

Supplementary Material

1 DEFINITION OF THE HEIGHT $h(x)$ OF THE TAPERED BEAM

In Figure S1 the function $h(x) = \frac{h_0}{l} [(1 - \alpha)l + 2\alpha x]$ is evaluated for some values of α .

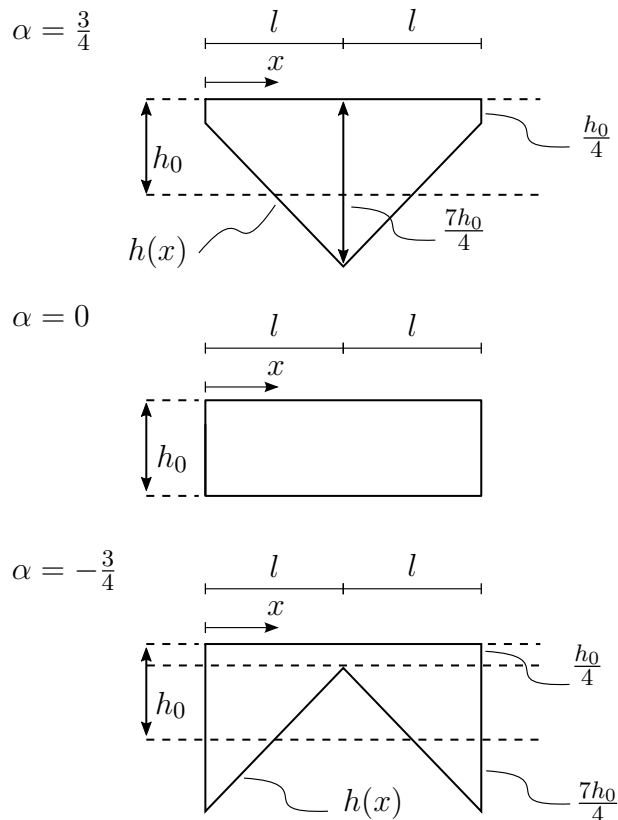


Figure S1. $h(x)$ for $\alpha = \{\frac{3}{4}, 0, -\frac{3}{4}\}$

2 DERIVATION FOR THE MID-SPAN DEFLECTION w_l OF THE TAPERED BEAM

The bending moment is introduced as the integration of the shear force:

$$Q(x) = -qx + C_1 \quad (\text{S1})$$

$$M(x) = -\frac{1}{2}qx^2 + C_1x + C_2 \quad (\text{S2})$$

The tapered beam has a rectangular cross section. The height $h(x)$ along the length is defined as

$$h(x) = \frac{h_0}{l} [(1 - \alpha)l + 2\alpha x]. \quad (\text{S3})$$

With these definitions the material law $\kappa = \frac{M}{EI}$ is introduced:

$$\kappa(x) = -\frac{12 \left(-\frac{1}{2}qx^2 + C_1x + C_2 \right) l^3}{Ebh_0^3 \left((\alpha - 1)l - 2\alpha x \right)^3} \quad (S4)$$

The integration of κ provides the rotation

$$\phi(x) = \frac{-6 \left((l - 2x)\alpha - l \right)^2 l^3 q \ln \left((-l + 2x)\alpha + l \right) + 8EbC_3h_0^3 (l - 2x)^2 \alpha^5}{8E \left((l - 2x)\alpha - l \right)^2 h_0^3 b \alpha^3} + \quad (S5)$$

$$+ \frac{8C_3E \alpha^3 b h_0^3 l^2 - 9l^3 \left(l^2 q + \left(-\frac{8qx}{3} - \frac{4C_1}{3} \right) l + \frac{16C_1x}{3} + \frac{8C_2}{3} \right) \alpha^2 - 9l^5 q}{8E \left((l - 2x)\alpha - l \right)^2 h_0^3 b \alpha^3} + \quad (S6)$$

$$+ \frac{18l^4 \left(ql - \frac{4}{3}qx - \frac{2}{3}C_1 \right) \alpha - 16EblC_3h_0^3 (l - 2x) \alpha^4}{8E \left((l - 2x)\alpha - l \right)^2 h_0^3 b \alpha^3}. \quad (S7)$$

The integration of ϕ leads to the deflection

$$w(x) = \frac{-18l^3 \left(\left(ql - \frac{2}{3}qx - \frac{4}{3}C_1 \right) \alpha - ql \right) \left((l - 2x)\alpha - l \right) \ln \left((-l + 2x)\alpha + l \right) + 3l^5 q}{16E \alpha^4 b h_0^3 \left((l - 2x)\alpha - l \right)} + \quad (S8)$$

$$+ \frac{8E h_0^3 b (l - 2x) (lC_3 - 2C_3x + 2C_4) \alpha^5 - 16E h_0^3 bl (lC_3 - 2C_3x + C_4) \alpha^4}{16E \alpha^4 b h_0^3 \left((l - 2x)\alpha - l \right)} + \quad (S9)$$

$$+ \frac{8C_3E \alpha^3 b h_0^3 l^2 + 3l^3 \left(l^2 q + (-8qx + 4C_1)l + 8qx^2 + 8C_2 \right) \alpha^2 - 6l^4 (ql - 4qx + 2C_1) \alpha}{16E \alpha^4 b h_0^3 \left((l - 2x)\alpha - l \right)}. \quad (S10)$$

