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Height systems calculations at Swabian Alb Test Area

Master Thesis

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Hereby it is confirmed, that this thesis was prepared independently, and that no sources and resources were used other than those explicitly listed in the thesis.

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Place, Date | Signature of thesis author

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Abstract

This project aims at the consolidation of the data from integrated fieldwork in Swabian Alb test area since 1996 to 2013 as well as the height systems computations. Reliable data were checked by height differences and gravity values, after that they were grouped into 5 closed loops with 56 out of 121 observed points. Potential differences were computed from height differences which acquired from spirit levelling and gravity value, then least square adjustment was adopted. Observation equation (A-matrix) and condition equation (B-matrix) were applied in the adjustment, a weight matrix was also assigned in the adjustment. Geopotential differences were computed based on the Helmert orthometric height at point 580, then geopotenial numbers of the other points were computed by adding the geopotential number with the adjusted potential differences, then height systems could be determined as well as height corrections. In the closed loop adjustment especially adjustment of several loops with many data, condition equation adjustment is preferred because of the smaller size of design matrix compared to the observation equations and the advantage of condition equations over observation equations is that loop misclosures can be determined by condition equation. The results from both equations are the same. The difference of the geopotential numbers between unweighted and weighted adjustment is up to 0.0129 m^2/s^2 and the difference of height systems between unweighted and weighted adjustment is up to 0.0013 meter or 1.3 millimeter, so the height system computations were not significantly affected by the assigned weight. The difference of height corrections between unweighted and weighted adjustment is up to 10^{-8} m so the assigned weights did not affect height correction results. Normal corrections give the smallest values while dynamic corrections give the largest values because the test area is located at latitude 48.485° instead of latitude 45° so the correction values are quite large.

Keywords: height system, height correction, geopotential number, geopotential difference, gravity, levelled height

Contents

Contents (con't)

List of Figures

List of Tables

List of tables (con't)

List of abbreviations and symbols

ΔH	Height difference from spirit levelling
ΔC	Geopotential difference
C	Geopotential number
g	Gravity value
$\overline{g}$	Mean gravity along plumb line
γ	Normal gravity
γ_0	Normal gravity at latitude 45°
$\overline{\gamma}$	Mean normal gravity along plumb line
φ	Geographic latitude
H^D	Dynamic height
H^O	Orthometric height
H^N	Normal height
DC	Dynamic correction
OC	Orthometric correction
NC	Normal correction

Chapter 1

Introduction

1.1 Background and statement of problems

Integrated fieldwork had been started at 1996 and it is carried out by Geomatics Engineering (GeoEngine) student and Geodesy and Geoinformatics Engineering (GuG) students every year in the SwabianAlb area, Glems as shown in figure 1-1. There are so many data that have been acquired from the fieldwork since 1996 until 2013 such as height differences, gravity values, and distances between each station but these data are not consolidated together because the surveying path is not the same, although some years the path was the same but the points were different instead that means there was no correlation between the points in each yearso the reliable reference points or benchmark (BM) could not be specified for the future work.

