Table S4. List of ordinary differential equations and rate expressions for the three distinct models

ODE/rate expression	
Model 1	
$\frac{dbiomass}{dt} = v_1 \cdot biomass - k_d \cdot biomass - D \cdot biomass,$	(1)
$\frac{dmAb}{dt} = v_2 - D \cdot mAb,$	(2)
$\frac{dpyr}{dt} = -2.534 \cdot v_1 - 8.560 \cdot v_2 + 2 \cdot v_3 - v_4 + v_{10} - v_{11} + D \cdot (pyr_i - pyr),$	(3)
$\frac{dglc}{dt} = -0.355 \cdot v_1 - v_3 - v_7 - m \cdot biomass + D \cdot (glc_i - glc),$	(4)
$\frac{dlac}{dt} = v_4 + D \cdot (lac_i - lac),$	(5)
$\frac{dnh4}{dt} = -1.384 \cdot v_1 - 22.890 \cdot v_2 + v_5 + v_{10} + D \cdot (nh4_i - nh4),$	(6)
$\frac{dasn}{dt} = -0.260 \cdot v_1 - 3.970 \cdot v_2 - v_5 + D \cdot (asn_i - asn),$	(7)
$\frac{dasp}{dt} = -0.398 \cdot v_1 - 3.460 \cdot v_2 + v_5 - v_6 + D \cdot (asp_i - asp),$	(8)
$\frac{dglu}{dt} = -2.834 \cdot v_1 - 62.060 \cdot v_2 + v_6 - 2 \cdot v_7 + v_8 + v_9 + v_{11} + D \cdot (glu_i - glu),$	(9)
$\frac{dile}{dt} = -0.278 \cdot v_1 - 2.500 \cdot v_2 - v_8 + D \cdot (ile_i - ile),$	(10)
$\frac{dleu}{dt} = -0.540 \cdot v_1 - 7.660 \cdot v_2 - v_9 + D \cdot (leu_i - leu),$	(11)
$\frac{dser}{dt} = -1.600 \cdot v_1 - 23.260 \cdot v_2 + 2 \cdot v_7 - v_{10} + D \cdot (ser_i - ser)$	(12)
$\frac{dV}{dt} = F_{in} - F_{out}$	(13)
$v_1 = v_{max,1} \cdot \frac{pyr}{K_{mpyr1}+pyr} \cdot \frac{glu}{K_{mglu1}+glu} \cdot \frac{asn}{K_{masn1}+asn} \cdot \frac{glc}{K_{mglc1}+glc} \cdot \frac{K_{dlac1}}{K_{dlac1}+lac} \cdot \frac{K_{dnh41}}{K_{dnh41}+nh4},$	(14)
$v_2 = a_2 \cdot biomass + b_2 \cdot v_1 \cdot biomass,$	(15)
$v_3 = v_{max,3} \cdot \frac{glc}{K_{mglc3}+glc} \cdot \frac{asn}{K_{masn3}+asn} + v_{3,base},$	(16)
$v_4 = v_{max,4f} \cdot \frac{pyr}{K_{mpyr4}+pyr} \cdot \frac{1}{1+\left(\frac{lac}{K_{ilac4}}\right)^{\gamma_4}} - v_{max,4r} \cdot \frac{lac}{K_{mlac4}+lac} \cdot \frac{1}{v_3},$	(17)
$v_5 = v_{max,5f} \cdot \frac{asn}{K_{masn5}+asn} - v_{max,5r} \cdot \frac{asp}{K_{masp5}+asp} \cdot \frac{nh4}{K_{mnh45}+nh4},$	(18)
$v_6 = v_{max,6f} \cdot \frac{asp}{K_{masp6}+asp} - v_{max,6r} \cdot \frac{glu}{K_{mglu6}+glu},$	(19)
$v_7 = v_{max,7} \cdot \frac{glu}{K_{mglu7}+glu} \cdot \frac{glc}{K_{mglc7}+glc},$	(20)

$$v_8 = v_{max,8} \cdot \frac{ile}{K_{mile8}+ile}, \quad (21)$$

$$v_9 = v_{max,9f} \cdot \frac{leu}{K_{mleu9}+leu} - v_{max,9r} \cdot \frac{glu}{K_{mglu9}+glu}, \quad (22)$$

$$v_{10} = v_{max,10} \cdot \frac{ser}{K_{mser10}+ser}, \quad (23)$$

$$v_{11} = v_{max,11f} \cdot \frac{pyr}{K_{mpyr11}+pyr} \cdot \frac{K_{iasn11}}{K_{iasn11}+asn} - v_{max,11r} \cdot \frac{glu}{K_{mglu11}+glu}, \quad (24)$$

