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RESEARCH ARTICLE

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On the construction of the Stokes flow in a domain with cylindrical ends

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Based on existence results for the Stokes operator and its solution properties in manifolds with cylindrical ends by Große et al. and Kohr et al., the Stokes flow in a three-dimensional compact domain Ω^+ with circular openings Σ_j ($j = 1, 2$) through which the fluid enters and leaves Ω^+ through unbounded cylindrical pipes the Stokes flow is modeled as a mixed boundary value problem Ω^+ whereas in the cylindrical ends, the velocities and pressures are constant on every straight line in the cylindrical directions with initial values from the openings Σ_j of Ω^+. These values equal the velocities and pressures which are obtained from the mixed boundary values' solution in Ω^+ at the openings Σ_j.

KEYWORDS
cylindrical ends, Stokes flow, Stokes operator

MSC CLASSIFICATION
35R01, 76M

1 | INTRODUCTION

In the works by Große et al. [1] and Kohr et al. [2], the analysis of the Stokes flow was investigated for manifolds with cylindrical ends. In this paper, for a three-dimensional compact domain Ω^+ with two cylindrical ends $\mathbf{N}_1, \mathbf{N}_2$ in the form of cylindrical pipes, the Stokes flow enters Ω^+ with the given velocity $\mathbf{u}_0$ through $\mathbf{N}_2$ and leaves Ω^+ through $\mathbf{N}_1$. The Lipschitz boundary $\partial\Omega^+$ contains two flat circular transmission surfaces $\boldsymbol{\Sigma}_2$ and $\boldsymbol{\Sigma}_1$, which are the hollow ends of the pipes where the flow enters and leaves Ω^+. In [2], the existence of a unique solution $\mathbf{u}$ for such a domain with cylindrical ends was shown and has rather particular properties:

- In Ω^+ the Stokes flow is modeled as a mixed boundary value problem in the Sobolov space $\mathbf{H}^1(\Omega^+)$.
- In the cylindrical ends, the velocities and pressures are constant on every straight half stream line in the cylindrical directions with initial values from the openings $\boldsymbol{\Sigma}_2$ and $\boldsymbol{\Sigma}_1$ of the domain Ω^+. These values equal velocities and pressures obtained from the solution in Ω^+.

For the construction of the solution and its numerical computation, domain and boundary potentials are used, which in engineering literature are called vortex or panel methods (see, e.g., Helmig [3, p. 76] , Helmig et al. [4] and Hess and Smith [5]). Software for these numerical methods is plenty available, for example, as OSTBEM developed by G. Of and O. Steinbach in Graz, Austria, or by S. Rjazanov and M. Bebendorf in Saarbrücken, Germany (Figure 1).

Dedicated to Prof. Dr.-Ing. Dr.h.c. Rainer Helmig on the occasion of his emeritation.

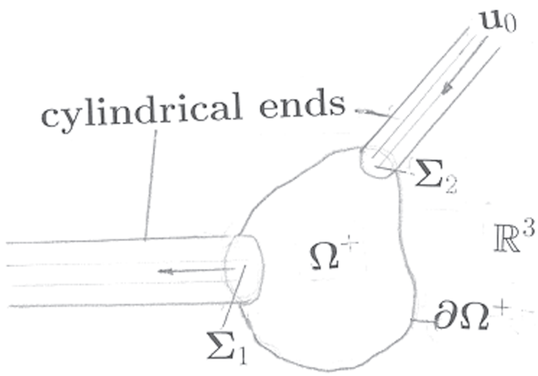

FIGURE 1 Compact body with two cylindrical ends.