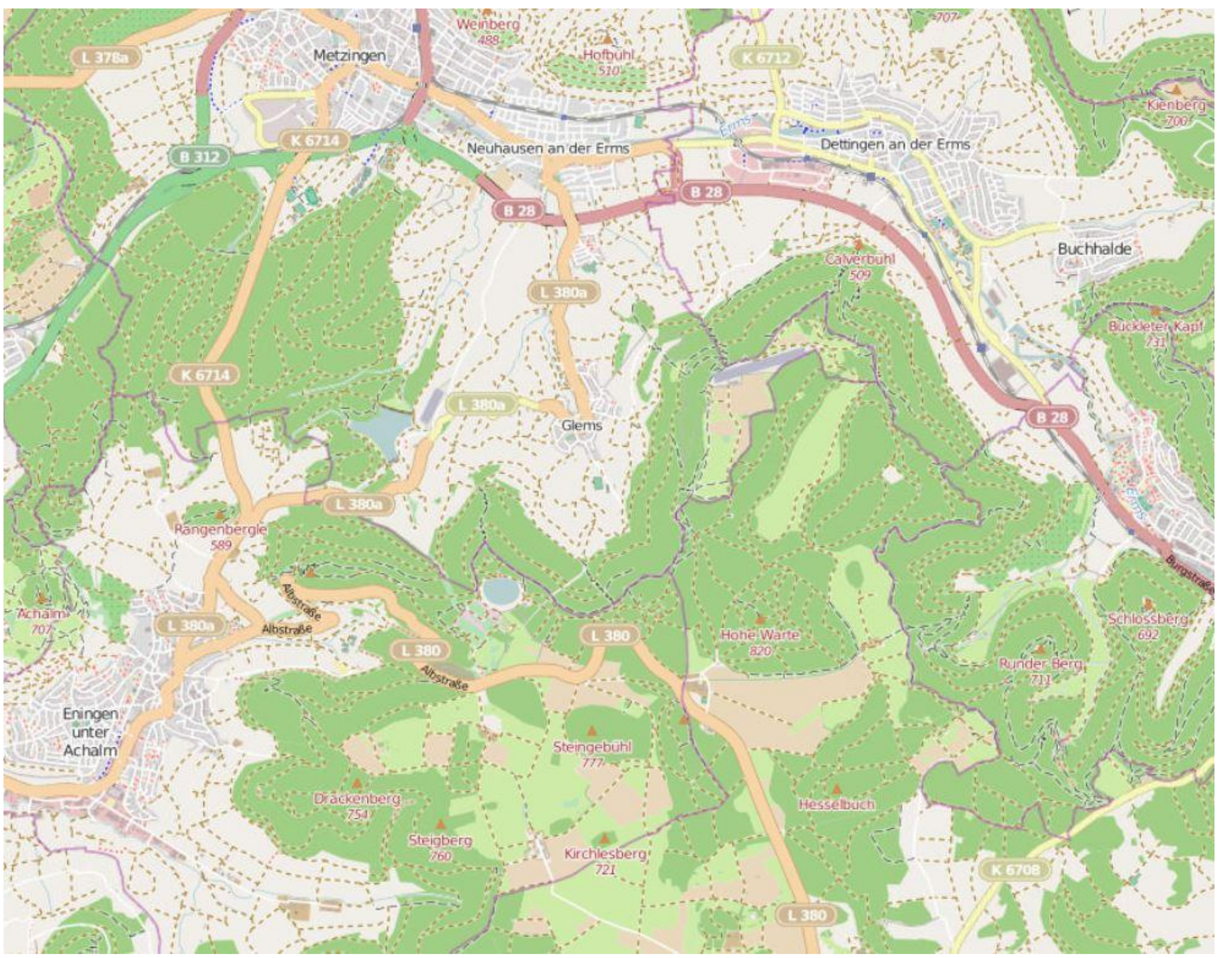

Figure 1.1 SwabianAlb test area

The height systems computations from each years were not equal, for instance, there were several height values at the same point, so it causes the confusing to choose the reliable heights for extension of the levelling path from the mentioned point, thus, the consolidation of the data is required.

If the fixed points or benchmarks with known heights were specified in the field, it will be convenient for the surveyors who want to extend the leveling path because they can select the nearest fixed point in the field as a start point and run the leveling to the desired path although the end point is not a fixed point, the surveying procedure called "Backward and Forward Run" could be adopted as shown in figure 1.2

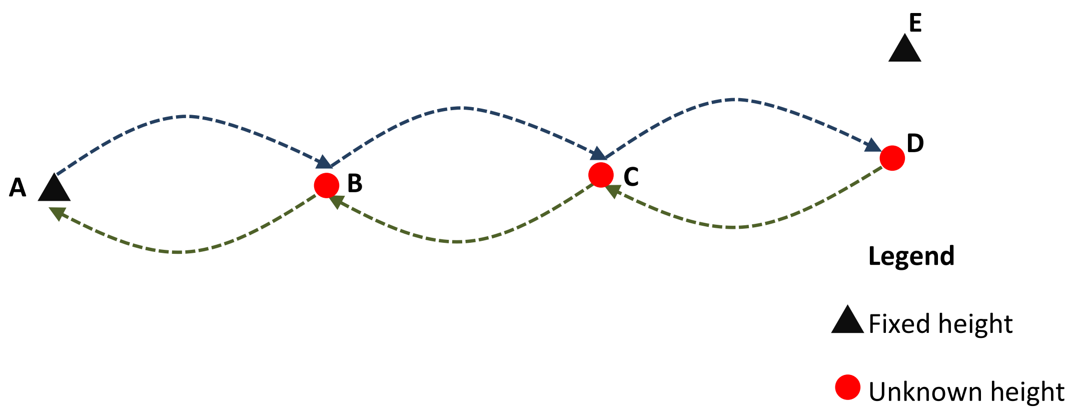

Figure 1.2 Backward and forward run

Figure 1.2 shows that point A with known height is start point of the leveling, the path ran to point B, C, and D, respectively. The heights of point B, C , and D are unknown although the leveling is finished at point D, the determined heights could be incorrect because the errors exist in the leveling process and they are not eliminated. This problem can be solved by setting point D as a start point and run the leveling back to point A, then the measurement errors could be determined. On the other hand, if point E is selected as the end point instead of point D, the measurement error could also be determined and heights at point B and C could also be adjusted. Thus, it is very important that the reliable heights have to be known in the field, especially for the extension of the leveling paths. Moreover, those fixed points could be used as the reference points for

1.2 Objectives

According to statement of problems that mentioned in section 1.1, to achieve the reliable fixed height values of the points that had been done in SwabianAlb test area, the objectives of this thesis are specified as follow:

- Consolidation of the data from integrated fieldwork.
- Adjustment of the geopotential differences
- Height systems computations and height correction computations.
- Visualization of the surveying path since 1996 to 2013 as well as the reliable reference points for future work.

