82(27) \cdot 10^{-2}$                | $1.28(1) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 497.2                | 295.01(4)    | 69.178(7200)   | 0.0082748               | $1.00(2) \cdot 10^{-1}$  | $1.03(4) \cdot 10^{-1}$                 | $1.31(1) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 497.2                | 295.000(45)  | 75.788(7800)   | 0.011702                | $1.01(3) \cdot 10^{-1}$  | $1.04(5) \cdot 10^{-1}$                 | $1.31(1) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 497.2                | 295.000(42)  | 88.047(4600)   | 0.018503                | $1.02(2) \cdot 10^{-1}$  | $1.05(3) \cdot 10^{-1}$                 | $1.31(1) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 497.2                | 294.980(55)  | 78.46(830)     | 0.026167                | $9.92(26) \cdot 10^{-2}$ | $1.05(3) \cdot 10^{-1}$                 | $1.30(1) \cdot 10^{-4}$                     |

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**Continued:** Results of propane at 295 K, calculated with rRESPA

| $\Delta t/\text{fs}$ | $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $\mathcal{T}/\text{fs}$ | $\eta/\text{mPa s}$      | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{cm}^2 \text{s}^{-1}$ |
|----------------------|----------------------|--------------|----------------|-------------------------|--------------------------|---|---|
| 1.0 / 0.1            | 497.2                | 295.010(48)  | 4.2512(76000)  | 0.037 006               | $9.33(15) \cdot 10^{-2}$ | $9.50(29) \cdot 10^{-2}$                | $1.26(1) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 497.2                | 295.000(45)  | 6.4127(27000)  | 0.052 334               | $9.61(20) \cdot 10^{-2}$ | $9.48(30) \cdot 10^{-2}$                | $1.25(1) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 497.2                | 295.000(51)  | 6.3677(18000)  | 0.058 511               | $9.57(18) \cdot 10^{-2}$ | $9.53(18) \cdot 10^{-2}$                | $1.26(1) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 497.2                | 294.990(48)  | 6.213(1100)    | 0.074 012               | $9.44(19) \cdot 10^{-2}$ | $9.31(24) \cdot 10^{-2}$                | $1.26(1) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 497.2                | 295.00(4)    | 6.0164(10000)  | 0.082 748               | $9.27(21) \cdot 10^{-2}$ | $9.40(17) \cdot 10^{-2}$                | $1.28(1) \cdot 10^{-4}$                     |
| 1.0 / 0.1            | 497.2                | 295.010(62)  | 5.6564(7000)   | 0.117 02                | $9.41(11) \cdot 10^{-2}$ | $9.63(14) \cdot 10^{-2}$                | $1.28(1) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 497.2                | 295.000(57)  | 69.491(6200)   | 0.008 274 8             | $1.02(1) \cdot 10^{-1}$  | $1.07(6) \cdot 10^{-1}$                 | $1.33(1) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 497.2                | 295.010(51)  | 74.571(8800)   | 0.011 702               | $1.01(2) \cdot 10^{-1}$  | $1.04(4) \cdot 10^{-1}$                 | $1.32(1) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 497.2                | 295.010(52)  | 81.646(5200)   | 0.018 503               | $1.01(2) \cdot 10^{-1}$  | $1.04(4) \cdot 10^{-1}$                 | $1.29(1) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 497.2                | 294.990(58)  | 93.123(6100)   | 0.026 167               | $1.02(2) \cdot 10^{-1}$  | $1.04(2) \cdot 10^{-1}$                 | $1.32(1) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 497.2                | 295.000(56)  | 6.9193(67000)  | 0.037 006               | $9.32(28) \cdot 10^{-2}$ | $9.71(44) \cdot 10^{-2}$                | $1.28(1) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 497.2                | 295.020(45)  | 7.2282(24000)  | 0.052 334               | $9.45(18) \cdot 10^{-2}$ | $9.53(25) \cdot 10^{-2}$                | $1.25(1) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 497.2                | 295.010(51)  | 7.4308(17000)  | 0.058 511               | $9.59(18) \cdot 10^{-2}$ | $1.00(8) \cdot 10^{-1}$                 | $1.26(1) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 497.2                | 294.970(48)  | 5.4418(19000)  | 0.074 012               | $9.57(18) \cdot 10^{-2}$ | $9.30(50) \cdot 10^{-2}$                | $1.26 \cdot 10^{-4}$                        |
| 1.0 / 0.5            | 497.2                | 295.000(56)  | 6.1665(7900)   | 0.082 748               | $9.11(16) \cdot 10^{-2}$ | $9.30(11) \cdot 10^{-2}$                | $1.27(1) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 497.2                | 294.990(51)  | 5.4778(5200)   | 0.117 02                | $9.12(12) \cdot 10^{-2}$ | $9.49(19) \cdot 10^{-2}$                | $1.27(1) \cdot 10^{-4}$                     |

**Table A.8:** Results of propane at 295 K, calculated with Velocity Verlet

| $\Delta t/\text{fs}$ | $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $\mathcal{T}/\text{fs}$ | $\eta/\text{mPa s}$      | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{cm}^2 \text{s}^{-1}$ |
|----------------------|----------------------|--------------|----------------|-------------------------|--------------------------|---|---|
| 2.0                  | 497.2                | 295.010(45)  | -111.63(900)   | 0.011 702               | $1.17(3) \cdot 10^{-1}$  | $1.14(3) \cdot 10^{-1}$                 | $1.31(1) \cdot 10^{-4}$                     |
| 2.0                  | 497.2                | 295.010(62)  | 54.482(8800)   | 0.018 503               | $1.03(4) \cdot 10^{-1}$  | $1.08(3) \cdot 10^{-1}$                 | $1.32(1) \cdot 10^{-4}$                     |
| 2.0                  | 497.2                | 294.990(56)  | 72.452(6800)   | 0.026 167               | $1.01(2) \cdot 10^{-1}$  | $1.07(5) \cdot 10^{-1}$                 | $1.31(1) \cdot 10^{-4}$                     |
| 2.0                  | 497.2                | 295.000(58)  | 7.5923(93000)  | 0.037 006               | $9.59(22) \cdot 10^{-2}$ | $9.78(31) \cdot 10^{-2}$                | $1.26(1) \cdot 10^{-4}$                     |
| 2.0                  | 497.2                | 295.010(54)  | 9.0637(29000)  | 0.052 334               | $9.54(14) \cdot 10^{-2}$ | $9.56(26) \cdot 10^{-2}$                | $1.27(1) \cdot 10^{-4}$                     |
| 2.0                  | 497.2                | 295.010(46)  | 8.6413(20000)  | 0.058 511               | $9.52(30) \cdot 10^{-2}$ | $9.53(22) \cdot 10^{-2}$                | $1.27(1) \cdot 10^{-4}$                     |
| 2.0                  | 497.2                | 294.99(5)    | 8.0245(17000)  | 0.074 012               | $9.48(20) \cdot 10^{-2}$ | $9.67(15) \cdot 10^{-2}$                | $1.27(1) \cdot 10^{-4}$                     |
| 2.0                  | 497.2                | 295.020(51)  | 7.5303(9200)   | 0.082 748               | $9.45(19) \cdot 10^{-2}$ | $9.78(21) \cdot 10^{-2}$                | $1.27(1) \cdot 10^{-4}$                     |
| 2.0                  | 497.2                | 295.000(47)  | 6.1831(8100)   | 0.117 02                | $9.22(25) \cdot 10^{-2}$ | $9.50(31) \cdot 10^{-2}$                | $1.28(1) \cdot 10^{-4}$                     |
| 0.5                  | 497.2                | 295.000(43)  | 33.707(8600)   | 0.008 274 8             | $1.03(3) \cdot 10^{-1}$  | $1.09(3) \cdot 10^{-1}$                 | $1.32(1) \cdot 10^{-4}$                     |
| 0.5                  | 497.2                | 295.010(36)  | 72.428(12000)  | 0.011 702               | $1.02(2) \cdot 10^{-1}$  | $1.04(3) \cdot 10^{-1}$                 | $1.31(1) \cdot 10^{-4}$                     |
| 0.5                  | 497.2                | 294.98(5)    | 88.425(9600)   | 0.018 503               | $1.00(2) \cdot 10^{-1}$  | $1.07(3) \cdot 10^{-1}$                 | $1.32(1) \cdot 10^{-4}$                     |
| 0.5                  | 497.2                | 295.020(59)  | 73.759(7700)   | 0.026 167               | $1.01(2) \cdot 10^{-1}$  | $1.02(2) \cdot 10^{-1}$                 | $1.31(1) \cdot 10^{-4}$                     |
| 0.5                  | 497.2                | 295.010(43)  | 7.9425(43000)  | 0.037 006               | $9.58(17) \cdot 10^{-2}$ | $9.66(36) \cdot 10^{-2}$                | $1.26(1) \cdot 10^{-4}$                     |
| 0.5                  | 497.2                | 294.990(63)  | 7.612(2600)    | 0.052 334               | $9.19(26) \cdot 10^{-2}$ | $9.72(60) \cdot 10^{-2}$                | $1.28(1) \cdot 10^{-4}$                     |
| 0.5                  | 497.2                | 295.010(49)  | 6.7469(16000)  | 0.058 511               | $9.54(12) \cdot 10^{-2}$ | $9.38(47) \cdot 10^{-2}$                | $1.26(1) \cdot 10^{-4}$                     |
| 0.5                  | 497.2                | 294.98(5)    | 6.9109(17000)  | 0.074 012               | $9.35(16) \cdot 10^{-2}$ | $9.72(45) \cdot 10^{-2}$                | $1.26(1) \cdot 10^{-4}$                     |
| 0.5                  | 497.2                | 295.010(48)  | 6.9764(12000)  | 0.082 748               | $9.46(14) \cdot 10^{-2}$ | $9.33(25) \cdot 10^{-2}$                | $1.26(1) \cdot 10^{-4}$                     |
| 0.5                  | 497.2                | 295.000(55)  | 5.7914(5500)   | 0.117 02                | $9.17(18) \cdot 10^{-2}$ | $9.82(35) \cdot 10^{-2}$                | $1.27(1) \cdot 10^{-4}$                     |

**Table A.9:** Results of dimethylether at 300 K, calculated with rRESPA

| $\Delta t/\text{fs}$ | $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $\mathcal{T}/\text{fs}$ | $\eta/\text{mPa s}$     | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{cm}^2 \text{s}^{-1}$ |
|----------------------|----------------------|--------------|----------------|-------------------------|-------------------------|---|---|
| 1.0 / 0.5            | 649.04               | 300.000(46)  | 121.25(1000)   | 0.012 094               | $1.45(2) \cdot 10^{-1}$ | $1.45(5) \cdot 10^{-1}$                 | $1.02(1) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 649.04               | 300.010(49)  | 100.53(1000)   | 0.019 123               | $1.40(2) \cdot 10^{-1}$ | $1.42(3) \cdot 10^{-1}$                 | $1.02(1) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 649.04               | 299.990(48)  | 100.34(780)    | 0.027 044               | $1.41(3) \cdot 10^{-1}$ | $1.42(6) \cdot 10^{-1}$                 | $1.01(1) \cdot 10^{-4}$                     |
| 1.0 / 0.5            | 649.04               | 300.000(45)  | 1.4783(64000)  | 0.038 245               | $1.34(2) \cdot 10^{-1}$ | $1.35(9) \cdot 10^{-1}$                 | $9.78(6) \cdot 10^{-5}$                     |
| 1.0 / 0.5            | 649.04               | 300.01(6)    | 1.5315(30000)  | 0.054 087               | $1.33(3) \cdot 10^{-1}$ | $1.32(4) \cdot 10^{-1}$                 | $9.73(8) \cdot 10^{-5}$                     |
| 1.0 / 0.5            | 649.04               | 300.010(67)  | 1.9463(18000)  | 0.060 471               | $1.35(3) \cdot 10^{-1}$ | $1.31(3) \cdot 10^{-1}$                 | $9.76(8) \cdot 10^{-5}$                     |
| 1.0 / 0.5            | 649.04               | 300.000(71)  | 2.0271(10000)  | 0.076 491               | $1.34(2) \cdot 10^{-1}$ | $1.33(5) \cdot 10^{-1}$                 | $9.85(8) \cdot 10^{-5}$                     |
| 1.0 / 0.5            | 649.04               | 300.010(54)  | 1.8268(17000)  | 0.085 519               | $1.34(3) \cdot 10^{-1}$ | $1.34(5) \cdot 10^{-1}$                 | $9.82(8) \cdot 10^{-5}$                     |
| 1.0 / 0.5            | 649.04               | 300.000(72)  | 2.7582(8100)   | 0.120 94                | $1.32(3) \cdot 10^{-1}$ | $1.37(7) \cdot 10^{-1}$                 | $9.93(5) \cdot 10^{-5}$                     |

**Table A.10:** Results of propane (vapor) at 360 K, calculated with rRESPA

| $\Delta t/\text{fs}$ | $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $\mathcal{T}/\text{fs}$ | $\eta/\text{mPa s}$      | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{cm}^2 \text{s}^{-1}$ |
|----------------------|----------------------|--------------|----------------|-------------------------|--------------------------|---|---|
| 1.0 / 0.5            | 94.323               | 360.000(43)  | 38.118(500)    | 0.008 274 8             | $1.37(12) \cdot 10^{-2}$ | $2.43(21) \cdot 10^{-2}$                | $1.57(1) \cdot 10^{-3}$                     |
| 1.0 / 0.5            | 94.323               | 359.990(54)  | 39.529(680)    | 0.011 702               | $1.35(8) \cdot 10^{-2}$  | $2.24(27) \cdot 10^{-2}$                | $1.58(2) \cdot 10^{-3}$                     |
| 1.0 / 0.5            | 94.323               | 359.960(25)  | 40.046(570)    | 0.018 503               | $1.24(12) \cdot 10^{-2}$ | $2.53(47) \cdot 10^{-2}$                | $1.57(1) \cdot 10^{-3}$                     |
| 1.0 / 0.5            | 94.323               | 360.000(48)  | 40.776(310)    | 0.026 167               | $1.42(15) \cdot 10^{-2}$ | $2.74(33) \cdot 10^{-2}$                | $1.60(1) \cdot 10^{-3}$                     |
| 1.0 / 0.5            | 94.323               | 359.980(46)  | 35.038(360)    | 0.037 006               | $1.24(8) \cdot 10^{-2}$  | $2.37(54) \cdot 10^{-2}$                | $1.50(1) \cdot 10^{-3}$                     |
| 1.0 / 0.5            | 94.323               | 360.010(47)  | 35.204(240)    | 0.052 334               | $1.17(10) \cdot 10^{-2}$ | $2.28(23) \cdot 10^{-2}$                | $1.50(1) \cdot 10^{-3}$                     |
| 1.0 / 0.5            | 94.323               | 360.020(37)  | 35.192(140)    | 0.058 511               | $1.11(12) \cdot 10^{-2}$ | $2.80(53) \cdot 10^{-2}$                | $1.49(1) \cdot 10^{-3}$                     |

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**Continued:** Results of propane (vapor) at 360 K, calculated with rRESPA

| $\Delta t/\text{fs}$ | $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $\mathcal{T}/\text{fs}$ | $\eta/\text{mPa s}$     | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{cm}^2 \text{s}^{-1}$ |
|----------------------|----------------------|--------------|----------------|-------------------------|-------------------------|---|---|
| 1.0 / 0.5            | 94.323               | 359.990(42)  | 35.227(99)     | 0.074012                | $1.17(6) \cdot 10^{-2}$ | $2.39(20) \cdot 10^{-2}$                | $1.50(2) \cdot 10^{-3}$                     |
| 1.0 / 0.5            | 94.323               | 360.000(41)  | 35.227(90)     | 0.082748                | $1.17(9) \cdot 10^{-2}$ | $2.52(47) \cdot 10^{-2}$                | $1.49(1) \cdot 10^{-3}$                     |
| 1.0 / 0.5            | 94.323               | 360.010(38)  | 35.340(69)     | 0.11702                 | $1.24(8) \cdot 10^{-2}$ | $2.09(38) \cdot 10^{-2}$                | $1.52(1) \cdot 10^{-3}$                     |





## **B Supporting Information for: Transferable Anisotropic United-Atom Mie (TAMie) Force-Field: Transport properties from Equilibrium Molecular Dynamic Simulations**

The content of this appendix is a literal quote of the supporting information:

M. Fischer, G. Bauer, J. Gross: Supporting Information for: Transferable Anisotropic United-Atom Mie (TAMie) Force Field: Transport Properties from Equilibrium-MD-Simulations. *Industrial & Engineering Chemistry Research*, **59**(18):8855–8869, 2020. doi:10.1021/acs.iecr.0c00848

## B.1 SAFT Parameters used in this Study

**Table B.1:** PC-SAFT Parameters used for the substances in this study

| CAS    | Substance     | Molar Weight / $\text{g mol}^{-1}$ | $m$    | $\sigma / \text{\AA}$ | $\epsilon/k/K$ | $\kappa_{ab}$ | $\epsilon_{ab}/k/K$ | $DM/D$ |
|--------|---------------|------------------------------------|--------|-----------------------|----------------|---------------|---------------------|--------|
| 74840  | ethane        | 30.07                              | 1.6069 | 3.5206                | 191.42         | 0             | 0                   | 0      |
| 74986  | propane       | 44.096                             | 2.002  | 3.6184                | 208.11         | 0             | 0                   | 0      |
| 110543 | hexane        | 86.177                             | 3.0576 | 3.7983                | 236.77         | 0             | 0                   | 0      |
| 111842 | nonane        | 128.25                             | 4.2079 | 3.8448                | 244.51         | 0             | 0                   | 0      |
| 106989 | 1-butene      | 56.107                             | 2.2864 | 3.6431                | 222            | 0             | 0                   | 0      |
| 592767 | 1-heptene     | 98.1861                            | 3.2863 | 3.8138                | 243.756        | 0             | 0                   | 0.6296 |
| 115106 | dimethylether | 46.069                             | 2.2634 | 3.2723                | 210.29         | 0             | 0                   | 1.3    |
| 75070  | ethanal       | 44.0526                            | 2.1188 | 3.246                 | 229.851        | 0             | 0                   | 2.6891 |
| 123728 | butanal       | 72.107                             | 2.8825 | 3.4698                | 247.09         | 0             | 0                   | 2.72   |
| 111717 | heptanal      | 114.185                            | 3.8527 | 3.66                  | 260.698        | 0             | 0                   | 2.5782 |
| 67641  | acetone       | 58.08                              | 2.7447 | 3.2742                | 232.99         | 0             | 0                   | 2.88   |
| 78933  | butanone      | 72.107                             | 2.9835 | 3.4239                | 244.99         | 0             | 0                   | 2.78   |

## B.2 Parameters of the TAMie force field

Tables B.2–B.5 provide the parameters of the TAMie force field as described in the *Molecular Model* section in the main text. The parameters were all published in previous work<sup>1–3,3</sup>, except for the bond constant  $k_l$  in table B.3, which was specified by the authors of this study.

| United atom $i$                 | $M_{ii}/(\text{g mol}^{-1})$ | $\varepsilon_{ii}/k_B/\text{K}$ | $\sigma_{ii}/\text{\AA}$ | $n_{ii}$ | $q_i/e$                 |
|---------------------------------|------------------------------|---------------------------------|--------------------------|----------|-------------------------|
| CH <sub>3</sub>                 | 15.035                       | 136.318                         | 3.6034                   | 14       | 0.175 (next to ether O) |
| CH <sub>3</sub> (ethane)        | 15.035                       | 130.780                         | 3.6463                   | 14       | —                       |
| CH <sub>2</sub>                 | 14.027                       | 52.913                          | 4.0400                   | 14       | —                       |
| CH <sub>2</sub> (olefins)       | 14.027                       | 100.681                         | 3.6005                   | 14       | —                       |
| CH (olefins)                    | 13.019                       | 53.951                          | 3.8234                   | 14       | —                       |
| O (dimethyl ether)              | 15.999                       | 54.36                           | 3.213                    | 12       | −0.35                   |
| O (aldehydes)                   | 15.999                       | 100.58                          | 3.0276                   | 12       | −0.422                  |
| CH <sub>x</sub> (neighbor ald.) |                              |                                 |                          |          | −0.038                  |
| CH (aldehydes)                  | 13.019                       | 68.934                          | 3.4941                   | 12       | 0.46                    |
| O (ketones)                     | 15.999                       | 65.55                           | 3.093                    | 12       | −0.49                   |
| C (ketones)                     | 12.011                       | 32.775                          | 3.919                    | 12       | 0.49                    |
| O (acetone)                     | 15.999                       | 69.184                          | 3.112                    | 12       | −0.49                   |
| C (acetone)                     | 12.011                       | 34.592                          | 3.942                    | 12       | 0.49                    |

**Table B.2:** Pair potential parameters of UA-groups used in this study: Energy parameter  $\varepsilon_{ii}$ , size parameter  $\sigma_{ii}$ , repulsive exponent  $n_{ii}$ , and point charge  $q_i$  for the TAMie force field

| Bond                                      | $r_0/\text{\AA}$ | $k_l/\text{K}\text{\AA}^{-2}$ |
|---|------------------|-------------------------------|
| CH <sub>3</sub> –CH <sub>3</sub> (ethane) | 1.94             | 100 644                       |
| CH <sub>3</sub> –CH <sub>x</sub>          | 1.74             | 100 644                       |
| CH <sub>x</sub> –CH <sub>y</sub>          | 1.54             | 100 644                       |
| CH=CH <sub>x</sub>                        | 1.33             | 100 644                       |
| CH <sub>3</sub> –CH[=O <sub>ald</sub> ]   | 1.74             | 100 644                       |
| CH <sub>2</sub> –CH[=O <sub>ald</sub> ]   | 1.54             | 100 644                       |
| CH=O <sub>ald</sub>                       | 1.217            | 100 644                       |
| CH <sub>3</sub> –C[=O <sub>ket</sub> ]    | 1.74             | 100 644                       |
| CH <sub>2</sub> –C[=O <sub>ket</sub> ]    | 1.54             | 100 644                       |
| C=O <sub>ket</sub>                        | 1.229            | 100 644                       |
| CH <sub>3</sub> –O <sub>ether</sub>       | 1.61             | 100 644                       |

**Table B.3:** Bond lengths and constants of the TAMie force field used in this study ( $x, y \in \{1, 2\}$ )

| Bending sites   | $\theta_0/\text{deg}$ | $k_\theta/k_B/\text{K}$ |
|---|-----------------------|-------------------------|
| $\text{CH}_x-\text{CH}_2-\text{CH}_y$                     | 114.0                 | 62 500                  |
| $\text{CH}_z=\text{CH}-\text{CH}_y$                       | 119.7                 | 70 420                  |
| $\text{CH}_x-\text{O}_{\text{ether}}-\text{CH}_y$         | 112.0                 | 60 400                  |
| $\text{CH}_x-\text{C}_{\text{ket}}-\text{CH}_y$           | 117.2                 | 62 500                  |
| $\text{C}_{\text{ket}}-\text{CH}_2-\text{CH}_x$           | 114.0                 | 62 500                  |
| $\text{CH}_x-\text{C}_{\text{ket}}=\text{O}_{\text{ket}}$ | 121.4                 | 62 500                  |
| $\text{CH}_x-\text{CH}=\text{O}_{\text{ald}}$             | 121.4                 | 62 500                  |

**Table B.4:** Bending angles and constants of the TAMie force field used in this study ( $x, y \in \{2, 3\}$  and  $z \in \{1, 2\}$ )

| Torsion sites   | $c_0/k_B/\text{K}$ | $c_1/k_B/\text{K}$ | $c_2/k_B/\text{K}$ | $c_3/k_B/\text{K}$ |
|---|--------------------|--------------------|--------------------|--------------------|
| $\text{CH}_x-\text{CH}_2-\text{CH}_2-\text{CH}_y$                       | 0.0                | 355.03             | -68.19             | 791.32             |
| $\text{CH}_x-\text{CH}_2-\text{O}_{\text{ether}}-\text{CH}_y$           | 0.0                | 725.35             | -163.75            | 558.2              |
| $\text{CH}_x-\text{CH}_2-\text{CH}_2-\text{CH}_y$                       | 0.0                | 176.62             | -53.34             | 769.93             |
| $\text{CH}_z=\text{CH}-\text{CH}_2-\text{CH}_y$                         | 688.5              | 86.36              | -109.77            | -282.24            |
| $\text{C}-\text{CH}_2-\text{CH}_2-\text{CH}_y$                          | 0.0                | 355.03             | -68.19             | 791.32             |
| $\text{CH}_x-\text{CH}_2-\text{C}=\text{O}_{\text{ket}}$                | 2035.58            | -736.90            | 57.84              | -293.23            |
| $\text{CH}_x-\text{C}-\text{CH}_2-\text{CH}_y$                          | -17.26             | 752.60             | 14.89              | 282.10             |
| $\text{CH}_x-\text{CH}_2-\text{CH}_2-\text{CH}[\text{=O}_{\text{ald}}]$ | 11.81              | 467.80             | -274.10            | 846.80             |
| $\text{CH}_x-\text{CH}_2-\text{CH}=\text{O}_{\text{ald}}$               | 1182.0             | -225.60            | 302.20             | -339.30            |

**Table B.5:** Torsion potential constants of the TAMie force field used in this study ( $x, y \in \{2, 3\}$  and  $z \in \{1, 2\}$ )

### B.3 Correlation Parameters from Simulation

Tables B.6–B.8 summarize the parameters of the entropy correlations for each transport property adjusted to the simulation results. The corresponding correlation functions can be found in Table 3 in the main text. Additional to the parameters, the AAD of each correlation to all available experiments is given for each substance, along with the number of experimental data points available. The sources of the experiments are given in the column *Lit.*

**Table B.6:** Entropy Scaling Parameters of the TAMie force field for viscosity

| CAS    | Substance     | $A_\eta$ | $B_\eta$ | $C_\eta$ | $D_\eta$  | $N_{\text{exp.}}$ | $AAD_{\text{exp.}}$ | Lit. |
|--------|---------------|----------|----------|----------|-----------|-------------------|---------------------|------|
| 74840  | ethane        | −0.58482 | −1.597   | −0.11957 | −0.015513 | 2606              | 5.93 %              | 4    |
| 74986  | propane       | −0.73904 | −1.9377  | −0.31713 | −0.051261 | 1752              | 9.47 %              | 4    |
| 110543 | hexane        | −1.3007  | −2.6303  | −0.50842 | −0.090849 | 1804              | 9.90 %              | 4    |
| 111842 | nonane        | −1.5238  | −3.0059  | −0.63805 | −0.13635  | 382               | 14.45 %             | 4    |
| 106989 | 1-butene      | −0.8661  | −2.207   | −0.44046 | −0.071512 | 22                | 5.59 %              | 4    |
| 592767 | 1-heptene     | −1.1781  | −2.5061  | −0.48927 | −0.10139  | 163               | 6.79 %              | 4    |
| 115106 | dimethylether | −0.8927  | −2.0926  | −0.37523 | −0.070398 | 211               | 6.31 %              | 4    |
| 75070  | ethanal       | −0.97854 | −1.8377  | −0.2819  | −0.072213 | 28                | 11.61 %             | 4    |
| 123728 | butanal       | −1.084   | −2.388   | −0.55646 | −0.13242  | 233               | 15.12 %             | 4    |
| 111717 | heptanal      | −1.4179  | −2.843   | −0.55104 | −0.13302  | 12                | 30.16 %             | 4    |
| 67641  | acetone       | −1.1162  | −2.3125  | −0.53579 | −0.14713  | 512               | 15.28 %             | 4    |
| 78933  | butanone      | −1.2108  | −2.4107  | −0.47133 | −0.12334  | 246               | 22.00 %             | 4    |

**Table B.7:** Entropy Scaling Parameters of the TAMie force field for thermal conductivity

| CAS    | Substance     | $A_\lambda$ | $B_\lambda$ | $C_\lambda$ | $D_\lambda$ | $N_{\text{exp.}}$ | $AAD_{\text{exp.}}$ | Lit. |
|--------|---------------|-------------|-------------|-------------|-------------|-------------------|---------------------|------|
| 74840  | ethane        | -0.41697    | -1.3177     | 0.53818     | -0.12065    | 1629              | 16.43 %             | 4    |
| 74986  | propane       | -0.3735     | -0.74596    | 1.2519      | -0.027495   | 3154              | 13.35 %             | 4    |
| 110543 | hexane        | -0.12834    | -0.83516    | 1.0578      | -0.077646   | 646               | 11.23 %             | 4    |
| 111842 | nonane        | -0.043518   | -0.38023    | 1.7543      | 0.03192     | 524               | 6.00 %              | 4    |
| 106989 | 1-butene      | -0.12792    | -1.06       | 0.44367     | -0.075675   | 31                | 11.03 %             | 4    |
| 592767 | 1-heptene     | -0.063358   | -0.65059    | 1.226       | -0.033603   | 415               | 7.69 %              | 4    |
| 115106 | dimethylether | -0.52864    | -0.73268    | 1.4949      | -0.026342   | 347               | 15.94 %             | 4    |
| 75070  | ethanal       | -0.54935    | -1.1198     | 0.82421     | -0.078127   | 96                | 11.57 %             | 4    |
| 123728 | butanal       | -0.24726    | 0.18717     | 2.4271      | 0.12488     | 82                | 10.35 %             | 4    |
| 111717 | heptanal      | -0.162      | 0.30738     | 2.7413      | 0.14491     | 12                | 1.98 %              | 4    |
| 67641  | acetone       | -0.56802    | -0.7099     | 1.3292      | -0.0017086  | 386               | 9.44 %              | 4    |
| 78933  | butanone      | -0.42298    | -0.13237    | 2.0665      | 0.0883      | 347               | 7.99 %              | 4    |

**Table B.8:** Entropy Scaling Parameters of the TAMie force field for diffusion

| CAS    | Substance     | $A_{D_{\text{self}}}$ | $B_{D_{\text{self}}}$ | $C_{D_{\text{self}}}$   | $N_{\text{exp.}}$ | $AAD_{\text{exp.}}$ | Lit.       |
|--------|---------------|-----------------------|-----------------------|-------------------------|-------------------|---------------------|------------|
| 74840  | ethane        | -0.37369              | 0.14859               | $2.4814 \cdot 10^{-13}$ | 154               | 14.40 %             | 5-9        |
| 74986  | propane       | -0.53649              | 0.15984               | $9.9011 \cdot 10^{-15}$ | 75                | 20.66 %             | 7,9        |
| 110543 | hexane        | -0.70772              | 0.26865               | $4.8284 \cdot 10^{-14}$ | 185               | 21.16 %             | 8-15       |
| 111842 | nonane        | -0.90991              | 0.39224               | $7.1876 \cdot 10^{-13}$ | 104               | 18.72 %             | 9,10,15-17 |
| 106989 | 1-butene      | -0.68283              | 0.14248               | $8.2794 \cdot 10^{-3}$  | 0                 | -                   | -          |
| 592767 | 1-heptene     | -0.79056              | 0.28369               | $1.9320 \cdot 10^{-14}$ | 0                 | -                   | -          |
| 115106 | dimethylether | -0.55305              | 0.20344               | $3.0468 \cdot 10^{-13}$ | 40                | 7.98 %              | 9          |
| 75070  | ethanal       | -0.4574               | 0.22717               | $1.3107 \cdot 10^{-13}$ | 0                 | -                   | -          |
| 123728 | butanal       | -0.6685               | 0.30023               | $7.3716 \cdot 10^{-14}$ | 0                 | -                   | -          |
| 111717 | heptanal      | -0.85365              | 0.43313               | $1.3380 \cdot 10^{-13}$ | 0                 | -                   | -          |
| 67641  | acetone       | -0.66809              | 0.31903               | $1.6004 \cdot 10^{-13}$ | 48                | 24.86 %             | 9          |
| 78933  | butanone      | -0.69477              | 0.33519               | $1.0136 \cdot 10^{-13}$ | 0                 | -                   | -          |

## B.4 Simulation Results of each Substance

This section contains tables summarizing temperatures  $T$ , pressures  $p$ , reduced entropies  $s^*$ , viscosities  $\eta$ , self-diffusion coefficients  $D_{\text{self}}$ , and thermal conductivities  $\lambda$  for all investigated substances. The first part of each table describes the simulated state points with the corresponding reduced entropy, calculated using PC-SAFT. The second part provides the simulation results for the transport properties. The respective state points can be assigned via  $s^*$ .