2 | THE STOKES PROBLEM

A Stokes flow with velocity $\mathbf{u} = (u^i)$, pressure π, and constant kinematic viscosity $\mu = 1$ satisfies the linear system of partial differential equations (Kohr et al. [2])

$$\mathcal{L}(\mathbf{u},\pi) = \Delta\mathbf{u} - \nabla\pi = \mathbf{f}\ ,\ \nabla\cdot\mathbf{u} = 0. \tag{2.1}$$

Fourier transform of (2.1) yields that the transformed system considered as a pseudodifferential operator has the matrix-valued symbol

$$\begin{pmatrix} |\xi|^2 & 0 & 0 & -i\xi_1 \\ 0 & |\xi|^2 & 0 & -i\xi_2 \\ 0 & 0 & |\xi|^2 & -i\xi_3 \\ -i\xi_1 & -i\xi_2 & -i\xi_3 & 0 \end{pmatrix} \begin{pmatrix} u^1 \\ u^2 \\ u^3 \\ \pi \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ 0 \end{pmatrix}. \tag{2.2}$$

Hence, (2.1) is a strongly elliptic system, and the inverse matrix symbol

$$\frac{1}{|\xi|^2}\begin{pmatrix} 1 & 0 & 0 & i\xi_1 \\ 0 & 1 & 0 & i\xi_2 \\ 0 & 0 & 1 & i\xi_3 \\ i\xi_1 & i\xi_2 & i\xi_3 & 2|\xi|^2 \end{pmatrix} \tag{2.3}$$

is the symbol of a pseudodifferential operator defining a pseudoinverse (here even the inverse).

3 | PRELIMINARIES

Stress tensor and conormal derivatives are defined as

$$\boldsymbol{\sigma}(\mathbf{u},\pi) = -\pi\mathbf{I} + 2\mathbb{E}(\mathbf{u}), \tag{3.1}$$

with the strain rate tensor $\mathbb{E}(\mathbf{u}) := \frac{1}{2}\left(\nabla u + (\nabla u)^{\top}\right)$. The hydrodynamic boundary stress then is defined by

$$\mathbf{t}^{\pm}(\mathbf{u},\pi) := \left(\gamma^{\pm}\boldsymbol{\sigma}(\mathbf{u},\pi)\right)\boldsymbol{\nu}\ \text{ on }\ \partial\Omega^{\pm}. \tag{3.2}$$

The weak definition is given with Green's theorem by the continuity of the right-hand side and the Riesz representation theorem in the Hilbert space $\mathbf{H}^{\frac{1}{2}}(\partial\Omega)$,

$$\pm\ \langle\mathbf{t}^{\pm}(\mathbf{u},\pi),\mathbf{v}\rangle_{\partial\Omega} = 2\langle\mathbb{E}(\mathbf{u}),\mathbb{E}(\mathbf{v})\rangle_{\Omega^{\pm}} - \langle\pi,\boldsymbol{\nabla}\cdot\mathbf{v}\rangle_{\Omega^{\pm}} + \langle\mathcal{L}(\mathbf{u},\pi),\mathbf{v}\rangle_{\Omega^{\pm}}\forall\mathbf{v}\in C_0^{\infty}(\mathbb{R}^3)^3 \tag{3.3}$$

defines $\mathbf{t}^{\pm}\in\mathbf{H}^{-1/2}(\partial\Omega)$.

The fundamental solution of the Stokes system (Kohr et al. [6]) is found by applying the inverse Fourier transform $\mathcal{F}^{-1}$ to (2.3) in $\mathbb{R}^3$:

$$\mathcal{G}_{j,k}(\mathbf{x}) = \frac{1}{8\pi}\left\{\frac{1}{|\mathbf{x}|}\delta_{jk} + \frac{x_j x_k}{|\mathbf{x}|^3}\right\}\ ,\ \Pi_k(\mathbf{x}) = \frac{1}{4\pi}\frac{x_k}{|\mathbf{x}|^3},\ j,k \in \{1,2,3\}. \tag{3.4}$$

