frontiers

Supplementary Material

1 SUPPLEMENTARY DATA

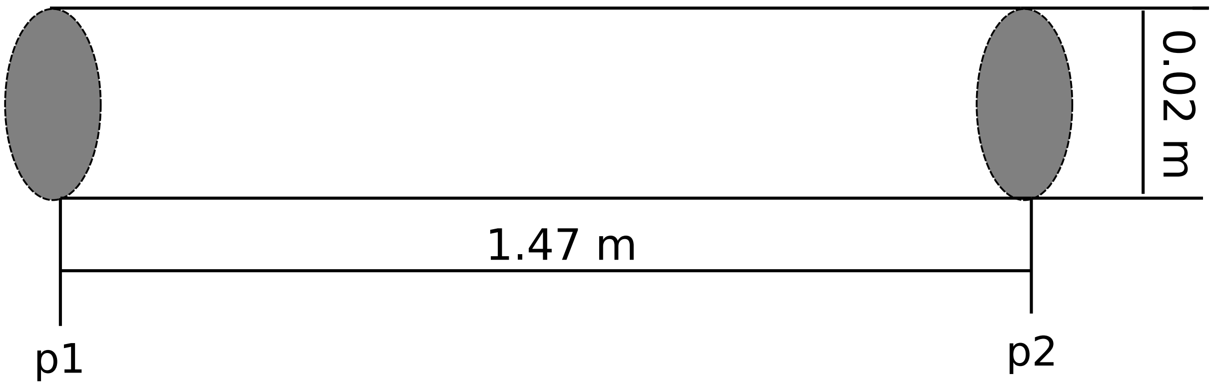

Figure S1: Setup of the cylindrical simulation geometry of length $1.47\,\mathrm{m}$ and diameter of $0.02\,\mathrm{m}$. The pressure drop is calculated between the pressure sensors p1 and p2. The suspension flows from left to right.

Table S1. Simulation setup for different software packages.

Abbreviation			AF-DD	AF-BS	OF-BS	OF-FG	COM [1]	SPH [2]
Solver			Pressure-based				direct linear solver (PARADISO)	Single Phase FlowNN
Model			Viscous laminar		Laminar		Viscous laminar	Bingham-Papanastasiou
Solution method	Pressure Velocity Coupling	scheme	SIMPLE				Fully coupled	-
	Discretization	Gradient	Least Square Cell Based		Gauss linear		-	-
		Pressure	Second order		Gauss linear (2.order central)		First/Second order	Tait EOS
		Momentum	Second order Upwind		Gauss linear (2.order central)		First/Second order	Tait EOS
		Time [3]	-		-		-	Velocity Verlet

[1] The basic Lagrange functions are 1st order for the Bingham model and 2nd order for Bingham-Papanastasiou model.
[2] Further information about SPH method is provided in section 2.2.5.4
[3] For the mesh-based methods, steady state simulations were used. Only SPH needs a transient time scheme.