**Table B.9:** Results of ethane

| $\rho/\text{kg/m}^3$ | $T/\text{K}$        | $p/\text{bar}$                          | $s^*$                              |
|----------------------|---------------------|---|------------------------------------|
| 598.62               | 135.55(4)           | 8.4677(130000)                          | -3.3338                            |
| 540.97               | 184.27(3)           | 7.9153(82000)                           | -2.4914                            |
| 450.66               | 247.380(47)         | 11.396(4200)                            | -1.6576                            |
| 308.42               | 310.470(61)         | 61.119(1100)                            | -0.91227                           |
| 16.067               | 246.960(46)         | 9.3787(510)                             | -0.064591                          |
| $s^*$                | $\eta/\text{mPa s}$ | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{mm s}^{-1}$ |
| -3.3338              | 0.388 15(1300)      | 0.222 94(1200)                          | $1.8987(163) \cdot 10^{-5}$        |
| -2.4914              | 0.175 62(530)       | 0.173 01(600)                           | $5.2744(447) \cdot 10^{-5}$        |
| -1.6576              | 0.082 168(1700)     | 0.117 82(860)                           | $1.3650(96) \cdot 10^{-4}$         |
| -0.91227             | 0.035 334(980)      | 0.061 351(2000)                         | $3.6130(305) \cdot 10^{-4}$        |
| -0.064591            | 0.008 186 9(8200)   | 0.015 338(2200)                         | $6.6563(388) \cdot 10^{-3}$        |

**Table B.10:** Results of propane

| $\rho/\text{kg/m}^3$ | $T/\text{K}$        | $p/\text{bar}$                          | $s^*$                              |
|----------------------|---------------------|---|------------------------------------|
| 721.87               | 94.076(17)          | 2.8624(110000)                          | -4.5467                            |
| 671.06               | 143.09(3)           | -4.5537(54000)                          | -3.4475                            |
| 592.59               | 220.060(44)         | 18.218(2700)                            | -2.3164                            |
| 432.67               | 334.470(59)         | 23.536(1100)                            | -1.1917                            |
| 1.1256               | 220.060(28)         | 0.461 98(360)                           | -0.005 494 6                       |
| $s^*$                | $\eta/\text{mPa s}$ | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{mm s}^{-1}$ |
| -4.5467              | 3.1202(1500)        | 0.219 45(850)                           | $1.5769(230) \cdot 10^{-6}$        |
| -3.4475              | 0.576 41(2100)      | 0.181 93(530)                           | $1.1712(104) \cdot 10^{-5}$        |
| -2.3164              | 0.202 68(840)       | 0.134 54(360)                           | $4.8891(474) \cdot 10^{-5}$        |
| -1.1917              | 0.062 521(1200)     | 0.076 826(4400)                         | $1.9923(191) \cdot 10^{-4}$        |
| -0.005 494 6         | 0.006 318 9(8300)   | 0.010 204(1200)                         | $7.4363(914) \cdot 10^{-2}$        |

**Table B.11:** Results of hexane

| $\rho/\text{kg/m}^3$ | $T/\text{K}$        | $p/\text{bar}$                          | $s^*$                              |
|----------------------|---------------------|---|------------------------------------|
| 723.12               | 207.730(36)         | 46.542(2100)                            | -2.7989                            |
| 668.69               | 276.560(43)         | 47.847(1400)                            | -2.1205                            |
| 575.01               | 383.300(84)         | 50.235(960)                             | -1.4148                            |
| 401.83               | 510.880(55)         | 48.334(340)                             | -0.74657                           |
| 7.4016               | 383.300(54)         | 2.4913(76)                              | -0.020003                          |
| $s^*$                | $\eta/\text{mPa s}$ | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{mm s}^{-1}$ |
| -2.7989              | 0.8528(420)         | 0.123 25(780)                           | $1.1487(124) \cdot 10^{-5}$        |
| -2.1205              | 0.334 26(2700)      | 0.112 66(620)                           | $3.4334(340) \cdot 10^{-5}$        |
| -1.4148              | 0.141 19(750)       | 0.093 022(9600)                         | $9.8579(909) \cdot 10^{-5}$        |
| -0.74657             | 0.050 907(1200)     | 0.064 095(5500)                         | $3.0435(304) \cdot 10^{-4}$        |
| -0.020003            | 0.007 597(600)      | 0.019 846(2200)                         | $1.5610(150) \cdot 10^{-2}$        |

**Table B.12:** Results of nonane

| $\rho/\text{kg/m}^3$ | $T/\text{K}$        | $p/\text{bar}$                          | $s^*$                              |
|----------------------|---------------------|---|------------------------------------|
| 761.68               | 219.700(36)         | 2.0632(28000)                           | -2.7275                            |
| 709.08               | 291.540(48)         | 8.9105(14000)                           | -2.0749                            |
| 616.51               | 409.07(6)           | 8.4399(6900)                            | -1.3849                            |
| 437.7                | 563.990(72)         | 13.847(300)                             | -0.723 24                          |
| 2.0972               | 409.100(52)         | 0.528 46(260)                           | -0.005 734                         |
| $s^*$                | $\eta/\text{mPa s}$ | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{mm s}^{-1}$ |
| -2.7275              | 1.973(110)          | 0.158 77(1200)                          | $4.2517(492) \cdot 10^{-6}$        |
| -2.0749              | 0.5737(220)         | 0.132 82(770)                           | $1.9450(153) \cdot 10^{-5}$        |
| -1.3849              | 0.197 07(550)       | 0.106 63(670)                           | $6.8291(520) \cdot 10^{-5}$        |
| -0.723 24            | 0.062 104(1900)     | 0.072 336(3700)                         | $2.4024(168) \cdot 10^{-4}$        |
| -0.005 734           | 0.007 307 9(7800)   | 0.017 784(3800)                         | $4.5883(674) \cdot 10^{-2}$        |



**Table B.13:** Results of 1-butene

| $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $s^*$        |
|----------------------|--------------|----------------|--------------|
| 799.96               | 96.575(20)   | 20.791(2000)   | -4.6778      |
| 749.86               | 147.330(25)  | 1.3977(24000)  | -3.5791      |
| 669.04               | 229.440(37)  | 9.9527(15000)  | -2.4138      |
| 496                  | 367.170(56)  | 19.509(610)    | -1.2184      |
| 0.423 44             | 229.440(22)  | 0.140 55(100)  | -0.002 004 2 |

| $s^*$        | $\eta/\text{mPa s}$ | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{mm s}^{-1}$ |
|--------------|---------------------|---|------------------------------------|
| -4.6778      | 7.7496(7000)        | 0.189 84(550)                           | $6.3405(941) \cdot 10^{-7}$        |
| -3.5791      | 0.947 02(4200)      | 0.176 15(1400)                          | $7.2819(674) \cdot 10^{-6}$        |
| -2.4138      | 0.2834(100)         | 0.135 18(920)                           | $3.6013(260) \cdot 10^{-5}$        |
| -1.2184      | 0.080 721(2700)     | 0.076 723(4400)                         | $1.7151(120) \cdot 10^{-4}$        |
| -0.002 004 2 | 0.006 214 3(8600)   | 0.011 827(3400)                         | $1.6299(350) \cdot 10^{-1}$        |

**Table B.14:** Results of 1-heptene

| $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $s^*$        |
|----------------------|--------------|----------------|--------------|
| 795.34               | 169.730(26)  | 8.0275(25000)  | -3.3766      |
| 745.48               | 232.640(39)  | 11.203(920)    | -2.5909      |
| 657.97               | 337.540(46)  | 9.5989(6700)   | -1.7376      |
| 476.21               | 497.05(8)    | 17.963(400)    | -0.890 39    |
| 1.1139               | 337.550(27)  | 0.307 33(100)  | -0.003 625 4 |

| $s^*$        | $\eta/\text{mPa s}$ | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{mm s}^{-1}$ |
|--------------|---------------------|---|------------------------------------|
| -3.3766      | 3.3615(3300)        | 0.132 87(480)                           | $2.3048(287) \cdot 10^{-6}$        |
| -2.5909      | 0.747 97(4200)      | 0.123 41(570)                           | $1.3433(144) \cdot 10^{-5}$        |
| -1.7376      | 0.232 09(740)       | 0.101 67(380)                           | $5.3644(340) \cdot 10^{-5}$        |
| -0.890 39    | 0.071 884(3500)     | 0.070 874(4500)                         | $2.1462(188) \cdot 10^{-4}$        |
| -0.003 625 4 | 0.007 649(940)      | 0.015 153(1600)                         | $7.9637(1088) \cdot 10^{-2}$       |

**Table B.15:** Results of dimethylether

| $\rho/\text{kg/m}^3$ | $T/\text{K}$        | $p/\text{bar}$                          | $s^*$                              |
|----------------------|---------------------|---|------------------------------------|
| 851.18               | 144.840(23)         | 9.2192(47000)                           | -3.4617                            |
| 782.33               | 202.410(41)         | 14.712(2600)                            | -2.6175                            |
| 670.6                | 286.67(5)           | 7.5628(21000)                           | -1.7364                            |
| 469.54               | 389.660(69)         | 52.462(750)                             | -0.9114                            |
| 6.9791               | 286.670(28)         | 3.3524(150)                             | -0.020621                          |
| $s^*$                | $\eta/\text{mPa s}$ | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{mm s}^{-1}$ |
| -3.4617              | 1.2459(930)         | 0.24626(1100)                           | $6.2692(747) \cdot 10^{-6}$        |
| -2.6175              | 0.3906(200)         | 0.20402(870)                            | $2.5581(237) \cdot 10^{-5}$        |
| -1.7364              | 0.15226(750)        | 0.14048(520)                            | $8.3276(715) \cdot 10^{-5}$        |
| -0.9114              | 0.058587(3000)      | 0.080419(5800)                          | $2.7345(259) \cdot 10^{-4}$        |
| -0.020621            | 0.0089533(6500)     | 0.014683(1300)                          | $1.7419(112) \cdot 10^{-2}$        |

**Table B.16:** Results of ethanal

| $\rho/\text{kg/m}^3$ | $T/\text{K}$        | $p/\text{bar}$                          | $s^*$                              |
|----------------------|---------------------|---|------------------------------------|
| 867.78               | 240.500(59)         | 11.111(4300)                            | -2.7034                            |
| 767.93               | 321.960(58)         | 2.847(2300)                             | -2.0673                            |
| 629.33               | 412.070(87)         | 23.247(1800)                            | -1.4241                            |
| 424.98               | 483.030(88)         | 69.886(590)                             | -0.82771                           |
| 28.14                | 412.06(5)           | 17.683(72)                              | -0.090534                          |
| $s^*$                | $\eta/\text{mPa s}$ | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{mm s}^{-1}$ |
| -2.7034              | 0.48334(1500)       | 0.25121(1300)                           | $2.4264(182) \cdot 10^{-5}$        |
| -2.0673              | 0.20761(490)        | 0.19695(1400)                           | $6.9555(513) \cdot 10^{-5}$        |
| -1.4241              | 0.10372(290)        | 0.13334(1100)                           | $1.6866(132) \cdot 10^{-4}$        |
| -0.82771             | 0.046199(1400)      | 0.076707(4500)                          | $4.0455(412) \cdot 10^{-4}$        |
| -0.090534            | 0.012409(1400)      | 0.023682(2600)                          | $5.8509(579) \cdot 10^{-3}$        |

**Table B.17:** Results of butanal

| $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $s^*$     |
|----------------------|--------------|----------------|-----------|
| 887.6                | 194.480(39)  | 3.9302(31000)  | -3.2082   |
| 816.6                | 270.560(39)  | 5.4477(18000)  | -2.4432   |
| 700.87               | 382.520(79)  | 5.9733(13000)  | -1.6255   |
| 491.26               | 519.630(89)  | 40.601(630)    | -0.85256  |
| 5.1518               | 382.520(36)  | 2.1243(65)     | -0.014342 |

| $s^*$     | $\eta/\text{mPa s}$ | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{mm s}^{-1}$ |
|-----------|---------------------|---|------------------------------------|
| -3.2082   | 2.7232(1400)        | 0.16809(1500)                           | $3.4614(314) \cdot 10^{-6}$        |
| -2.4432   | 0.56526(2000)       | 0.14787(680)                            | $1.9936(205) \cdot 10^{-5}$        |
| -1.6255   | 0.20144(1300)       | 0.11326(690)                            | $7.4787(753) \cdot 10^{-5}$        |
| -0.85256  | 0.067213(4400)      | 0.085947(12000)                         | $2.6619(278) \cdot 10^{-4}$        |
| -0.014342 | 0.010027(1100)      | 0.019798(2100)                          | $2.4747(231) \cdot 10^{-2}$        |

**Table B.18:** Results of heptanal

| $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $s^*$     |
|----------------------|--------------|----------------|-----------|
| 840.85               | 252.130(36)  | 11.371(1500)   | -2.5729   |
| 776.89               | 332.03(5)    | 10.004(1100)   | -1.9492   |
| 666.84               | 455.240(69)  | 9.449(690)     | -1.3006   |
| 470.79               | 604.82(8)    | 30.874(430)    | -0.69529  |
| 5.3559               | 455.220(41)  | 1.6501(57)     | -0.012129 |

| $s^*$     | $\eta/\text{mPa s}$ | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{mm s}^{-1}$ |
|-----------|---------------------|---|------------------------------------|
| -2.5729   | 1.9113(1200)        | 0.1404(60)                              | $5.3695(512) \cdot 10^{-6}$        |
| -1.9492   | 0.55395(4600)       | 0.12885(620)                            | $2.3664(192) \cdot 10^{-5}$        |
| -1.3006   | 0.19292(720)        | 0.10234(390)                            | $8.0727(427) \cdot 10^{-5}$        |
| -0.69529  | 0.064829(2500)      | 0.079916(7100)                          | $2.6343(194) \cdot 10^{-4}$        |
| -0.012129 | 0.0092368(9400)     | 0.020528(1700)                          | $2.2672(221) \cdot 10^{-2}$        |

**Table B.19:** Results of acetone

| $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $s^*$     |
|----------------------|--------------|----------------|-----------|
| 867.62               | 196.340(36)  | 3.6394(40000)  | -2.877    |
| 785.39               | 279.09(5)    | 6.8071(28000)  | -2.1969   |
| 660.67               | 389.350(57)  | 9.1299(14000)  | -1.4677   |
| 458.77               | 508.55(10)   | 58.237(540)    | -0.800 93 |
| 8.7917               | 389.320(34)  | 4.4061(80)     | -0.025 45 |

| $s^*$     | $\eta/\text{mPa s}$ | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{mm s}^{-1}$ |
|-----------|---------------------|---|------------------------------------|
| -2.877    | 1.5422(570)         | 0.205 21(1200)                          | $6.0940(641) \cdot 10^{-6}$        |
| -2.1969   | 0.398 05(1400)      | 0.168 46(950)                           | $2.9230(228) \cdot 10^{-5}$        |
| -1.4677   | 0.154 58(860)       | 0.118 94(400)                           | $9.8146(844) \cdot 10^{-5}$        |
| -0.800 93 | 0.057 889(2100)     | 0.072 404(3900)                         | $3.0273(155) \cdot 10^{-4}$        |
| -0.025 45 | 0.010 419(1300)     | 0.018 319(2100)                         | $1.4738(103) \cdot 10^{-2}$        |

**Table B.20:** Results of butanone

| $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $s^*$      |
|----------------------|--------------|----------------|------------|
| 848.2                | 205.160(42)  | 8.6126(19000)  | -2.8524    |
| 775.85               | 281.70(6)    | 10.668(2000)   | -2.167     |
| 657.08               | 393.700(55)  | 5.8554(8300)   | -1.4307    |
| 431.71               | 527.400(71)  | 43.736(400)    | -0.708 43  |
| 6.0661               | 393.710(44)  | 2.5430(69)     | -0.016 154 |

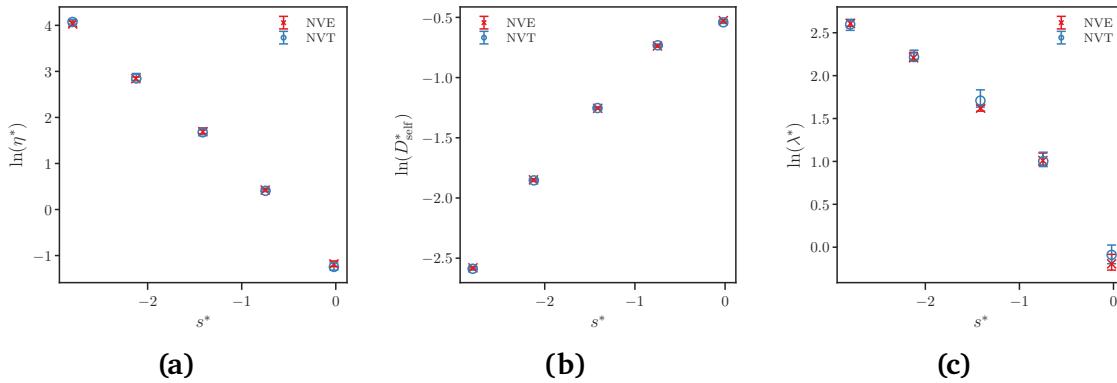
  

| $s^*$      | $\eta/\text{mPa s}$ | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{mm s}^{-1}$ |
|------------|---------------------|---|------------------------------------|
| -2.8524    | 1.7406(810)         | 0.162 44(1100)                          | $5.6837(614) \cdot 10^{-6}$        |
| -2.167     | 0.454 37(1800)      | 0.1351(130)                             | $2.6547(263) \cdot 10^{-5}$        |
| -1.4307    | 0.1593(42)          | 0.102 49(710)                           | $9.1637(928) \cdot 10^{-5}$        |
| -0.708 43  | 0.051 395(1400)     | 0.066 298(5900)                         | $3.2439(255) \cdot 10^{-4}$        |
| -0.016 154 | 0.009 358 4(7300)   | 0.017 866(1700)                         | $2.0597(213) \cdot 10^{-2}$        |

## B.5 Verification of the Nosé-Hoover thermostat

We repeated the simulations of hexane with a switched off thermostat in production to check the consistency with the results we obtained with the Nose-Hoover thermostat. Figure B.1 shows the results for all transport properties in the entropy scaled depiction. The results are equivalent within statistical uncertainties, whether the thermostat is switched on or not.

The results are summarized in table form in table B.21.



**Figure B.1:** Logarithmic depiction of reduced viscosity  $\eta^*$  (a), self-diffusion  $D_{\text{self}}^*$  (b), and thermal conductivity  $\lambda^*$  (c) over residual entropy  $s^*$  for hexane. The figure compares the results of simulations with an Nosé-Hoover thermostat (NVE) and simulations without any thermostat (NVT)

**Table B.21:** Results of hexane (NVE)

| $\rho/\text{kg/m}^3$ | $T/\text{K}$ | $p/\text{bar}$ | $s^*$     |
|----------------------|--------------|----------------|-----------|
| 723.12               | 208.160(66)  | 52.036(2200)   | -2.796    |
| 668.69               | 275.410(35)  | 36.59(110)     | -2.1252   |
| 575.01               | 384.620(17)  | 56.929(810)    | -1.4124   |
| 401.83               | 504.480(29)  | 38.499(410)    | -0.75066  |
| 7.4016               | 386.47(11)   | 2.5158(83)     | -0.019807 |

| $s^*$     | $\eta/\text{mPa s}$ | $\lambda/\text{W m}^{-1} \text{K}^{-1}$ | $D_{\text{self}}/\text{mm s}^{-1}$ |
|-----------|---------------------|---|------------------------------------|
| -2.796    | 0.81781(4600)       | 0.12482(500)                            | $1.1598(126) \cdot 10^{-5}$        |
| -2.1252   | 0.331(13)           | 0.11118(560)                            | $3.4336(312) \cdot 10^{-5}$        |
| -1.4124   | 0.1433(90)          | 0.085467(2900)                          | $9.8678(711) \cdot 10^{-5}$        |
| -0.75066  | 0.051185(1700)      | 0.063674(4600)                          | $2.9936(253) \cdot 10^{-4}$        |
| -0.019807 | 0.0081651(5100)     | 0.018118(1700)                          | $1.5940(158) \cdot 10^{-2}$        |

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## C Supporting Information for: Assessing Entropy Scaling for Mixture Viscosities using Molecular Simulations

This supporting information contains the results of all simulations shown in the figures of chapter 3, in tabulated form. The SI is divided into results of the viscosity calculations in appendix C.1, the results of static properties along isotherms from the GCMC simulations (appendix C.2), and results of the vapour-liquid equilibria of the model mixtures at different temperatures, also from GCMC simulations (appendix C.3).

### C.1 Tabulated Results of the MD Simulations

This chapter provides the viscosities of the investigated model mixtures at various state points as result of the MD simulations. Additionally, all information that is needed for an entropy scaled analysis of the data are given. Table C.1 thereby shows the results of the model mixture consisting of two fluids with different energy parameters (Mixture I:  $\varepsilon_{22}/\varepsilon_{11} = 0.7$ ), table C.2 presents the results of Mixture II with different diameters ( $\sigma_{22}/\sigma_{11} = 1.5$ ), and table C.3 gives the results of the non-ideal mixture of two identical LJ-fluids ( $k_{ij} = 0.25$ ).

The particle fraction  $x_1$  of the standard LJ fluid with  $\sigma_{11} = 1$ ,  $\varepsilon_{11} = 1$ , as well as the number density  $\rho^*$  and the temperature  $T^*$  are simulation parameters that are held constant during the MD simulation. Pressure  $p^*$  and viscosity  $\eta^*$  are direct results of the MD simulations. Entropy  $s^*$  and the CE reference viscosity  $\eta_{\text{ref,CE}}^*$  are needed for the entropy scaled depiction and to calculate the reduced viscosity  $\ln(\eta^\#) = \eta^*/\eta_{\text{ref,CE}}^*$ . Entropy  $s^*$  is a result of the GCMC simulations (see appendix C.2),  $\eta_{\text{ref,CE}}^*$  is calculated according to eq. (4.4). All results are given in reduced LJ units, according to table 4.1. The reported statistical uncertainties are the 95 % confidence interval, calculated as described in the main text in section 4.2.4.



**Table C.1:** Results of the MD-Simulations of Mixture I

| $x_1$ | $T^*$   | $p^*$       | $\rho^*$ | $s^*$    | $\eta^*$      | $\eta_{\text{ref,CE}}^*$ | $\ln(\eta^\#)$ |
|-------|---------|-------------|----------|----------|---------------|--------------------------|----------------|
| 0.00  | 1.00000 | 0.19986(18) | 0.4776   | -1.24985 | 0.44494(1206) | 0.060331                 | 1.9981(270)    |
| 0.00  | 1.05000 | 0.20471(15) | 0.41404  | -1.05426 | 0.36864(988)  | 0.063392                 | 1.7605(270)    |
| 0.00  | 1.10001 | 0.19822(11) | 0.33134  | -0.81221 | 0.28862(743)  | 0.066438                 | 1.4688(260)    |
| 0.00  | 1.15001 | 0.19840(7)  | 0.27815  | -0.66042 | 0.26440(929)  | 0.069459                 | 1.3367(350)    |
| 0.10  | 1.00000 | 0.20423(21) | 0.52161  | -1.41985 | 0.53276(1582) | 0.060331                 | 2.1782(300)    |
| 0.10  | 1.05000 | 0.20219(17) | 0.45681  | -1.19992 | 0.43310(1366) | 0.063392                 | 1.9216(310)    |
| 0.10  | 1.10001 | 0.19759(11) | 0.37432  | -0.94156 | 0.33159(1115) | 0.066438                 | 1.6076(330)    |
| 0.10  | 1.15001 | 0.19751(9)  | 0.30635  | -0.74522 | 0.28053(950)  | 0.069459                 | 1.3959(340)    |
| 0.30  | 1.00000 | 0.19953(25) | 0.58453  | -1.69998 | 0.66000(1481) | 0.060331                 | 2.3924(220)    |
| 0.30  | 1.05000 | 0.19899(25) | 0.53611  | -1.49812 | 0.57192(1634) | 0.063392                 | 2.1997(290)    |
| 0.30  | 1.10000 | 0.19797(21) | 0.47545  | -1.27269 | 0.48965(2919) | 0.066438                 | 1.9974(590)    |
| 0.30  | 1.15000 | 0.19722(14) | 0.39901  | -1.02404 | 0.37397(1361) | 0.069459                 | 1.6834(360)    |
| 0.50  | 1.00000 | 0.19195(36) | 0.63408  | -1.94971 | 0.85432(2082) | 0.060331                 | 2.6505(240)    |
| 0.50  | 1.05000 | 0.19813(29) | 0.59841  | -1.77171 | 0.74111(2416) | 0.063392                 | 2.4588(320)    |
| 0.50  | 1.10000 | 0.19600(25) | 0.55339  | -1.57328 | 0.64848(2580) | 0.066438                 | 2.2784(390)    |
| 0.50  | 1.15000 | 0.19612(24) | 0.49918  | -1.36287 | 0.54698(3687) | 0.069459                 | 2.0637(660)    |
| 0.70  | 1.00000 | 0.20143(30) | 0.67839  | -2.19793 | 1.06525(4621) | 0.060331                 | 2.8711(430)    |
| 0.70  | 1.05000 | 0.19706(34) | 0.6466   | -2.01667 | 0.91253(2894) | 0.063392                 | 2.6669(320)    |
| 0.70  | 1.10000 | 0.19701(34) | 0.61224  | -1.84053 | 0.81766(2866) | 0.066438                 | 2.5102(350)    |
| 0.70  | 1.15000 | 0.19480(30) | 0.57193  | -1.65602 | 0.70469(2861) | 0.069459                 | 2.317(40)      |
| 0.90  | 1.00000 | 0.20074(41) | 0.71264  | -2.41455 | 1.27297(4640) | 0.060331                 | 3.0493(360)    |

Continued on next page

**Continued:** Results of the MD-Simulations of Mixture I

| $x_1$ | $T^*$    | $p^*$        | $\rho^*$ | $s^*$     | $\eta^*$       | $\eta_{\text{ref,CE}}^*$ | $\ln(\eta^\#)$ |
|-------|----------|--------------|----------|-----------|----------------|--------------------------|----------------|
| 0.90  | 1.050 00 | 0.201 59(43) | 0.6867   | -2.244 71 | 1.123 22(2790) | 0.063 392                | 2.8746(250)    |
| 0.90  | 1.100 00 | 0.190 17(34) | 0.655 99 | -2.068 62 | 0.988 31(3683) | 0.066 438                | 2.6997(370)    |
| 0.90  | 1.150 00 | 0.196 62(33) | 0.626 28 | -1.910 91 | 0.863 99(2088) | 0.069 459                | 2.5208(240)    |
| 1.00  | 1.000 00 | 0.180 32(41) | 0.725 11 | -2.502 20 | 1.395 30(5196) | 0.060 331                | 3.141(37)      |
| 1.00  | 1.050 01 | 0.189 44(39) | 0.7016   | -2.338 66 | 1.209 65(3620) | 0.063 392                | 2.9487(300)    |
| 1.00  | 1.100 01 | 0.193 66(41) | 0.675 97 | -2.182 34 | 1.077 59(2629) | 0.066 438                | 2.7862(240)    |
| 1.00  | 1.150 01 | 0.194 34(34) | 0.647 68 | -2.024 77 | 0.982 02(3031) | 0.069 459                | 2.6489(310)    |

**Table C.2:** Results of the MD-Simulations of Mixture II

| $x_1$ | $T^*$    | $p^*$        | $\rho^*$ | $s^*$     | $\eta^*$       | $\eta_{\text{ref,CE}}^*$ | $\ln(\eta^\#)$ |
|-------|----------|--------------|----------|-----------|----------------|--------------------------|----------------|
| 0.00  | 0.900 66 | 0.386 38(11) | 0.125 45 | -3.391 73 | 1.302 99(3714) | 0.024 085                | 3.9908(280)    |
| 0.00  | 1.000 63 | 0.434 54(9)  | 0.122 41 | -3.123 80 | 1.090 51(3846) | 0.026 814                | 3.7055(350)    |
| 0.00  | 1.100 60 | 0.413 67(10) | 0.117 61 | -2.843 98 | 0.857 85(2187) | 0.029 528                | 3.3691(260)    |
| 0.00  | 1.150 58 | 0.380 30(9)  | 0.114 39 | -2.680 80 | 0.776 96(2803) | 0.030 871                | 3.2256(360)    |
| 0.00  | 1.200 58 | 0.410 21(10) | 0.1133   | -2.599 66 | 0.749 67(2846) | 0.0322                   | 3.1477(380)    |
| 0.25  | 0.900 54 | 0.397 40(13) | 0.151 95 | -3.252 38 | 1.389 62(3805) | 0.029 086                | 3.8665(270)    |
| 0.25  | 1.000 48 | 0.394 70(12) | 0.146 31 | -2.948 73 | 1.113 43(3409) | 0.032 381                | 3.5376(300)    |
| 0.25  | 1.100 45 | 0.395 53(10) | 0.140 76 | -2.682 82 | 0.927 88(3249) | 0.035 659                | 3.2589(350)    |
| 0.25  | 1.150 44 | 0.392 96(9)  | 0.137 88 | -2.559 77 | 0.843 78(2860) | 0.037 281                | 3.1194(340)    |
| 0.25  | 1.200 43 | 0.395 60(9)  | 0.135 19 | -2.448 60 | 0.768 82(2897) | 0.038 885                | 2.9842(370)    |