With the fundamental solution (3.4), the solution of (2.1) can be represented in Ω^+ (Wendland and Zhu [7]) by the potentials:

$$\mathbf{u}(\mathbf{x}) = \int_\Gamma \mathcal{G}(\mathbf{x}-\mathbf{y})\left(\mathbf{t}^+(\mathbf{u})(\mathbf{y})\right) ds_y - \left(\mathbf{t}_y^{+\prime}\mathcal{G}(\mathbf{x}-\mathbf{y})\right)^\top \mathbf{u}(\mathbf{y}) ds_y + \int_{\Omega^+} \mathcal{G}(\mathbf{x}-\mathbf{y})\mathbf{f}(\mathbf{y}) d\mathbf{y}, \tag{3.5}$$

$$\pi(\mathbf{x}) = \int_\Gamma \Pi(\mathbf{x}-\mathbf{y})\cdot\left(\mathbf{t}^+(\mathbf{u})\right) ds_y - 2\int_\Gamma \frac{\partial \Pi(\mathbf{x}-\mathbf{y})}{\partial n_y}\cdot \mathbf{u}(\mathbf{y}) ds_y + \int_{\Omega^+} \Pi(\mathbf{x}-\mathbf{y})\cdot \mathbf{f}(\mathbf{y}) dy, \tag{3.6}$$

where $\partial\Omega^+ =: \Gamma$.

4 | CYLINDRICAL ENDS

Transmission between $\Omega^+, \partial\Omega^+$ and the cylinders through the openings $\mathbf{\Sigma}_j = (x_j^1, x_j^2, 0)^\top$ with $0 \le (x_j^1)^2 + (x_j^2)^2 \le \varrho_j^2 > 0$ and the cylindrical domains (Große et al. [1], Kohr et al. [8]):

$$\begin{aligned} &\mathbf{N}_j = (r_j \sin\varphi, r_j\cos\varphi, t_j)^\top\ ,\ \partial\mathbf{N}_j = (\varrho_j \sin\varphi, \varrho_j\cos\varphi, t_j)^\top \\ &0 \le \varphi \le 2\pi\ ,\ 0 \le r_j < \varrho_j\ ,\ t_j \in (-\infty, 0]\ ,\ j = 1 \text{ or } 2. \end{aligned} \tag{4.1}$$

Transmission at the openings $\mathbf{\Sigma}_j$:

$$\begin{aligned} \mathbf{u}|_{\Sigma_j} &= \mathbf{u}(\mathbf{x}) \text{ for } \mathbf{x} \in \Sigma_j \cap \partial\Omega^+, \\ \mathbf{t}^+(\mathbf{u})|_{\Sigma_j} &= \mathbf{t}^+|_{\partial\Omega^+\cap\Sigma_j} = \mathbf{0}\ ,\ \mathbf{u}|_{\Sigma_2} = \mathbf{u}_0 \text{ given.} \end{aligned} \tag{4.2}$$

4.1 | Outflow $\mathbf{N}_1$

At the flat surface $\mathbf{\Sigma}_1$, the fluid flows with the velocity $\mathbf{u}_1(\mathbf{x})$ from Ω^+ into the pipe $\mathbf{N}_1$, a cylindrical end. From (4.2), there hold the transmission conditions

$$\begin{aligned} \mathbf{u}_1(\mathbf{x}) &= \mathbf{u}|_{\mathbf{\Sigma}_1}(\mathbf{x}), \\ \mathbf{t}^+(\mathbf{u})(\mathbf{x}) &= \mathbf{t}^+_{\partial\Omega^+\cap\mathbf{\Sigma}_1}(\mathbf{x}) \text{ for } \mathbf{x} \in \overline{\Omega^+} \cap \mathbf{\Sigma}_1. \end{aligned} \tag{4.3}$$

Since $\mathbf{N}_1$ is a cylindrical end, the Stokes flow $\mathbf{u}_1$ in the cylindrical x_3–direction is constant,

$$\frac{\partial}{\partial x_3}\mathbf{u}_1(\mathbf{x}) = \mathbf{0}, \tag{4.4}$$

along the straight lines parallel to the x_3-axis and, hence,

$$\mathbf{u}_1(\mathbf{x}) = \mathbf{v}_1(x_1, x_2) = \mathbf{v}_1(r\cos\varphi, r\sin\varphi), \varrho > 0 \le r \le \varrho\ ,\ \varphi \in [0, 2\pi]. \tag{4.5}$$