1.3 Scopes of the work

To achieve the reliable height values from several data from several years, the scopes of the work have to be specified. The scopes of the work are as follow:

- Collection of the data from integrated fieldwork since 1996 to 2013
- Height differences, gravity values and distances between each point in the test area were collected but trigonomeric leveling data were neglected because of its rough accuracy compared to spirit levelling.
- Comparison of the results betweenunweighted and weighted least square adjustment
- Adjustment of the geopotential differences are done simultaneously for every closed loop.
- Observation equation and condition equation were adopted for geopotential difference correction.
- Geopotential numbers were calculated based on Helmertorthometric height.
- Computation of dynamic height, orthometric height and normal height
- Computation of dynamic correction, orthometric correction and normal correction

Chapter 2

Theoritical Backgrounds

2.1 Levelling

2.1.1 Differential levelling

Leveling is the determination of relative height differences of points of interest on or below the earth surface so leveling deals with measurement in the vertical plane. Height difference on gentle slope or plain area can be done by differential levelling. The principle of differential levelling is measuring height of each point by the levelling rod, the difference between two rods (A and B) is the height difference as shown in figure 2.1

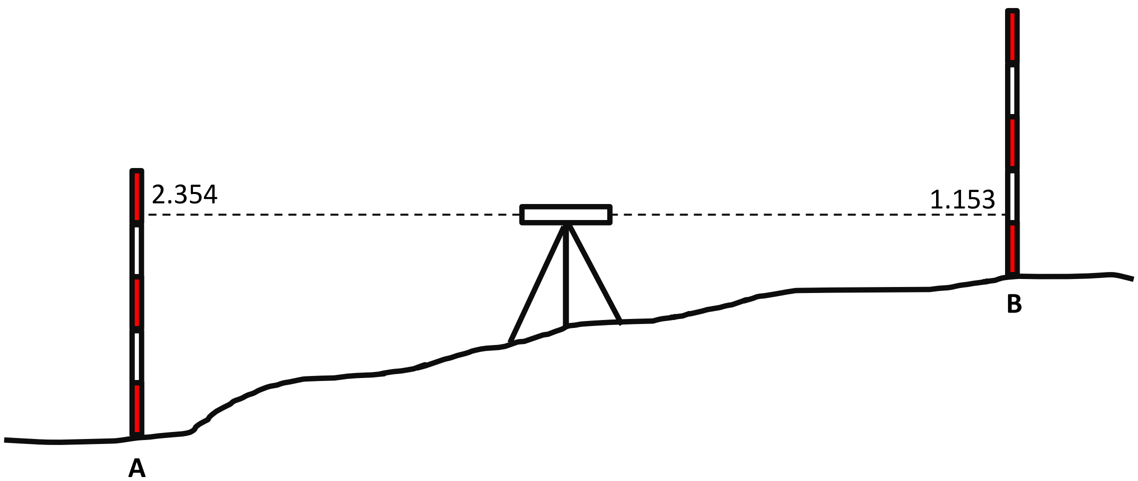

Figure 2.1 Differential leveling

From figure 2.1, the height difference between A and B can be calculated as follows

$$\Delta H_{AB} = 2.354 - 1.153 = 1.201$$

That means point B is 1.201 meters higher than point A

In differential leveling, the distances between the level and the rods should be made approximately equal by pacing or stadia method to eliminate errors due to improper adjustment of the instrument as well as the curvature effect of the earth and refraction.

Consider figure 2.1, if the distance between the instrument to point A is approximately equal to the distance from the instrument to point B, the error due to earth curvature and refraction from both sides of the rods cancel each other. Thus, it is very important to keep the distance between the instrument and adjacent points equal.