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**Continued:** Results of the MD-Simulations of Mixture II

| $x_1$ | $T^*$   | $p^*$       | $\rho^*$ | $s^*$    | $\eta^*$      | $\eta_{\text{ref,CE}}^*$ | $\ln(\eta^\#)$ |
|-------|---------|-------------|----------|----------|---------------|--------------------------|----------------|
| 0.50  | 0.90037 | 0.40375(14) | 0.19206  | -3.14469 | 1.50269(4414) | 0.035362                 | 3.7494(290)    |
| 0.50  | 1.00032 | 0.38806(13) | 0.18395  | -2.82471 | 1.20366(3018) | 0.039369                 | 3.4201(250)    |
| 0.50  | 1.10031 | 0.39948(12) | 0.17669  | -2.56561 | 0.99305(4972) | 0.043353                 | 3.1314(490)    |
| 0.50  | 1.15030 | 0.39846(13) | 0.17277  | -2.44109 | 0.89990(3217) | 0.045325                 | 2.9884(360)    |
| 0.50  | 1.20027 | 0.40035(12) | 0.16895  | -2.32657 | 0.83481(2403) | 0.047276                 | 2.8712(290)    |
| 0.75  | 0.90018 | 0.43838(20) | 0.2607   | -3.06282 | 1.76497(5746) | 0.043444                 | 3.7044(320)    |
| 0.75  | 1.00016 | 0.41450(15) | 0.2487   | -2.73989 | 1.35419(3713) | 0.048366                 | 3.3322(270)    |
| 0.75  | 1.10015 | 0.40317(18) | 0.23662  | -2.46254 | 1.09831(3805) | 0.053262                 | 3.0263(340)    |
| 0.75  | 1.15013 | 0.39766(15) | 0.23035  | -2.32595 | 0.99220(3061) | 0.055684                 | 2.8802(310)    |
| 0.75  | 1.20014 | 0.39677(15) | 0.22419  | -2.20199 | 0.91333(1927) | 0.058081                 | 2.7553(210)    |
| 1.00  | 0.90000 | 0.21772(19) | 0.38733  | -2.73233 | 1.80848(6025) | 0.054191                 | 3.5077(330)    |
| 1.00  | 1.00000 | 0.35698(23) | 0.37291  | -2.54346 | 1.58366(7875) | 0.060331                 | 3.2676(490)    |
| 1.00  | 1.10000 | 0.34691(20) | 0.35024  | -2.28767 | 1.22764(4357) | 0.066438                 | 2.9166(350)    |
| 1.00  | 1.15000 | 0.44760(19) | 0.34611  | -2.21785 | 1.15217(2175) | 0.069459                 | 2.8087(190)    |
| 1.00  | 1.20000 | 0.39259(19) | 0.33031  | -2.04685 | 1.01169(2472) | 0.072449                 | 2.6365(240)    |

**Table C.3:** Results of the MD-Simulations of Mixture III

| $x_1$ | $T^*$   | $p^*$       | $\rho^*$ | $s^*$    | $\eta^*$      | $\eta_{\text{ref,CE}}^*$ | $\ln(\eta^\#)$ |
|-------|---------|-------------|----------|----------|---------------|--------------------------|----------------|
| 0.00  | 0.95000 | 0.19698(33) | 0.74982  | -2.68675 | 1.58675(3633) | 0.057263                 | 3.3218(230)    |
| 0.00  | 1.00000 | 0.18760(32) | 0.72539  | -2.50800 | 1.36899(3395) | 0.060331                 | 3.122(25)      |

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**Continued:** Results of the MD-Simulations of Mixture III

| $x_1$ | $T^*$   | $p^*$       | $\rho^*$ | $s^*$    | $\eta^*$      | $\eta_{\text{ref,CE}}^*$ | $\ln(\eta^\#)$ |
|-------|---------|-------------|----------|----------|---------------|--------------------------|----------------|
| 0.00  | 1.05000 | 0.21071(25) | 0.70419  | -2.35726 | 1.27043(3656) | 0.063392                 | 2.9978(290)    |
| 0.00  | 1.10000 | 0.22152(27) | 0.68035  | -2.21039 | 1.13288(3913) | 0.066438                 | 2.8363(340)    |
| 0.00  | 1.15000 | 0.20760(24) | 0.65003  | -2.03084 | 0.95903(2755) | 0.069459                 | 2.6252(290)    |
| 0.10  | 0.95000 | 0.20897(42) | 0.7332   | -2.56611 | 1.42877(4973) | 0.057263                 | 3.2169(350)    |
| 0.10  | 1.00000 | 0.19821(40) | 0.70646  | -2.38136 | 1.21576(3090) | 0.060331                 | 3.0033(250)    |
| 0.10  | 1.05000 | 0.19818(36) | 0.67938  | -2.20602 | 1.05415(2739) | 0.063392                 | 2.8111(260)    |
| 0.10  | 1.10000 | 0.20004(38) | 0.6506   | -2.04614 | 0.97030(2979) | 0.066438                 | 2.6813(310)    |
| 0.10  | 1.15000 | 0.19019(33) | 0.61533  | -1.85824 | 0.83945(2823) | 0.069459                 | 2.492(33)      |
| 0.10  | 1.20000 | 0.19774(25) | 0.58073  | -1.69687 | 0.75048(3503) | 0.072449                 | 2.3378(470)    |
| 0.20  | 0.95000 | 0.27747(46) | 0.72751  | -2.64662 | 1.34436(3972) | 0.057263                 | 3.156(29)      |
| 0.20  | 1.00000 | 0.19698(38) | 0.68871  | -2.28103 | 1.14809(4245) | 0.060331                 | 2.946(37)      |
| 0.20  | 1.05000 | 0.20097(38) | 0.66003  | -2.10467 | 0.99472(3738) | 0.063392                 | 2.7531(370)    |
| 0.20  | 1.10000 | 0.19701(35) | 0.62677  | -1.92638 | 0.87104(3101) | 0.066438                 | 2.5734(350)    |
| 0.20  | 1.15000 | 0.20014(36) | 0.59121  | -1.74662 | 0.75780(2721) | 0.069459                 | 2.3897(360)    |
| 0.30  | 1.05000 | 0.19954(40) | 0.64498  | -2.03594 | 0.89919(2202) | 0.063392                 | 2.6522(250)    |
| 0.30  | 1.10000 | 0.19685(40) | 0.60891  | -1.84382 | 0.79311(3453) | 0.066438                 | 2.4797(430)    |
| 0.30  | 1.15000 | 0.19764(31) | 0.56859  | -1.65227 | 0.67894(1962) | 0.069459                 | 2.2798(290)    |
| 0.50  | 1.05000 | 0.20443(44) | 0.63441  | -1.99636 | 0.87584(2532) | 0.063392                 | 2.6258(290)    |
| 0.50  | 1.10000 | 0.19854(31) | 0.59474  | -1.78226 | 0.77162(3386) | 0.066438                 | 2.4522(430)    |
| 0.50  | 1.15000 | 0.19899(30) | 0.55071  | -1.58050 | 0.65542(2546) | 0.069459                 | 2.2445(390)    |
| 0.70  | 1.05000 | 0.19996(42) | 0.64498  | -2.03563 | 0.91755(3160) | 0.063392                 | 2.6724(340)    |

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Continued: Results of the MD-Simulations of Mixture III

| $x_1$ | $T^*$   | $p^*$       | $\rho^*$ | $s^*$    | $\eta^*$      | $\eta_{\text{ref,CE}}^*$ | $\ln(\eta^\#)$ |
|-------|---------|-------------|----------|----------|---------------|--------------------------|----------------|
| 0.70  | 1.10001 | 0.19708(34) | 0.60891  | -1.84252 | 0.78062(1952) | 0.066438                 | 2.4638(250)    |
| 0.70  | 1.15001 | 0.19755(32) | 0.56859  | -1.65448 | 0.67656(1763) | 0.069459                 | 2.2763(260)    |
| 0.80  | 0.95000 | 0.27718(37) | 0.72751  | -2.63361 | 1.36954(5619) | 0.057263                 | 3.1746(410)    |
| 0.80  | 1.00000 | 0.19707(29) | 0.68871  | -2.28280 | 1.11720(2031) | 0.060331                 | 2.9187(180)    |
| 0.80  | 1.05000 | 0.20114(24) | 0.66003  | -2.10537 | 0.99109(3205) | 0.063392                 | 2.7495(320)    |
| 0.80  | 1.10000 | 0.19752(25) | 0.62677  | -1.92203 | 0.85801(2340) | 0.066438                 | 2.5583(270)    |
| 0.80  | 1.15000 | 0.20006(22) | 0.59121  | -1.74319 | 0.75206(2681) | 0.069459                 | 2.3821(350)    |
| 0.80  | 1.20000 | 0.19572(19) | 0.54587  | -1.55518 | 0.63985(1569) | 0.072449                 | 2.1783(240)    |
| 0.90  | 0.95000 | 0.20899(32) | 0.7332   | -2.53710 | 1.38814(4001) | 0.057263                 | 3.1881(290)    |
| 0.90  | 1.00000 | 0.19855(30) | 0.70646  | -2.37278 | 1.23233(3229) | 0.060331                 | 3.0168(260)    |
| 0.90  | 1.05000 | 0.19807(28) | 0.67938  | -2.20897 | 1.10742(4178) | 0.063392                 | 2.8605(370)    |
| 0.90  | 1.10000 | 0.20012(27) | 0.6506   | -2.03433 | 0.98208(3831) | 0.066438                 | 2.6934(390)    |
| 0.90  | 1.15000 | 0.19027(25) | 0.61533  | -1.87363 | 0.81993(1984) | 0.069459                 | 2.4685(240)    |
| 0.90  | 1.20000 | 0.19788(21) | 0.58073  | -1.69065 | 0.73611(2067) | 0.072449                 | 2.3185(280)    |

## C.2 Tabulated Results of the isothermal GCMC Simulations

This chapter provides static energetic and entropic properties of the investigated model mixtures at various state points as result of the isothermal GCMC simulations. The tables are structured by mixture and temperature. Tables C.4–C.7 thereby show the results of the model mixture consisting of two fluids with different energy parameters (Mixture I:  $\varepsilon_{22}/\varepsilon_{11} = 0.7$ ), tables C.8–C.13 present the results of Mixture II with different diameters ( $\sigma_{22}/\sigma_{11} = 1.5$ ), and tables C.14–C.20 give the results of the non-ideal mixture of two identical LJ-fluids ( $k_{ij} = 0.25$ ).

The particle fraction  $x_1$  of the standard LJ fluid with  $\sigma_{11} = 1, \varepsilon_{11} = 1$ , as well as the temperature  $T^*$  are simulation parameters that are held constant during the simulations. The specific volume  $v^*$  and the number density  $\rho^*$ , the specific internal energy  $u_{\text{res}}^*$ , the specific enthalpy  $h_{\text{res}}^*$ , and the specific Gibbs energy  $g_{\text{res}}^*$  are calculated using histogram reweighting as described in section 4.2.3. Entropy  $s^*$  is calculated with eq. (4.19). All results are given in reduced LJ units, according to table 4.1.

**Table C.4:** Results of the GCMC-Simulations of Mixture I at  $p^* = 0.20$  and  $T^* = 1.00$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.001 | 1.00  | 2.093 78 | 0.477 60 | -2.245 86          | -1.577 25          | -2.827 10          | -1.249 85 |
| 0.050 | 1.00  | 2.007 21 | 0.498 20 | -2.393 26          | -1.663 95          | -2.991 82          | -1.327 87 |
| 0.100 | 1.00  | 1.917 15 | 0.521 61 | -2.558 44          | -1.755 16          | -3.175 01          | -1.419 85 |
| 0.150 | 1.00  | 1.856 43 | 0.538 67 | -2.699 39          | -1.835 27          | -3.328 10          | -1.492 83 |
| 0.200 | 1.00  | 1.799 55 | 0.555 70 | -2.842 99          | -1.917 34          | -3.483 08          | -1.565 74 |
| 0.250 | 1.00  | 1.750 23 | 0.571 35 | -2.984 50          | -1.997 86          | -3.634 45          | -1.636 59 |
| 0.300 | 1.00  | 1.710 78 | 0.584 53 | -3.117 31          | -2.075 17          | -3.775 15          | -1.699 98 |
| 0.350 | 1.00  | 1.675 28 | 0.596 91 | -3.249 18          | -2.153 55          | -3.914 12          | -1.760 57 |
| 0.400 | 1.00  | 1.637 53 | 0.610 68 | -3.391 75          | -2.236 81          | -4.064 24          | -1.827 44 |
| 0.450 | 1.00  | 1.603 21 | 0.623 75 | -3.534 34          | -2.319 69          | -4.213 69          | -1.894 01 |
| 0.500 | 1.00  | 1.577 09 | 0.634 08 | -3.664 85          | -2.399 72          | -4.349 43          | -1.949 71 |
| 0.550 | 1.00  | 1.546 76 | 0.646 51 | -3.810 12          | -2.484 48          | -4.500 76          | -2.016 29 |
| 0.600 | 1.00  | 1.520 94 | 0.657 49 | -3.950 82          | -2.569 02          | -4.646 63          | -2.077 61 |
| 0.650 | 1.00  | 1.497 91 | 0.667 60 | -4.089 52          | -2.654 68          | -4.789 94          | -2.135 26 |
| 0.700 | 1.00  | 1.474 08 | 0.678 39 | -4.235 38          | -2.742 64          | -4.940 57          | -2.197 93 |
| 0.750 | 1.00  | 1.453 33 | 0.688 08 | -4.377 35          | -2.830 49          | -5.086 68          | -2.256 19 |

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**Continued:** Results of the GCMC-Simulations of Mixture I at  $p^* = 0.20$  and  $T^* = 1.00$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.800 | 1.00  | 1.435 46 | 0.696 64 | -4.515 07          | -2.918 90          | -5.227 98          | -2.309 08 |
| 0.850 | 1.00  | 1.422 16 | 0.703 16 | -4.643 38          | -3.005 78          | -5.358 95          | -2.353 17 |
| 0.900 | 1.00  | 1.403 23 | 0.712 64 | -4.792 89          | -3.097 69          | -5.512 25          | -2.414 55 |
| 0.950 | 1.00  | 1.387 97 | 0.720 48 | -4.934 23          | -3.189 39          | -5.656 64          | -2.467 25 |
| 0.999 | 1.00  | 1.379 11 | 0.725 11 | -5.053 85          | -3.275 83          | -5.778 03          | -2.502 20 |

**Table C.5:** Results of the GCMC-Simulations of Mixture I at  $p^* = 0.20$  and  $T^* = 1.05$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.001 | 1.05  | 2.415 22 | 0.414 04 | -1.947 75          | -1.407 73          | -2.514 71          | -1.054 26 |
| 0.050 | 1.05  | 2.268 12 | 0.440 89 | -2.114 33          | -1.512 42          | -2.710 71          | -1.141 23 |
| 0.100 | 1.05  | 2.189 11 | 0.456 81 | -2.238 34          | -1.590 60          | -2.850 52          | -1.199 92 |
| 0.150 | 1.05  | 2.097 65 | 0.476 72 | -2.385 08          | -1.679 88          | -3.015 55          | -1.272 07 |
| 0.200 | 1.05  | 2.017 94 | 0.495 55 | -2.530 46          | -1.767 81          | -3.176 87          | -1.341 96 |
| 0.250 | 1.05  | 1.934 11 | 0.517 03 | -2.692 84          | -1.862 17          | -3.356 02          | -1.422 72 |
| 0.300 | 1.05  | 1.865 27 | 0.536 11 | -2.848 46          | -1.952 38          | -3.525 41          | -1.498 12 |
| 0.350 | 1.05  | 1.811 91 | 0.551 90 | -2.991 91          | -2.037 28          | -3.679 53          | -1.564 05 |
| 0.400 | 1.05  | 1.758 08 | 0.568 80 | -3.144 77          | -2.125 54          | -3.843 16          | -1.635 82 |
| 0.450 | 1.05  | 1.711 67 | 0.584 23 | -3.294 78          | -2.211 68          | -4.002 44          | -1.705 49 |
| 0.500 | 1.05  | 1.671 09 | 0.598 41 | -3.441 36          | -2.296 84          | -4.157 14          | -1.771 71 |
| 0.550 | 1.05  | 1.636 55 | 0.611 04 | -3.583 13          | -2.380 75          | -4.305 82          | -1.833 40 |
| 0.600 | 1.05  | 1.604 92 | 0.623 08 | -3.725 05          | -2.465 18          | -4.454 06          | -1.894 17 |
| 0.650 | 1.05  | 1.574 55 | 0.635 10 | -3.869 84          | -2.551 37          | -4.604 93          | -1.955 77 |
| 0.700 | 1.05  | 1.546 55 | 0.646 60 | -4.015 23          | -2.638 42          | -4.755 92          | -2.016 67 |
| 0.750 | 1.05  | 1.520 74 | 0.657 58 | -4.161 12          | -2.726 24          | -4.906 97          | -2.076 89 |
| 0.800 | 1.05  | 1.497 82 | 0.667 64 | -4.303 89          | -2.814 55          | -5.054 32          | -2.133 12 |
| 0.850 | 1.05  | 1.477 04 | 0.677 03 | -4.446 43          | -2.903 42          | -5.201 03          | -2.188 19 |
| 0.900 | 1.05  | 1.456 24 | 0.686 70 | -4.592 95          | -2.994 75          | -5.351 70          | -2.244 71 |
| 0.950 | 1.05  | 1.435 23 | 0.696 75 | -4.745 48          | -3.088 46          | -5.508 44          | -2.304 73 |
| 0.999 | 1.05  | 1.425 31 | 0.701 60 | -4.864 10          | -3.173 44          | -5.629 04          | -2.338 66 |

**Table C.6:** Results of the GCMC-Simulations of Mixture I at  $p^* = 0.20$  and  $T^* = 1.10$

| $x_1$ | $T^*$ | $\nu^*$ | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$    |
|-------|-------|---------|----------|--------------------|--------------------|--------------------|----------|
| 0.001 | 1.10  | 3.01807 | 0.33134  | -1.56786           | -1.17081           | -2.06424           | -0.81221 |
| 0.050 | 1.10  | 2.86492 | 0.34905  | -1.68464           | -1.26032           | -2.21166           | -0.86485 |
| 0.100 | 1.10  | 2.67151 | 0.37432  | -1.84284           | -1.37282           | -2.40854           | -0.94156 |
| 0.150 | 1.10  | 2.50945 | 0.39849  | -2.00110           | -1.47976           | -2.59921           | -1.01768 |
| 0.200 | 1.10  | 2.35474 | 0.42468  | -2.17241           | -1.59053           | -2.80146           | -1.10085 |
| 0.250 | 1.10  | 2.21473 | 0.45152  | -2.35276           | -1.70117           | -3.00982           | -1.18968 |
| 0.300 | 1.10  | 2.10329 | 0.47545  | -2.52440           | -1.80379           | -3.20374           | -1.27269 |
| 0.350 | 1.10  | 2.01105 | 0.49725  | -2.69057           | -1.90130           | -3.38836           | -1.35187 |
| 0.400 | 1.10  | 1.93141 | 0.51776  | -2.85518           | -1.99653           | -3.56889           | -1.42942 |
| 0.450 | 1.10  | 1.86420 | 0.53642  | -3.01520           | -2.08897           | -3.74236           | -1.50309 |
| 0.500 | 1.10  | 1.80705 | 0.55339  | -3.17076           | -2.17875           | -3.90935           | -1.57328 |
| 0.550 | 1.10  | 1.75489 | 0.56984  | -3.32759           | -2.26810           | -4.07661           | -1.64409 |
| 0.600 | 1.10  | 1.71129 | 0.58436  | -3.47772           | -2.35531           | -4.23547           | -1.70924 |
| 0.650 | 1.10  | 1.67271 | 0.59783  | -3.62571           | -2.44204           | -4.39117           | -1.77193 |
| 0.700 | 1.10  | 1.63335 | 0.61224  | -3.78330           | -2.53205           | -4.55663           | -1.84053 |
| 0.750 | 1.10  | 1.59870 | 0.62551  | -3.93816           | -2.62229           | -4.71842           | -1.90557 |
| 0.800 | 1.10  | 1.57012 | 0.63690  | -4.08499           | -2.71142           | -4.87097           | -1.96322 |
| 0.850 | 1.10  | 1.54703 | 0.64640  | -4.22357           | -2.79832           | -5.01416           | -2.01440 |
| 0.900 | 1.10  | 1.52440 | 0.65599  | -4.36579           | -2.88542           | -5.16091           | -2.06862 |
| 0.950 | 1.10  | 1.49968 | 0.66681  | -4.51797           | -2.97583           | -5.31803           | -2.12928 |
| 0.999 | 1.10  | 1.47935 | 0.67597  | -4.66009           | -3.06364           | -5.46422           | -2.18234 |

**Table C.7:** Results of the GCMC-Simulations of Mixture I at  $p^* = 0.20$  and  $T^* = 1.15$

| $x_1$ | $T^*$ | $\nu^*$ | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$    |
|-------|-------|---------|----------|--------------------|--------------------|--------------------|----------|
| 0.001 | 1.15  | 3.59524 | 0.27815  | -1.31491           | -0.98638           | -1.74586           | -0.66042 |
| 0.001 | 1.15  | 3.59524 | 0.27815  | -1.31491           | -0.98638           | -1.74586           | -0.66042 |
| 0.050 | 1.15  | 3.44182 | 0.29054  | -1.40571           | -1.06417           | -1.86735           | -0.69841 |
| 0.050 | 1.15  | 3.44182 | 0.29054  | -1.40571           | -1.06417           | -1.86735           | -0.69841 |
| 0.100 | 1.15  | 3.26428 | 0.30635  | -1.51462           | -1.15477           | -2.01177           | -0.74522 |
| 0.100 | 1.15  | 3.26428 | 0.30635  | -1.51462           | -1.15477           | -2.01177           | -0.74522 |

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**Continued:** Results of the GCMC-Simulations of Mixture I at  $p^* = 0.20$  and  $T^* = 1.15$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.150 | 1.15  | 3.076 07 | 0.325 09 | -1.640 23          | -1.254 56          | -2.175 02          | -0.800 40 |
| 0.150 | 1.15  | 3.076 07 | 0.325 09 | -1.640 23          | -1.254 56          | -2.175 02          | -0.800 40 |
| 0.200 | 1.15  | 2.882 10 | 0.346 97 | -1.785 52          | -1.363 22          | -2.359 10          | -0.865 98 |
| 0.200 | 1.15  | 2.882 10 | 0.346 97 | -1.785 52          | -1.363 22          | -2.359 10          | -0.865 98 |
| 0.250 | 1.15  | 2.689 48 | 0.371 82 | -1.948 63          | -1.479 08          | -2.560 74          | -0.940 57 |
| 0.250 | 1.15  | 2.689 48 | 0.371 82 | -1.948 63          | -1.479 08          | -2.560 74          | -0.940 57 |
| 0.300 | 1.15  | 2.506 21 | 0.399 01 | -2.127 85          | -1.598 96          | -2.776 60          | -1.024 04 |
| 0.300 | 1.15  | 2.506 21 | 0.399 01 | -2.127 85          | -1.598 96          | -2.776 60          | -1.024 04 |
| 0.350 | 1.15  | 2.346 17 | 0.426 23 | -2.313 11          | -1.716 26          | -2.993 88          | -1.110 97 |
| 0.350 | 1.15  | 2.346 17 | 0.426 23 | -2.313 11          | -1.716 26          | -2.993 88          | -1.110 97 |
| 0.400 | 1.15  | 2.215 24 | 0.451 42 | -2.493 38          | -1.826 30          | -3.200 33          | -1.194 80 |
| 0.400 | 1.15  | 2.215 24 | 0.451 42 | -2.493 38          | -1.826 30          | -3.200 33          | -1.194 80 |
| 0.450 | 1.15  | 2.100 50 | 0.476 08 | -2.676 07          | -1.934 26          | -3.405 97          | -1.279 75 |
| 0.450 | 1.15  | 2.100 50 | 0.476 08 | -2.676 07          | -1.934 26          | -3.405 97          | -1.279 75 |
| 0.500 | 1.15  | 2.003 29 | 0.499 18 | -2.856 33          | -2.038 38          | -3.605 67          | -1.362 87 |
| 0.500 | 1.15  | 2.003 29 | 0.499 18 | -2.856 33          | -2.038 38          | -3.605 67          | -1.362 87 |
| 0.550 | 1.15  | 1.923 50 | 0.519 89 | -3.029 48          | -2.137 33          | -3.794 78          | -1.441 26 |
| 0.550 | 1.15  | 1.923 50 | 0.519 89 | -3.029 48          | -2.137 33          | -3.794 78          | -1.441 26 |
| 0.600 | 1.15  | 1.857 69 | 0.538 30 | -3.194 33          | -2.231 69          | -3.972 79          | -1.514 00 |
| 0.600 | 1.15  | 1.857 69 | 0.538 30 | -3.194 33          | -2.231 69          | -3.972 79          | -1.514 00 |
| 0.650 | 1.15  | 1.798 38 | 0.556 06 | -3.360 39          | -2.325 36          | -4.150 71          | -1.587 27 |
| 0.650 | 1.15  | 1.798 38 | 0.556 06 | -3.360 39          | -2.325 36          | -4.150 71          | -1.587 27 |
| 0.700 | 1.15  | 1.748 48 | 0.571 93 | -3.520 40          | -2.416 28          | -4.320 71          | -1.656 02 |
| 0.700 | 1.15  | 1.748 48 | 0.571 93 | -3.520 40          | -2.416 28          | -4.320 71          | -1.656 02 |
| 0.750 | 1.15  | 1.701 48 | 0.587 72 | -3.684 09          | -2.508 53          | -4.493 79          | -1.726 32 |
| 0.750 | 1.15  | 1.701 48 | 0.587 72 | -3.684 09          | -2.508 53          | -4.493 79          | -1.726 32 |
| 0.800 | 1.15  | 1.659 51 | 0.602 59 | -3.846 77          | -2.601 33          | -4.664 87          | -1.794 38 |
| 0.800 | 1.15  | 1.659 51 | 0.602 59 | -3.846 77          | -2.601 33          | -4.664 87          | -1.794 38 |
| 0.850 | 1.15  | 1.622 97 | 0.616 15 | -4.006 16          | -2.693 71          | -4.831 57          | -1.859 01 |
| 0.850 | 1.15  | 1.622 97 | 0.616 15 | -4.006 16          | -2.693 71          | -4.831 57          | -1.859 01 |
| 0.900 | 1.15  | 1.596 74 | 0.626 28 | -4.147 43          | -2.780 53          | -4.978 08          | -1.910 91 |
| 0.900 | 1.15  | 1.596 74 | 0.626 28 | -4.147 43          | -2.780 53          | -4.978 08          | -1.910 91 |

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**Continued:** Results of the GCMC-Simulations of Mixture I at  $p^* = 0.20$  and  $T^* = 1.15$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.950 | 1.15  | 1.571 52 | 0.636 33 | -4.290 99          | -2.868 32          | -5.126 68          | -1.963 79 |
| 0.950 | 1.15  | 1.571 52 | 0.636 33 | -4.290 99          | -2.868 32          | -5.126 68          | -1.963 79 |
| 0.999 | 1.15  | 1.543 98 | 0.647 68 | -4.445 34          | -2.958 05          | -5.286 54          | -2.024 77 |
| 0.999 | 1.15  | 1.543 98 | 0.647 68 | -4.445 34          | -2.958 05          | -5.286 54          | -2.024 77 |

**Table C.8:** Results of the GCMC-Simulations of Mixture II at  $p^* = 0.40$  and  $T^* = 0.90$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.001 | 0.90  | 3.985 80 | 0.250 89 | -5.895 36          | -2.148 48          | -5.201 04          | -3.391 73 |
| 0.050 | 0.90  | 3.847 77 | 0.259 89 | -5.870 44          | -2.204 44          | -5.231 34          | -3.363 22 |
| 0.100 | 0.90  | 3.721 77 | 0.268 69 | -5.825 27          | -2.259 77          | -5.236 56          | -3.307 54 |
| 0.150 | 0.90  | 3.582 22 | 0.279 16 | -5.798 70          | -2.316 28          | -5.265 81          | -3.277 26 |
| 0.200 | 0.90  | 3.431 12 | 0.291 45 | -5.790 70          | -2.373 83          | -5.318 25          | -3.271 58 |
| 0.250 | 0.90  | 3.290 50 | 0.303 91 | -5.771 61          | -2.428 28          | -5.355 42          | -3.252 38 |
| 0.300 | 0.90  | 3.156 22 | 0.316 83 | -5.742 48          | -2.480 97          | -5.379 99          | -3.221 13 |
| 0.350 | 0.90  | 3.017 50 | 0.331 40 | -5.722 42          | -2.535 81          | -5.415 42          | -3.199 57 |
| 0.400 | 0.90  | 2.879 68 | 0.347 26 | -5.702 21          | -2.590 47          | -5.450 34          | -3.177 64 |
| 0.450 | 0.90  | 2.741 98 | 0.364 70 | -5.684 89          | -2.644 40          | -5.488 09          | -3.159 66 |
| 0.500 | 0.90  | 2.603 41 | 0.384 11 | -5.671 72          | -2.700 14          | -5.530 36          | -3.144 69 |
| 0.550 | 0.90  | 2.468 28 | 0.405 14 | -5.653 25          | -2.755 98          | -5.565 94          | -3.122 18 |
| 0.600 | 0.90  | 2.331 24 | 0.428 96 | -5.640 86          | -2.813 80          | -5.608 37          | -3.105 08 |
| 0.650 | 0.90  | 2.197 88 | 0.454 98 | -5.623 36          | -2.872 27          | -5.644 21          | -3.079 93 |
| 0.700 | 0.90  | 2.058 99 | 0.485 68 | -5.620 50          | -2.934 54          | -5.696 90          | -3.069 29 |
| 0.750 | 0.90  | 1.917 92 | 0.521 40 | -5.627 16          | -3.003 45          | -5.759 99          | -3.062 82 |
| 0.800 | 0.90  | 1.778 67 | 0.562 22 | -5.635 29          | -3.081 34          | -5.823 82          | -3.047 20 |
| 0.850 | 0.90  | 1.656 83 | 0.603 56 | -5.596 54          | -3.155 35          | -5.833 81          | -2.976 07 |
| 0.900 | 0.90  | 1.536 43 | 0.650 86 | -5.553 71          | -3.226 47          | -5.839 14          | -2.902 97 |
| 0.950 | 0.90  | 1.426 44 | 0.701 04 | -5.463 49          | -3.270 39          | -5.792 91          | -2.802 80 |
| 0.999 | 0.90  | 1.290 89 | 0.774 66 | -5.444 47          | -3.369 01          | -5.828 11          | -2.732 33 |