On the pipes cylindrical boundary part $\partial\mathbf{N}_1 \in (\varrho\cos\varphi, \varrho\sin\varphi, x_3), x_3 \le 0$, we assume homogeneous boundary conditions

$$\begin{aligned} \mathbf{v}_1(\varrho\cos\varphi, \varrho\sin\varphi) &= \mathbf{0}, \\ \mathbf{t}_1^+(\mathbf{v}_1)(\varrho\cos\varphi, \varrho\sin\varphi) &= \mathbf{0}. \end{aligned} \tag{4.6}$$

4.2 | Inflow through $\mathbf{N}_2$

The inflow region, the second cylindrical end $\mathbf{N}_2$ is again supposed to be a rotational symmetric pipe in the direction of the unit vector $\mathbf{e_{03}}$. Along $\mathbf{N}_2$, let us introduce new Euclidean coordinates $\mathbf{x}_0 = (x_{01}\mathbf{e_{01}}, x_{02}\mathbf{e_{02}}, x_{03}\mathbf{e_{03}})$. The inflow velocity $\mathbf{u}_0$ is given as the trace of a Stokes flow on the circular cross section of the pipe which is parallel to the circular flat opening $\boldsymbol{\Sigma}_2$ of Ω^+. In $\mathbf{N}_2$ is $\mathbf{u}_0$ constant along the straight lines parallel to $\mathbf{e}_{03}$ as the solution of the Stokes flow in the cylindrical end. Therefore,

$$\frac{\partial}{\partial x_{03}}\mathbf{u}_0(\mathbf{x}_0) = \boldsymbol{0}, \tag{4.7}$$

and at the opening $\boldsymbol{\Sigma}_2$ of Ω^+, the inflow transmission properties read

$$\begin{aligned} \mathbf{u}_0(\mathbf{x}_0) &= \mathbf{u}|_{\boldsymbol{\Sigma}_2}(\mathbf{x}_0), \\ \mathbf{t}^+(\mathbf{u}_0)(\mathbf{x}_0) &= \mathbf{t}^+|_{\partial\Omega^+\cap\boldsymbol{\Sigma}_2}(\mathbf{x}_0)\ ,\ \mathbf{x}_0 \in \overline{\Omega^+} \cap \boldsymbol{\Sigma}_2. \end{aligned} \tag{4.8}$$

Since $\mathbf{u}_0(\mathbf{x}_0)$ is constant along straight lines on $\partial\mathbf{N}_0$ parallel to $\mathbf{e}_{03}$, to $\mathbf{u}_0$, there exists $\mathbf{v}_0(x_{01}, x_{02})$ such that

$$\mathbf{u}_0(\mathbf{x}_0) = \mathbf{v}_0(x_{01}, x_{02}) = \mathbf{v}_0(r_0\cos\varphi, r_0\sin\varphi), \tag{4.9}$$

with $\varrho_0 > 0 \le r_0 \le \varrho_0\ ,\ \varphi \in [0, 2\pi]$.

On the cylindrical boundary part

$$\partial\mathbf{N}_2 \in (\varrho_0\cos\varphi, \varrho_0\sin\varphi)\ \text{ for }\ x_{03} \le 0,$$

we require homogeneous boundary conditions

$$\begin{aligned} \mathbf{v}_0(\varrho_0\cos\varphi, \varrho_0\sin\varphi) &= \boldsymbol{0}, \\ \mathbf{t}_1^+(\mathbf{v}_0)(\varrho_0\cos\varphi, \varrho_0\sin\varphi) &= \boldsymbol{0}. \end{aligned} \tag{4.10}$$

5 | THE RESULTING SYSTEM OF EQUATIONS

Summarizing the equations in Ω^+ and $\mathbf{N}_j$ and the transmission conditions, we obtain the following coupled system of equations:

$\mathbf{u}$ in Ω^+ is the solution of the mixed problem (Costabel and Stephan [9]).