2.1.2 Alternative Leveling

If the required accuracy is higher than commonly achievable by simple leveling with a construction level, then an alternative leveling is used in conjunction with a self-checking method, such as leveling with double scale rods. If both scales are read, there is the check that their difference has to be constant to within ±1 mm. If, in addition, the spirit level is newly centered or the compensator activated prior to the second reading, then the leveling is checked as well. In order to counteract possible systematic errors. It is advisable to use two rods and to observe in the sequence B1, F1, F2, B2, as shown in figure 2.2

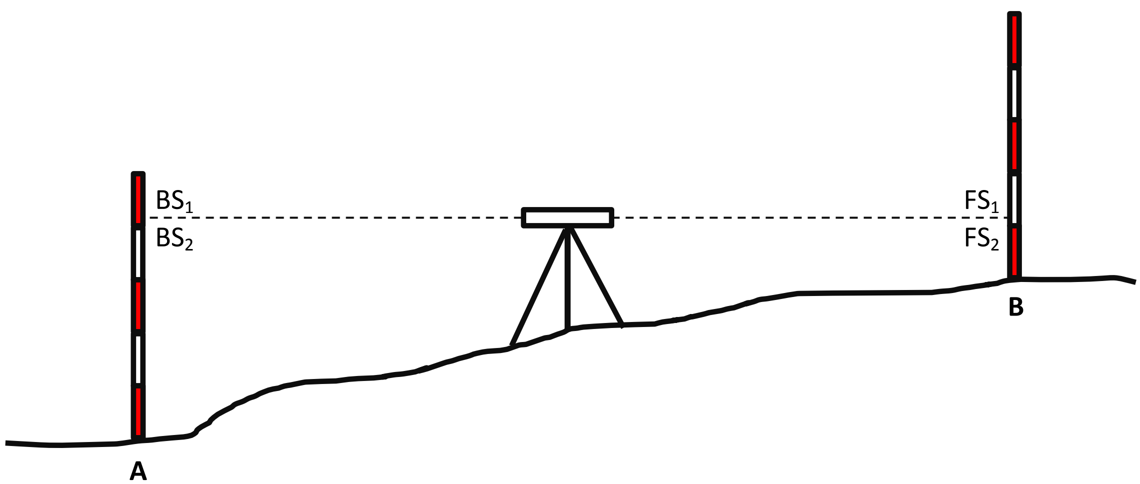

Figure 2.2 Alternative leveling

The height differences can be calculated as follows

$$\Delta H_1 = BS_1 - FS_1 \qquad (2.1a)$$

$$\Delta H_2 = BS_2 - FS_2 \qquad (2.1b)$$

$$\Delta H_{AB} = \frac{\Delta H_1 + \Delta H_2}{2} \qquad (2.1c)$$

As an independent check, the elevation differences are formed for scales I and II separately and then compared. Differences should be less than ±1.0 mm by carefully operating, a standard deviation of ±2 to ±3 mm per kilometer can be achieved with this method.

2.1.3 Continuous Leveling

In case of the points in the field are a great distance apart and it is impossible to finish the leveling between the points by setting the level in just one time so continuous leveling is adopted.

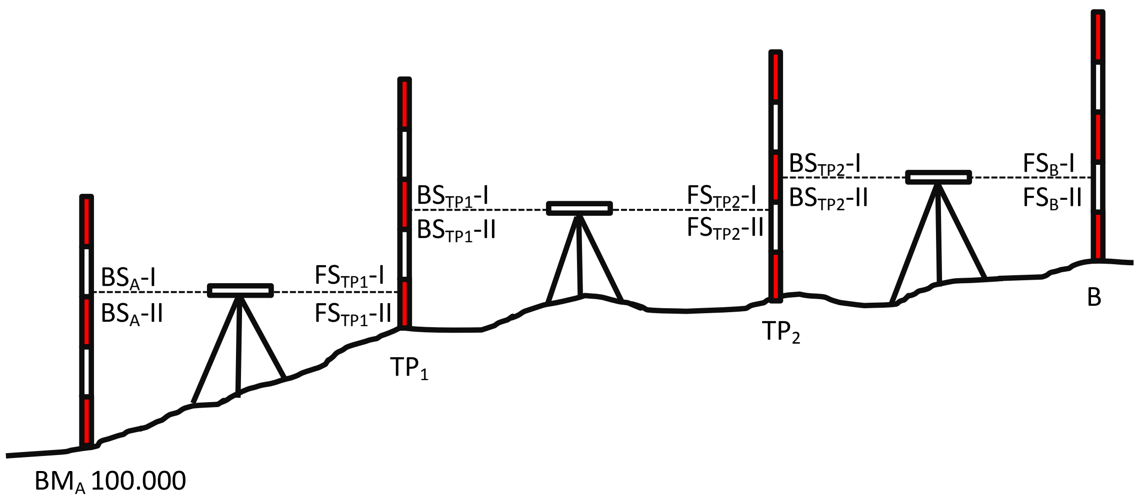

Figure 2.3 Continuous leveling

Consider figure 2.3, point B is a great distance apart from point BM_A so the height difference between these two points could not be done by setting up the level between the points because it exceeds the capacity of the level so turning points (TP) have to be put as additional points for transferring the height differences from point BM_A to point B. In order to achieve a high accuracy of the leveling, alternative leveling as mentioned in section 2.2 has to be adopted by reading the rods as follows:

Sequence of rod reading

BM_A to TP_1 : BS_A-I ⟶ FS_{TP1}-I ⟶ FS_{TP1}-II ⟶ BS_A-II

TP_1 to TP_2 : FS_{TP2}-I ⟶ BS_{TP1}-I ⟶ BS_{TP1}-II ⟶ FS_{TP2}-II

TP_2 to B : BS_{TP2}-I ⟶ FS_B-I ⟶ FS_B-II ⟶ BS_{TP2}-II

2.1.4 Errors in leveling

Leveling measurements are subjected to the sources of error as follows

Instrument errors

- *Line of sight*. Since the instrument is properly adjusted with the bubble centered, a horizontal plane, rather than a conical surface, is generated as the telescope is revolved. Also, if the compensators of automatic levels are operating properly, a truly horizontal line of sight is always produced. If the mentioned conditions are not met then an error of line of sight so-called “collimation error” occurred which resulted in serious errors in rod reading. These errors are systematic, but they are canceled if the distance between the level and adjacent rods are kept equal.

- *Cross hair not exactly horizontal*. This type of error can be eliminated or minimized by reading the rod near the center of the horizontal cross.

- *Rod not correct length*. Inaccurate divisions on a rod cause errors in elevation difference measurement. Thus, rod graduations should be checked by comparing them with standardized tape.

- Tripod legs loose. Tripod legs that are too loose or too tight cause the movement or strain that effect the instrument as well as unstable set up of the instrument.

Natural errors

- *Curvature of the earth*. A level surface curves away from a horizontal plane at the rate of $0.0785K^2$ (mm) where K is the distance in kilometer, which is about 8 cm/km. The effect of the earth curvature cause increasing of rod reading but it can be eliminated by keeping the distance between the level and adjacent points equal.

- *Refraction*. Light rays coming from an object to the telescope are bent which leads to the line of sight a curve concave to the earth’s surface, which decrease the rod readings. Balancing the distance between the level and the rods could eliminate the errors due to refraction. Error due to refraction tend to be random over a long period of time, they could be systematic on one day’s run.

- *Wind*. Strong wind causes the vibration of instrument and makes the rod unsteady. Precise leveling should not be done on excessively windy days.

- Settlement of the instrument. Settlement of the instrument makes the elevation of the next point too high and this error is cumulative in a series of setups on soft material. Therefore, set up the instrument on soft ground should be avoided.

- Settlement of turning point. This condition causes an error as settlement of instrument. Can be avoided by selecting a firm turning point or, if none are available, using a steel turning pin set firmly in the ground. A footplate can also be used in most situations.

Personal errors

- *Bubble not centered*. In working with levels that employ level vials, errors caused by bubble not being exactly centered at the time of sighting are the most important of any particularly on long sights.

- *Parallax*. This error caused by improper focusing of the objective or eyepiece lens results in incorrect rod readings. This can be eliminated by careful focusing the objects.

- *Faulty rod readings*. Incorrect rod readings result from parallax, poor weather conditions, long sights, improper target settings, and others causes, including mistakes such as those due to careless interpolation and transposition of figures. Short sight selected to accommodate weather and instrument conditions reduce the magnitude of reading errors.

- *Rod handling*. Improper plumbing of the rod causes serious error in rod reading. It can be eliminated by using a rod level that is in adjustment or holding the rod parallel to a plumb bob string.

Errors in leveling cannot be eliminated but it can be minimized by carefully adjusting and manipulating the instrument and rods and establishing standard field method routines. The following routines prevent large errors: (1) make sure that he bubble of the instrument and the rods are centered before and after each reading (if an automatic level is not being used), (2) using a rod level all the time while reading the rod, (3) keeping the distance between the instrument and the adjacent rods equal, (4) making the useful field-book arithmetic check, (5) method of red trousers has to be adopted, the principle is using the same rod for the first observation (the man with red trousers). In principle, the BFFB method is adopted for the first set up and then FBBF for the next one in rotation as explained in section 2.1.3

2.2 Gravimetry

Any instrument that needs to be leveled in order to measure the horizontal and vertical angles depends on gravity for orientation. Thus, many surveying measurements are affected by gravity. The gravity around the earth is a combination of the forces produced by the earth's mass and the rotation of the earth itself. The force due to the earth mass is called "gravitational" and the force that produced by rotation of the earth is called "centrifugal force". The combination of these forces is called "gravity"

An instrument for measuring gravity is known as “gravimeter”. It is a type of accelerometer for measuring the acceleration of gravity over the earth surface. The gravimeter displays the unit of the gravity measurement in Gals (cm/s^2) which named after Galileo Galilei, thus 1 mGal is equal to 10^{-5} m/s^2. There are two types of gravimeter: absolute gravimeter and relative gravimeter.