**Table C.9:** Results of the GCMC-Simulations of Mixture II at  $p^* = 0.40$  and  $T^* = 1.00$

| $x_1$ | $T^*$ | $v^*$   | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$    |
|-------|-------|---------|----------|--------------------|--------------------|--------------------|----------|
| 0.001 | 1.00  | 4.08472 | 0.24481  | -5.67755           | -1.91987           | -5.04366           | -3.12380 |
| 0.050 | 1.00  | 3.96186 | 0.25241  | -5.63469           | -1.96821           | -5.04995           | -3.08174 |
| 0.100 | 1.00  | 3.83080 | 0.26104  | -5.59259           | -2.01787           | -5.06027           | -3.04240 |
| 0.150 | 1.00  | 3.70117 | 0.27019  | -5.54976           | -2.07020           | -5.06929           | -2.99909 |
| 0.200 | 1.00  | 3.56167 | 0.28077  | -5.51916           | -2.12552           | -5.09450           | -2.96897 |
| 0.250 | 1.00  | 3.41739 | 0.29262  | -5.49775           | -2.18206           | -5.13079           | -2.94873 |
| 0.300 | 1.00  | 3.27473 | 0.30537  | -5.47393           | -2.23776           | -5.16404           | -2.92628 |
| 0.350 | 1.00  | 3.13344 | 0.31914  | -5.45002           | -2.29364           | -5.19664           | -2.90301 |
| 0.400 | 1.00  | 2.99152 | 0.33428  | -5.42852           | -2.35055           | -5.23191           | -2.88136 |
| 0.450 | 1.00  | 2.85292 | 0.35052  | -5.40294           | -2.40623           | -5.26177           | -2.85554 |
| 0.500 | 1.00  | 2.71817 | 0.36789  | -5.37202           | -2.46004           | -5.28475           | -2.82471 |
| 0.550 | 1.00  | 2.57883 | 0.38777  | -5.35057           | -2.51612           | -5.31903           | -2.80292 |
| 0.600 | 1.00  | 2.43353 | 0.41093  | -5.34281           | -2.57633           | -5.36940           | -2.79307 |
| 0.650 | 1.00  | 2.29194 | 0.43631  | -5.33023           | -2.63612           | -5.41345           | -2.77733 |
| 0.700 | 1.00  | 2.14558 | 0.46608  | -5.33059           | -2.70180           | -5.47236           | -2.77056 |
| 0.750 | 1.00  | 2.01047 | 0.49740  | -5.30827           | -2.76419           | -5.50408           | -2.73989 |
| 0.800 | 1.00  | 1.87780 | 0.53254  | -5.28246           | -2.82838           | -5.53134           | -2.70297 |
| 0.850 | 1.00  | 1.74236 | 0.57393  | -5.26338           | -2.89756           | -5.56643           | -2.66888 |
| 0.900 | 1.00  | 1.59658 | 0.62634  | -5.27619           | -2.97130           | -5.63756           | -2.66625 |
| 0.950 | 1.00  | 1.47344 | 0.67869  | -5.22335           | -3.01878           | -5.63397           | -2.61519 |
| 0.999 | 1.00  | 1.34081 | 0.74582  | -5.20172           | -3.12194           | -5.66540           | -2.54346 |

**Table C.10:** Results of the GCMC-Simulations of Mixture II at  $p^* = 0.40$  and  $T^* = 1.05$

| $x_1$ | $T^*$ | $v^*$   | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$    |
|-------|-------|---------|----------|--------------------|--------------------|--------------------|----------|
| 0.001 | 1.05  | 4.19191 | 0.23855  | -5.51686           | -1.80071           | -4.89010           | -2.94228 |
| 0.050 | 1.05  | 4.05588 | 0.24656  | -5.48038           | -1.85194           | -4.90802           | -2.91056 |
| 0.100 | 1.05  | 3.91173 | 0.25564  | -5.45057           | -1.90504           | -4.93588           | -2.88651 |
| 0.150 | 1.05  | 3.76770 | 0.26541  | -5.42255           | -1.95835           | -4.96547           | -2.86392 |
| 0.200 | 1.05  | 3.62552 | 0.27582  | -5.39346           | -2.01237           | -4.99326           | -2.83894 |
| 0.250 | 1.05  | 3.48148 | 0.28723  | -5.36728           | -2.06789           | -5.02469           | -2.81600 |

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**Continued:** Results of the GCMC-Simulations of Mixture II at  $p^* = 0.40$  and  $T^* = 1.05$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.300 | 1.05  | 3.338 75 | 0.299 51 | -5.339 92          | -2.123 77          | -5.054 42          | -2.791 10 |
| 0.350 | 1.05  | 3.197 45 | 0.312 75 | -5.312 31          | -2.179 99          | -5.083 33          | -2.765 08 |
| 0.400 | 1.05  | 3.054 68 | 0.327 37 | -5.287 72          | -2.237 05          | -5.115 84          | -2.741 71 |
| 0.450 | 1.05  | 2.912 57 | 0.343 34 | -5.263 61          | -2.293 62          | -5.148 58          | -2.719 01 |
| 0.500 | 1.05  | 2.770 67 | 0.360 92 | -5.239 90          | -2.351 01          | -5.181 63          | -2.695 82 |
| 0.550 | 1.05  | 2.629 64 | 0.380 28 | -5.216 62          | -2.408 48          | -5.214 77          | -2.672 65 |
| 0.600 | 1.05  | 2.488 61 | 0.401 83 | -5.195 04          | -2.466 18          | -5.249 60          | -2.650 87 |
| 0.650 | 1.05  | 2.343 57 | 0.426 70 | -5.183 48          | -2.527 29          | -5.296 05          | -2.636 92 |
| 0.700 | 1.05  | 2.203 39 | 0.453 85 | -5.163 22          | -2.586 85          | -5.331 86          | -2.614 29 |
| 0.750 | 1.05  | 2.059 81 | 0.485 48 | -5.152 65          | -2.651 80          | -5.378 73          | -2.597 08 |
| 0.800 | 1.05  | 1.925 65 | 0.519 30 | -5.121 68          | -2.713 18          | -5.401 42          | -2.560 22 |
| 0.850 | 1.05  | 1.781 88 | 0.561 21 | -5.115 76          | -2.776 25          | -5.453 01          | -2.549 29 |
| 0.900 | 1.05  | 1.640 84 | 0.609 44 | -5.105 61          | -2.840 63          | -5.499 27          | -2.532 04 |
| 0.950 | 1.05  | 1.524 25 | 0.656 06 | -5.020 75          | -2.884 20          | -5.461 05          | -2.454 15 |
| 0.999 | 1.05  | 1.373 46 | 0.728 09 | -5.049 69          | -2.979 33          | -5.550 30          | -2.448 54 |

**Table C.11:** Results of the GCMC-Simulations of Mixture II at  $p^* = 0.40$  and  $T^* = 1.10$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.001 | 1.10  | 4.251 21 | 0.235 23 | -5.410 80          | -1.681 93          | -4.810 32          | -2.843 98 |
| 0.050 | 1.10  | 4.118 98 | 0.242 78 | -5.365 31          | -1.736 29          | -4.817 72          | -2.801 31 |
| 0.100 | 1.10  | 3.983 90 | 0.251 01 | -5.324 46          | -1.790 44          | -4.830 90          | -2.764 06 |
| 0.150 | 1.10  | 3.842 07 | 0.260 28 | -5.290 81          | -1.846 06          | -4.853 98          | -2.734 47 |
| 0.200 | 1.10  | 3.697 43 | 0.270 46 | -5.260 33          | -1.903 07          | -4.881 35          | -2.707 53 |
| 0.250 | 1.10  | 3.552 08 | 0.281 53 | -5.231 98          | -1.960 05          | -4.911 14          | -2.682 82 |
| 0.300 | 1.10  | 3.406 31 | 0.293 57 | -5.205 70          | -2.017 15          | -4.943 18          | -2.660 02 |
| 0.350 | 1.10  | 3.261 11 | 0.306 64 | -5.180 11          | -2.074 47          | -4.975 66          | -2.637 45 |
| 0.400 | 1.10  | 3.118 20 | 0.320 70 | -5.152 21          | -2.131 21          | -5.004 93          | -2.612 47 |
| 0.450 | 1.10  | 2.974 59 | 0.336 18 | -5.125 39          | -2.188 80          | -5.035 55          | -2.587 96 |
| 0.500 | 1.10  | 2.829 85 | 0.353 38 | -5.101 74          | -2.247 63          | -5.069 80          | -2.565 61 |
| 0.550 | 1.10  | 2.686 22 | 0.372 27 | -5.078 03          | -2.306 19          | -5.103 54          | -2.543 04 |

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**Continued:** Results of the GCMC-Simulations of Mixture II at  $p^* = 0.40$  and  $T^* = 1.10$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.600 | 1.10  | 2.542 73 | 0.393 28 | -5.055 86          | -2.364 18          | -5.138 77          | -2.522 35 |
| 0.650 | 1.10  | 2.398 32 | 0.416 96 | -5.037 21          | -2.423 14          | -5.177 89          | -2.504 31 |
| 0.700 | 1.10  | 2.256 86 | 0.443 09 | -5.012 80          | -2.481 47          | -5.210 06          | -2.480 54 |
| 0.750 | 1.10  | 2.113 05 | 0.473 25 | -4.995 17          | -2.541 16          | -5.249 95          | -2.462 54 |
| 0.800 | 1.10  | 1.971 32 | 0.507 27 | -4.974 37          | -2.603 88          | -5.285 84          | -2.438 15 |
| 0.850 | 1.10  | 1.831 98 | 0.545 86 | -4.948 50          | -2.662 08          | -5.315 71          | -2.412 39 |
| 0.900 | 1.10  | 1.683 67 | 0.593 94 | -4.947 97          | -2.728 76          | -5.374 50          | -2.405 22 |
| 0.950 | 1.10  | 1.544 41 | 0.647 49 | -4.923 94          | -2.789 04          | -5.406 17          | -2.379 21 |
| 0.999 | 1.10  | 1.427 58 | 0.700 49 | -4.835 79          | -2.848 31          | -5.364 76          | -2.287 67 |

**Table C.12:** Results of the GCMC-Simulations of Mixture II at  $p^* = 0.40$  and  $T^* = 1.15$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.001 | 1.15  | 4.371 09 | 0.228 78 | -5.235 41          | -1.554 05          | -4.636 97          | -2.680 80 |
| 0.050 | 1.15  | 4.194 36 | 0.238 42 | -5.241 99          | -1.619 63          | -4.714 25          | -2.690 97 |
| 0.100 | 1.15  | 4.063 58 | 0.246 09 | -5.194 04          | -1.676 41          | -4.718 61          | -2.645 39 |
| 0.150 | 1.15  | 3.917 73 | 0.255 25 | -5.162 23          | -1.737 07          | -4.745 14          | -2.615 72 |
| 0.200 | 1.15  | 3.772 00 | 0.265 11 | -5.129 99          | -1.796 53          | -4.771 19          | -2.586 66 |
| 0.250 | 1.15  | 3.626 33 | 0.275 76 | -5.099 02          | -1.854 75          | -4.798 48          | -2.559 77 |
| 0.300 | 1.15  | 3.479 09 | 0.287 43 | -5.070 83          | -1.913 32          | -4.829 20          | -2.535 54 |
| 0.350 | 1.15  | 3.331 74 | 0.300 14 | -5.044 65          | -1.972 25          | -4.861 95          | -2.512 78 |
| 0.400 | 1.15  | 3.185 94 | 0.313 88 | -5.015 61          | -2.030 35          | -4.891 23          | -2.487 72 |
| 0.450 | 1.15  | 3.040 30 | 0.328 92 | -4.987 37          | -2.088 41          | -4.921 25          | -2.463 33 |
| 0.500 | 1.15  | 2.894 05 | 0.345 54 | -4.961 96          | -2.147 08          | -4.954 33          | -2.441 09 |
| 0.550 | 1.15  | 2.749 13 | 0.363 75 | -4.934 95          | -2.205 16          | -4.985 30          | -2.417 51 |
| 0.600 | 1.15  | 2.605 73 | 0.383 77 | -4.906 40          | -2.263 21          | -5.014 10          | -2.392 08 |
| 0.650 | 1.15  | 2.461 95 | 0.406 18 | -4.879 40          | -2.322 25          | -5.044 62          | -2.367 27 |
| 0.700 | 1.15  | 2.315 41 | 0.431 89 | -4.858 63          | -2.382 47          | -5.082 46          | -2.347 82 |
| 0.750 | 1.15  | 2.170 58 | 0.460 71 | -4.835 68          | -2.442 60          | -5.117 44          | -2.325 95 |
| 0.800 | 1.15  | 2.025 46 | 0.493 71 | -4.814 96          | -2.503 59          | -5.154 78          | -2.305 38 |
| 0.850 | 1.15  | 1.881 73 | 0.531 43 | -4.791 05          | -2.565 49          | -5.188 36          | -2.280 76 |

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**Continued:** Results of the GCMC-Simulations of Mixture II at  $p^* = 0.40$  and  $T^* = 1.15$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.900 | 1.15  | 1.741 39 | 0.574 25 | -4.758 54          | -2.623 20          | -5.211 99          | -2.251 12 |
| 0.950 | 1.15  | 1.600 23 | 0.624 91 | -4.728 40          | -2.683 29          | -5.238 30          | -2.221 75 |
| 0.999 | 1.15  | 1.444 61 | 0.692 23 | -4.751 74          | -2.773 37          | -5.323 90          | -2.217 85 |

**Table C.13:** Results of the GCMC-Simulations of Mixture II at  $p^* = 0.40$  and  $T^* = 1.20$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.001 | 1.20  | 4.412 91 | 0.226 61 | -5.156 85          | -1.472 10          | -4.591 69          | -2.599 66 |
| 0.050 | 1.20  | 4.280 87 | 0.233 60 | -5.112 62          | -1.522 95          | -4.600 27          | -2.564 44 |
| 0.100 | 1.20  | 4.136 42 | 0.241 76 | -5.075 16          | -1.579 70          | -4.620 59          | -2.534 08 |
| 0.150 | 1.20  | 3.992 09 | 0.250 50 | -5.038 63          | -1.637 15          | -4.641 80          | -2.503 87 |
| 0.200 | 1.20  | 3.845 10 | 0.260 07 | -5.005 85          | -1.696 19          | -4.667 81          | -2.476 36 |
| 0.250 | 1.20  | 3.698 58 | 0.270 37 | -4.973 50          | -1.755 75          | -4.694 07          | -2.448 60 |
| 0.300 | 1.20  | 3.550 35 | 0.281 66 | -4.943 23          | -1.815 98          | -4.723 08          | -2.422 59 |
| 0.350 | 1.20  | 3.401 39 | 0.294 00 | -4.914 73          | -1.875 47          | -4.754 17          | -2.398 92 |
| 0.400 | 1.20  | 3.252 77 | 0.307 43 | -4.886 72          | -1.934 32          | -4.785 61          | -2.376 08 |
| 0.450 | 1.20  | 3.106 07 | 0.321 95 | -4.856 42          | -1.992 33          | -4.813 99          | -2.351 38 |
| 0.500 | 1.20  | 2.959 41 | 0.337 91 | -4.826 19          | -2.050 54          | -4.842 43          | -2.326 57 |
| 0.550 | 1.20  | 2.814 53 | 0.355 30 | -4.794 30          | -2.108 88          | -4.868 48          | -2.299 67 |
| 0.600 | 1.20  | 2.669 03 | 0.374 67 | -4.764 44          | -2.168 44          | -4.896 83          | -2.273 66 |
| 0.650 | 1.20  | 2.522 16 | 0.396 49 | -4.736 68          | -2.227 77          | -4.927 81          | -2.250 03 |
| 0.700 | 1.20  | 2.376 90 | 0.420 72 | -4.706 69          | -2.285 84          | -4.955 93          | -2.225 07 |
| 0.750 | 1.20  | 2.230 23 | 0.448 39 | -4.680 33          | -2.345 86          | -4.988 24          | -2.201 99 |
| 0.800 | 1.20  | 2.080 91 | 0.480 56 | -4.659 69          | -2.407 00          | -5.027 33          | -2.183 60 |
| 0.850 | 1.20  | 1.929 25 | 0.518 34 | -4.645 67          | -2.473 12          | -5.073 98          | -2.167 38 |
| 0.900 | 1.20  | 1.794 96 | 0.557 12 | -4.591 60          | -2.525 75          | -5.073 62          | -2.123 22 |
| 0.950 | 1.20  | 1.652 45 | 0.605 16 | -4.553 91          | -2.582 28          | -5.092 93          | -2.092 20 |
| 0.999 | 1.20  | 1.513 74 | 0.660 62 | -4.510 52          | -2.648 81          | -5.105 02          | -2.046 85 |

**Table C.14:** Results of the GCMC-Simulations of Mixture III at  $p^* = 0.20$  and  $T^* = 0.90$

| $x_1$ | $T^*$ | $v^*$   | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$    |
|-------|-------|---------|----------|--------------------|--------------------|--------------------|----------|
| 0.001 | 0.90  | 1.31751 | 0.75901  | -5.35649           | -3.49814           | -5.99299           | -2.77205 |
| 0.050 | 0.90  | 1.31057 | 0.76303  | -5.24620           | -3.38021           | -5.88408           | -2.78208 |
| 0.100 | 0.90  | 1.31844 | 0.75847  | -5.14531           | -3.26513           | -5.78163           | -2.79611 |
| 0.150 | 0.90  | 1.31871 | 0.75831  | -5.14293           | -3.17822           | -5.77919           | -2.88997 |
| 0.200 | 0.90  | 1.31890 | 0.75821  | -5.14158           | -3.10923           | -5.77780           | -2.96508 |
| 0.250 | 0.90  | 1.31904 | 0.75812  | -5.14057           | -3.05441           | -5.77676           | -3.02484 |
| 0.300 | 0.90  | 1.31917 | 0.75805  | -5.13975           | -3.01163           | -5.77592           | -3.07143 |
| 0.350 | 0.90  | 1.31929 | 0.75798  | -5.13906           | -2.97958           | -5.77520           | -3.10625 |
| 0.400 | 0.90  | 1.31940 | 0.75792  | -5.13847           | -2.95742           | -5.77459           | -3.13019 |
| 0.450 | 0.90  | 1.31950 | 0.75786  | -5.13797           | -2.94465           | -5.77407           | -3.14380 |
| 0.500 | 0.90  | 1.31959 | 0.75781  | -5.13755           | -2.94096           | -5.77363           | -3.14741 |
| 0.550 | 0.90  | 1.31967 | 0.75776  | -5.13720           | -2.94627           | -5.77326           | -3.14110 |
| 0.600 | 0.90  | 1.31975 | 0.75772  | -5.13692           | -2.96067           | -5.77297           | -3.12477 |
| 0.650 | 0.90  | 1.31981 | 0.75768  | -5.13672           | -2.98445           | -5.77276           | -3.09812 |
| 0.700 | 0.90  | 1.31167 | 0.76238  | -5.25888           | -3.11215           | -5.89655           | -3.09378 |
| 0.750 | 0.90  | 1.31433 | 0.76084  | -5.19337           | -3.12923           | -5.83050           | -3.00141 |
| 0.800 | 0.90  | 1.31602 | 0.75987  | -5.20031           | -3.14403           | -5.83710           | -2.99231 |
| 0.850 | 0.90  | 1.31516 | 0.76037  | -5.21407           | -3.21127           | -5.85103           | -2.93308 |
| 0.900 | 0.90  | 1.30519 | 0.76617  | -5.31229           | -3.31449           | -5.95125           | -2.92973 |
| 0.950 | 0.90  | 1.31392 | 0.76108  | -5.23286           | -3.39287           | -5.87007           | -2.75244 |
| 0.999 | 0.90  | 1.31831 | 0.75854  | -5.35240           | -3.51080           | -5.98874           | -2.75326 |

**Table C.15:** Results of the GCMC-Simulations of Mixture III at  $p^* = 0.20$  and  $T^* = 0.95$

| $x_1$ | $T^*$ | $v^*$   | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$    |
|-------|-------|---------|----------|--------------------|--------------------|--------------------|----------|
| 0.001 | 0.95  | 1.33365 | 0.74982  | -5.25700           | -3.38785           | -5.94027           | -2.68675 |
| 0.050 | 0.95  | 1.34772 | 0.74199  | -5.07050           | -3.25874           | -5.75096           | -2.62339 |
| 0.100 | 0.95  | 1.36388 | 0.73320  | -4.90072           | -3.14013           | -5.57794           | -2.56611 |
| 0.150 | 0.95  | 1.37401 | 0.72780  | -4.81333           | -3.04281           | -5.48853           | -2.57444 |
| 0.200 | 0.95  | 1.37455 | 0.72751  | -4.80915           | -2.96995           | -5.48424           | -2.64662 |
| 0.250 | 0.95  | 1.37483 | 0.72736  | -4.80709           | -2.91215           | -5.48212           | -2.70523 |

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**Continued:** Results of the GCMC-Simulations of Mixture III at  $p^* = 0.20$  and  $T^* = 0.95$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.300 | 0.95  | 1.375 02 | 0.727 26 | -4.805 78          | -2.867 09          | -5.480 77          | -2.751 25 |
| 0.350 | 0.95  | 1.375 16 | 0.727 19 | -4.804 90          | -2.833 37          | -5.479 87          | -2.785 79 |
| 0.400 | 0.95  | 1.375 25 | 0.727 14 | -4.804 34          | -2.810 11          | -5.479 29          | -2.809 67 |
| 0.450 | 0.95  | 1.375 31 | 0.727 11 | -4.804 04          | -2.796 75          | -5.478 97          | -2.823 39 |
| 0.500 | 0.95  | 1.375 35 | 0.727 09 | -4.803 95          | -2.792 99          | -5.478 88          | -2.827 25 |
| 0.550 | 0.95  | 1.375 36 | 0.727 08 | -4.804 07          | -2.798 74          | -5.479 00          | -2.821 32 |
| 0.600 | 0.95  | 1.375 34 | 0.727 09 | -4.804 40          | -2.814 09          | -5.479 33          | -2.805 52 |
| 0.650 | 0.95  | 1.375 29 | 0.727 12 | -4.804 96          | -2.839 34          | -5.479 90          | -2.779 54 |
| 0.700 | 0.95  | 1.375 20 | 0.727 16 | -4.805 79          | -2.875 04          | -5.480 75          | -2.742 85 |
| 0.750 | 0.95  | 1.375 08 | 0.727 23 | -4.806 98          | -2.922 08          | -5.481 96          | -2.694 61 |
| 0.800 | 0.95  | 1.374 88 | 0.727 33 | -4.808 74          | -2.981 84          | -5.483 77          | -2.633 61 |
| 0.850 | 0.95  | 1.374 52 | 0.727 53 | -4.811 99          | -3.056 60          | -5.487 09          | -2.558 41 |
| 0.900 | 0.95  | 1.367 25 | 0.731 40 | -4.887 38          | -3.153 68          | -5.563 93          | -2.537 10 |
| 0.950 | 0.95  | 1.355 14 | 0.737 93 | -5.044 19          | -3.268 21          | -5.723 16          | -2.584 15 |
| 0.999 | 0.95  | 1.340 43 | 0.746 03 | -5.230 18          | -3.395 36          | -5.912 09          | -2.649 19 |

**Table C.16:** Results of the GCMC-Simulations of Mixture III at  $p^* = 0.20$  and  $T^* = 1.00$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.001 | 1.00  | 1.378 57 | 0.725 39 | -5.056 14          | -3.272 43          | -5.780 43          | -2.508 00 |
| 0.050 | 1.00  | 1.394 51 | 0.717 10 | -4.873 82          | -3.146 12          | -5.594 92          | -2.448 81 |
| 0.100 | 1.00  | 1.415 50 | 0.706 46 | -4.693 23          | -3.028 77          | -5.410 13          | -2.381 36 |
| 0.150 | 1.00  | 1.435 41 | 0.696 66 | -4.538 20          | -2.927 53          | -5.251 12          | -2.323 59 |
| 0.200 | 1.00  | 1.451 99 | 0.688 71 | -4.414 12          | -2.842 69          | -5.123 72          | -2.281 03 |
| 0.250 | 1.00  | 1.462 41 | 0.683 80 | -4.339 96          | -2.775 07          | -5.047 47          | -2.272 40 |
| 0.300 | 1.00  | 1.465 55 | 0.682 34 | -4.318 68          | -2.725 15          | -5.025 57          | -2.300 43 |
| 0.350 | 1.00  | 1.466 89 | 0.681 71 | -4.309 65          | -2.688 21          | -5.016 28          | -2.328 07 |
| 0.400 | 1.00  | 1.467 45 | 0.681 45 | -4.305 79          | -2.662 70          | -5.012 30          | -2.349 60 |
| 0.450 | 1.00  | 1.467 51 | 0.681 43 | -4.305 23          | -2.647 88          | -5.011 73          | -2.363 85 |
| 0.500 | 1.00  | 1.467 17 | 0.681 59 | -4.307 29          | -2.643 37          | -5.013 86          | -2.370 49 |
| 0.550 | 1.00  | 1.466 46 | 0.681 91 | -4.311 74          | -2.649 05          | -5.018 45          | -2.369 40 |

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**Continued:** Results of the GCMC-Simulations of Mixture III at  $p^* = 0.20$  and  $T^* = 1.00$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.600 | 1.00  | 1.465 38 | 0.682 42 | -4.318 65          | -2.665 02          | -5.025 57          | -2.360 55 |
| 0.650 | 1.00  | 1.463 86 | 0.683 12 | -4.328 45          | -2.691 65          | -5.035 68          | -2.344 03 |
| 0.700 | 1.00  | 1.461 74 | 0.684 12 | -4.342 30          | -2.729 60          | -5.049 95          | -2.320 35 |
| 0.750 | 1.00  | 1.458 47 | 0.685 65 | -4.363 97          | -2.780 13          | -5.072 28          | -2.292 15 |
| 0.800 | 1.00  | 1.450 44 | 0.689 44 | -4.419 79          | -2.846 90          | -5.129 70          | -2.282 80 |
| 0.850 | 1.00  | 1.434 66 | 0.697 03 | -4.540 59          | -2.932 21          | -5.253 66          | -2.321 45 |
| 0.900 | 1.00  | 1.416 92 | 0.705 76 | -4.688 09          | -3.031 92          | -5.404 70          | -2.372 78 |
| 0.950 | 1.00  | 1.397 26 | 0.715 69 | -4.863 99          | -3.147 21          | -5.584 54          | -2.437 32 |
| 0.999 | 1.00  | 1.377 82 | 0.725 78 | -5.059 74          | -3.275 33          | -5.784 17          | -2.508 85 |

**Table C.17:** Results of the GCMC-Simulations of Mixture III at  $p^* = 0.20$  and  $T^* = 1.05$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.001 | 1.05  | 1.420 06 | 0.704 19 | -4.882 07          | -3.172 93          | -5.648 06          | -2.357 26 |
| 0.050 | 1.05  | 1.449 38 | 0.689 95 | -4.663 45          | -3.041 28          | -5.423 57          | -2.268 85 |
| 0.100 | 1.05  | 1.471 93 | 0.679 38 | -4.487 91          | -2.927 21          | -5.243 53          | -2.206 02 |
| 0.150 | 1.05  | 1.494 97 | 0.668 91 | -4.332 12          | -2.826 26          | -5.083 12          | -2.149 40 |
| 0.200 | 1.05  | 1.515 07 | 0.660 03 | -4.203 37          | -2.740 45          | -4.950 35          | -2.104 67 |
| 0.250 | 1.05  | 1.534 26 | 0.651 78 | -4.094 42          | -2.668 01          | -4.837 57          | -2.066 25 |
| 0.300 | 1.05  | 1.550 45 | 0.644 98 | -4.007 69          | -2.609 87          | -4.747 61          | -2.035 94 |
| 0.350 | 1.05  | 1.562 26 | 0.640 10 | -3.945 82          | -2.566 05          | -4.683 36          | -2.016 49 |
| 0.400 | 1.05  | 1.570 05 | 0.636 92 | -3.905 33          | -2.535 66          | -4.641 32          | -2.005 40 |
| 0.450 | 1.05  | 1.574 64 | 0.635 07 | -3.881 65          | -2.517 72          | -4.616 73          | -1.999 05 |
| 0.500 | 1.05  | 1.576 27 | 0.634 41 | -3.873 23          | -2.511 80          | -4.607 98          | -1.996 36 |
| 0.550 | 1.05  | 1.574 83 | 0.634 99 | -3.880 53          | -2.517 87          | -4.615 56          | -1.997 80 |
| 0.600 | 1.05  | 1.570 13 | 0.636 89 | -3.904 54          | -2.536 11          | -4.640 51          | -2.004 19 |
| 0.650 | 1.05  | 1.562 03 | 0.640 19 | -3.946 35          | -2.566 86          | -4.683 94          | -2.016 27 |
| 0.700 | 1.05  | 1.550 31 | 0.645 03 | -4.008 16          | -2.610 69          | -4.748 10          | -2.035 63 |
| 0.750 | 1.05  | 1.534 59 | 0.651 64 | -4.093 49          | -2.668 44          | -4.836 57          | -2.064 88 |
| 0.800 | 1.05  | 1.514 61 | 0.660 23 | -4.204 85          | -2.741 29          | -4.951 92          | -2.105 37 |
| 0.850 | 1.05  | 1.491 57 | 0.670 43 | -4.341 29          | -2.829 73          | -5.092 97          | -2.155 47 |

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**Continued:** Results of the GCMC-Simulations of Mixture III at  $p^* = 0.20$  and  $T^* = 1.05$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.900 | 1.05  | 1.469 11 | 0.680 68 | -4.495 38          | -2.932 14          | -5.251 56          | -2.208 97 |
| 0.950 | 1.05  | 1.448 50 | 0.690 37 | -4.666 13          | -3.045 73          | -5.426 43          | -2.267 33 |
| 0.999 | 1.05  | 1.425 32 | 0.701 60 | -4.863 68          | -3.172 23          | -5.628 62          | -2.339 42 |

**Table C.18:** Results of the GCMC-Simulations of Mixture III at  $p^* = 0.20$  and  $T^* = 1.10$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.001 | 1.10  | 1.469 84 | 0.680 35 | -4.690 27          | -3.064 87          | -5.496 30          | -2.210 39 |
| 0.050 | 1.10  | 1.505 80 | 0.664 10 | -4.464 98          | -2.932 42          | -5.263 82          | -2.119 46 |
| 0.100 | 1.10  | 1.537 05 | 0.650 60 | -4.274 69          | -2.816 52          | -5.067 27          | -2.046 14 |
| 0.150 | 1.10  | 1.568 63 | 0.637 50 | -4.106 59          | -2.715 32          | -4.892 86          | -1.979 59 |
| 0.200 | 1.10  | 1.595 48 | 0.626 77 | -3.968 48          | -2.630 36          | -4.749 38          | -1.926 38 |
| 0.250 | 1.10  | 1.620 30 | 0.617 17 | -3.851 84          | -2.558 41          | -4.627 78          | -1.881 24 |
| 0.300 | 1.10  | 1.642 27 | 0.608 91 | -3.756 21          | -2.499 56          | -4.527 76          | -1.843 82 |
| 0.350 | 1.10  | 1.660 34 | 0.602 29 | -3.681 91          | -2.453 89          | -4.449 84          | -1.814 50 |
| 0.400 | 1.10  | 1.673 32 | 0.597 61 | -3.629 53          | -2.421 67          | -4.394 86          | -1.793 81 |
| 0.450 | 1.10  | 1.680 08 | 0.595 21 | -3.601 16          | -2.403 18          | -4.365 15          | -1.783 61 |
| 0.500 | 1.10  | 1.681 40 | 0.594 74 | -3.594 45          | -2.397 68          | -4.358 17          | -1.782 26 |
| 0.550 | 1.10  | 1.678 15 | 0.595 90 | -3.606 18          | -2.404 64          | -4.370 55          | -1.787 19 |
| 0.600 | 1.10  | 1.670 40 | 0.598 66 | -3.636 41          | -2.424 12          | -4.402 33          | -1.798 37 |
| 0.650 | 1.10  | 1.658 35 | 0.603 01 | -3.686 17          | -2.456 15          | -4.454 50          | -1.816 68 |
| 0.700 | 1.10  | 1.642 10 | 0.608 98 | -3.756 26          | -2.501 07          | -4.527 84          | -1.842 52 |
| 0.750 | 1.10  | 1.621 21 | 0.616 82 | -3.848 90          | -2.559 79          | -4.624 66          | -1.877 15 |
| 0.800 | 1.10  | 1.596 39 | 0.626 41 | -3.965 95          | -2.632 43          | -4.746 67          | -1.922 03 |
| 0.850 | 1.10  | 1.571 03 | 0.636 52 | -4.099 90          | -2.717 24          | -4.885 69          | -1.971 32 |
| 0.900 | 1.10  | 1.542 06 | 0.648 48 | -4.262 44          | -2.816 26          | -5.054 03          | -2.034 33 |
| 0.950 | 1.10  | 1.505 07 | 0.664 42 | -4.466 94          | -2.935 48          | -5.265 93          | -2.118 59 |
| 0.999 | 1.10  | 1.475 86 | 0.677 57 | -4.671 56          | -3.062 84          | -5.476 39          | -2.194 14 |