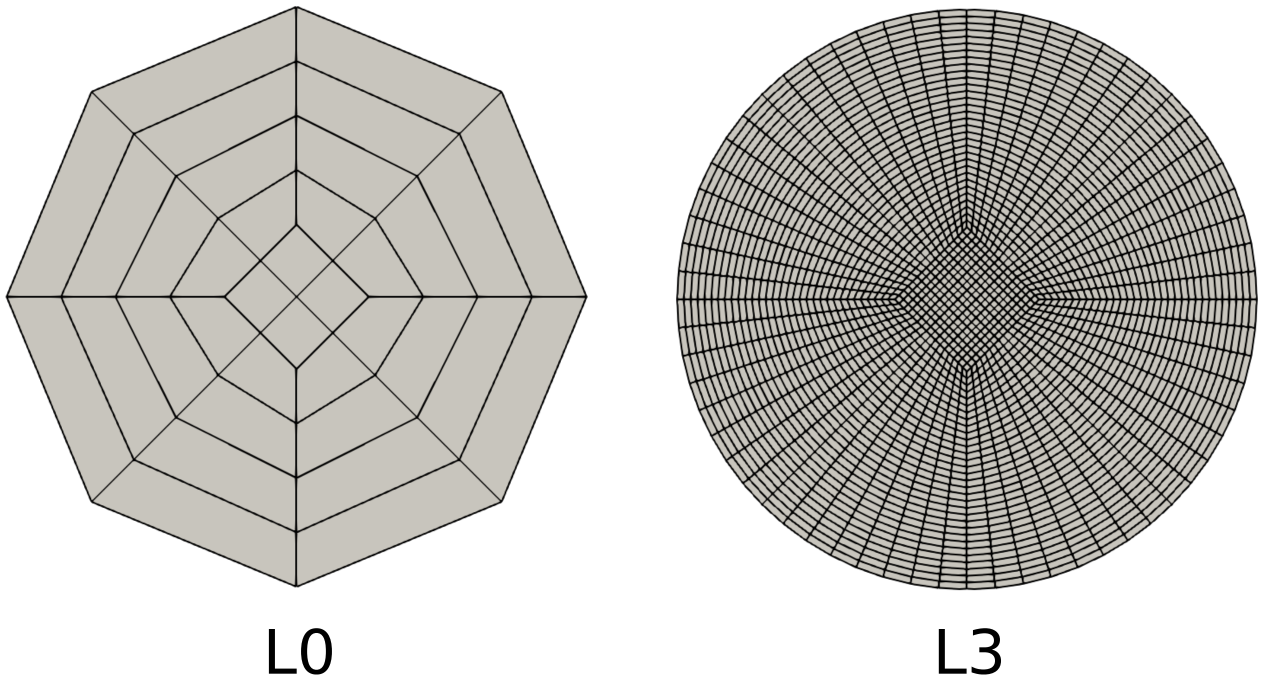

Figure S2: Mesh of pipe cross-section created with Ansys ICEM CFD for coarsest (L0) and finest resolution (L3).

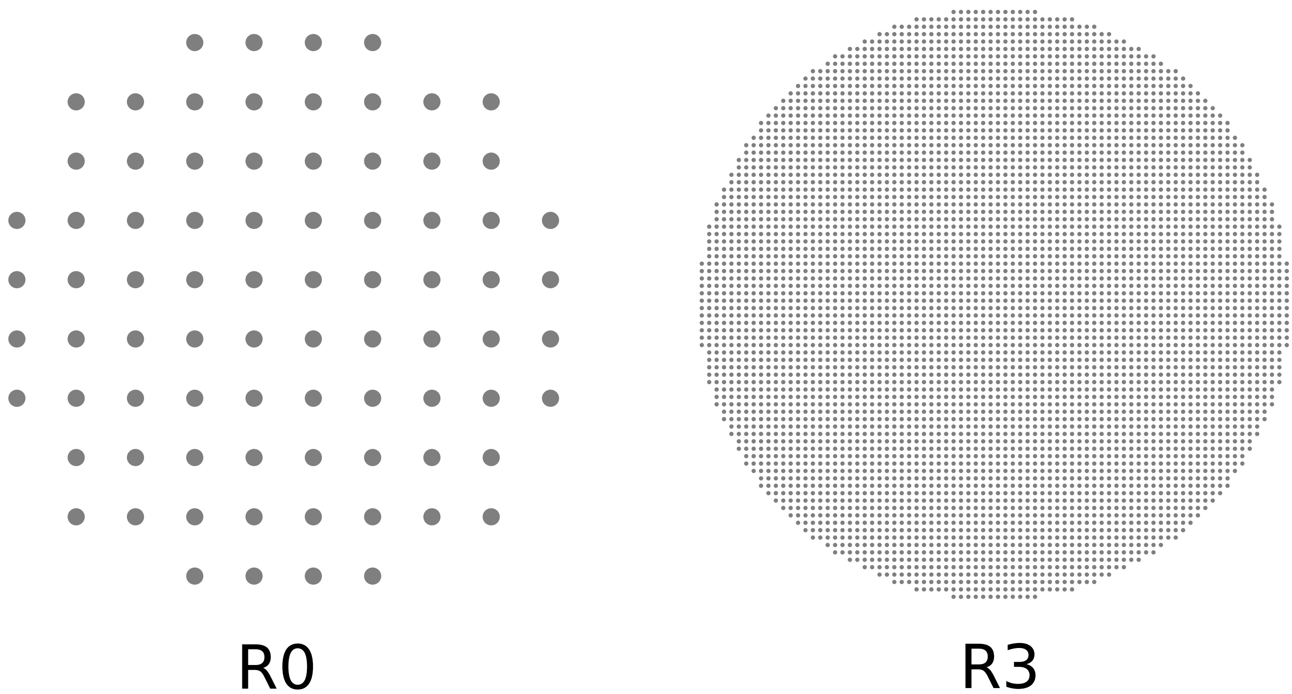

Figure S3: Initial fluid particle distribution for coarsest (R0) and finest resolution (R3) used in the SPH method.