Model 2

$$\frac{dbiomass}{dt} = v_1 \cdot biomass - k_d \cdot biomass - D \cdot biomass, \quad (25)$$

$$\frac{dmAb}{dt} = v_2 - D \cdot mAb, \quad (26)$$

$$\frac{dpyr}{dt} = -2.918 \cdot v_1 + 2 \cdot v_3 - v_4 + v_{10} - 2 \cdot v_{11} + D \cdot (pyr_i - pyr), \quad (27)$$

$$\frac{dglc}{dt} = -0.355 \cdot v_1 - v_3 - v_7 - m \cdot biomass + D \cdot (glc_i - glc), \quad (28)$$

$$\frac{dlac}{dt} = v_4 + D \cdot (lac_i - lac), \quad (29)$$

$$\frac{dnh4}{dt} = -1.223 \cdot v_1 - 19.390 \cdot v_2 + v_5 + v_{10} + D \cdot (nh4_i - nh4), \quad (30)$$

$$\frac{dasn}{dt} = -0.260 \cdot v_1 - 3.970 \cdot v_2 - v_5 + D \cdot (asn_i - asn), \quad (31)$$

$$\frac{dasp}{dt} = -0.398 \cdot v_1 - 3.460 \cdot v_2 + v_5 - v_6 + D \cdot (asp_i - asp), \quad (32)$$

$$\frac{dglu}{dt} = -3.184 \cdot v_1 - 71.550 \cdot v_2 + v_6 + v_8 + v_9 + v_{11} + D \cdot (glu_i - glu), \quad (33)$$

$$\frac{dile}{dt} = -0.278 \cdot v_1 - 2.500 \cdot v_2 - v_8 + D \cdot (ile_i - ile), \quad (34)$$

$$\frac{dleu}{dt} = -0.540 \cdot v_1 - 7.660 \cdot v_2 - v_9 + D \cdot (leu_i - leu), \quad (35)$$

$$\frac{dser}{dt} = -1.515 \cdot v_1 - 24.500 \cdot v_2 + 2 \cdot v_7 - v_{10} + D \cdot (ser_i - ser), \quad (36)$$

$$\frac{dV}{dt} = F_{in} - F_{out} \quad (37)$$

$$v_1 = v_{max,1} \cdot \frac{pyr}{K_{mpyr1}+pyr} \cdot \frac{glu}{K_{mglu1}+glu} \cdot \frac{asn}{K_{masn1}+asn} \cdot \frac{glc}{K_{mglc1}+glc} \cdot \frac{K_{dlac1}}{K_{dlac1}+lac} \cdot \frac{K_{dnh41}}{K_{dnh41}+nh4}, \quad (38)$$

$$v_2 = a_2 \cdot biomass + b_2 \cdot v_1 \cdot biomass, \quad (39)$$

$$v_3 = v_{max,3} \cdot \frac{glc}{K_{mglc3}+glc} \cdot \frac{asn}{K_{masn3}+asn} + v_{3,base}, \quad (40)$$

$$v_4 = v_{max,4f} \cdot \frac{pyr}{K_{mpyr4}+pyr} \cdot \frac{1}{1+\left(\frac{lac}{K_{ilac4}}\right)^{\gamma_4}} - v_{max,4r} \cdot \frac{lac}{K_{mlac4}+lac} \cdot \frac{1}{v_3}, \quad (41)$$

$$v_5 = v_{max,5f} \cdot \frac{asn}{K_{masn5}+asn} - v_{max,5r} \cdot \frac{asp}{K_{masp5}+asp} \cdot \frac{nh4}{K_{mnh45}+nh4}, \quad (42)$$

$$v_6 = v_{max,6f} \cdot \frac{asp}{K_{masp6}+asp} - v_{max,6r} \cdot \frac{glu}{K_{mglu6}+glu}, \quad (43)$$

$$v_7 = v_{max,7} \cdot \frac{glc}{K_{mglc7}+glc}, \quad (44)$$

$$v_8 = v_{max,8} \cdot \frac{ile}{K_{mile8}+ile}, \quad (45)$$

$$v_9 = v_{max,9f} \cdot \frac{leu}{K_{mleu9}+leu} - v_{max,9r} \cdot \frac{glu}{K_{mglu9}+glu}, \quad (46)$$

$$v_{10} = v_{max,10} \cdot \frac{ser}{K_{mser10}+ser}, \quad (47)$$

$$v_{11} = v_{max,11f} \cdot \frac{pyr}{K_{mpyr11}+pyr} \cdot \frac{K_{iasn11}}{K_{iasn11}+asn}, \quad (48)$$