**Table C.19:** Results of the GCMC-Simulations of Mixture III at  $p^* = 0.20$  and  $T^* = 1.15$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.001 | 1.15  | 1.538 38 | 0.650 03 | -4.457 71          | -2.964 57          | -5.300 03          | -2.030 84 |
| 0.050 | 1.15  | 1.577 34 | 0.633 98 | -4.241 08          | -2.835 02          | -5.075 61          | -1.948 34 |
| 0.100 | 1.15  | 1.625 15 | 0.615 33 | -4.024 56          | -2.712 55          | -4.849 53          | -1.858 24 |
| 0.150 | 1.15  | 1.657 95 | 0.603 15 | -3.864 84          | -2.614 59          | -4.683 25          | -1.798 84 |
| 0.200 | 1.15  | 1.691 46 | 0.591 21 | -3.723 79          | -2.526 88          | -4.535 49          | -1.746 62 |
| 0.250 | 1.15  | 1.727 43 | 0.578 90 | -3.594 09          | -2.449 56          | -4.398 60          | -1.694 82 |
| 0.300 | 1.15  | 1.758 73 | 0.568 59 | -3.488 65          | -2.386 80          | -4.286 91          | -1.652 27 |
| 0.350 | 1.15  | 1.783 57 | 0.560 67 | -3.408 37          | -2.338 55          | -4.201 66          | -1.620 10 |
| 0.400 | 1.15  | 1.801 24 | 0.555 17 | -3.352 67          | -2.304 52          | -4.142 42          | -1.598 17 |
| 0.450 | 1.15  | 1.811 64 | 0.551 99 | -3.320 12          | -2.284 35          | -4.107 79          | -1.585 60 |
| 0.500 | 1.15  | 1.815 82 | 0.550 71 | -3.307 89          | -2.277 14          | -4.094 72          | -1.580 50 |
| 0.550 | 1.15  | 1.813 55 | 0.551 41 | -3.316 11          | -2.282 94          | -4.103 40          | -1.583 01 |
| 0.600 | 1.15  | 1.802 76 | 0.554 71 | -3.349 28          | -2.303 08          | -4.138 73          | -1.596 22 |
| 0.650 | 1.15  | 1.783 47 | 0.560 70 | -3.408 15          | -2.337 92          | -4.201 45          | -1.620 47 |
| 0.700 | 1.15  | 1.757 41 | 0.569 02 | -3.491 08          | -2.386 94          | -4.289 60          | -1.654 48 |
| 0.750 | 1.15  | 1.727 34 | 0.578 92 | -3.594 82          | -2.449 07          | -4.399 35          | -1.695 90 |
| 0.800 | 1.15  | 1.694 54 | 0.590 13 | -3.717 71          | -2.524 14          | -4.528 80          | -1.743 19 |
| 0.850 | 1.15  | 1.656 29 | 0.603 76 | -3.869 62          | -2.614 33          | -4.688 36          | -1.803 51 |
| 0.900 | 1.15  | 1.615 59 | 0.618 97 | -4.046 85          | -2.719 06          | -4.873 74          | -1.873 63 |
| 0.950 | 1.15  | 1.579 93 | 0.632 94 | -4.234 30          | -2.834 02          | -5.068 32          | -1.942 87 |
| 0.999 | 1.15  | 1.544 15 | 0.647 61 | -4.443 04          | -2.960 12          | -5.284 21          | -2.020 95 |

**Table C.20:** Results of the GCMC-Simulations of Mixture III at  $p^* = 0.20$  and  $T^* = 1.20$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.001 | 1.20  | 1.608 33 | 0.621 76 | -4.244 17          | -2.862 73          | -5.122 51          | -1.883 15 |
| 0.050 | 1.20  | 1.655 34 | 0.604 10 | -4.022 61          | -2.732 05          | -4.891 54          | -1.799 57 |
| 0.100 | 1.20  | 1.721 96 | 0.580 73 | -3.781 54          | -2.600 91          | -4.637 15          | -1.696 87 |
| 0.150 | 1.20  | 1.774 96 | 0.563 39 | -3.596 49          | -2.494 43          | -4.441 50          | -1.622 55 |
| 0.200 | 1.20  | 1.831 95 | 0.545 87 | -3.426 07          | -2.397 69          | -4.259 68          | -1.551 65 |
| 0.250 | 1.20  | 1.882 07 | 0.531 33 | -3.287 53          | -2.317 16          | -4.111 12          | -1.494 96 |

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**Continued:** Results of the GCMC-Simulations of Mixture III at  $p^* = 0.20$  and  $T^* = 1.20$

| $x_1$ | $T^*$ | $v^*$    | $\rho^*$ | $u_{\text{res}}^*$ | $h_{\text{res}}^*$ | $g_{\text{res}}^*$ | $s^*$     |
|-------|-------|----------|----------|--------------------|--------------------|--------------------|-----------|
| 0.300 | 1.20  | 1.928 20 | 0.518 62 | -3.171 35          | -2.249 65          | -3.985 71          | -1.446 71 |
| 0.350 | 1.20  | 1.967 08 | 0.508 37 | -3.080 22          | -2.196 38          | -3.886 80          | -1.408 68 |
| 0.400 | 1.20  | 1.996 32 | 0.500 92 | -3.015 11          | -2.158 08          | -3.815 84          | -1.381 47 |
| 0.450 | 1.20  | 2.016 37 | 0.495 94 | -2.973 40          | -2.133 91          | -3.770 13          | -1.363 52 |
| 0.500 | 1.20  | 2.024 41 | 0.493 97 | -2.957 50          | -2.125 20          | -3.752 62          | -1.356 18 |
| 0.550 | 1.20  | 2.017 05 | 0.495 77 | -2.971 87          | -2.133 62          | -3.768 46          | -1.362 36 |
| 0.600 | 1.20  | 1.996 82 | 0.500 80 | -3.014 06          | -2.157 77          | -3.814 69          | -1.380 77 |
| 0.650 | 1.20  | 1.966 98 | 0.508 39 | -3.080 13          | -2.196 37          | -3.886 73          | -1.408 63 |
| 0.700 | 1.20  | 1.927 53 | 0.518 80 | -3.172 17          | -2.250 00          | -3.986 67          | -1.447 22 |
| 0.750 | 1.20  | 1.880 87 | 0.531 67 | -3.289 54          | -2.317 98          | -4.113 37          | -1.496 16 |
| 0.800 | 1.20  | 1.828 75 | 0.546 82 | -3.432 35          | -2.400 39          | -4.266 60          | -1.555 18 |
| 0.850 | 1.20  | 1.775 64 | 0.563 18 | -3.595 73          | -2.495 28          | -4.440 60          | -1.621 10 |
| 0.900 | 1.20  | 1.725 89 | 0.579 41 | -3.773 75          | -2.599 79          | -4.628 57          | -1.690 65 |
| 0.950 | 1.20  | 1.669 98 | 0.598 81 | -3.988 77          | -2.721 82          | -4.854 78          | -1.777 46 |
| 0.999 | 1.20  | 1.616 95 | 0.618 45 | -4.222 59          | -2.855 03          | -5.099 20          | -1.870 14 |

### C.3 Tabulated Results of the Vapor Liquid Equilibrium GCMC Simulations

This supporting information contains tables summarizing vapor liquid equilibrium data for the investigated model mixtures at several temperatures. The tables are structured by mixture and temperature. Tables C.21–C.25 thereby show the results of the model mixture consisting of two fluids with different energy parameters (Mixture I:  $\varepsilon_{22}/\varepsilon_{11} = 0.7$ ), tables C.26–C.30 present the results of Mixture II with different diameters ( $\sigma_{22}/\sigma_{11} = 1.5$ ), and tables C.31–C.34 give the results of the non-ideal mixture of two identical LJ-fluids ( $k_{ij} = 0.25$ ).

The tables show the composition of the liquid  $x_2^{\text{liq}}$  and the vapor phase  $x_2^{\text{vap}}$ , the vapor pressure  $p_{\text{sat}}^*$ , and the liquid and vapor densities ( $\rho^{\text{liq}}$  &  $\rho^{\text{vap}}$ ). All results are given in reduced LJ units, according to table 4.1.

**Table C.21:** VLE-Data from GCMC-Simulations of Mixture I at  $T^* = 0.90$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $p_{\text{sat}}^*$ | $\rho^{\text{vap}}$ | $\rho^{\text{liq}}$ |
|--------------------|--------------------|--------------------|---------------------|---------------------|
| 0.006 21           | 0.001 00           | 0.010 10           | 0.012 18            | 0.739 21            |
| 0.008 20           | 0.025 00           | 0.010 11           | 0.012 19            | 0.703 46            |
| 0.010 05           | 0.050 00           | 0.010 03           | 0.012 10            | 0.673 65            |
| 0.018 88           | 0.075 00           | 0.009 70           | 0.011 68            | 0.642 22            |
| 0.039 26           | 0.100 00           | 0.009 26           | 0.011 14            | 0.619 78            |
| 0.046 54           | 0.125 00           | 0.009 13           | 0.010 99            | 0.592 76            |
| 0.072 18           | 0.150 00           | 0.008 77           | 0.010 55            | 0.571 32            |
| 0.088 86           | 0.175 00           | 0.008 59           | 0.010 35            | 0.548 61            |
| 0.096 67           | 0.200 00           | 0.008 51           | 0.010 26            | 0.525 80            |
| 0.103 03           | 0.225 00           | 0.008 43           | 0.010 17            | 0.504 88            |
| 0.110 63           | 0.250 00           | 0.008 33           | 0.010 05            | 0.485 85            |
| 0.119 55           | 0.275 00           | 0.008 21           | 0.009 91            | 0.467 81            |
| 0.128 52           | 0.300 00           | 0.008 09           | 0.009 76            | 0.450 64            |
| 0.141 66           | 0.325 00           | 0.007 91           | 0.009 54            | 0.435 09            |
| 0.162 70           | 0.350 00           | 0.007 64           | 0.009 22            | 0.421 38            |
| 0.178 42           | 0.375 00           | 0.007 46           | 0.009 00            | 0.408 11            |
| 0.192 64           | 0.400 00           | 0.007 30           | 0.008 81            | 0.395 35            |
| 0.211 09           | 0.425 00           | 0.007 11           | 0.008 57            | 0.383 36            |
| 0.232 49           | 0.450 00           | 0.006 89           | 0.008 32            | 0.372 15            |
| 0.253 01           | 0.475 00           | 0.006 71           | 0.008 09            | 0.361 47            |

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**Continued:** VLE-Data from GCMC-Simulations of Mixture I at  $T^* = 0.90$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.27470            | 0.50000            | 0.00652            | 0.00787              | 0.35137              |
| 0.29819            | 0.52500            | 0.00633            | 0.00764              | 0.34190              |
| 0.32150            | 0.55000            | 0.00615            | 0.00743              | 0.33293              |
| 0.34535            | 0.57500            | 0.00599            | 0.00723              | 0.32440              |
| 0.37191            | 0.60000            | 0.00581            | 0.00702              | 0.31631              |
| 0.40108            | 0.62500            | 0.00563            | 0.00681              | 0.30851              |
| 0.43064            | 0.65000            | 0.00546            | 0.00660              | 0.30096              |
| 0.46002            | 0.67500            | 0.00531            | 0.00642              | 0.29369              |
| 0.49143            | 0.70000            | 0.00515            | 0.00623              | 0.28670              |
| 0.52552            | 0.72500            | 0.00499            | 0.00604              | 0.27991              |
| 0.55979            | 0.75000            | 0.00484            | 0.00587              | 0.27338              |
| 0.59430            | 0.77500            | 0.00470            | 0.00570              | 0.26727              |
| 0.63095            | 0.80000            | 0.00456            | 0.00553              | 0.26157              |
| 0.67130            | 0.82500            | 0.00441            | 0.00536              | 0.25611              |
| 0.71448            | 0.85000            | 0.00427            | 0.00519              | 0.25075              |
| 0.75848            | 0.87500            | 0.00413            | 0.00503              | 0.24550              |
| 0.80343            | 0.90000            | 0.00401            | 0.00488              | 0.24037              |
| 0.84992            | 0.92500            | 0.00388            | 0.00474              | 0.23536              |
| 0.89818            | 0.95000            | 0.00376            | 0.00460              | 0.23036              |
| 0.94633            | 0.97500            | 0.00365            | 0.00446              | 0.22541              |
| 0.99785            | 0.99900            | 0.00354            | 0.00433              | 0.22090              |

**Table C.22:** VLE-Data from GCMC-Simulations of Mixture I at  $T^* = 1.00$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.00517            | 0.00100            | 0.02240            | 0.02596              | 0.71515              |
| 0.00894            | 0.02500            | 0.02240            | 0.02600              | 0.66704              |
| 0.01055            | 0.05000            | 0.02227            | 0.02583              | 0.62729              |
| 0.01963            | 0.07500            | 0.02142            | 0.02475              | 0.60686              |
| 0.03499            | 0.10000            | 0.02055            | 0.02369              | 0.58064              |
| 0.04494            | 0.12500            | 0.02007            | 0.02311              | 0.55639              |

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C.3 Tabulated Results of the Vapor Liquid Equilibrium GCMC Simulations

Continued: VLE-Data from GCMC-Simulations of Mixture I at  $T^* = 1.00$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.05978            | 0.15000            | 0.01946            | 0.02240              | 0.53459              |
| 0.07008            | 0.17500            | 0.01907            | 0.02194              | 0.51098              |
| 0.08372            | 0.20000            | 0.01858            | 0.02136              | 0.48966              |
| 0.09557            | 0.22500            | 0.01817            | 0.02089              | 0.47058              |
| 0.10889            | 0.25000            | 0.01773            | 0.02038              | 0.45309              |
| 0.12787            | 0.27500            | 0.01717            | 0.01973              | 0.43766              |
| 0.14180            | 0.30000            | 0.01678            | 0.01930              | 0.42223              |
| 0.15321            | 0.32500            | 0.01648            | 0.01894              | 0.40729              |
| 0.16637            | 0.35000            | 0.01612            | 0.01853              | 0.39344              |
| 0.18444            | 0.37500            | 0.01565            | 0.01800              | 0.38088              |
| 0.20629            | 0.40000            | 0.01514            | 0.01741              | 0.36932              |
| 0.22626            | 0.42500            | 0.01471            | 0.01692              | 0.35829              |
| 0.24488            | 0.45000            | 0.01433            | 0.01649              | 0.34775              |
| 0.26554            | 0.47500            | 0.01394            | 0.01604              | 0.33773              |
| 0.29013            | 0.50000            | 0.01350            | 0.01554              | 0.32830              |
| 0.31625            | 0.52500            | 0.01307            | 0.01506              | 0.31950              |
| 0.34287            | 0.55000            | 0.01267            | 0.01461              | 0.31116              |
| 0.36998            | 0.57500            | 0.01229            | 0.01419              | 0.30312              |
| 0.39656            | 0.60000            | 0.01195            | 0.01381              | 0.29530              |
| 0.42334            | 0.62500            | 0.01162            | 0.01344              | 0.28773              |
| 0.45132            | 0.65000            | 0.01130            | 0.01308              | 0.28048              |
| 0.48109            | 0.67500            | 0.01097            | 0.01272              | 0.27361              |
| 0.51307            | 0.70000            | 0.01064            | 0.01236              | 0.26707              |
| 0.54639            | 0.72500            | 0.01033            | 0.01200              | 0.26088              |
| 0.58039            | 0.75000            | 0.01002            | 0.01167              | 0.25504              |
| 0.61588            | 0.77500            | 0.00972            | 0.01134              | 0.24945              |
| 0.65307            | 0.80000            | 0.00943            | 0.01102              | 0.24396              |
| 0.69135            | 0.82500            | 0.00915            | 0.01071              | 0.23860              |
| 0.73134            | 0.85000            | 0.00888            | 0.01040              | 0.23342              |
| 0.77365            | 0.87500            | 0.00861            | 0.01011              | 0.22841              |
| 0.81672            | 0.90000            | 0.00835            | 0.00983              | 0.22352              |
| 0.86144            | 0.92500            | 0.00810            | 0.00956              | 0.21878              |

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**Continued:** VLE-Data from GCMC-Simulations of Mixture I at  $T^* = 1.00$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.90757            | 0.95000            | 0.00787            | 0.00930              | 0.21404              |
| 0.95148            | 0.97500            | 0.00765            | 0.00907              | 0.20978              |
| 0.99806            | 0.99900            | 0.00743            | 0.00883              | 0.20584              |

**Table C.23:** VLE-Data from GCMC-Simulations of Mixture I at  $T^* = 1.05$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.00318            | 0.00100            | 0.03188            | 0.03697              | 0.66996              |
| 0.00722            | 0.02500            | 0.03183            | 0.03698              | 0.63622              |
| 0.01421            | 0.05000            | 0.03106            | 0.03598              | 0.60691              |
| 0.02261            | 0.07500            | 0.03020            | 0.03487              | 0.58329              |
| 0.03299            | 0.10000            | 0.02931            | 0.03374              | 0.55891              |
| 0.04131            | 0.12500            | 0.02864            | 0.03291              | 0.53174              |
| 0.05160            | 0.15000            | 0.02783            | 0.03191              | 0.50822              |
| 0.06562            | 0.17500            | 0.02686            | 0.03073              | 0.48885              |
| 0.08453            | 0.20000            | 0.02579            | 0.02945              | 0.47186              |
| 0.10125            | 0.22500            | 0.02500            | 0.02852              | 0.45474              |
| 0.11425            | 0.25000            | 0.02443            | 0.02786              | 0.43730              |
| 0.12634            | 0.27500            | 0.02390            | 0.02725              | 0.42045              |
| 0.13976            | 0.30000            | 0.02334            | 0.02659              | 0.40487              |
| 0.15676            | 0.32500            | 0.02265            | 0.02580              | 0.39106              |
| 0.17748            | 0.35000            | 0.02190            | 0.02493              | 0.37863              |
| 0.19866            | 0.37500            | 0.02121            | 0.02414              | 0.36672              |
| 0.21788            | 0.40000            | 0.02062            | 0.02348              | 0.35517              |
| 0.23606            | 0.42500            | 0.02010            | 0.02289              | 0.34430              |
| 0.25638            | 0.45000            | 0.01955            | 0.02227              | 0.33424              |
| 0.28039            | 0.47500            | 0.01894            | 0.02158              | 0.32479              |
| 0.30503            | 0.50000            | 0.01836            | 0.02094              | 0.31562              |
| 0.32864            | 0.52500            | 0.01785            | 0.02037              | 0.30677              |
| 0.35269            | 0.55000            | 0.01736            | 0.01982              | 0.29845              |
| 0.37891            | 0.57500            | 0.01685            | 0.01927              | 0.29066              |

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**Continued:** VLE-Data from GCMC-Simulations of Mixture I at  $T^* = 1.05$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.40774            | 0.60000            | 0.01633            | 0.01870              | 0.28327              |
| 0.43842            | 0.62500            | 0.01582            | 0.01815              | 0.27622              |
| 0.46946            | 0.65000            | 0.01535            | 0.01763              | 0.26951              |
| 0.49976            | 0.67500            | 0.01492            | 0.01717              | 0.26308              |
| 0.53013            | 0.70000            | 0.01451            | 0.01673              | 0.25689              |
| 0.56178            | 0.72500            | 0.01411            | 0.01630              | 0.25095              |
| 0.59463            | 0.75000            | 0.01371            | 0.01587              | 0.24524              |
| 0.62801            | 0.77500            | 0.01333            | 0.01546              | 0.23975              |
| 0.66218            | 0.80000            | 0.01296            | 0.01507              | 0.23450              |
| 0.69808            | 0.82500            | 0.01259            | 0.01467              | 0.22951              |
| 0.73600            | 0.85000            | 0.01222            | 0.01427              | 0.22475              |
| 0.77551            | 0.87500            | 0.01185            | 0.01387              | 0.22016              |
| 0.81797            | 0.90000            | 0.01149            | 0.01347              | 0.21573              |
| 0.86222            | 0.92500            | 0.01113            | 0.01309              | 0.21128              |
| 0.90658            | 0.95000            | 0.01079            | 0.01273              | 0.20670              |
| 0.95259            | 0.97500            | 0.01047            | 0.01238              | 0.20261              |
| 0.99809            | 0.99900            | 0.01016            | 0.01205              | 0.19948              |

**Table C.24:** VLE-Data from GCMC-Simulations of Mixture I at  $T^* = 1.10$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.00164            | 0.00100            | 0.04386            | 0.05148              | 0.65230              |
| 0.00733            | 0.02500            | 0.04340            | 0.05096              | 0.61686              |
| 0.01436            | 0.05000            | 0.04236            | 0.04954              | 0.58395              |
| 0.02429            | 0.07500            | 0.04095            | 0.04762              | 0.55537              |
| 0.03456            | 0.10000            | 0.03974            | 0.04604              | 0.53222              |
| 0.04461            | 0.12500            | 0.03865            | 0.04462              | 0.50866              |
| 0.05627            | 0.15000            | 0.03747            | 0.04311              | 0.48650              |
| 0.07205            | 0.17500            | 0.03608            | 0.04138              | 0.46908              |
| 0.08675            | 0.20000            | 0.03497            | 0.04002              | 0.45041              |
| 0.10015            | 0.22500            | 0.03402            | 0.03889              | 0.43193              |

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Continued: VLE-Data from GCMC-Simulations of Mixture I at  $T^* = 1.10$ 

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $p_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.11405            | 0.25000            | 0.03310            | 0.03777              | 0.41561              |
| 0.13058            | 0.27500            | 0.03207            | 0.03655              | 0.40105              |
| 0.14894            | 0.30000            | 0.03103            | 0.03533              | 0.38749              |
| 0.16670            | 0.32500            | 0.03012            | 0.03427              | 0.37458              |
| 0.18422            | 0.35000            | 0.02927            | 0.03329              | 0.36230              |
| 0.20335            | 0.37500            | 0.02840            | 0.03229              | 0.35082              |
| 0.22450            | 0.40000            | 0.02752            | 0.03128              | 0.34003              |
| 0.24680            | 0.42500            | 0.02666            | 0.03031              | 0.32966              |
| 0.27020            | 0.45000            | 0.02583            | 0.02938              | 0.31981              |
| 0.29399            | 0.47500            | 0.02505            | 0.02852              | 0.31053              |
| 0.31808            | 0.50000            | 0.02432            | 0.02771              | 0.30180              |
| 0.34297            | 0.52500            | 0.02361            | 0.02694              | 0.29351              |
| 0.36861            | 0.55000            | 0.02293            | 0.02619              | 0.28559              |
| 0.39527            | 0.57500            | 0.02227            | 0.02547              | 0.27799              |
| 0.42315            | 0.60000            | 0.02162            | 0.02477              | 0.27071              |
| 0.45198            | 0.62500            | 0.02099            | 0.02410              | 0.26377              |
| 0.48145            | 0.65000            | 0.02039            | 0.02345              | 0.25723              |
| 0.51158            | 0.67500            | 0.01981            | 0.02284              | 0.25104              |
| 0.54261            | 0.70000            | 0.01925            | 0.02224              | 0.24512              |
| 0.57492            | 0.72500            | 0.01870            | 0.02166              | 0.23938              |
| 0.60885            | 0.75000            | 0.01816            | 0.02109              | 0.23381              |
| 0.64394            | 0.77500            | 0.01764            | 0.02054              | 0.22844              |
| 0.67938            | 0.80000            | 0.01714            | 0.02002              | 0.22331              |
| 0.71514            | 0.82500            | 0.01666            | 0.01953              | 0.21842              |
| 0.75167            | 0.85000            | 0.01620            | 0.01905              | 0.21374              |
| 0.78975            | 0.87500            | 0.01575            | 0.01857              | 0.20930              |
| 0.83016            | 0.90000            | 0.01529            | 0.01810              | 0.20508              |
| 0.87151            | 0.92500            | 0.01486            | 0.01764              | 0.20104              |
| 0.91263            | 0.95000            | 0.01445            | 0.01722              | 0.19716              |
| 0.95474            | 0.97500            | 0.01404            | 0.01680              | 0.19348              |
| 0.99816            | 0.99900            | 0.01363            | 0.01636              | 0.19003              |

**Table C.25:** VLE-Data from GCMC-Simulations of Mixture I at  $T^* = 1.15$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.001 20           | 0.001 00           | 0.057 49           | 0.069 07             | 0.631 73             |
| 0.009 02           | 0.025 00           | 0.056 46           | 0.067 71             | 0.585 50             |
| 0.016 98           | 0.050 00           | 0.055 15           | 0.065 86             | 0.554 47             |
| 0.025 92           | 0.075 00           | 0.053 60           | 0.063 60             | 0.526 94             |
| 0.037 67           | 0.100 00           | 0.051 76           | 0.060 99             | 0.505 54             |
| 0.050 62           | 0.125 00           | 0.050 05           | 0.058 65             | 0.484 33             |
| 0.064 20           | 0.150 00           | 0.048 45           | 0.056 53             | 0.463 55             |
| 0.077 90           | 0.175 00           | 0.046 98           | 0.054 63             | 0.444 64             |
| 0.092 65           | 0.200 00           | 0.045 54           | 0.052 79             | 0.427 37             |
| 0.107 07           | 0.225 00           | 0.044 22           | 0.051 15             | 0.410 62             |
| 0.122 56           | 0.250 00           | 0.042 90           | 0.049 51             | 0.395 01             |
| 0.139 72           | 0.275 00           | 0.041 54           | 0.047 85             | 0.380 78             |
| 0.159 17           | 0.300 00           | 0.040 14           | 0.046 18             | 0.368 01             |
| 0.179 53           | 0.325 00           | 0.038 82           | 0.044 64             | 0.356 06             |
| 0.200 07           | 0.350 00           | 0.037 61           | 0.043 24             | 0.344 63             |
| 0.221 01           | 0.375 00           | 0.036 47           | 0.041 95             | 0.333 54             |
| 0.242 08           | 0.400 00           | 0.035 41           | 0.040 75             | 0.322 88             |
| 0.263 76           | 0.425 00           | 0.034 38           | 0.039 60             | 0.312 95             |
| 0.286 41           | 0.450 00           | 0.033 38           | 0.038 49             | 0.303 70             |
| 0.309 80           | 0.475 00           | 0.032 41           | 0.037 42             | 0.294 88             |
| 0.334 54           | 0.500 00           | 0.031 45           | 0.036 37             | 0.286 41             |
| 0.360 52           | 0.525 00           | 0.030 52           | 0.035 36             | 0.278 33             |
| 0.386 79           | 0.550 00           | 0.029 63           | 0.034 41             | 0.270 66             |
| 0.413 74           | 0.575 00           | 0.028 78           | 0.033 50             | 0.263 46             |
| 0.441 80           | 0.600 00           | 0.027 95           | 0.032 61             | 0.256 65             |
| 0.470 45           | 0.625 00           | 0.027 16           | 0.031 77             | 0.250 10             |
| 0.499 66           | 0.650 00           | 0.026 40           | 0.030 97             | 0.243 79             |
| 0.529 98           | 0.675 00           | 0.025 65           | 0.030 18             | 0.237 75             |
| 0.561 49           | 0.700 00           | 0.024 92           | 0.029 42             | 0.231 96             |
| 0.593 90           | 0.725 00           | 0.024 22           | 0.028 69             | 0.226 41             |
| 0.627 07           | 0.750 00           | 0.023 54           | 0.028 00             | 0.221 07             |
| 0.661 13           | 0.775 00           | 0.022 89           | 0.027 33             | 0.215 93             |

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**Continued:** VLE-Data from GCMC-Simulations of Mixture I at  $T^* = 1.15$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.696 09           | 0.800 00           | 0.022 25           | 0.026 69             | 0.210 96             |
| 0.731 57           | 0.825 00           | 0.021 65           | 0.026 08             | 0.206 12             |
| 0.767 50           | 0.850 00           | 0.021 06           | 0.025 50             | 0.201 44             |
| 0.804 41           | 0.875 00           | 0.020 50           | 0.024 94             | 0.196 93             |
| 0.842 27           | 0.900 00           | 0.019 95           | 0.024 39             | 0.192 58             |
| 0.881 65           | 0.925 00           | 0.019 41           | 0.023 87             | 0.188 41             |
| 0.920 64           | 0.950 00           | 0.018 92           | 0.023 39             | 0.184 38             |
| 0.959 47           | 0.975 00           | 0.018 44           | 0.022 94             | 0.180 67             |
| 0.998 33           | 0.999 00           | 0.017 97           | 0.022 48             | 0.177 42             |

**Table C.26:** VLE-Data from GCMC-Simulations of Mixture II at  $T^* = 0.90$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.006 21           | 0.001 00           | 0.010 10           | 0.012 18             | 0.739 21             |
| 0.008 20           | 0.025 00           | 0.010 11           | 0.012 19             | 0.703 46             |
| 0.010 05           | 0.050 00           | 0.010 03           | 0.012 10             | 0.673 65             |
| 0.018 88           | 0.075 00           | 0.009 70           | 0.011 68             | 0.642 22             |
| 0.039 26           | 0.100 00           | 0.009 26           | 0.011 14             | 0.619 78             |
| 0.046 54           | 0.125 00           | 0.009 13           | 0.010 99             | 0.592 76             |
| 0.072 18           | 0.150 00           | 0.008 77           | 0.010 55             | 0.571 32             |
| 0.088 86           | 0.175 00           | 0.008 59           | 0.010 35             | 0.548 61             |
| 0.096 67           | 0.200 00           | 0.008 51           | 0.010 26             | 0.525 80             |
| 0.103 03           | 0.225 00           | 0.008 43           | 0.010 17             | 0.504 88             |
| 0.110 63           | 0.250 00           | 0.008 33           | 0.010 05             | 0.485 85             |
| 0.119 55           | 0.275 00           | 0.008 21           | 0.009 91             | 0.467 81             |
| 0.128 52           | 0.300 00           | 0.008 09           | 0.009 76             | 0.450 64             |
| 0.141 66           | 0.325 00           | 0.007 91           | 0.009 54             | 0.435 09             |
| 0.162 70           | 0.350 00           | 0.007 64           | 0.009 22             | 0.421 38             |
| 0.178 42           | 0.375 00           | 0.007 46           | 0.009 00             | 0.408 11             |
| 0.192 64           | 0.400 00           | 0.007 30           | 0.008 81             | 0.395 35             |
| 0.211 09           | 0.425 00           | 0.007 11           | 0.008 57             | 0.383 36             |