- Absolute gravimeter, nowadays it is made compact such that it can be used in the field. Gravity can be measured directly. Normally, this type of gravimeter is preferred in calibration of relative gravimeters, gravity anomaly surveying, and for establishing the vertical control network.

- Relative gravimeter, this type of gravimeter compares the difference between gravity at different locations such that one absolute gravity value in the field has to be known as a start point. Most common relative gravimeters are spring-based. Lacoste developed the concept of zero-length spring in gravimeter design, so called “astatic or astatized spring” as shown in figure 2.4

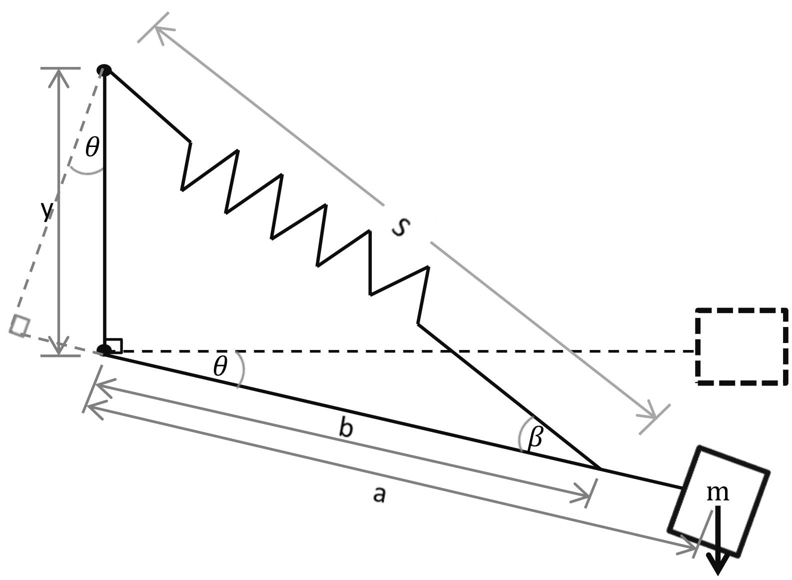

Figure 2.4 Schematic diagram of zero length spring gravimeter

From figure 2.4, Force due to mass is mg

Force due to spring is $k(S-Z)$,

where k is spring constant

Z is unstretched length of spring

Moment balance about pivot gives

$$mga\cdot\cos\theta = k\left(S-Z\right)b\cdot\sin\beta \tag{2.2a}$$

sine law : $\frac{y}{\sin\beta} = \frac{S}{\sin(90+\theta)}$

thus, $\sin\beta = \frac{y \cdot \cos\theta}{S}$ (2.2b)

Substitute (2.2b) into (2.2a), then

$$mga \cdot \cos\theta = k(S-Z)b\frac{y \cdot \cos\theta}{S} \quad (2.2c)$$

$$g = \frac{k}{m}\frac{b}{a}\left(1-\frac{Z}{S}\right)y \quad (2.2d)$$

$\frac{\partial g}{\partial S}$ has to be small, thus $\partial g = \frac{k}{m}\frac{b}{a}\frac{Z}{S^2}y \cdot \partial S$ (2.2e)

Set $\frac{k}{m}\frac{b}{a}\frac{Z}{S^2}y$ as κ, then equation 2.2e is simplified as $\partial g = \kappa \cdot \partial S$ (2.2f)

κ is the effective spring constant and it is the square of oscillation frequency. Thus all parameters can be tuned to produce a required frequency (ω).

Equation 2.2f is inverted yields:

$$\partial S = \frac{\partial g}{\kappa} \quad (2.2g)$$

All the parameters above can be tuned to produce a certain ∂S for a given change in gravity (∂g). The spring with nearly zero-length is selected to prevent infinitely sensitivity but the equilibrium condition in 2.2a is slightly changed.