$$\mathcal{L}(\mathbf{u}, \pi) = \Delta\mathbf{u} - \nabla\pi = \mathbf{f}\ \text{ and }\ \nabla\cdot\mathbf{u} = 0\ \text{ in }\ \Omega^+,$$

$$\begin{aligned} \mathbf{u}|_{\boldsymbol{\Sigma}_1}(\mathbf{x}) &= \mathbf{u}_1(\mathbf{x})\ ,\ \mathbf{x} \in \boldsymbol{\Sigma}_1, \\ \mathbf{t}^+|_{\partial\Omega^+\cap\boldsymbol{\Sigma}_1}(\mathbf{u})(\mathbf{x}) &= \mathbf{t}^+_{\mathbf{N}_1}(\mathbf{u}_1)(\mathbf{x})\ ,\ \mathbf{x} \in \overline{\Omega^+} \cap \boldsymbol{\Sigma}_1, \\ \mathbf{u}|_{\boldsymbol{\Sigma}_2}(\mathbf{x}_0) &= \mathbf{u}_0(\mathbf{x}_0)\ \text{ for }\ \mathbf{x}_0 \in \boldsymbol{\Sigma}_2\ \text{ with given }\ \mathbf{u}_0, \\ \mathbf{t}^+|_{\partial\Omega^+\cap\boldsymbol{\Sigma}_2}(\mathbf{x}_0) &= \mathbf{t}^+_{\mathbf{N}_2}(\mathbf{u}_0)(\mathbf{x}_0)\ \text{ for }\ \mathbf{x}_0 \in \overline{\Omega^+} \cap \boldsymbol{\Sigma}_2. \end{aligned} \tag{5.1}$$

6 | COMPUTATION OF THE SOLUTION

The computation of $\mathbf{u}$ in Ω^+ with Stokes potentials leads to systems (3.5) and (3.6):

$$\begin{aligned} \mathbf{u}(\mathbf{x}) &= \mathcal{N}\mathbf{f}(\mathbf{x}) + \mathcal{V}\mathbf{t}^+(\mathbf{u})(\mathbf{x}) - \mathcal{W}\mathbf{u}(\mathbf{x}), \\ \pi(\mathbf{x}) &= \Pi\mathbf{f}(\mathbf{x}) + \Pi_\Gamma\mathbf{t}^+(\mathbf{u})(\mathbf{x}) - 2\partial_n\Pi_\Gamma(\mathbf{u})(\mathbf{x})\ \text{ for }\ \mathbf{x} \in \Omega^+. \end{aligned} \tag{6.1}$$

Applying the boundary trace operator to the representation equations (6.1) results together with the jump relations at the system of boundary integral equations:

$$\mathbf{V}\mathbf{t}^{+}|_{\partial\Omega+}(\mathbf{u})(\mathbf{x})+\left(\frac{1}{2}\mathbf{I}-\mathbf{K}\right)\mathbf{u}(\mathbf{x})+\mathcal{N}\mathbf{f}(\mathbf{x})=\mathbf{0} \text{ for } \mathbf{x}\in\partial\Omega^{+}\setminus\bigcup_{j=1}^{2}\Sigma_{j}, \tag{6.2}$$

$$\mathbf{t}^{+}|_{\partial\Omega^{+}}(\mathbf{u})(\mathbf{x})=\mathbf{D}\mathbf{u}(\mathbf{x})+\left(\frac{1}{2}\mathbf{I}+\mathbf{K}'\right)\mathbf{t}^{+}(\mathbf{x})+\mathbf{t}^{+}(\mathcal{V}\mathbf{f})(\mathbf{x})=\boldsymbol{0},\mathbf{x}\in\Sigma_{j}\ ,\ j=1,2. \tag{6.3}$$

For the pressure equation (3.6), we use operator relations going back to Mitrea and Nistor [10]:

$$\int_{\Gamma}\Pi(\mathbf{x}-\mathbf{y})\cdot\left(\mathbf{t}^{+}(\mathbf{u})\right)(\mathbf{y})ds_{y}=\frac{1}{4\pi}\partial_{x_{k}}\int_{\Gamma}\frac{1}{|\mathbf{x}-\mathbf{y}|}\left(t_{k}^{+}(\mathbf{u})\right)(\mathbf{y})ds_{y}, \tag{6.4}$$

$$2\int_{\Gamma}\frac{\partial\Pi(\mathbf{x}-\mathbf{y})}{\partial n_{y}}\cdot\mathbf{u}(\mathbf{y})ds_{y}=\frac{1}{2\pi}\partial_{x_{k}}\int_{\Gamma}\left(\frac{\partial}{\partial n_{y}}\frac{1}{|\mathbf{x}-\mathbf{y}|}\right)u^{k}(\mathbf{y})ds_{y}\ ,\ k=1,2. \tag{6.5}$$