Model 3

$$\frac{dbiomass}{dt} = v_1 \cdot biomass - k_d \cdot biomass - D \cdot biomass, \quad (49)$$

$$\frac{dmAb}{dt} = v_2 - D \cdot mAb, \quad (50)$$

$$\frac{dpyr}{dt} = -2.918 \cdot v_1 + 2 \cdot v_3 - v_4 + v_{10} - 2 \cdot v_{11} + D \cdot (pyr_i - pyr), \quad (51)$$

$$\frac{dglc}{dt} = -0.355 \cdot v_1 - v_3 - v_7 - m \cdot biomass + D \cdot (glc_i - glc), \quad (52)$$

$$\frac{dlac}{dt} = v_4 - \frac{1}{Y_{\frac{lac}{biomass}}} \cdot biomass + D \cdot (lac_i - lac), \quad (53)$$

$$\frac{dnh4}{dt} = -1.223 \cdot v_1 - 19.390 \cdot v_2 + v_5 + v_{10} + D \cdot (nh4_i - nh4), \quad (54)$$

$$\frac{dasn}{dt} = -0.260 \cdot v_1 - 3.970 \cdot v_2 - v_5 + D \cdot (asn_i - asn), \quad (55)$$

$$\frac{dasp}{dt} = -0.398 \cdot v_1 - 3.460 \cdot v_2 + v_5 - v_6 + D \cdot (asp_i - asp), \quad (56)$$

$$\frac{dglu}{dt} = -3.184 \cdot v_1 - 71.550 \cdot v_2 + v_6 + v_8 + v_9 + v_{11} + D \cdot (glu_i - glu), \quad (57)$$

$$\frac{dile}{dt} = -0.278 \cdot v_1 - 2.500 \cdot v_2 - v_8 + D \cdot (ile_i - ile), \quad (58)$$

$$\frac{dleu}{dt} = -0.540 \cdot v_1 - 7.660 \cdot v_2 - v_9 + D \cdot (leu_i - leu), \quad (59)$$

$$\frac{dser}{dt} = -1.515 \cdot v_1 - 24.500 \cdot v_2 + 2 \cdot v_7 - v_{10} + D \cdot (ser_i - ser), \quad (60)$$

$$\frac{dV}{dt} = F_{in} - F_{out} \quad (61)$$

$$v_1 = v_{max,1} \cdot \frac{pyr}{K_{mpyr1}+pyr} \cdot \frac{glu}{K_{mglu1}+glu} \cdot \frac{asn}{K_{masn1}+asn} \cdot \frac{glc}{K_{mglc1}+glc} \cdot \frac{K_{dlac1}}{K_{dlac1}+lac} \cdot \frac{K_{dnh41}}{K_{dnh41}+nh4}, \quad (62)$$

$$v_2 = a_2 \cdot biomass + b_2 \cdot v_1 \cdot biomass, \quad (63)$$

$$v_3 = v_{max,3} \cdot \frac{glc}{K_{mglc3}+glc} \cdot \frac{asn}{K_{masn3}+asn} + v_{3,base}, \quad (64)$$

$$v_4 = v_{max,4f} \cdot \frac{pyr}{K_{mpyr4}+pyr} \cdot \frac{1}{1+\left(\frac{lac}{K_{ilac4}}\right)^{\gamma_4}} - v_{max,4r} \cdot \frac{lac}{K_{mlac4}+lac} \cdot \frac{1}{v_3}, \quad (65)$$

$$v_5 = v_{max,5f} \cdot \frac{asn}{K_{masn5}+asn} - v_{max,5r} \cdot \frac{asp}{K_{masp5}+asp} \cdot \frac{nh4}{K_{mnh45}+nh4}, \quad (66)$$

$$v_6 = v_{max,6f} \cdot \frac{asp}{K_{masp6}+asp} - v_{max,6r} \cdot \frac{glu}{K_{mglu6}+glu}, \quad (67)$$

$$v_7 = v_{max,7} \cdot \frac{glc}{K_{mglc7}+glc}, \quad (68)$$

$$v_8 = v_{max,8} \cdot \frac{ile}{K_{mile8}+ile}, \quad (69)$$

$$v_9 = v_{max,9f} \cdot \frac{leu}{K_{mleu9}+leu} - v_{max,9r} \cdot \frac{glu}{K_{mglu9}+glu}, \quad (70)$$

$$v_{10} = v_{max,10} \cdot \frac{ser}{K_{mser10}+ser}, \quad (71)$$

$$v_{11} = v_{max,11f} \cdot \frac{pyr}{K_{mpyr11}+pyr} \cdot \frac{K_{iasn11}}{K_{iasn11}+asn}, \quad (72)$$