The performance of this type of gravimeter depends on the material properties of the spring. Drift in the measurement is occurred by a creep rate of the spring. A metallic spring is used in Lacoste-Romberg gravimeters, while quartz spring is used in Scintrex gravimeter. The advantages and disadvantages of those two types of springs are listed in table 2.1

Properties of the spring	Type of spring	
	Metallic	**Quartz**
Thermal expansion	high, protection required	low and linear
Magnetic influence	yes, shielding needed	no
Weight	high	low
Drift rate	low	high

Table 2.1 Advantaged and disadvantage of metallic and quartz springs

Lacoste and Romberg gravimeter model G is shown in figure 2.5 and the specification is shown in table 2.2 as follows:

Figure 2.5 Relative gravimeter (Lacoste and Romberg gravimeter, model G759)

System Performance	Sensor type	Zero length metal spring
	Data resolution	0.005 mGal
	Repeatability	In field conditions *Repeatability depends completely on care in handing meter) 0.01 to 0.02 mGal
	Accuracy	0.04 mGal or better
	Temp. Range	0° to + 45°C
	Absolute drift	< 0.1 mGal per month
		< 0.5 mGal per month after aging
Physical Dimension	Size and weight	19.7x17.8x25.1 cm,7.75x7.0x9.875 in
	Meter	7 lbs; 3.2 kg
	Battery	5 lbs; 2.3 kg
	Meter, battery and case	22 lbs; 10 kg

Table 2.2 Specification of Lacoste and Romberg gravimeter mode G

Scintrex gravimeter model CG-5 is shown in figure 2.6 and the specification is shown in table 2.3 as follows:

Figure 2.6 Relative gravimeter (Scintrex model CG-5)

Sensor type	Fused quartz using electrostatic nulling
Reading resolution	1 microGal
Standard field repeatability	< 5 microGal
Operating range	8,000 mGal without resetting
Residual long-term drift	Less than 0.02 mGal/day (static)
Automatic tilt compensation	±200 arc sec
Tares	Typically less than 5 microGals for shock up to 20G
Automated corrections	Tide, instrument tilt, temperature, drift, near terrain, noisy sample, seismic noise filter
Operating temperature	-40° to +45°C (-40°F to 113°F)
Ambient temperature coefficient	0.2 microGal/°C (typical)
Pressure coefficient	0.15 microGa/kPa (typical)
Magnetic field coefficient	1 microGal/Gauss (typical)
Memory	Flash technology (data security)
Dimensions	30(H) x 22 x 21 cm, 12(H) x 8.5 x 8 in
Weight (including batteries)	8 kg (17.5 lbs)

Table 2.3 Specification of Scintrex gravimeter mode CG-5

The sensing element of Scintrex CG-5gravimeter is based on a fused quartz elastic system. The gravitationa force on the proof mass is balanced by a spring and a relatively small electrostatic restoring force. The position of the mass, which is sensed by a capacitive dispacemnt transducer, is altered by a change in gravity. An automatic feedback circuit applies DC voltage to the capacitor plates producing an electrostatic force on the mass which brings it back to a null position. The feedback votage, which is a measure of the relative value of gravity is converted to a digital signal and then transmitted to the instrument's data acquisition system for processing, display on the screen and storage.

Because the gravity field of the earth is not constant, the equipotential surfaces of the earth are not parallel to each other and the sum of height differences in a closed loop does not equal to zero. Gravimetric corrections are needed and therefore a network of gravity points is set up by measuring the gravity differences between two points. Each point will be measured three times for one minute by the gravimeter called "Scintrex CG-5" but we can get only gravity differences from the instrument so at least one datum point with known absolute gravity value is required.

The profile of gravity measurement using Scintrex CG-5 to determine gravity differences is shown in figure 2.7 where each station is measured two times. The first gravity observation should be on the absolute gravity point

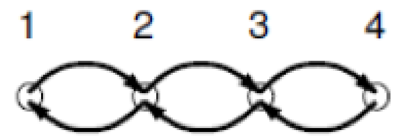

Figure 2.7 Gravimetry measurement profile

After the measurement in the field is finished, a tidal correction and air pressure correction are needed to correct to the observations.

Free Air Reduction

$$K_{FA} = -0.3086\left[\frac{mGal}{m}\right]\cdot\left(\Delta h - \Delta H\right) \tag{2.3}$$

where Δh is the difference between top edge of gravimeter and measuring system (0.211 m)

ΔH is the difference between top edge of gravimeter and the station

Air pressure correction

$$K_{PC} = 3.1\times10^{-4}\left(P_{meas} - P_{norm}\right) \tag{2.4}$$

where P_{meas} is the air pressure value at the measuring station

$$P_{norm} = 1013.25\left[mbar\right]\cdot\left(1-\frac{0.0065H}{288.15}\right)^{5.2559} \tag{2.5}$$

Tidal correction

Tide causes he changing of gravity during the day in a known way. Tidal correction can be computed precisely if the time is known and Scintrex CG-5 Autograv can automatically calculate and apply a real time tidal correction to each reading.