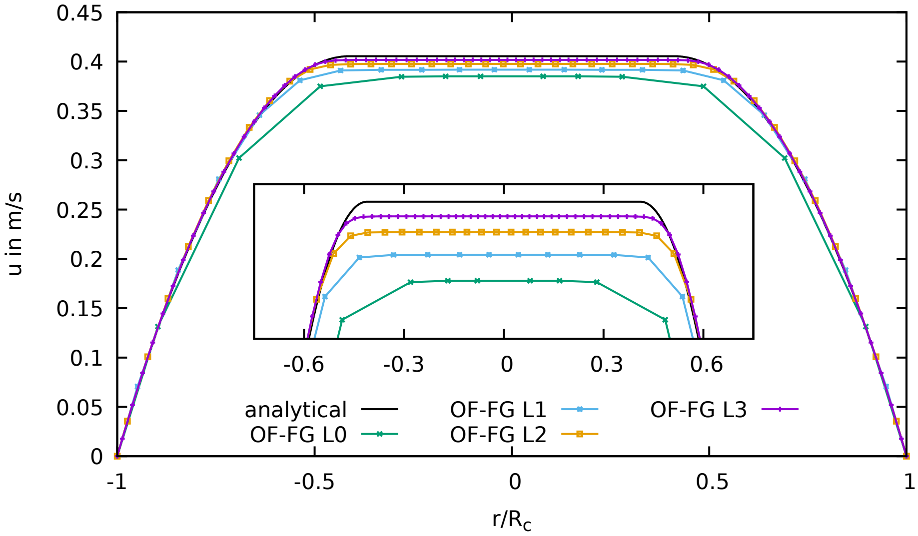

Figure S4: Velocity profiles for different mesh refinements modeled with the software OpenFOAM (FG). The data is shown for the bi-viscous model in the *cv* case. The lines connect the numerical points as guidance of the eye. The plug-flow region is also shown enlarged for $u = 0.37 - 0.41\,\mathrm{m\,s^{-1}}$.

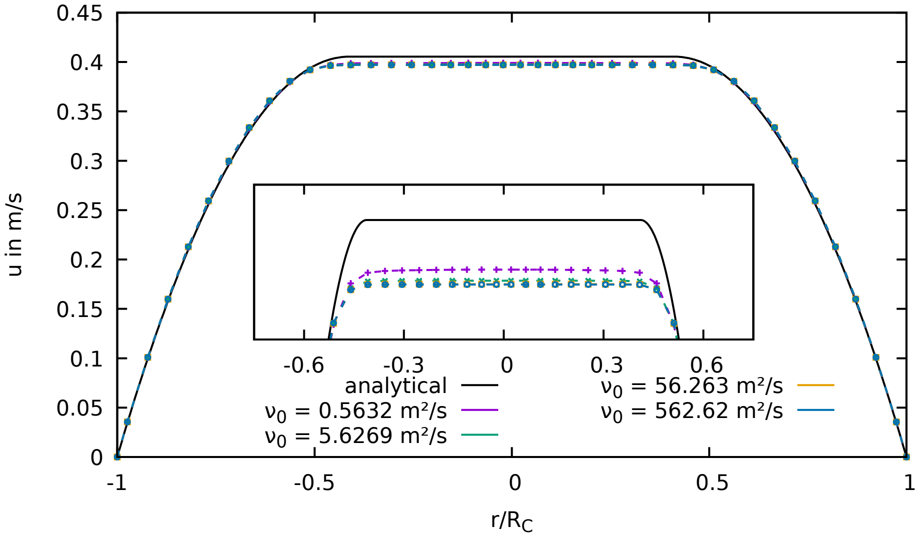

Figure S5: Velocity profile $u(r)$ for different regularization parameters ν_0 in the OpenFOAM solution for the bi-viscous model (*cv*). The analytical solution is drawn as continuous line, the numerical results at the center of the grid cells are shown as points. The dashed lines connect the numerical points as guidance of the eye. The plug-flow region is also shown enlarged for $u = 0.39 - 0.41\,\mathrm{m\,s^{-1}}$.