With these relations, one obtains the pressure for $\mathbf{x}\in\Sigma_j$:

$$\begin{aligned}\pi(\mathbf{x})=\ &\gamma^{+}\mathbf{div}V_{\Delta}\left(\mathbf{t}^{+}(\mathbf{u})\right)|_{\Sigma_{j}}+\gamma^{+}\mathbf{div}\left(\frac{1}{2}\mathbf{I}-\mathbf{K}_{\Delta}\right)\mathbf{u}|_{\Sigma_{j}}\\ &+\gamma^{+}\int_{\Omega^{+}}\Pi(\mathbf{x}-\mathbf{y})\cdot\mathbf{f}(\mathbf{y})d\mathbf{y}|_{\Sigma_{j}}\ ,\ j=1\ \text{or}\ 2.\end{aligned}$$

7 | MAPPING PROPERTIES AND SPACES: $\Gamma := \partial\Omega^{+}$:

$$\begin{aligned}&\mathbf{u}\in\mathbf{H}^{1}(\Omega^{+}),\gamma^{+}\mathbf{u}\in\mathbf{H}^{1/2}(\Gamma),\mathbf{t}^{+}\in\mathbf{H}^{-1/2}(\Gamma),\mathbf{f}\in\mathbf{L}^{2}(\Omega^{+}),\\ &\pi\in L^{2}(\Omega^{+}),\gamma^{+}\pi\in H^{-1/2}(\Gamma).\end{aligned} \tag{7.1}$$

$$\begin{aligned}\mathbf{V}&:\mathbf{H}^{-1/2}(\Gamma)\to\mathbf{H}^{1/2}(\Gamma),\mathcal{N}:\mathbf{L}^{2}(\Omega^{+})\to\mathbf{H}^{1/2}(\Gamma),\\ \frac{1}{2}\mathbf{\Pi}^{\pm}\mp\mathbf{K}&:\mathbf{H}^{1/2}(\Gamma)\to\mathbf{H}^{1/2}(\Gamma),\mathbf{t}^{+}:\mathbf{H}^{1}(\Omega^{+})\to\mathbf{H}^{-1/2}(\Gamma),\\ \frac{1}{2}\mathbf{\Pi}^{\pm}\pm\mathbf{K}'&:\mathbf{H}^{-1/2}(\Gamma)\to\mathbf{H}^{-1/2}(\Gamma),\\ \mathbf{D}&:\mathbf{H}^{1/2}(\Gamma)\to\mathbf{H}^{-1/2}(\Gamma).\end{aligned} \tag{7.2}$$

The Fredholmness of the system of Equations (4.5), (4.6), and (4.9) and the mapping properties corresponding to (4.10) follow with the Calderon projection properties (see Wendland and Zhu [7]): Particularly, $\nabla\cdot\mathbf{u}=0$ in (2.1) implies with $\mathbf{u}=\boldsymbol{0}$ on $\partial\Omega^{+}\setminus\bigcup_{j=1}^{2}\Sigma_{j}$.

$$\int_{\partial\Omega^{+}}\mathbf{u}\cdot\boldsymbol{v}ds_{\partial\Omega^{+}}=\int_{\Sigma_{1}}\mathbf{u}\cdot\mathbf{e}_{3}ds_{\Sigma_{1}}+\int_{\Sigma_{2}}\mathbf{u}_{0}\cdot\mathbf{e}_{03}ds_{\Sigma_{2}}=0, \tag{7.3}$$

since $\boldsymbol{v}|_{\Sigma_1}=\mathbf{e}_{03}$ and $\boldsymbol{v}|_{\Sigma_2}=\mathbf{e}_3$. This is an additional equation for the solution $\mathbf{u}$ in Ω^{+} at $\Sigma_1\cup\Sigma_2$.