By applying the following boundary conditions

$$w(x = 0) = 0 \quad (S11)$$

$$M(x = 0) = 0 \quad (S12)$$

$$\phi(x = l) = 0 \quad (S13)$$

$$Q(x = l) = 0 \quad (S14)$$

the integration constants evaluate to:

$$C_1 = lq \quad (S15)$$

$$C_2 = 0 \quad (S16)$$

$$C_3 = \frac{3l^3 \left((\alpha + 1)^2 \ln(l(\alpha + 1)) + \frac{7\alpha^2}{2} + 3\alpha + \frac{3}{2} \right) q}{4\alpha^3 Eb h_0^3 (\alpha + 1)^2} \quad (S17)$$

$$C_4 = -\frac{3l^4 q \left((\alpha + 3)(\alpha + 1)^2 \ln(-(\alpha - 1)l) + (\alpha - 1)(\alpha + 1)^2 \ln(l(\alpha + 1)) + 6\alpha^3 + 4\alpha^2 - 2 \right)}{8\alpha^4 h_0^3 b E (\alpha + 1)^2} \quad (S18)$$

With these results, the mid-span deflection $w(x = l) = w_l$ yields to

$$w_l = \frac{3ql^4 \left((\alpha + 3)(\alpha + 1)^2 \ln\left(\frac{1+\alpha}{1-\alpha}\right) - 12\alpha^3 - 14\alpha^2 - 6\alpha \right)}{8(\alpha + 1)^2 Ebh_0^3 \alpha^4}. \quad (\text{S19})$$

3 DERIVATION FOR THE ACTUATION FORCE F_{act} OF THE TAPERED BEAM

The actuation force F_{act} is derived in the same way as the mid-span deflection w_l . Only different boundary conditions are applied:

$$w(x = 0) = 0 \quad (\text{S20})$$

$$M(x = 0) = 0 \quad (\text{S21})$$

$$\phi(x = l) = 0 \quad (\text{S22})$$

$$Q(x = l) = -F_{\text{act}} \quad (\text{S23})$$

The integration constants evaluate to:

$$C_1 = -F_{\text{act}} + 25 \quad (\text{S24})$$

$$C_2 = 0 \quad (\text{S25})$$

$$C_3 = \frac{50(1 + \alpha)^2 \ln(1 + \alpha) + 50(1 + \alpha)^2 \ln(5) + 75 + (-12F_{\text{act}} + 175)\alpha^2 + (-4F_{\text{act}} + 150)\alpha}{12800\alpha^3(1 + \alpha)^2} \quad (\text{S26})$$

$$C_4 = \frac{4(1 + \alpha)^2 \left(-\frac{75}{4} + (F_{\text{act}} - \frac{25}{4})\alpha\right) \ln(-\alpha + 1) - 25(\alpha - 1)(1 + \alpha)^2 \ln(1 + \alpha)}{2560\alpha^4(1 + \alpha)^2} + \quad (\text{S27})$$

$$+ \frac{+4\left(-\frac{25}{2} + (F_{\text{act}} - \frac{25}{2})\alpha\right)(1 + \alpha)^2 \ln(5) + 50 + (8F_{\text{act}} - 150)\alpha^3 - 100\alpha^2}{2560\alpha^4(1 + \alpha)^2} \quad (\text{S28})$$

So the deflection $w(x = l) = w_l$ at mid span is:

$$w_l = \frac{4(1 + \alpha)^2 \left(-\frac{75}{4} + (F_{\text{act}} - \frac{25}{4})\alpha\right) \ln(-\alpha + 1) - 4(1 + \alpha)^2 \left(-\frac{75}{4} + (F_{\text{act}} - \frac{25}{4})\alpha\right) \ln(1 + \alpha)}{2560\alpha^4(1 + \alpha)^2} + \quad (\text{S29})$$

$$+ \frac{(16F_{\text{act}} - 300)\alpha^3 + (8F_{\text{act}} - 350)\alpha^2 - 150\alpha}{2560\alpha^4(1 + \alpha)^2} \quad (\text{S30})$$

F_{act} is expressed analytically with respect to the constraint $w_l = \hat{w}$:

$$F_{\text{act}}(\alpha) = \frac{2Ebh_0^3 \hat{w} \alpha^3 (\alpha^2 + 2\alpha + 1)}{3l^3 (4\alpha^2 + \Lambda + 2\alpha)} + \frac{ql (12\alpha^3 + 2\alpha(7\alpha + 3) + \Lambda(\alpha + 3))}{4\alpha(4\alpha^2 + \Lambda + 2\alpha)}, \quad (\text{S31})$$

$$\text{with } \Lambda = (\alpha + 1)^2 \ln \frac{1 - \alpha}{1 + \alpha}. \quad (\text{S32})$$