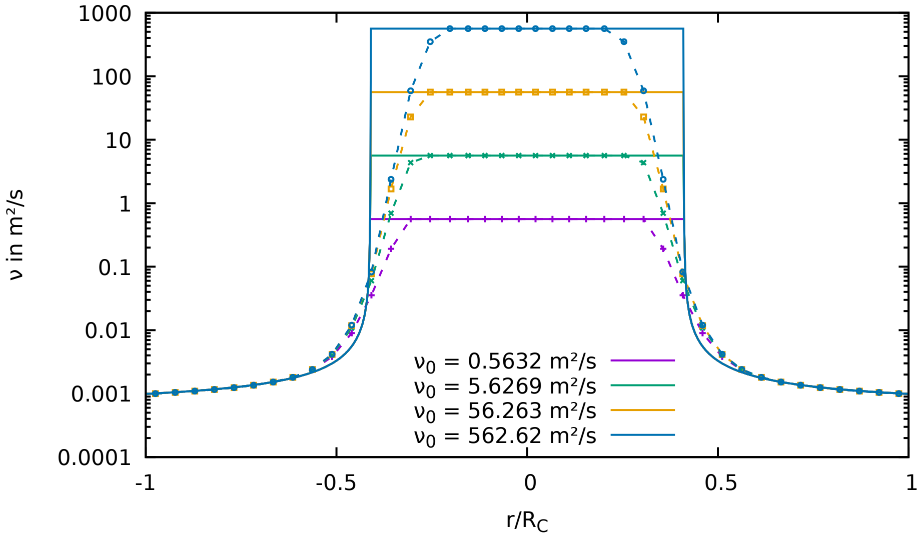

Figure S6: Viscosity profile $\nu(r)$ for different regularization parameters ν_0 in the OpenFOAM solution for the bi-viscous model (*cv*). The analytical solution is drawn as continuous lines, the numerical results at the center of the grid cells are shown as points. The dashed lines connect the numerical points as guidance of the eye.

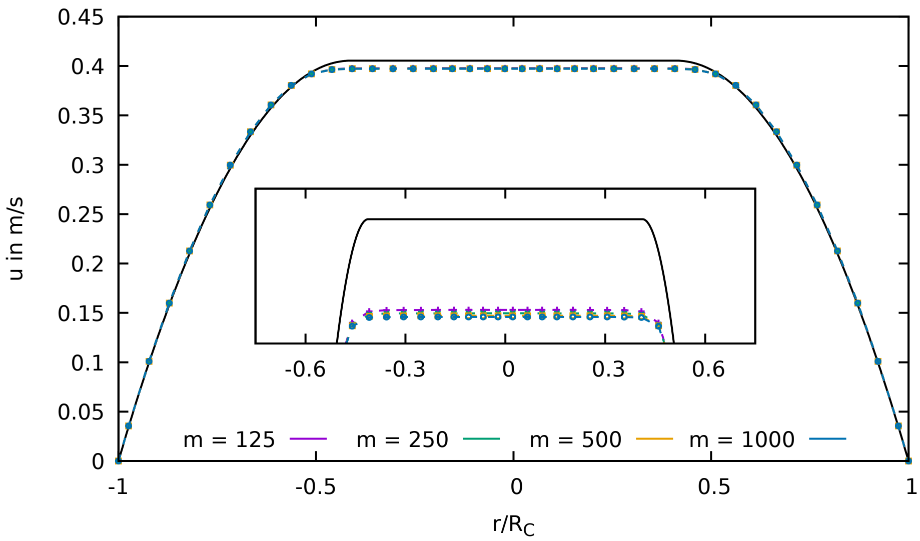

Figure S7: Velocity profile $u(r)$ for different regularization parameters m in the OpenFOAM solution for the Papanastasiou model (*cv*). The analytical solution is drawn as continuous lines, the numerical results at the center of the grid cells are shown as points. The dashed lines connect the numerical points as guidance of the eye. The plug-flow region is also shown enlarged for $u = 0.395 - 0.408\,\mathrm{m\,s^{-1}}$.

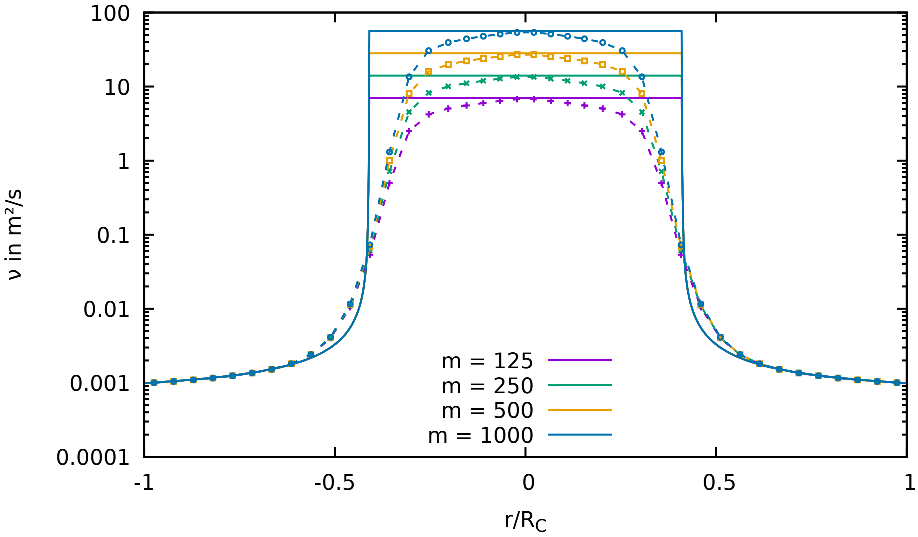

Figure S8: Viscosity profile $\nu(r)$ for different regularization parameters m in the OpenFOAM solution for the Papanastasiou model (*cv*). The analytical solution is drawn as continuous lines, the numerical results at the center of the grid cells are shown as points. The dashed lines connect the numerical points as guidance of the eye.