Concerning the regularity of $\mathbf{u}$ and $\mathbf{t}^{+}$ on $\partial\Omega^{+}$, at the boundary curves σ_j of Σ_j, as (distance to $\sigma_j)^{1/2}\to 0$ on Σ_j since due to applying the Kelvin transform, the openings Σ_j become screens whose stresses $\boldsymbol{\sigma}_j$ have a singularity in the form (distance to $\sigma_j)^{-1/2}$, see (Costabel and Stephan [9], Stephan [11], and Dauge [12]). Therefore, we append additional equations on the curves σ_j (see Wendland and Zhu [7]),

$$\int_{\sigma_{j}}(\text{distance to }\sigma_{j})^{1/2}\mathbf{t}^{+}|_{\sigma_{j}}ds_{\sigma_{j}}=0\ ,\ j=1\text{ and }2. \tag{7.4}$$

With $\mathbf{u}, \mathbf{t}^{+}$ and π in $\overline{\Omega^{+}}$ on Σ_j, these define the constant values in the direction of the x_3-axis on every straight line parallel to the x_3-axis and the contact values in the direction of $\mathbf{e}_{03}$ on every straight line parallel to $\mathbf{e}_{03}$, in both straight cylindrical ends.

Further analysis is possible, extending the geometry to a third or finitely many additional cylindrical ends.

AUTHOR CONTRIBUTIONS

Wolfgang L. Wendland: Conceptualization; writing—review and editing; methodology.

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REFERENCES

1. N. Große, M. Kohr, and V. Nistor, *General regularity on manifolds with bounded geometry and applications to the deformation operator*, 2023. arXiv:2304.10943v1 [math AP] 21.
2. M. Kohr, V. Nistor, and W. L. Wendland, *Layer potentials and essentially translation invariant pseudodifferential operators on manifolds with cylindrical ends. arXiv:2308.06308v3 [math.AP]*. Analysis of partial differential equations. Cornell University, Ithaca, New York, USA. To be published in Postpandemic Operator Theory, 61–115 C Theta 2024.
3. R. Helmig, *Numerische Methoden in der Hydromchanik*, Institut für Wasserbau, Lehrstuhl für Hydraulik und Grundwasser, Stuttgart, Germany, 1995.
4. R. Helmig, A. Mielke, and B. Wohlmuth, *Multifield problems in solid and fluid mechanics*, Technik, Springer, Berlin, 2006.
5. D. Hess and A. M. O. Smith, *Calculation of potential flow about arbitrary bodies*, Progr. Aeronaut. Sci. **8**, no. 966, 1–138.
6. M. Kohr, S. E. Mikhailov, and W. L. Wendland, *On some mixed-transmission problems for the anisotropic Stokes and Navier–Stokes systems in Lipschitz domains with transversal interfaces*, J. Math. Anal. Appl. **516** (2022), no. 1, 126464.
7. W. L. Wendland and J. Zhu, *The boundary element method for three-dimensional Stokes flows exterior to an open surface*, Math. Comp. Modelling **15** (1991), no. 6, 19–41.
8. M. Kohr, C. Pintea, and W. L. Wendland, *Brinkman-type operators on Riemannian manifolds: transmission problems in Lipschitz and C^1 domains*, Potential Anal. **32** (2010), 229–273.
9. M. Costabel and E. P. Stephan, *Boundary integral equation for mixed boundary value problem in polygonal domains and Galerkin approximation*, Mathematical Models and Methods in Mechanics, vol. **15**, Banach Center Publications, Warsaw, Poland, 1985, pp. 175–251.
10. M. Mitrea and V. Nistor, *Boundary value problems and layer potentials on manifolds with cylindrical ends*, Czechoslovak Math. J. **57** (2007), no. 132, 1151–1197.
11. E. P. Stephan, *Boundary integral equations for screen problems in $\mathbb{R}^3$*, Integral Equ. Oper. Theory **10** (1987), 236–257.
12. M. Dauge, *Elliptic boundary value problems on corner domains. Smoothness and asymptotics of solutions*, Lecture Notes in Mathematics, vol. **1341**, Springer, Berlin, 1988.

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