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Continued: VLE-Data from GCMC-Simulations of Mixture II at  $T^* = 0.90$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.23249            | 0.45000            | 0.00689            | 0.00832              | 0.37215              |
| 0.25301            | 0.47500            | 0.00671            | 0.00809              | 0.36147              |
| 0.27470            | 0.50000            | 0.00652            | 0.00787              | 0.35137              |
| 0.29819            | 0.52500            | 0.00633            | 0.00764              | 0.34190              |
| 0.32150            | 0.55000            | 0.00615            | 0.00743              | 0.33293              |
| 0.34535            | 0.57500            | 0.00599            | 0.00723              | 0.32440              |
| 0.37191            | 0.60000            | 0.00581            | 0.00702              | 0.31631              |
| 0.40108            | 0.62500            | 0.00563            | 0.00681              | 0.30851              |
| 0.43064            | 0.65000            | 0.00546            | 0.00660              | 0.30096              |
| 0.46002            | 0.67500            | 0.00531            | 0.00642              | 0.29369              |
| 0.49143            | 0.70000            | 0.00515            | 0.00623              | 0.28670              |
| 0.52552            | 0.72500            | 0.00499            | 0.00604              | 0.27991              |
| 0.55979            | 0.75000            | 0.00484            | 0.00587              | 0.27338              |
| 0.59430            | 0.77500            | 0.00470            | 0.00570              | 0.26727              |
| 0.63095            | 0.80000            | 0.00456            | 0.00553              | 0.26157              |
| 0.67130            | 0.82500            | 0.00441            | 0.00536              | 0.25611              |
| 0.71448            | 0.85000            | 0.00427            | 0.00519              | 0.25075              |
| 0.75848            | 0.87500            | 0.00413            | 0.00503              | 0.24550              |
| 0.80343            | 0.90000            | 0.00401            | 0.00488              | 0.24037              |
| 0.84992            | 0.92500            | 0.00388            | 0.00474              | 0.23536              |
| 0.89818            | 0.95000            | 0.00376            | 0.00460              | 0.23036              |
| 0.94633            | 0.97500            | 0.00365            | 0.00446              | 0.22541              |
| 0.99785            | 0.99900            | 0.00354            | 0.00433              | 0.22090              |

Table C.27: VLE-Data from GCMC-Simulations of Mixture II at  $T^* = 1.00$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.00517            | 0.00100            | 0.02240            | 0.02596              | 0.71515              |
| 0.00894            | 0.02500            | 0.02240            | 0.02600              | 0.66704              |
| 0.01055            | 0.05000            | 0.02227            | 0.02583              | 0.62729              |
| 0.01963            | 0.07500            | 0.02142            | 0.02475              | 0.60686              |

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Continued: VLE-Data from GCMC-Simulations of Mixture II at  $T^* = 1.00$ 

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.03499            | 0.10000            | 0.02055            | 0.02369              | 0.58064              |
| 0.04494            | 0.12500            | 0.02007            | 0.02311              | 0.55639              |
| 0.05978            | 0.15000            | 0.01946            | 0.02240              | 0.53459              |
| 0.07008            | 0.17500            | 0.01907            | 0.02194              | 0.51098              |
| 0.08372            | 0.20000            | 0.01858            | 0.02136              | 0.48966              |
| 0.09557            | 0.22500            | 0.01817            | 0.02089              | 0.47058              |
| 0.10889            | 0.25000            | 0.01773            | 0.02038              | 0.45309              |
| 0.12787            | 0.27500            | 0.01717            | 0.01973              | 0.43766              |
| 0.14180            | 0.30000            | 0.01678            | 0.01930              | 0.42223              |
| 0.15321            | 0.32500            | 0.01648            | 0.01894              | 0.40729              |
| 0.16637            | 0.35000            | 0.01612            | 0.01853              | 0.39344              |
| 0.18444            | 0.37500            | 0.01565            | 0.01800              | 0.38088              |
| 0.20629            | 0.40000            | 0.01514            | 0.01741              | 0.36932              |
| 0.22626            | 0.42500            | 0.01471            | 0.01692              | 0.35829              |
| 0.24488            | 0.45000            | 0.01433            | 0.01649              | 0.34775              |
| 0.26554            | 0.47500            | 0.01394            | 0.01604              | 0.33773              |
| 0.29013            | 0.50000            | 0.01350            | 0.01554              | 0.32830              |
| 0.31625            | 0.52500            | 0.01307            | 0.01506              | 0.31950              |
| 0.34287            | 0.55000            | 0.01267            | 0.01461              | 0.31116              |
| 0.36998            | 0.57500            | 0.01229            | 0.01419              | 0.30312              |
| 0.39656            | 0.60000            | 0.01195            | 0.01381              | 0.29530              |
| 0.42334            | 0.62500            | 0.01162            | 0.01344              | 0.28773              |
| 0.45132            | 0.65000            | 0.01130            | 0.01308              | 0.28048              |
| 0.48109            | 0.67500            | 0.01097            | 0.01272              | 0.27361              |
| 0.51307            | 0.70000            | 0.01064            | 0.01236              | 0.26707              |
| 0.54639            | 0.72500            | 0.01033            | 0.01200              | 0.26088              |
| 0.58039            | 0.75000            | 0.01002            | 0.01167              | 0.25504              |
| 0.61588            | 0.77500            | 0.00972            | 0.01134              | 0.24945              |
| 0.65307            | 0.80000            | 0.00943            | 0.01102              | 0.24396              |
| 0.69135            | 0.82500            | 0.00915            | 0.01071              | 0.23860              |
| 0.73134            | 0.85000            | 0.00888            | 0.01040              | 0.23342              |
| 0.77365            | 0.87500            | 0.00861            | 0.01011              | 0.22841              |

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Continued: VLE-Data from GCMC-Simulations of Mixture II at  $T^* = 1.00$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.816 72           | 0.900 00           | 0.008 35           | 0.009 83             | 0.223 52             |
| 0.861 44           | 0.925 00           | 0.008 10           | 0.009 56             | 0.218 78             |
| 0.907 57           | 0.950 00           | 0.007 87           | 0.009 30             | 0.214 04             |
| 0.951 48           | 0.975 00           | 0.007 65           | 0.009 07             | 0.209 78             |
| 0.998 06           | 0.999 00           | 0.007 43           | 0.008 83             | 0.205 84             |

**Table C.28:** VLE-Data from GCMC-Simulations of Mixture II at  $T^* = 1.05$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.003 18           | 0.001 00           | 0.031 88           | 0.036 97             | 0.669 96             |
| 0.007 22           | 0.025 00           | 0.031 83           | 0.036 98             | 0.636 22             |
| 0.014 21           | 0.050 00           | 0.031 06           | 0.035 98             | 0.606 91             |
| 0.022 61           | 0.075 00           | 0.030 20           | 0.034 87             | 0.583 29             |
| 0.032 99           | 0.100 00           | 0.029 31           | 0.033 74             | 0.558 91             |
| 0.041 31           | 0.125 00           | 0.028 64           | 0.032 91             | 0.531 74             |
| 0.051 60           | 0.150 00           | 0.027 83           | 0.031 91             | 0.508 22             |
| 0.065 62           | 0.175 00           | 0.026 86           | 0.030 73             | 0.488 85             |
| 0.084 53           | 0.200 00           | 0.025 79           | 0.029 45             | 0.471 86             |
| 0.101 25           | 0.225 00           | 0.025 00           | 0.028 52             | 0.454 74             |
| 0.114 25           | 0.250 00           | 0.024 43           | 0.027 86             | 0.437 30             |
| 0.126 34           | 0.275 00           | 0.023 90           | 0.027 25             | 0.420 45             |
| 0.139 76           | 0.300 00           | 0.023 34           | 0.026 59             | 0.404 87             |
| 0.156 76           | 0.325 00           | 0.022 65           | 0.025 80             | 0.391 06             |
| 0.177 48           | 0.350 00           | 0.021 90           | 0.024 93             | 0.378 63             |
| 0.198 66           | 0.375 00           | 0.021 21           | 0.024 14             | 0.366 72             |
| 0.217 88           | 0.400 00           | 0.020 62           | 0.023 48             | 0.355 17             |
| 0.236 06           | 0.425 00           | 0.020 10           | 0.022 89             | 0.344 30             |
| 0.256 38           | 0.450 00           | 0.019 55           | 0.022 27             | 0.334 24             |
| 0.280 39           | 0.475 00           | 0.018 94           | 0.021 58             | 0.324 79             |
| 0.305 03           | 0.500 00           | 0.018 36           | 0.020 94             | 0.315 62             |
| 0.328 64           | 0.525 00           | 0.017 85           | 0.020 37             | 0.306 77             |

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**Continued:** VLE-Data from GCMC-Simulations of Mixture II at  $T^* = 1.05$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.35269            | 0.55000            | 0.01736            | 0.01982              | 0.29845              |
| 0.37891            | 0.57500            | 0.01685            | 0.01927              | 0.29066              |
| 0.40774            | 0.60000            | 0.01633            | 0.01870              | 0.28327              |
| 0.43842            | 0.62500            | 0.01582            | 0.01815              | 0.27622              |
| 0.46946            | 0.65000            | 0.01535            | 0.01763              | 0.26951              |
| 0.49976            | 0.67500            | 0.01492            | 0.01717              | 0.26308              |
| 0.53013            | 0.70000            | 0.01451            | 0.01673              | 0.25689              |
| 0.56178            | 0.72500            | 0.01411            | 0.01630              | 0.25095              |
| 0.59463            | 0.75000            | 0.01371            | 0.01587              | 0.24524              |
| 0.62801            | 0.77500            | 0.01333            | 0.01546              | 0.23975              |
| 0.66218            | 0.80000            | 0.01296            | 0.01507              | 0.23450              |
| 0.69808            | 0.82500            | 0.01259            | 0.01467              | 0.22951              |
| 0.73600            | 0.85000            | 0.01222            | 0.01427              | 0.22475              |
| 0.77551            | 0.87500            | 0.01185            | 0.01387              | 0.22016              |
| 0.81797            | 0.90000            | 0.01149            | 0.01347              | 0.21573              |
| 0.86222            | 0.92500            | 0.01113            | 0.01309              | 0.21128              |
| 0.90658            | 0.95000            | 0.01079            | 0.01273              | 0.20670              |
| 0.95259            | 0.97500            | 0.01047            | 0.01238              | 0.20261              |
| 0.99809            | 0.99900            | 0.01016            | 0.01205              | 0.19948              |

**Table C.29:** VLE-Data from GCMC-Simulations of Mixture II at  $T^* = 1.10$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.00164            | 0.00100            | 0.04386            | 0.05148              | 0.65230              |
| 0.00733            | 0.02500            | 0.04340            | 0.05096              | 0.61686              |
| 0.01436            | 0.05000            | 0.04236            | 0.04954              | 0.58395              |
| 0.02429            | 0.07500            | 0.04095            | 0.04762              | 0.55537              |
| 0.03456            | 0.10000            | 0.03974            | 0.04604              | 0.53222              |
| 0.04461            | 0.12500            | 0.03865            | 0.04462              | 0.50866              |
| 0.05627            | 0.15000            | 0.03747            | 0.04311              | 0.48650              |
| 0.07205            | 0.17500            | 0.03608            | 0.04138              | 0.46908              |

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C.3 Tabulated Results of the Vapor Liquid Equilibrium GCMC Simulations

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Continued: VLE-Data from GCMC-Simulations of Mixture II at  $T^* = 1.10$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.08675            | 0.20000            | 0.03497            | 0.04002              | 0.45041              |
| 0.10015            | 0.22500            | 0.03402            | 0.03889              | 0.43193              |
| 0.11405            | 0.25000            | 0.03310            | 0.03777              | 0.41561              |
| 0.13058            | 0.27500            | 0.03207            | 0.03655              | 0.40105              |
| 0.14894            | 0.30000            | 0.03103            | 0.03533              | 0.38749              |
| 0.16670            | 0.32500            | 0.03012            | 0.03427              | 0.37458              |
| 0.18422            | 0.35000            | 0.02927            | 0.03329              | 0.36230              |
| 0.20335            | 0.37500            | 0.02840            | 0.03229              | 0.35082              |
| 0.22450            | 0.40000            | 0.02752            | 0.03128              | 0.34003              |
| 0.24680            | 0.42500            | 0.02666            | 0.03031              | 0.32966              |
| 0.27020            | 0.45000            | 0.02583            | 0.02938              | 0.31981              |
| 0.29399            | 0.47500            | 0.02505            | 0.02852              | 0.31053              |
| 0.31808            | 0.50000            | 0.02432            | 0.02771              | 0.30180              |
| 0.34297            | 0.52500            | 0.02361            | 0.02694              | 0.29351              |
| 0.36861            | 0.55000            | 0.02293            | 0.02619              | 0.28559              |
| 0.39527            | 0.57500            | 0.02227            | 0.02547              | 0.27799              |
| 0.42315            | 0.60000            | 0.02162            | 0.02477              | 0.27071              |
| 0.45198            | 0.62500            | 0.02099            | 0.02410              | 0.26377              |
| 0.48145            | 0.65000            | 0.02039            | 0.02345              | 0.25723              |
| 0.51158            | 0.67500            | 0.01981            | 0.02284              | 0.25104              |
| 0.54261            | 0.70000            | 0.01925            | 0.02224              | 0.24512              |
| 0.57492            | 0.72500            | 0.01870            | 0.02166              | 0.23938              |
| 0.60885            | 0.75000            | 0.01816            | 0.02109              | 0.23381              |
| 0.64394            | 0.77500            | 0.01764            | 0.02054              | 0.22844              |
| 0.67938            | 0.80000            | 0.01714            | 0.02002              | 0.22331              |
| 0.71514            | 0.82500            | 0.01666            | 0.01953              | 0.21842              |
| 0.75167            | 0.85000            | 0.01620            | 0.01905              | 0.21374              |
| 0.78975            | 0.87500            | 0.01575            | 0.01857              | 0.20930              |
| 0.83016            | 0.90000            | 0.01529            | 0.01810              | 0.20508              |
| 0.87151            | 0.92500            | 0.01486            | 0.01764              | 0.20104              |
| 0.91263            | 0.95000            | 0.01445            | 0.01722              | 0.19716              |
| 0.95474            | 0.97500            | 0.01404            | 0.01680              | 0.19348              |

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**Continued:** VLE-Data from GCMC-Simulations of Mixture II at  $T^* = 1.10$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.998 16           | 0.999 00           | 0.013 63           | 0.016 36             | 0.190 03             |

**Table C.30:** VLE-Data from GCMC-Simulations of Mixture II at  $T^* = 1.15$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.001 20           | 0.001 00           | 0.057 49           | 0.069 07             | 0.631 73             |
| 0.009 02           | 0.025 00           | 0.056 46           | 0.067 71             | 0.585 50             |
| 0.016 98           | 0.050 00           | 0.055 15           | 0.065 86             | 0.554 47             |
| 0.025 92           | 0.075 00           | 0.053 60           | 0.063 60             | 0.526 94             |
| 0.037 67           | 0.100 00           | 0.051 76           | 0.060 99             | 0.505 54             |
| 0.050 62           | 0.125 00           | 0.050 05           | 0.058 65             | 0.484 33             |
| 0.064 20           | 0.150 00           | 0.048 45           | 0.056 53             | 0.463 55             |
| 0.077 90           | 0.175 00           | 0.046 98           | 0.054 63             | 0.444 64             |
| 0.092 65           | 0.200 00           | 0.045 54           | 0.052 79             | 0.427 37             |
| 0.107 07           | 0.225 00           | 0.044 22           | 0.051 15             | 0.410 62             |
| 0.122 56           | 0.250 00           | 0.042 90           | 0.049 51             | 0.395 01             |
| 0.139 72           | 0.275 00           | 0.041 54           | 0.047 85             | 0.380 78             |
| 0.159 17           | 0.300 00           | 0.040 14           | 0.046 18             | 0.368 01             |
| 0.179 53           | 0.325 00           | 0.038 82           | 0.044 64             | 0.356 06             |
| 0.200 07           | 0.350 00           | 0.037 61           | 0.043 24             | 0.344 63             |
| 0.221 01           | 0.375 00           | 0.036 47           | 0.041 95             | 0.333 54             |
| 0.242 08           | 0.400 00           | 0.035 41           | 0.040 75             | 0.322 88             |
| 0.263 76           | 0.425 00           | 0.034 38           | 0.039 60             | 0.312 95             |
| 0.286 41           | 0.450 00           | 0.033 38           | 0.038 49             | 0.303 70             |
| 0.309 80           | 0.475 00           | 0.032 41           | 0.037 42             | 0.294 88             |
| 0.334 54           | 0.500 00           | 0.031 45           | 0.036 37             | 0.286 41             |
| 0.360 52           | 0.525 00           | 0.030 52           | 0.035 36             | 0.278 33             |
| 0.386 79           | 0.550 00           | 0.029 63           | 0.034 41             | 0.270 66             |
| 0.413 74           | 0.575 00           | 0.028 78           | 0.033 50             | 0.263 46             |
| 0.441 80           | 0.600 00           | 0.027 95           | 0.032 61             | 0.256 65             |
| 0.470 45           | 0.625 00           | 0.027 16           | 0.031 77             | 0.250 10             |

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Continued: VLE-Data from GCMC-Simulations of Mixture II at  $T^* = 1.15$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.49966            | 0.65000            | 0.02640            | 0.03097              | 0.24379              |
| 0.52998            | 0.67500            | 0.02565            | 0.03018              | 0.23775              |
| 0.56149            | 0.70000            | 0.02492            | 0.02942              | 0.23196              |
| 0.59390            | 0.72500            | 0.02422            | 0.02869              | 0.22641              |
| 0.62707            | 0.75000            | 0.02354            | 0.02800              | 0.22107              |
| 0.66113            | 0.77500            | 0.02289            | 0.02733              | 0.21593              |
| 0.69609            | 0.80000            | 0.02225            | 0.02669              | 0.21096              |
| 0.73157            | 0.82500            | 0.02165            | 0.02608              | 0.20612              |
| 0.76750            | 0.85000            | 0.02106            | 0.02550              | 0.20144              |
| 0.80441            | 0.87500            | 0.02050            | 0.02494              | 0.19693              |
| 0.84227            | 0.90000            | 0.01995            | 0.02439              | 0.19258              |
| 0.88165            | 0.92500            | 0.01941            | 0.02387              | 0.18841              |
| 0.92064            | 0.95000            | 0.01892            | 0.02339              | 0.18438              |
| 0.95947            | 0.97500            | 0.01844            | 0.02294              | 0.18067              |
| 0.99833            | 0.99900            | 0.01797            | 0.02248              | 0.17742              |

Table C.31: VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.00$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.00445            | 0.00100            | 0.02476            | 0.02925              | 0.70050              |
| 0.10171            | 0.02500            | 0.02729            | 0.03253              | 0.69654              |
| 0.18860            | 0.05000            | 0.03009            | 0.03626              | 0.69297              |
| 0.25602            | 0.07500            | 0.03265            | 0.03975              | 0.68751              |
| 0.31153            | 0.10000            | 0.03508            | 0.04315              | 0.68194              |
| 0.36491            | 0.12500            | 0.03778            | 0.04702              | 0.67864              |
| 0.41041            | 0.15000            | 0.04043            | 0.05096              | 0.67571              |
| 0.44989            | 0.17500            | 0.04306            | 0.05500              | 0.67122              |
| 0.48225            | 0.20000            | 0.04548            | 0.05883              | 0.66475              |
| 0.51177            | 0.22500            | 0.04791            | 0.06280              | 0.65832              |
| 0.53957            | 0.25000            | 0.05043            | 0.06704              | 0.65273              |
| 0.56208            | 0.27500            | 0.05265            | 0.07087              | 0.64705              |

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**Continued:** VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.00$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.583 98           | 0.300 00           | 0.054 95           | 0.074 98             | 0.641 36             |
| 0.606 23           | 0.325 00           | 0.057 49           | 0.079 64             | 0.635 67             |
| 0.626 57           | 0.350 00           | 0.060 02           | 0.084 42             | 0.629 35             |
| 0.643 63           | 0.375 00           | 0.062 30           | 0.088 86             | 0.622 31             |
| 0.658 77           | 0.400 00           | 0.064 44           | 0.093 13             | 0.614 65             |
| 0.672 90           | 0.425 00           | 0.066 53           | 0.097 41             | 0.606 51             |
| 0.686 50           | 0.450 00           | 0.068 62           | 0.101 86             | 0.598 69             |
| 0.700 61           | 0.475 00           | 0.070 88           | 0.106 91             | 0.591 63             |
| 0.715 31           | 0.500 00           | 0.073 37           | 0.112 86             | 0.584 23             |
| 0.729 04           | 0.525 00           | 0.075 87           | 0.119 32             | 0.575 43             |
| 0.741 15           | 0.550 00           | 0.078 22           | 0.125 94             | 0.566 10             |
| 0.752 47           | 0.575 00           | 0.080 55           | 0.133 15             | 0.556 77             |
| 0.763 50           | 0.600 00           | 0.083 00           | 0.141 60             | 0.546 67             |
| 0.773 51           | 0.625 00           | 0.085 41           | 0.151 14             | 0.534 50             |
| 0.782 53           | 0.650 00           | 0.087 73           | 0.161 34             | 0.519 62             |
| 0.790 82           | 0.675 00           | 0.090 05           | 0.173 25             | 0.502 94             |
| 0.681 54           | 0.681 54           | 0.013 19           | 0.013 88             | 0.013 88             |
| 0.799 27           | 0.700 00           | 0.092 39           | 0.185 62             | 0.484 50             |
| 0.614 76           | 0.725 00           | 0.012 27           | 0.012 88             | 0.018 80             |

**Table C.32:** VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.05$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.004 08           | 0.001 08           | 0.033 89           | 0.039 86             | 0.674 21             |
| 0.010 74           | 0.002 84           | 0.034 11           | 0.040 15             | 0.674 18             |
| 0.017 33           | 0.004 59           | 0.034 33           | 0.040 45             | 0.674 18             |
| 0.023 83           | 0.006 34           | 0.034 56           | 0.040 74             | 0.674 20             |
| 0.030 27           | 0.008 09           | 0.034 78           | 0.041 04             | 0.674 21             |
| 0.036 62           | 0.009 83           | 0.035 00           | 0.041 33             | 0.674 21             |
| 0.042 91           | 0.011 57           | 0.035 22           | 0.041 63             | 0.674 18             |
| 0.049 13           | 0.013 34           | 0.035 45           | 0.041 93             | 0.674 11             |

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C.3 Tabulated Results of the Vapor Liquid Equilibrium GCMC Simulations

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Continued: VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.05$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.055 27           | 0.015 14           | 0.035 67           | 0.042 23             | 0.674 00             |
| 0.061 35           | 0.016 96           | 0.035 89           | 0.042 53             | 0.673 85             |
| 0.067 36           | 0.018 81           | 0.036 12           | 0.042 83             | 0.673 67             |
| 0.073 31           | 0.020 66           | 0.036 34           | 0.043 13             | 0.673 45             |
| 0.079 19           | 0.022 50           | 0.036 56           | 0.043 43             | 0.673 20             |
| 0.085 01           | 0.024 28           | 0.036 79           | 0.043 73             | 0.672 92             |
| 0.090 76           | 0.026 00           | 0.037 01           | 0.044 03             | 0.672 64             |
| 0.096 46           | 0.027 63           | 0.037 24           | 0.044 34             | 0.672 34             |
| 0.102 08           | 0.029 16           | 0.037 46           | 0.044 65             | 0.672 03             |
| 0.107 65           | 0.030 59           | 0.037 69           | 0.044 95             | 0.671 71             |
| 0.113 15           | 0.031 94           | 0.037 92           | 0.045 27             | 0.671 38             |
| 0.118 59           | 0.033 24           | 0.038 14           | 0.045 58             | 0.671 03             |
| 0.123 97           | 0.034 51           | 0.038 37           | 0.045 90             | 0.670 66             |
| 0.129 29           | 0.035 79           | 0.038 61           | 0.046 22             | 0.670 26             |
| 0.134 55           | 0.037 12           | 0.038 84           | 0.046 54             | 0.669 83             |
| 0.139 75           | 0.038 53           | 0.039 07           | 0.046 86             | 0.669 35             |
| 0.144 89           | 0.040 07           | 0.039 31           | 0.047 19             | 0.668 83             |
| 0.149 98           | 0.041 75           | 0.039 54           | 0.047 51             | 0.668 25             |
| 0.155 01           | 0.043 58           | 0.039 78           | 0.047 84             | 0.667 63             |
| 0.160 00           | 0.045 57           | 0.040 01           | 0.048 17             | 0.666 96             |
| 0.164 93           | 0.047 69           | 0.040 25           | 0.048 50             | 0.666 27             |
| 0.169 81           | 0.049 91           | 0.040 48           | 0.048 83             | 0.665 58             |
| 0.174 65           | 0.052 19           | 0.040 71           | 0.049 15             | 0.664 89             |
| 0.179 45           | 0.054 48           | 0.040 95           | 0.049 48             | 0.664 22             |
| 0.184 20           | 0.056 74           | 0.041 18           | 0.049 81             | 0.663 59             |
| 0.188 90           | 0.058 96           | 0.041 41           | 0.050 13             | 0.663 01             |
| 0.193 56           | 0.061 10           | 0.041 64           | 0.050 46             | 0.662 48             |
| 0.198 18           | 0.063 18           | 0.041 87           | 0.050 79             | 0.661 99             |
| 0.202 75           | 0.065 18           | 0.042 10           | 0.051 11             | 0.661 54             |
| 0.207 28           | 0.067 13           | 0.042 33           | 0.051 44             | 0.661 14             |
| 0.211 77           | 0.069 03           | 0.042 56           | 0.051 77             | 0.660 76             |
| 0.216 22           | 0.070 89           | 0.042 80           | 0.052 10             | 0.660 41             |

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Continued: VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.05$ 

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $p_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.22062            | 0.07272            | 0.04303            | 0.05243              | 0.66008              |
| 0.22498            | 0.07454            | 0.04326            | 0.05277              | 0.65976              |
| 0.22931            | 0.07635            | 0.04349            | 0.05310              | 0.65944              |
| 0.23359            | 0.07817            | 0.04372            | 0.05344              | 0.65913              |
| 0.23783            | 0.07999            | 0.04396            | 0.05377              | 0.65881              |
| 0.24203            | 0.08182            | 0.04419            | 0.05411              | 0.65849              |
| 0.24620            | 0.08366            | 0.04442            | 0.05445              | 0.65815              |
| 0.25033            | 0.08552            | 0.04466            | 0.05479              | 0.65781              |
| 0.25442            | 0.08740            | 0.04489            | 0.05513              | 0.65745              |
| 0.25847            | 0.08929            | 0.04512            | 0.05548              | 0.65707              |
| 0.26249            | 0.09119            | 0.04536            | 0.05582              | 0.65668              |
| 0.26648            | 0.09311            | 0.04559            | 0.05617              | 0.65627              |
| 0.27043            | 0.09503            | 0.04583            | 0.05651              | 0.65585              |
| 0.27434            | 0.09695            | 0.04606            | 0.05686              | 0.65540              |
| 0.27822            | 0.09888            | 0.04630            | 0.05721              | 0.65494              |
| 0.28207            | 0.10082            | 0.04653            | 0.05756              | 0.65447              |
| 0.28589            | 0.10275            | 0.04677            | 0.05792              | 0.65398              |
| 0.28967            | 0.10469            | 0.04701            | 0.05827              | 0.65347              |
| 0.29342            | 0.10663            | 0.04724            | 0.05863              | 0.65296              |
| 0.29714            | 0.10857            | 0.04748            | 0.05898              | 0.65242              |
| 0.30083            | 0.11052            | 0.04772            | 0.05934              | 0.65188              |
| 0.30449            | 0.11248            | 0.04795            | 0.05970              | 0.65133              |
| 0.30812            | 0.11444            | 0.04819            | 0.06006              | 0.65077              |
| 0.31172            | 0.11642            | 0.04843            | 0.06042              | 0.65021              |
| 0.31528            | 0.11840            | 0.04867            | 0.06079              | 0.64964              |
| 0.31882            | 0.12039            | 0.04890            | 0.06115              | 0.64908              |
| 0.32233            | 0.12239            | 0.04914            | 0.06152              | 0.64852              |
| 0.32581            | 0.12439            | 0.04938            | 0.06189              | 0.64796              |
| 0.32926            | 0.12639            | 0.04962            | 0.06225              | 0.64742              |
| 0.33268            | 0.12840            | 0.04986            | 0.06262              | 0.64688              |
| 0.33607            | 0.13039            | 0.05010            | 0.06300              | 0.64636              |
| 0.33944            | 0.13238            | 0.05034            | 0.06337              | 0.64586              |

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C.3 Tabulated Results of the Vapor Liquid Equilibrium GCMC Simulations

Continued: VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.05$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.34278            | 0.13435            | 0.05058            | 0.06374              | 0.64537              |
| 0.34609            | 0.13631            | 0.05082            | 0.06412              | 0.64489              |
| 0.34937            | 0.13824            | 0.05106            | 0.06450              | 0.64444              |
| 0.35263            | 0.14016            | 0.05130            | 0.06488              | 0.64399              |
| 0.35586            | 0.14206            | 0.05154            | 0.06526              | 0.64356              |
| 0.35906            | 0.14394            | 0.05178            | 0.06564              | 0.64314              |
| 0.36223            | 0.14581            | 0.05202            | 0.06602              | 0.64273              |
| 0.36538            | 0.14767            | 0.05226            | 0.06640              | 0.64233              |
| 0.36851            | 0.14952            | 0.05251            | 0.06679              | 0.64193              |
| 0.37160            | 0.15137            | 0.05275            | 0.06718              | 0.64153              |
| 0.37467            | 0.15323            | 0.05299            | 0.06757              | 0.64113              |
| 0.37772            | 0.15510            | 0.05324            | 0.06796              | 0.64073              |
| 0.38074            | 0.15699            | 0.05348            | 0.06835              | 0.64032              |
| 0.38373            | 0.15890            | 0.05373            | 0.06874              | 0.63991              |
| 0.38670            | 0.16085            | 0.05397            | 0.06914              | 0.63949              |
| 0.38965            | 0.16282            | 0.05422            | 0.06953              | 0.63906              |
| 0.39258            | 0.16483            | 0.05446            | 0.06993              | 0.63862              |
| 0.39548            | 0.16687            | 0.05471            | 0.07033              | 0.63818              |
| 0.39836            | 0.16894            | 0.05495            | 0.07073              | 0.63772              |
| 0.40121            | 0.17103            | 0.05520            | 0.07114              | 0.63725              |
| 0.40405            | 0.17315            | 0.05545            | 0.07154              | 0.63678              |
| 0.40686            | 0.17529            | 0.05569            | 0.07194              | 0.63630              |
| 0.40966            | 0.17743            | 0.05594            | 0.07235              | 0.63582              |
| 0.41243            | 0.17958            | 0.05619            | 0.07276              | 0.63532              |
| 0.41518            | 0.18172            | 0.05643            | 0.07317              | 0.63483              |
| 0.41792            | 0.18385            | 0.05668            | 0.07358              | 0.63433              |
| 0.42063            | 0.18596            | 0.05693            | 0.07399              | 0.63383              |
| 0.42332            | 0.18806            | 0.05718            | 0.07440              | 0.63332              |
| 0.42600            | 0.19015            | 0.05742            | 0.07482              | 0.63281              |
| 0.42865            | 0.19222            | 0.05767            | 0.07524              | 0.63230              |
| 0.43129            | 0.19427            | 0.05792            | 0.07565              | 0.63178              |
| 0.43391            | 0.19632            | 0.05817            | 0.07607              | 0.63125              |

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Continued: VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.05$ 

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.436 51           | 0.198 36           | 0.058 42           | 0.076 50             | 0.630 72             |
| 0.439 09           | 0.200 40           | 0.058 67           | 0.076 92             | 0.630 19             |
| 0.441 65           | 0.202 46           | 0.058 92           | 0.077 35             | 0.629 64             |
| 0.444 19           | 0.204 53           | 0.059 17           | 0.077 78             | 0.629 08             |
| 0.446 72           | 0.206 62           | 0.059 42           | 0.078 21             | 0.628 51             |
| 0.449 23           | 0.208 73           | 0.059 67           | 0.078 64             | 0.627 93             |
| 0.451 72           | 0.210 88           | 0.059 92           | 0.079 07             | 0.627 33             |
| 0.454 19           | 0.213 06           | 0.060 17           | 0.079 51             | 0.626 72             |
| 0.456 65           | 0.215 27           | 0.060 42           | 0.079 95             | 0.626 11             |
| 0.459 09           | 0.217 51           | 0.060 67           | 0.080 38             | 0.625 48             |
| 0.461 52           | 0.219 79           | 0.060 93           | 0.080 83             | 0.624 84             |
| 0.463 93           | 0.222 09           | 0.061 18           | 0.081 27             | 0.624 20             |
| 0.466 32           | 0.224 41           | 0.061 43           | 0.081 72             | 0.623 55             |
| 0.468 71           | 0.226 76           | 0.061 68           | 0.082 16             | 0.622 89             |
| 0.471 07           | 0.229 11           | 0.061 94           | 0.082 61             | 0.622 24             |
| 0.473 42           | 0.231 46           | 0.062 19           | 0.083 06             | 0.621 59             |
| 0.475 76           | 0.233 82           | 0.062 44           | 0.083 52             | 0.620 95             |
| 0.478 08           | 0.236 16           | 0.062 69           | 0.083 97             | 0.620 31             |
| 0.480 39           | 0.238 50           | 0.062 95           | 0.084 43             | 0.619 67             |
| 0.482 68           | 0.240 82           | 0.063 20           | 0.084 89             | 0.619 05             |
| 0.484 96           | 0.243 12           | 0.063 45           | 0.085 35             | 0.618 44             |
| 0.487 23           | 0.245 40           | 0.063 71           | 0.085 81             | 0.617 83             |
| 0.489 48           | 0.247 65           | 0.063 96           | 0.086 27             | 0.617 24             |
| 0.491 72           | 0.249 89           | 0.064 22           | 0.086 74             | 0.616 66             |
| 0.493 95           | 0.252 10           | 0.064 47           | 0.087 21             | 0.616 08             |
| 0.496 16           | 0.254 30           | 0.064 72           | 0.087 68             | 0.615 52             |
| 0.498 35           | 0.256 48           | 0.064 98           | 0.088 16             | 0.614 96             |
| 0.500 54           | 0.258 64           | 0.065 24           | 0.088 63             | 0.614 41             |
| 0.502 70           | 0.260 79           | 0.065 49           | 0.089 11             | 0.613 86             |
| 0.504 86           | 0.262 92           | 0.065 75           | 0.089 60             | 0.613 32             |
| 0.507 00           | 0.265 05           | 0.066 00           | 0.090 08             | 0.612 77             |
| 0.509 13           | 0.267 18           | 0.066 26           | 0.090 57             | 0.612 23             |

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C.3 Tabulated Results of the Vapor Liquid Equilibrium GCMC Simulations

Continued: VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.05$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.51124            | 0.26929            | 0.06652            | 0.09106              | 0.61168              |
| 0.51335            | 0.27141            | 0.06677            | 0.09155              | 0.61114              |
| 0.51543            | 0.27353            | 0.06703            | 0.09205              | 0.61058              |
| 0.51751            | 0.27565            | 0.06729            | 0.09255              | 0.61003              |
| 0.51957            | 0.27778            | 0.06755            | 0.09305              | 0.60946              |
| 0.52162            | 0.27991            | 0.06781            | 0.09355              | 0.60889              |
| 0.52366            | 0.28206            | 0.06807            | 0.09406              | 0.60830              |
| 0.52568            | 0.28421            | 0.06833            | 0.09457              | 0.60771              |
| 0.52769            | 0.28638            | 0.06859            | 0.09508              | 0.60711              |
| 0.52969            | 0.28857            | 0.06885            | 0.09560              | 0.60650              |
| 0.53168            | 0.29077            | 0.06911            | 0.09611              | 0.60587              |
| 0.53365            | 0.29299            | 0.06937            | 0.09664              | 0.60523              |
| 0.53561            | 0.29522            | 0.06963            | 0.09716              | 0.60458              |
| 0.53756            | 0.29748            | 0.06990            | 0.09769              | 0.60392              |
| 0.53950            | 0.29976            | 0.07016            | 0.09822              | 0.60325              |
| 0.54143            | 0.30206            | 0.07042            | 0.09875              | 0.60256              |
| 0.54335            | 0.30438            | 0.07068            | 0.09929              | 0.60187              |
| 0.54525            | 0.30672            | 0.07095            | 0.09983              | 0.60116              |
| 0.54715            | 0.30908            | 0.07121            | 0.10037              | 0.60045              |
| 0.54903            | 0.31146            | 0.07147            | 0.10091              | 0.59973              |
| 0.55090            | 0.31386            | 0.07174            | 0.10146              | 0.59900              |
| 0.55277            | 0.31626            | 0.07200            | 0.10201              | 0.59826              |
| 0.55462            | 0.31868            | 0.07227            | 0.10257              | 0.59752              |
| 0.55646            | 0.32112            | 0.07253            | 0.10312              | 0.59677              |
| 0.55829            | 0.32355            | 0.07280            | 0.10368              | 0.59602              |
| 0.56011            | 0.32600            | 0.07306            | 0.10425              | 0.59527              |
| 0.56192            | 0.32844            | 0.07333            | 0.10481              | 0.59451              |
| 0.56372            | 0.33089            | 0.07360            | 0.10538              | 0.59376              |
| 0.56552            | 0.33333            | 0.07386            | 0.10595              | 0.59300              |
| 0.56730            | 0.33577            | 0.07413            | 0.10653              | 0.59224              |
| 0.56907            | 0.33821            | 0.07439            | 0.10711              | 0.59149              |
| 0.57083            | 0.34064            | 0.07466            | 0.10769              | 0.59073              |

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**Continued:** VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.05$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.57258            | 0.34306            | 0.07493            | 0.10828              | 0.58997              |
| 0.57432            | 0.34547            | 0.07520            | 0.10887              | 0.58921              |
| 0.57605            | 0.34788            | 0.07547            | 0.10946              | 0.58846              |
| 0.57777            | 0.35028            | 0.07573            | 0.11005              | 0.58770              |
| 0.57948            | 0.35267            | 0.07600            | 0.11065              | 0.58694              |
| 0.58118            | 0.35505            | 0.07627            | 0.11126              | 0.58618              |
| 0.58287            | 0.35743            | 0.07654            | 0.11186              | 0.58541              |
| 0.58455            | 0.35981            | 0.07681            | 0.11247              | 0.58465              |
| 0.58622            | 0.36218            | 0.07708            | 0.11309              | 0.58387              |
| 0.58788            | 0.36456            | 0.07735            | 0.11371              | 0.58310              |
| 0.58953            | 0.36693            | 0.07762            | 0.11433              | 0.58232              |
| 0.59117            | 0.36930            | 0.07789            | 0.11496              | 0.58153              |
| 0.59280            | 0.37168            | 0.07816            | 0.11559              | 0.58073              |
| 0.59443            | 0.37406            | 0.07843            | 0.11622              | 0.57993              |
| 0.59604            | 0.37644            | 0.07871            | 0.11686              | 0.57911              |
| 0.59764            | 0.37884            | 0.07898            | 0.11751              | 0.57829              |
| 0.59923            | 0.38123            | 0.07925            | 0.11815              | 0.57746              |
| 0.60081            | 0.38364            | 0.07952            | 0.11881              | 0.57662              |
| 0.60239            | 0.38606            | 0.07980            | 0.11946              | 0.57577              |
| 0.60395            | 0.38848            | 0.08007            | 0.12013              | 0.57490              |
| 0.60550            | 0.39091            | 0.08035            | 0.12079              | 0.57403              |
| 0.60705            | 0.39335            | 0.08062            | 0.12147              | 0.57315              |
| 0.60858            | 0.39580            | 0.08090            | 0.12214              | 0.57226              |
| 0.61011            | 0.39826            | 0.08117            | 0.12282              | 0.57136              |
| 0.61163            | 0.40072            | 0.08145            | 0.12351              | 0.57044              |
| 0.61314            | 0.40319            | 0.08172            | 0.12420              | 0.56953              |
| 0.61464            | 0.40567            | 0.08200            | 0.12490              | 0.56860              |
| 0.61613            | 0.40815            | 0.08227            | 0.12561              | 0.56766              |
| 0.61761            | 0.41064            | 0.08255            | 0.12631              | 0.56672              |
| 0.61909            | 0.41313            | 0.08283            | 0.12703              | 0.56577              |
| 0.62056            | 0.41563            | 0.08311            | 0.12775              | 0.56482              |
| 0.62201            | 0.41813            | 0.08338            | 0.12848              | 0.56386              |

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C.3 Tabulated Results of the Vapor Liquid Equilibrium GCMC Simulations

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Continued: VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.05$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.62346            | 0.42063            | 0.08366            | 0.12921              | 0.56289              |
| 0.62490            | 0.42313            | 0.08394            | 0.12995              | 0.56193              |
| 0.62634            | 0.42564            | 0.08422            | 0.13070              | 0.56095              |
| 0.62776            | 0.42815            | 0.08450            | 0.13145              | 0.55997              |
| 0.62919            | 0.43066            | 0.08478            | 0.13220              | 0.55898              |
| 0.63060            | 0.43317            | 0.08506            | 0.13297              | 0.55799              |
| 0.63199            | 0.43569            | 0.08534            | 0.13375              | 0.55700              |
| 0.63338            | 0.43820            | 0.08562            | 0.13453              | 0.55601              |
| 0.63477            | 0.44072            | 0.08590            | 0.13533              | 0.55501              |
| 0.63614            | 0.44323            | 0.08618            | 0.13613              | 0.55400              |
| 0.63750            | 0.44575            | 0.08646            | 0.13694              | 0.55299              |
| 0.63886            | 0.44826            | 0.08674            | 0.13777              | 0.55198              |
| 0.64021            | 0.45078            | 0.08703            | 0.13859              | 0.55096              |
| 0.64156            | 0.45330            | 0.08731            | 0.13942              | 0.54991              |
| 0.64290            | 0.45581            | 0.08759            | 0.14027              | 0.54887              |
| 0.64422            | 0.45833            | 0.08788            | 0.14112              | 0.54782              |
| 0.64553            | 0.46084            | 0.08816            | 0.14200              | 0.54677              |
| 0.64683            | 0.46335            | 0.08844            | 0.14288              | 0.54571              |
| 0.64816            | 0.46587            | 0.08873            | 0.14375              | 0.54461              |
| 0.64945            | 0.46838            | 0.08902            | 0.14465              | 0.54352              |
| 0.65073            | 0.47089            | 0.08930            | 0.14557              | 0.54242              |
| 0.65200            | 0.47340            | 0.08959            | 0.14649              | 0.54131              |
| 0.65324            | 0.47590            | 0.08987            | 0.14746              | 0.54021              |
| 0.65449            | 0.47841            | 0.09016            | 0.14841              | 0.53908              |
| 0.65572            | 0.48093            | 0.09045            | 0.14938              | 0.53793              |
| 0.65698            | 0.48345            | 0.09074            | 0.15034              | 0.53674              |
| 0.65820            | 0.48597            | 0.09103            | 0.15134              | 0.53557              |
| 0.65941            | 0.48850            | 0.09132            | 0.15235              | 0.53437              |
| 0.66059            | 0.49102            | 0.09161            | 0.15340              | 0.53318              |
| 0.66178            | 0.49356            | 0.09190            | 0.15444              | 0.53195              |
| 0.66293            | 0.49610            | 0.09219            | 0.15553              | 0.53074              |
| 0.66412            | 0.49867            | 0.09248            | 0.15658              | 0.52945              |

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**Continued:** VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.05$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.665 23           | 0.501 22           | 0.092 77           | 0.157 73             | 0.528 22             |
| 0.666 40           | 0.503 81           | 0.093 07           | 0.158 81             | 0.526 89             |
| 0.667 57           | 0.506 41           | 0.093 36           | 0.159 90             | 0.525 54             |
| 0.668 86           | 0.509 07           | 0.093 65           | 0.160 86             | 0.524 02             |
| 0.669 92           | 0.511 67           | 0.093 95           | 0.162 09             | 0.522 73             |
| 0.671 03           | 0.514 30           | 0.094 24           | 0.163 25             | 0.521 34             |
| 0.672 13           | 0.516 95           | 0.094 54           | 0.164 44             | 0.519 95             |
| 0.673 23           | 0.519 61           | 0.094 83           | 0.165 63             | 0.518 51             |
| 0.674 21           | 0.522 25           | 0.095 13           | 0.166 97             | 0.517 20             |
| 0.675 28           | 0.524 93           | 0.095 43           | 0.168 21             | 0.515 74             |
| 0.676 29           | 0.527 61           | 0.095 73           | 0.169 53             | 0.514 33             |
| 0.677 34           | 0.530 31           | 0.096 03           | 0.170 81             | 0.512 85             |
| 0.678 47           | 0.533 06           | 0.096 32           | 0.172 00             | 0.511 24             |
| 0.679 50           | 0.535 78           | 0.096 62           | 0.173 31             | 0.509 73             |
| 0.680 78           | 0.538 61           | 0.096 92           | 0.174 33             | 0.507 89             |
| 0.681 81           | 0.541 34           | 0.097 22           | 0.175 65             | 0.506 33             |
| 0.682 85           | 0.544 09           | 0.097 52           | 0.176 96             | 0.504 75             |
| 0.683 89           | 0.546 83           | 0.097 82           | 0.178 29             | 0.503 14             |
| 0.684 68           | 0.549 46           | 0.098 13           | 0.179 95             | 0.501 87             |
| 0.685 68           | 0.552 18           | 0.098 43           | 0.181 34             | 0.500 29             |
| 0.687 01           | 0.555 05           | 0.098 72           | 0.182 31             | 0.498 25             |
| 0.688 02           | 0.557 77           | 0.099 03           | 0.183 69             | 0.496 63             |
| 0.688 93           | 0.560 43           | 0.099 33           | 0.185 23             | 0.495 15             |
| 0.689 94           | 0.563 12           | 0.099 64           | 0.186 63             | 0.493 53             |
| 0.691 08           | 0.565 87           | 0.099 94           | 0.187 87             | 0.491 71             |
| 0.691 97           | 0.568 49           | 0.100 24           | 0.189 44             | 0.490 24             |
| 0.692 99           | 0.571 16           | 0.100 55           | 0.190 84             | 0.488 59             |
| 0.693 58           | 0.573 59           | 0.100 86           | 0.192 87             | 0.487 57             |
| 0.694 57           | 0.576 21           | 0.101 17           | 0.194 34             | 0.485 98             |
| 0.696 59           | 0.579 34           | 0.101 45           | 0.194 32             | 0.482 79             |
| 0.697 62           | 0.581 95           | 0.101 76           | 0.195 74             | 0.481 12             |
| 0.698 65           | 0.584 55           | 0.102 07           | 0.197 15             | 0.479 46             |

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**Continued:** VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.05$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.69969            | 0.58716            | 0.10237            | 0.19853              | 0.47776              |
| 0.70073            | 0.58974            | 0.10268            | 0.19993              | 0.47607              |
| 0.70161            | 0.59222            | 0.10299            | 0.20158              | 0.47468              |
| 0.70283            | 0.59488            | 0.10330            | 0.20272              | 0.47271              |
| 0.70405            | 0.59753            | 0.10360            | 0.20386              | 0.47075              |
| 0.70530            | 0.60019            | 0.10390            | 0.20495              | 0.46872              |
| 0.70637            | 0.60274            | 0.10421            | 0.20631              | 0.46702              |
| 0.70704            | 0.60501            | 0.10453            | 0.20833              | 0.46610              |
| 0.70811            | 0.60753            | 0.10485            | 0.20971              | 0.46445              |
| 0.70939            | 0.61018            | 0.10515            | 0.21073              | 0.46239              |
| 0.71049            | 0.61272            | 0.10546            | 0.21204              | 0.46068              |
| 0.71325            | 0.61642            | 0.10571            | 0.21063              | 0.45556              |
| 0.71439            | 0.61898            | 0.10602            | 0.21186              | 0.45377              |
| 0.71529            | 0.62134            | 0.10634            | 0.21351              | 0.45255              |
| 0.71595            | 0.62351            | 0.10667            | 0.21557              | 0.45188              |
| 0.71752            | 0.62639            | 0.10696            | 0.21605              | 0.44917              |
| 0.71868            | 0.62892            | 0.10727            | 0.21727              | 0.44745              |
| 0.71985            | 0.63146            | 0.10758            | 0.21845              | 0.44571              |
| 0.72098            | 0.63394            | 0.10789            | 0.21973              | 0.44412              |
| 0.72217            | 0.63647            | 0.10820            | 0.22088              | 0.44239              |
| 0.72358            | 0.63921            | 0.10850            | 0.22163              | 0.44013              |
| 0.72322            | 0.64036            | 0.10889            | 0.22564              | 0.44213              |
| 0.72429            | 0.64275            | 0.10921            | 0.22703              | 0.44079              |
| 0.68291            | 0.68291            | 0.10985            | 0.33766              | 0.33766              |

**Table C.33:** VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.10$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.00336            | 0.00100            | 0.04567            | 0.05432              | 0.64125              |
| 0.07360            | 0.02500            | 0.04908            | 0.05915              | 0.63660              |
| 0.13811            | 0.05000            | 0.05267            | 0.06440              | 0.62987              |

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**Continued:** VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.10$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.18871            | 0.07500            | 0.05583            | 0.06917              | 0.62131              |
| 0.23784            | 0.10000            | 0.05924            | 0.07447              | 0.61506              |
| 0.28822            | 0.12500            | 0.06326            | 0.08102              | 0.61007              |
| 0.32975            | 0.15000            | 0.06710            | 0.08756              | 0.60590              |
| 0.36499            | 0.17500            | 0.07077            | 0.09409              | 0.60026              |
| 0.39614            | 0.20000            | 0.07438            | 0.10083              | 0.59359              |
| 0.42487            | 0.22500            | 0.07808            | 0.10817              | 0.58612              |
| 0.44804            | 0.25000            | 0.08135            | 0.11517              | 0.57786              |
| 0.46735            | 0.27500            | 0.08427            | 0.12184              | 0.56821              |
| 0.48573            | 0.30000            | 0.08722            | 0.12907              | 0.55770              |
| 0.50421            | 0.32500            | 0.09043            | 0.13760              | 0.54707              |
| 0.52176            | 0.35000            | 0.09379            | 0.14742              | 0.53622              |
| 0.53739            | 0.37500            | 0.09718            | 0.15842              | 0.52456              |
| 0.55076            | 0.40000            | 0.10051            | 0.17076              | 0.51180              |
| 0.56278            | 0.42500            | 0.10373            | 0.18329              | 0.49761              |
| 0.57400            | 0.45000            | 0.10689            | 0.19607              | 0.48293              |
| 0.58481            | 0.47500            | 0.11010            | 0.20981              | 0.46885              |
| 0.59808            | 0.50000            | 0.11308            | 0.21870              | 0.45166              |
| 0.60992            | 0.52500            | 0.11685            | 0.23612              | 0.44124              |
| 0.56097            | 0.56097            | 0.11619            | 0.34038              | 0.34038              |

**Table C.34:** VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.15$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.00692            | 0.00256            | 0.06009            | 0.07354              | 0.60515              |
| 0.00692            | 0.00256            | 0.06009            | 0.07354              | 0.60515              |
| 0.01108            | 0.00417            | 0.06034            | 0.07392              | 0.60444              |
| 0.01108            | 0.00417            | 0.06034            | 0.07392              | 0.60444              |
| 0.01521            | 0.00578            | 0.06059            | 0.07430              | 0.60384              |
| 0.01521            | 0.00578            | 0.06059            | 0.07430              | 0.60384              |
| 0.01931            | 0.00738            | 0.06085            | 0.07468              | 0.60331              |

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**Continued:** VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.15$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.01931            | 0.00738            | 0.06085            | 0.07468              | 0.60331              |
| 0.02338            | 0.00896            | 0.06110            | 0.07507              | 0.60284              |
| 0.02338            | 0.00896            | 0.06110            | 0.07507              | 0.60284              |
| 0.02742            | 0.01053            | 0.06135            | 0.07546              | 0.60239              |
| 0.02742            | 0.01053            | 0.06135            | 0.07546              | 0.60239              |
| 0.03143            | 0.01210            | 0.06161            | 0.07585              | 0.60197              |
| 0.03143            | 0.01210            | 0.06161            | 0.07585              | 0.60197              |
| 0.03541            | 0.01367            | 0.06186            | 0.07624              | 0.60156              |
| 0.03541            | 0.01367            | 0.06186            | 0.07624              | 0.60156              |
| 0.03936            | 0.01524            | 0.06212            | 0.07663              | 0.60117              |
| 0.03936            | 0.01524            | 0.06212            | 0.07663              | 0.60117              |
| 0.04329            | 0.01682            | 0.06237            | 0.07703              | 0.60078              |
| 0.04329            | 0.01682            | 0.06237            | 0.07703              | 0.60078              |
| 0.04718            | 0.01840            | 0.06263            | 0.07743              | 0.60040              |
| 0.04718            | 0.01840            | 0.06263            | 0.07743              | 0.60040              |
| 0.05104            | 0.01998            | 0.06289            | 0.07783              | 0.60002              |
| 0.05104            | 0.01998            | 0.06289            | 0.07783              | 0.60002              |
| 0.05488            | 0.02156            | 0.06314            | 0.07823              | 0.59965              |
| 0.05488            | 0.02156            | 0.06314            | 0.07823              | 0.59965              |
| 0.05868            | 0.02313            | 0.06340            | 0.07863              | 0.59929              |
| 0.05868            | 0.02313            | 0.06340            | 0.07863              | 0.59929              |
| 0.06246            | 0.02471            | 0.06366            | 0.07904              | 0.59894              |
| 0.06246            | 0.02471            | 0.06366            | 0.07904              | 0.59894              |
| 0.06621            | 0.02628            | 0.06392            | 0.07945              | 0.59859              |
| 0.06621            | 0.02628            | 0.06392            | 0.07945              | 0.59859              |
| 0.06993            | 0.02784            | 0.06417            | 0.07985              | 0.59824              |
| 0.06993            | 0.02784            | 0.06417            | 0.07985              | 0.59824              |
| 0.07362            | 0.02940            | 0.06443            | 0.08026              | 0.59790              |
| 0.07362            | 0.02940            | 0.06443            | 0.08026              | 0.59790              |
| 0.07729            | 0.03096            | 0.06469            | 0.08068              | 0.59756              |
| 0.07729            | 0.03096            | 0.06469            | 0.08068              | 0.59756              |
| 0.08093            | 0.03251            | 0.06495            | 0.08109              | 0.59722              |

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**Continued:** VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.15$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.08093            | 0.03251            | 0.06495            | 0.08109              | 0.59722              |
| 0.08455            | 0.03405            | 0.06521            | 0.08151              | 0.59688              |
| 0.08455            | 0.03405            | 0.06521            | 0.08151              | 0.59688              |
| 0.08813            | 0.03560            | 0.06547            | 0.08192              | 0.59654              |
| 0.08813            | 0.03560            | 0.06547            | 0.08192              | 0.59654              |
| 0.09170            | 0.03715            | 0.06573            | 0.08234              | 0.59619              |
| 0.09170            | 0.03715            | 0.06573            | 0.08234              | 0.59619              |
| 0.09523            | 0.03871            | 0.06599            | 0.08277              | 0.59584              |
| 0.09523            | 0.03871            | 0.06599            | 0.08277              | 0.59584              |
| 0.09875            | 0.04028            | 0.06625            | 0.08319              | 0.59548              |
| 0.09875            | 0.04028            | 0.06625            | 0.08319              | 0.59548              |
| 0.10224            | 0.04186            | 0.06651            | 0.08362              | 0.59511              |
| 0.10224            | 0.04186            | 0.06651            | 0.08362              | 0.59511              |
| 0.10570            | 0.04346            | 0.06678            | 0.08404              | 0.59473              |
| 0.10570            | 0.04346            | 0.06678            | 0.08404              | 0.59473              |
| 0.10914            | 0.04508            | 0.06704            | 0.08447              | 0.59434              |
| 0.10914            | 0.04508            | 0.06704            | 0.08447              | 0.59434              |
| 0.11256            | 0.04671            | 0.06730            | 0.08490              | 0.59395              |
| 0.11256            | 0.04671            | 0.06730            | 0.08490              | 0.59395              |
| 0.11595            | 0.04836            | 0.06756            | 0.08533              | 0.59354              |
| 0.11595            | 0.04836            | 0.06756            | 0.08533              | 0.59354              |
| 0.11932            | 0.05001            | 0.06783            | 0.08577              | 0.59313              |
| 0.11932            | 0.05001            | 0.06783            | 0.08577              | 0.59313              |
| 0.12267            | 0.05167            | 0.06809            | 0.08620              | 0.59272              |
| 0.12267            | 0.05167            | 0.06809            | 0.08620              | 0.59272              |
| 0.12600            | 0.05334            | 0.06835            | 0.08664              | 0.59230              |
| 0.12600            | 0.05334            | 0.06835            | 0.08664              | 0.59230              |
| 0.12931            | 0.05500            | 0.06861            | 0.08707              | 0.59189              |
| 0.12931            | 0.05500            | 0.06861            | 0.08707              | 0.59189              |
| 0.13259            | 0.05665            | 0.06888            | 0.08751              | 0.59147              |
| 0.13259            | 0.05665            | 0.06888            | 0.08751              | 0.59147              |
| 0.13585            | 0.05830            | 0.06914            | 0.08795              | 0.59106              |

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C.3 Tabulated Results of the Vapor Liquid Equilibrium GCMC Simulations

Continued: VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.15$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.135 85           | 0.058 30           | 0.069 14           | 0.087 95             | 0.591 06             |
| 0.139 09           | 0.059 94           | 0.069 40           | 0.088 39             | 0.590 65             |
| 0.139 09           | 0.059 94           | 0.069 40           | 0.088 39             | 0.590 65             |
| 0.142 31           | 0.061 56           | 0.069 67           | 0.088 84             | 0.590 25             |
| 0.142 31           | 0.061 56           | 0.069 67           | 0.088 84             | 0.590 25             |
| 0.145 51           | 0.063 16           | 0.069 93           | 0.089 28             | 0.589 85             |
| 0.145 51           | 0.063 16           | 0.069 93           | 0.089 28             | 0.589 85             |
| 0.148 69           | 0.064 75           | 0.070 20           | 0.089 73             | 0.589 46             |
| 0.148 69           | 0.064 75           | 0.070 20           | 0.089 73             | 0.589 46             |
| 0.151 84           | 0.066 32           | 0.070 46           | 0.090 18             | 0.589 08             |
| 0.151 84           | 0.066 32           | 0.070 46           | 0.090 18             | 0.589 08             |
| 0.154 98           | 0.067 88           | 0.070 73           | 0.090 63             | 0.588 70             |
| 0.154 98           | 0.067 88           | 0.070 73           | 0.090 63             | 0.588 70             |
| 0.158 10           | 0.069 42           | 0.070 99           | 0.091 09             | 0.588 33             |
| 0.158 10           | 0.069 42           | 0.070 99           | 0.091 09             | 0.588 33             |
| 0.161 19           | 0.070 94           | 0.071 26           | 0.091 55             | 0.587 96             |
| 0.161 19           | 0.070 94           | 0.071 26           | 0.091 55             | 0.587 96             |
| 0.164 27           | 0.072 45           | 0.071 52           | 0.092 01             | 0.587 60             |
| 0.164 27           | 0.072 45           | 0.071 52           | 0.092 01             | 0.587 60             |
| 0.167 32           | 0.073 94           | 0.071 79           | 0.092 47             | 0.587 24             |
| 0.167 32           | 0.073 94           | 0.071 79           | 0.092 47             | 0.587 24             |
| 0.170 35           | 0.075 43           | 0.072 06           | 0.092 94             | 0.586 88             |
| 0.170 35           | 0.075 43           | 0.072 06           | 0.092 94             | 0.586 88             |
| 0.173 37           | 0.076 91           | 0.072 33           | 0.093 41             | 0.586 52             |
| 0.173 37           | 0.076 91           | 0.072 33           | 0.093 41             | 0.586 52             |
| 0.176 36           | 0.078 38           | 0.072 60           | 0.093 88             | 0.586 15             |
| 0.176 36           | 0.078 38           | 0.072 60           | 0.093 88             | 0.586 15             |
| 0.179 34           | 0.079 86           | 0.072 87           | 0.094 36             | 0.585 79             |
| 0.179 34           | 0.079 86           | 0.072 87           | 0.094 36             | 0.585 79             |
| 0.182 29           | 0.081 34           | 0.073 14           | 0.094 84             | 0.585 42             |
| 0.182 29           | 0.081 34           | 0.073 14           | 0.094 84             | 0.585 42             |
| 0.185 22           | 0.082 83           | 0.073 41           | 0.095 32             | 0.585 04             |

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**Continued:** VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.15$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.185 22           | 0.082 83           | 0.073 41           | 0.095 32             | 0.585 04             |
| 0.188 14           | 0.084 33           | 0.073 68           | 0.095 81             | 0.584 65             |
| 0.188 14           | 0.084 33           | 0.073 68           | 0.095 81             | 0.584 65             |
| 0.191 04           | 0.085 84           | 0.073 96           | 0.096 30             | 0.584 25             |
| 0.191 04           | 0.085 84           | 0.073 96           | 0.096 30             | 0.584 25             |
| 0.193 92           | 0.087 38           | 0.074 23           | 0.096 79             | 0.583 85             |
| 0.193 92           | 0.087 38           | 0.074 23           | 0.096 79             | 0.583 85             |
| 0.196 77           | 0.088 93           | 0.074 50           | 0.097 29             | 0.583 43             |
| 0.196 77           | 0.088 93           | 0.074 50           | 0.097 29             | 0.583 43             |
| 0.199 62           | 0.090 51           | 0.074 78           | 0.097 79             | 0.583 00             |
| 0.199 62           | 0.090 51           | 0.074 78           | 0.097 79             | 0.583 00             |
| 0.202 44           | 0.092 12           | 0.075 05           | 0.098 29             | 0.582 55             |
| 0.202 44           | 0.092 12           | 0.075 05           | 0.098 29             | 0.582 55             |
| 0.205 24           | 0.093 75           | 0.075 33           | 0.098 79             | 0.582 10             |
| 0.205 24           | 0.093 75           | 0.075 33           | 0.098 79             | 0.582 10             |
| 0.208 03           | 0.095 40           | 0.075 60           | 0.099 30             | 0.581 63             |
| 0.208 03           | 0.095 40           | 0.075 60           | 0.099 30             | 0.581 63             |
| 0.210 80           | 0.097 07           | 0.075 88           | 0.099 81             | 0.581 15             |
| 0.210 80           | 0.097 07           | 0.075 88           | 0.099 81             | 0.581 15             |
| 0.213 55           | 0.098 77           | 0.076 15           | 0.100 33             | 0.580 66             |
| 0.213 55           | 0.098 77           | 0.076 15           | 0.100 33             | 0.580 66             |
| 0.216 29           | 0.100 48           | 0.076 43           | 0.100 84             | 0.580 16             |
| 0.216 29           | 0.100 48           | 0.076 43           | 0.100 84             | 0.580 16             |
| 0.219 01           | 0.102 21           | 0.076 71           | 0.101 36             | 0.579 65             |
| 0.219 01           | 0.102 21           | 0.076 71           | 0.101 36             | 0.579 65             |
| 0.221 71           | 0.103 94           | 0.076 98           | 0.101 88             | 0.579 13             |
| 0.221 71           | 0.103 94           | 0.076 98           | 0.101 88             | 0.579 13             |
| 0.224 39           | 0.105 68           | 0.077 26           | 0.102 40             | 0.578 60             |
| 0.224 39           | 0.105 68           | 0.077 26           | 0.102 40             | 0.578 60             |
| 0.227 06           | 0.107 43           | 0.077 54           | 0.102 93             | 0.578 07             |
| 0.227 06           | 0.107 43           | 0.077 54           | 0.102 93             | 0.578 07             |
| 0.229 71           | 0.109 17           | 0.077 81           | 0.103 46             | 0.577 54             |

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C.3 Tabulated Results of the Vapor Liquid Equilibrium GCMC Simulations

Continued: VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.15$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.22971            | 0.10917            | 0.07781            | 0.10346              | 0.57754              |
| 0.23235            | 0.11090            | 0.07809            | 0.10399              | 0.57701              |
| 0.23235            | 0.11090            | 0.07809            | 0.10399              | 0.57701              |
| 0.23496            | 0.11263            | 0.07837            | 0.10452              | 0.57647              |
| 0.23496            | 0.11263            | 0.07837            | 0.10452              | 0.57647              |
| 0.23756            | 0.11435            | 0.07865            | 0.10505              | 0.57593              |
| 0.23756            | 0.11435            | 0.07865            | 0.10505              | 0.57593              |
| 0.24015            | 0.11606            | 0.07893            | 0.10559              | 0.57539              |
| 0.24015            | 0.11606            | 0.07893            | 0.10559              | 0.57539              |
| 0.24272            | 0.11775            | 0.07921            | 0.10613              | 0.57486              |
| 0.24272            | 0.11775            | 0.07921            | 0.10613              | 0.57486              |
| 0.24527            | 0.11944            | 0.07948            | 0.10667              | 0.57432              |
| 0.24527            | 0.11944            | 0.07948            | 0.10667              | 0.57432              |
| 0.24780            | 0.12111            | 0.07976            | 0.10722              | 0.57379              |
| 0.24780            | 0.12111            | 0.07976            | 0.10722              | 0.57379              |
| 0.25032            | 0.12277            | 0.08004            | 0.10777              | 0.57326              |
| 0.25032            | 0.12277            | 0.08004            | 0.10777              | 0.57326              |
| 0.25282            | 0.12443            | 0.08032            | 0.10832              | 0.57273              |
| 0.25282            | 0.12443            | 0.08032            | 0.10832              | 0.57273              |
| 0.25530            | 0.12608            | 0.08061            | 0.10888              | 0.57220              |
| 0.25530            | 0.12608            | 0.08061            | 0.10888              | 0.57220              |
| 0.25777            | 0.12772            | 0.08089            | 0.10944              | 0.57168              |
| 0.25777            | 0.12772            | 0.08089            | 0.10944              | 0.57168              |
| 0.26022            | 0.12937            | 0.08117            | 0.11000              | 0.57115              |
| 0.26022            | 0.12937            | 0.08117            | 0.11000              | 0.57115              |
| 0.26265            | 0.13101            | 0.08145            | 0.11056              | 0.57062              |
| 0.26265            | 0.13101            | 0.08145            | 0.11056              | 0.57062              |
| 0.26507            | 0.13266            | 0.08173            | 0.11113              | 0.57010              |
| 0.26507            | 0.13266            | 0.08173            | 0.11113              | 0.57010              |
| 0.26747            | 0.13432            | 0.08202            | 0.11170              | 0.56956              |
| 0.26747            | 0.13432            | 0.08202            | 0.11170              | 0.56956              |
| 0.26985            | 0.13599            | 0.08230            | 0.11228              | 0.56903              |

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**Continued:** VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.15$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.26985            | 0.13599            | 0.08230            | 0.11228              | 0.56903              |
| 0.27222            | 0.13767            | 0.08258            | 0.11286              | 0.56849              |
| 0.27222            | 0.13767            | 0.08258            | 0.11286              | 0.56849              |
| 0.27457            | 0.13936            | 0.08287            | 0.11344              | 0.56794              |
| 0.27457            | 0.13936            | 0.08287            | 0.11344              | 0.56794              |
| 0.27691            | 0.14107            | 0.08315            | 0.11403              | 0.56738              |
| 0.27691            | 0.14107            | 0.08315            | 0.11403              | 0.56738              |
| 0.27923            | 0.14279            | 0.08344            | 0.11462              | 0.56682              |
| 0.27923            | 0.14279            | 0.08344            | 0.11462              | 0.56682              |
| 0.28153            | 0.14453            | 0.08372            | 0.11521              | 0.56624              |
| 0.28153            | 0.14453            | 0.08372            | 0.11521              | 0.56624              |
| 0.28382            | 0.14629            | 0.08401            | 0.11581              | 0.56565              |
| 0.28382            | 0.14629            | 0.08401            | 0.11581              | 0.56565              |
| 0.28610            | 0.14807            | 0.08429            | 0.11641              | 0.56505              |
| 0.28610            | 0.14807            | 0.08429            | 0.11641              | 0.56505              |
| 0.28836            | 0.14986            | 0.08458            | 0.11702              | 0.56444              |
| 0.28836            | 0.14986            | 0.08458            | 0.11702              | 0.56444              |
| 0.29060            | 0.15166            | 0.08487            | 0.11762              | 0.56381              |
| 0.29060            | 0.15166            | 0.08487            | 0.11762              | 0.56381              |
| 0.29284            | 0.15348            | 0.08515            | 0.11824              | 0.56316              |
| 0.29284            | 0.15348            | 0.08515            | 0.11824              | 0.56316              |
| 0.29505            | 0.15531            | 0.08544            | 0.11885              | 0.56250              |
| 0.29505            | 0.15531            | 0.08544            | 0.11885              | 0.56250              |
| 0.29726            | 0.15715            | 0.08573            | 0.11947              | 0.56182              |
| 0.29726            | 0.15715            | 0.08573            | 0.11947              | 0.56182              |
| 0.29944            | 0.15900            | 0.08601            | 0.12010              | 0.56113              |
| 0.29944            | 0.15900            | 0.08601            | 0.12010              | 0.56113              |
| 0.30162            | 0.16086            | 0.08630            | 0.12072              | 0.56042              |
| 0.30162            | 0.16086            | 0.08630            | 0.12072              | 0.56042              |
| 0.30378            | 0.16272            | 0.08659            | 0.12136              | 0.55969              |
| 0.30378            | 0.16272            | 0.08659            | 0.12136              | 0.55969              |
| 0.30593            | 0.16459            | 0.08688            | 0.12199              | 0.55895              |

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C.3 Tabulated Results of the Vapor Liquid Equilibrium GCMC Simulations

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Continued: VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.15$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.30593            | 0.16459            | 0.08688            | 0.12199              | 0.55895              |
| 0.30806            | 0.16646            | 0.08717            | 0.12263              | 0.55820              |
| 0.30806            | 0.16646            | 0.08717            | 0.12263              | 0.55820              |
| 0.31018            | 0.16833            | 0.08745            | 0.12328              | 0.55743              |
| 0.31018            | 0.16833            | 0.08745            | 0.12328              | 0.55743              |
| 0.31229            | 0.17021            | 0.08774            | 0.12393              | 0.55665              |
| 0.31229            | 0.17021            | 0.08774            | 0.12393              | 0.55665              |
| 0.31438            | 0.17208            | 0.08803            | 0.12458              | 0.55586              |
| 0.31438            | 0.17208            | 0.08803            | 0.12458              | 0.55586              |
| 0.31646            | 0.17396            | 0.08832            | 0.12524              | 0.55506              |
| 0.31646            | 0.17396            | 0.08832            | 0.12524              | 0.55506              |
| 0.31853            | 0.17583            | 0.08861            | 0.12591              | 0.55425              |
| 0.31853            | 0.17583            | 0.08861            | 0.12591              | 0.55425              |
| 0.32058            | 0.17771            | 0.08890            | 0.12658              | 0.55343              |
| 0.32058            | 0.17771            | 0.08890            | 0.12658              | 0.55343              |
| 0.32264            | 0.17959            | 0.08919            | 0.12723              | 0.55258              |
| 0.32264            | 0.17959            | 0.08919            | 0.12723              | 0.55258              |
| 0.32467            | 0.18147            | 0.08948            | 0.12791              | 0.55175              |
| 0.32467            | 0.18147            | 0.08948            | 0.12791              | 0.55175              |
| 0.32669            | 0.18335            | 0.08977            | 0.12859              | 0.55091              |
| 0.32669            | 0.18335            | 0.08977            | 0.12859              | 0.55091              |
| 0.32869            | 0.18523            | 0.09006            | 0.12928              | 0.55006              |
| 0.32869            | 0.18523            | 0.09006            | 0.12928              | 0.55006              |
| 0.33068            | 0.18712            | 0.09035            | 0.12998              | 0.54922              |
| 0.33068            | 0.18712            | 0.09035            | 0.12998              | 0.54922              |
| 0.33265            | 0.18901            | 0.09064            | 0.13069              | 0.54837              |
| 0.33265            | 0.18901            | 0.09064            | 0.13069              | 0.54837              |
| 0.33461            | 0.19091            | 0.09093            | 0.13140              | 0.54751              |
| 0.33461            | 0.19091            | 0.09093            | 0.13140              | 0.54751              |
| 0.33656            | 0.19281            | 0.09123            | 0.13211              | 0.54665              |
| 0.33656            | 0.19281            | 0.09123            | 0.13211              | 0.54665              |
| 0.33850            | 0.19472            | 0.09152            | 0.13283              | 0.54578              |

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**Continued:** VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.15$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.338 50           | 0.194 72           | 0.091 52           | 0.132 83             | 0.545 78             |
| 0.340 42           | 0.196 63           | 0.091 81           | 0.133 56             | 0.544 91             |
| 0.340 42           | 0.196 63           | 0.091 81           | 0.133 56             | 0.544 91             |
| 0.342 33           | 0.198 56           | 0.092 10           | 0.134 29             | 0.544 03             |
| 0.342 33           | 0.198 56           | 0.092 10           | 0.134 29             | 0.544 03             |
| 0.344 22           | 0.200 49           | 0.092 40           | 0.135 04             | 0.543 15             |
| 0.344 22           | 0.200 49           | 0.092 40           | 0.135 04             | 0.543 15             |
| 0.346 10           | 0.202 43           | 0.092 69           | 0.135 79             | 0.542 26             |
| 0.346 10           | 0.202 43           | 0.092 69           | 0.135 79             | 0.542 26             |
| 0.347 97           | 0.204 39           | 0.092 98           | 0.136 55             | 0.541 37             |
| 0.347 97           | 0.204 39           | 0.092 98           | 0.136 55             | 0.541 37             |
| 0.349 83           | 0.206 35           | 0.093 28           | 0.137 32             | 0.540 47             |
| 0.349 83           | 0.206 35           | 0.093 28           | 0.137 32             | 0.540 47             |
| 0.351 67           | 0.208 33           | 0.093 57           | 0.138 09             | 0.539 55             |
| 0.351 67           | 0.208 33           | 0.093 57           | 0.138 09             | 0.539 55             |
| 0.353 56           | 0.210 33           | 0.093 86           | 0.138 80             | 0.538 57             |
| 0.353 56           | 0.210 33           | 0.093 86           | 0.138 80             | 0.538 57             |
| 0.355 38           | 0.212 33           | 0.094 16           | 0.139 59             | 0.537 63             |
| 0.355 38           | 0.212 33           | 0.094 16           | 0.139 59             | 0.537 63             |
| 0.357 19           | 0.214 35           | 0.094 45           | 0.140 38             | 0.536 69             |
| 0.357 19           | 0.214 35           | 0.094 45           | 0.140 38             | 0.536 69             |
| 0.358 99           | 0.216 37           | 0.094 75           | 0.141 17             | 0.535 71             |
| 0.358 99           | 0.216 37           | 0.094 75           | 0.141 17             | 0.535 71             |
| 0.360 78           | 0.218 41           | 0.095 04           | 0.141 98             | 0.534 74             |
| 0.360 78           | 0.218 41           | 0.095 04           | 0.141 98             | 0.534 74             |
| 0.362 54           | 0.220 46           | 0.095 34           | 0.142 79             | 0.533 76             |
| 0.362 54           | 0.220 46           | 0.095 34           | 0.142 79             | 0.533 76             |
| 0.364 28           | 0.222 52           | 0.095 63           | 0.143 64             | 0.532 79             |
| 0.364 28           | 0.222 52           | 0.095 63           | 0.143 64             | 0.532 79             |
| 0.366 04           | 0.224 59           | 0.095 93           | 0.144 46             | 0.531 76             |
| 0.366 04           | 0.224 59           | 0.095 93           | 0.144 46             | 0.531 76             |
| 0.367 74           | 0.226 67           | 0.096 23           | 0.145 32             | 0.530 77             |

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C.3 Tabulated Results of the Vapor Liquid Equilibrium GCMC Simulations

Continued: VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.15$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.36774            | 0.22667            | 0.09623            | 0.14532              | 0.53077              |
| 0.36948            | 0.22876            | 0.09652            | 0.14616              | 0.52972              |
| 0.36948            | 0.22876            | 0.09652            | 0.14616              | 0.52972              |
| 0.37118            | 0.23086            | 0.09682            | 0.14702              | 0.52868              |
| 0.37118            | 0.23086            | 0.09682            | 0.14702              | 0.52868              |
| 0.37287            | 0.23297            | 0.09711            | 0.14789              | 0.52764              |
| 0.37287            | 0.23297            | 0.09711            | 0.14789              | 0.52764              |
| 0.37454            | 0.23508            | 0.09741            | 0.14878              | 0.52658              |
| 0.37454            | 0.23508            | 0.09741            | 0.14878              | 0.52658              |
| 0.37619            | 0.23719            | 0.09771            | 0.14969              | 0.52554              |
| 0.37619            | 0.23719            | 0.09771            | 0.14969              | 0.52554              |
| 0.37782            | 0.23931            | 0.09801            | 0.15061              | 0.52449              |
| 0.37782            | 0.23931            | 0.09801            | 0.15061              | 0.52449              |
| 0.37945            | 0.24143            | 0.09830            | 0.15153              | 0.52342              |
| 0.37945            | 0.24143            | 0.09830            | 0.15153              | 0.52342              |
| 0.38111            | 0.24357            | 0.09860            | 0.15241              | 0.52231              |
| 0.38111            | 0.24357            | 0.09860            | 0.15241              | 0.52231              |
| 0.38276            | 0.24570            | 0.09890            | 0.15330              | 0.52120              |
| 0.38276            | 0.24570            | 0.09890            | 0.15330              | 0.52120              |
| 0.38468            | 0.24792            | 0.09919            | 0.15387              | 0.51978              |
| 0.38468            | 0.24792            | 0.09919            | 0.15387              | 0.51978              |
| 0.38629            | 0.25005            | 0.09949            | 0.15480              | 0.51869              |
| 0.38629            | 0.25005            | 0.09949            | 0.15480              | 0.51869              |
| 0.38789            | 0.25218            | 0.09979            | 0.15574              | 0.51759              |
| 0.38789            | 0.25218            | 0.09979            | 0.15574              | 0.51759              |
| 0.38946            | 0.25430            | 0.10009            | 0.15670              | 0.51651              |
| 0.38946            | 0.25430            | 0.10009            | 0.15670              | 0.51651              |
| 0.39107            | 0.25643            | 0.10038            | 0.15761              | 0.51537              |
| 0.39107            | 0.25643            | 0.10038            | 0.15761              | 0.51537              |
| 0.39263            | 0.25855            | 0.10068            | 0.15857              | 0.51428              |
| 0.39263            | 0.25855            | 0.10068            | 0.15857              | 0.51428              |
| 0.39422            | 0.26067            | 0.10098            | 0.15951              | 0.51315              |

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Continued: VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.15$ 

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.39422            | 0.26067            | 0.10098            | 0.15951              | 0.51315              |
| 0.39569            | 0.26276            | 0.10128            | 0.16058              | 0.51215              |
| 0.39569            | 0.26276            | 0.10128            | 0.16058              | 0.51215              |
| 0.39721            | 0.26486            | 0.10158            | 0.16158              | 0.51107              |
| 0.39721            | 0.26486            | 0.10158            | 0.16158              | 0.51107              |
| 0.39870            | 0.26694            | 0.10188            | 0.16262              | 0.51002              |
| 0.39870            | 0.26694            | 0.10188            | 0.16262              | 0.51002              |
| 0.40024            | 0.26904            | 0.10218            | 0.16359              | 0.50891              |
| 0.40024            | 0.26904            | 0.10218            | 0.16359              | 0.50891              |
| 0.40173            | 0.27112            | 0.10248            | 0.16463              | 0.50785              |
| 0.40173            | 0.27112            | 0.10248            | 0.16463              | 0.50785              |
| 0.40335            | 0.27324            | 0.10278            | 0.16551              | 0.50662              |
| 0.40335            | 0.27324            | 0.10278            | 0.16551              | 0.50662              |
| 0.40472            | 0.27528            | 0.10308            | 0.16668              | 0.50568              |
| 0.40472            | 0.27528            | 0.10308            | 0.16668              | 0.50568              |
| 0.40625            | 0.27736            | 0.10338            | 0.16766              | 0.50453              |
| 0.40625            | 0.27736            | 0.10338            | 0.16766              | 0.50453              |
| 0.40767            | 0.27942            | 0.10368            | 0.16877              | 0.50350              |
| 0.40767            | 0.27942            | 0.10368            | 0.16877              | 0.50350              |
| 0.40942            | 0.28157            | 0.10398            | 0.16947              | 0.50206              |
| 0.40942            | 0.28157            | 0.10398            | 0.16947              | 0.50206              |
| 0.41085            | 0.28363            | 0.10428            | 0.17056              | 0.50099              |
| 0.41085            | 0.28363            | 0.10428            | 0.17056              | 0.50099              |
| 0.41225            | 0.28568            | 0.10459            | 0.17169              | 0.49994              |
| 0.41225            | 0.28568            | 0.10459            | 0.17169              | 0.49994              |
| 0.41366            | 0.28773            | 0.10489            | 0.17281              | 0.49887              |
| 0.41366            | 0.28773            | 0.10489            | 0.17281              | 0.49887              |
| 0.41506            | 0.28978            | 0.10520            | 0.17394              | 0.49779              |
| 0.41506            | 0.28978            | 0.10520            | 0.17394              | 0.49779              |
| 0.41652            | 0.29186            | 0.10550            | 0.17499              | 0.49662              |
| 0.41652            | 0.29186            | 0.10550            | 0.17499              | 0.49662              |
| 0.41786            | 0.29390            | 0.10580            | 0.17619              | 0.49557              |

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**Continued:** VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.15$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.41786            | 0.29390            | 0.10580            | 0.17619              | 0.49557              |
| 0.41923            | 0.29596            | 0.10611            | 0.17735              | 0.49447              |
| 0.41923            | 0.29596            | 0.10611            | 0.17735              | 0.49447              |
| 0.42059            | 0.29802            | 0.10642            | 0.17853              | 0.49338              |
| 0.42059            | 0.29802            | 0.10642            | 0.17853              | 0.49338              |
| 0.42193            | 0.30008            | 0.10672            | 0.17973              | 0.49228              |
| 0.42193            | 0.30008            | 0.10672            | 0.17973              | 0.49228              |
| 0.42318            | 0.30212            | 0.10703            | 0.18105              | 0.49127              |
| 0.42318            | 0.30212            | 0.10703            | 0.18105              | 0.49127              |
| 0.42461            | 0.30422            | 0.10734            | 0.18214              | 0.49004              |
| 0.42461            | 0.30422            | 0.10734            | 0.18214              | 0.49004              |
| 0.42607            | 0.30635            | 0.10764            | 0.18318              | 0.48874              |
| 0.42607            | 0.30635            | 0.10764            | 0.18318              | 0.48874              |
| 0.42737            | 0.30843            | 0.10795            | 0.18442              | 0.48761              |
| 0.42737            | 0.30843            | 0.10795            | 0.18442              | 0.48761              |
| 0.42867            | 0.31052            | 0.10826            | 0.18567              | 0.48647              |
| 0.42867            | 0.31052            | 0.10826            | 0.18567              | 0.48647              |
| 0.43003            | 0.31263            | 0.10857            | 0.18685              | 0.48525              |
| 0.43003            | 0.31263            | 0.10857            | 0.18685              | 0.48525              |
| 0.43131            | 0.31473            | 0.10888            | 0.18812              | 0.48411              |
| 0.43131            | 0.31473            | 0.10888            | 0.18812              | 0.48411              |
| 0.43260            | 0.31684            | 0.10919            | 0.18939              | 0.48295              |
| 0.43260            | 0.31684            | 0.10919            | 0.18939              | 0.48295              |
| 0.43458            | 0.31924            | 0.10949            | 0.18968              | 0.48085              |
| 0.43458            | 0.31924            | 0.10949            | 0.18968              | 0.48085              |
| 0.43562            | 0.32126            | 0.10980            | 0.19128              | 0.48000              |
| 0.43562            | 0.32126            | 0.10980            | 0.19128              | 0.48000              |
| 0.43689            | 0.32339            | 0.11012            | 0.19257              | 0.47884              |
| 0.43689            | 0.32339            | 0.11012            | 0.19257              | 0.47884              |
| 0.43815            | 0.32551            | 0.11043            | 0.19387              | 0.47768              |
| 0.43815            | 0.32551            | 0.11043            | 0.19387              | 0.47768              |
| 0.43912            | 0.32753            | 0.11075            | 0.19558              | 0.47690              |

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Continued: VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.15$ 

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.43912            | 0.32753            | 0.11075            | 0.19558              | 0.47690              |
| 0.44057            | 0.32974            | 0.11106            | 0.19661              | 0.47548              |
| 0.44057            | 0.32974            | 0.11106            | 0.19661              | 0.47548              |
| 0.44181            | 0.33188            | 0.11137            | 0.19792              | 0.47433              |
| 0.44181            | 0.33188            | 0.11137            | 0.19792              | 0.47433              |
| 0.44549            | 0.33516            | 0.11161            | 0.19566              | 0.46953              |
| 0.44549            | 0.33516            | 0.11161            | 0.19566              | 0.46953              |
| 0.44672            | 0.33730            | 0.11193            | 0.19696              | 0.46840              |
| 0.44672            | 0.33730            | 0.11193            | 0.19696              | 0.46840              |
| 0.44802            | 0.33948            | 0.11224            | 0.19816              | 0.46717              |
| 0.44802            | 0.33948            | 0.11224            | 0.19816              | 0.46717              |
| 0.44931            | 0.34165            | 0.11255            | 0.19938              | 0.46596              |
| 0.44931            | 0.34165            | 0.11255            | 0.19938              | 0.46596              |
| 0.45074            | 0.34389            | 0.11286            | 0.20039              | 0.46453              |
| 0.45074            | 0.34389            | 0.11286            | 0.20039              | 0.46453              |
| 0.45180            | 0.34594            | 0.11318            | 0.20193              | 0.46371              |
| 0.45180            | 0.34594            | 0.11318            | 0.20193              | 0.46371              |
| 0.45299            | 0.34805            | 0.11350            | 0.20330              | 0.46271              |
| 0.45299            | 0.34805            | 0.11350            | 0.20330              | 0.46271              |
| 0.45417            | 0.35014            | 0.11381            | 0.20468              | 0.46172              |
| 0.45417            | 0.35014            | 0.11381            | 0.20468              | 0.46172              |
| 0.45534            | 0.35223            | 0.11413            | 0.20605              | 0.46076              |
| 0.45534            | 0.35223            | 0.11413            | 0.20605              | 0.46076              |
| 0.45623            | 0.35417            | 0.11446            | 0.20786              | 0.46025              |
| 0.45623            | 0.35417            | 0.11446            | 0.20786              | 0.46025              |
| 0.45737            | 0.35621            | 0.11478            | 0.20930              | 0.45936              |
| 0.45737            | 0.35621            | 0.11478            | 0.20930              | 0.45936              |
| 0.45822            | 0.35811            | 0.11511            | 0.21118              | 0.45894              |
| 0.45822            | 0.35811            | 0.11511            | 0.21118              | 0.45894              |
| 0.46039            | 0.36070            | 0.11539            | 0.21096              | 0.45643              |
| 0.46039            | 0.36070            | 0.11539            | 0.21096              | 0.45643              |
| 0.46180            | 0.36287            | 0.11570            | 0.21195              | 0.45514              |

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Continued: VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.15$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $P_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.461 80           | 0.362 87           | 0.115 70           | 0.211 95             | 0.455 14             |
| 0.463 04           | 0.364 95           | 0.116 02           | 0.213 19             | 0.454 15             |
| 0.463 04           | 0.364 95           | 0.116 02           | 0.213 19             | 0.454 15             |
| 0.464 16           | 0.366 94           | 0.116 34           | 0.214 63             | 0.453 37             |
| 0.464 16           | 0.366 94           | 0.116 34           | 0.214 63             | 0.453 37             |
| 0.465 69           | 0.369 18           | 0.116 65           | 0.215 35             | 0.451 89             |
| 0.465 69           | 0.369 18           | 0.116 65           | 0.215 35             | 0.451 89             |
| 0.467 10           | 0.371 32           | 0.116 96           | 0.216 31             | 0.450 66             |
| 0.467 10           | 0.371 32           | 0.116 96           | 0.216 31             | 0.450 66             |
| 0.468 33           | 0.373 34           | 0.117 28           | 0.217 54             | 0.449 75             |
| 0.468 33           | 0.373 34           | 0.117 28           | 0.217 54             | 0.449 75             |
| 0.469 12           | 0.375 09           | 0.117 61           | 0.219 52             | 0.449 61             |
| 0.469 12           | 0.375 09           | 0.117 61           | 0.219 52             | 0.449 61             |
| 0.470 33           | 0.377 08           | 0.117 93           | 0.220 78             | 0.448 75             |
| 0.470 33           | 0.377 08           | 0.117 93           | 0.220 78             | 0.448 75             |
| 0.472 07           | 0.379 37           | 0.118 23           | 0.221 16             | 0.447 02             |
| 0.472 07           | 0.379 37           | 0.118 23           | 0.221 16             | 0.447 02             |
| 0.473 94           | 0.381 77           | 0.118 52           | 0.221 27             | 0.445 01             |
| 0.473 94           | 0.381 77           | 0.118 52           | 0.221 27             | 0.445 01             |
| 0.475 17           | 0.383 74           | 0.118 84           | 0.222 48             | 0.444 16             |
| 0.475 17           | 0.383 74           | 0.118 84           | 0.222 48             | 0.444 16             |
| 0.476 40           | 0.385 71           | 0.119 16           | 0.223 68             | 0.443 32             |
| 0.476 40           | 0.385 71           | 0.119 16           | 0.223 68             | 0.443 32             |
| 0.477 34           | 0.387 49           | 0.119 49           | 0.225 35             | 0.443 01             |
| 0.477 34           | 0.387 49           | 0.119 49           | 0.225 35             | 0.443 01             |
| 0.478 20           | 0.389 20           | 0.119 83           | 0.227 18             | 0.442 85             |
| 0.478 20           | 0.389 20           | 0.119 83           | 0.227 18             | 0.442 85             |
| 0.479 41           | 0.391 14           | 0.120 16           | 0.228 38             | 0.442 07             |
| 0.479 41           | 0.391 14           | 0.120 16           | 0.228 38             | 0.442 07             |
| 0.478 75           | 0.391 91           | 0.120 58           | 0.232 82             | 0.444 41             |
| 0.478 75           | 0.391 91           | 0.120 58           | 0.232 82             | 0.444 41             |
| 0.481 32           | 0.394 64           | 0.120 83           | 0.231 68             | 0.441 44             |

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**Continued:** VLE-Data from GCMC-Simulations of Mixture III at  $T^* = 1.15$

| $x_2^{\text{vap}}$ | $x_2^{\text{liq}}$ | $p_{\text{sat}}^*$ | $\rho^{*\text{vap}}$ | $\rho^{*\text{liq}}$ |
|--------------------|--------------------|--------------------|----------------------|----------------------|
| 0.48132            | 0.39464            | 0.12083            | 0.23168              | 0.44144              |
| 0.48273            | 0.39668            | 0.12114            | 0.23252              | 0.44035              |
| 0.48273            | 0.39668            | 0.12114            | 0.23252              | 0.44035              |
| 0.44144            | 0.44144            | 0.12159            | 0.33378              | 0.33378              |
| 0.44144            | 0.44144            | 0.12159            | 0.33378              | 0.33378              |