Correction of observed gravity value is

$$g_i^{'} = g_i + K_{FA} + K_{PC} \tag{2.6}$$

After performing the measurements, the preliminary gravity values can be calculated but those values contain the unknown offset as shown in equation below

$$y_n(t_k) = g_n + b + d \cdot t_k + \varepsilon \tag{2.7}$$

where g_n is gravity value at measured point

b is unknown bias or offset

d is drift which linear with time (Gal/h)

ε is noise in measurement

The bias (b) can be removed by subtracting the measured gravity value with another measured gravity at different station as follows:

$$\begin{aligned} \Delta y_n(t_k) &= y_n(t_k) - y_1(t_1) \\ &= g_n - g_1 + d(t_k - t_1) + \varepsilon \\ &= \Delta g_n + d \cdot \Delta t_k + \varepsilon \end{aligned} \tag{2.8}$$

According to the Linear observation model $\mathbf{y} = \mathbf{Ax} + \mathbf{e}$: adjusted gravity differences and drift value can be determined.

2.3 Geopotential Number

Geopotential number (C) is the potential energy difference between two different equipotential surface as shown in figure 2.8.

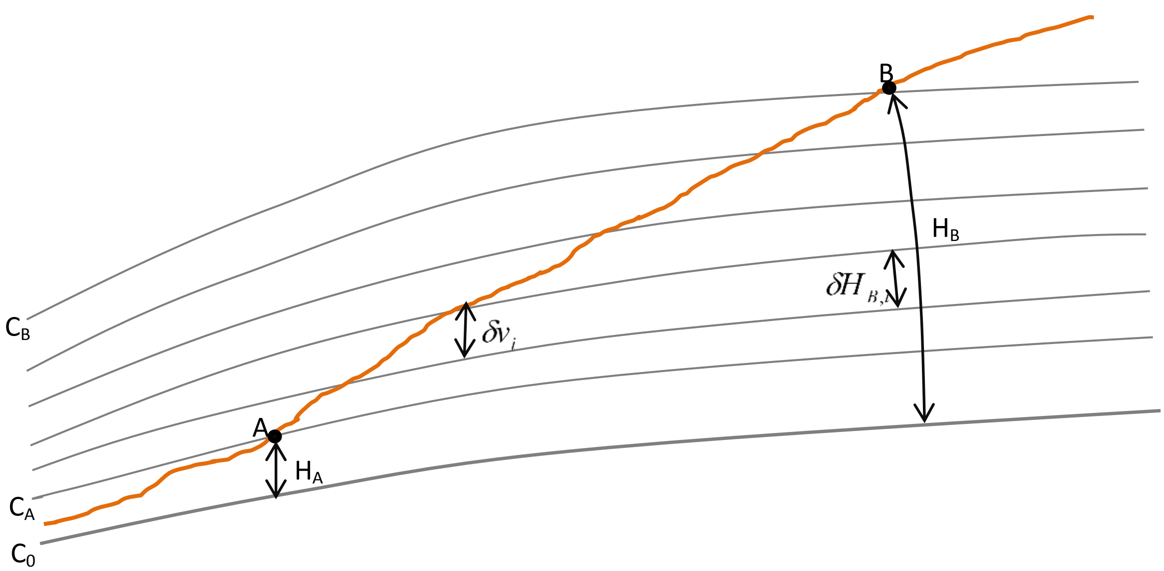

Figure 2.8 Comparison of differential leveling and orthometric height

The levelling line from point A to point B is shown in figure 2.8. The uncorrected height difference between point A and B can be determined by the sum of vertical geometric separation δvi , whereas the sum of vertical geometric separation of two equipotential surfaces along plumb line is $\delta H_{B,i}$. Because the equipotential surfaces are not parallel so $\delta v_i \neq \delta H_{B,i}$. The orthometric height at point B can be written as $H_B = \sum_i \delta H_{B,i}$ so it could be inferred that $H_B \neq \sum_i \delta v_i$. It is obvious that gravity plays an important role in height determination. By analogy we claimed that gravity force is the result of a change in gravity potential over a finite separation (Thomas H. Meyer et al, 2007) as equation below

$$g = -\frac{dW}{dH} \tag{2.9}$$

where g is the gravity force

W is the geopotential

H is the orthometric height

Equation (2.9) is rearranged as $dW = -gdH$, then

$$\int_{W_0}^{W_A} dW = -\int_0^{H_A} gdH$$

$$W_A - W_0 = -\int_0^{H_A} gdH$$

$$W_0 - W_A = \int_0^{H_A} gdH$$

$$C_A = \int_0^{H_A} gdH \tag{2.10}$$

If the right side of equation 2.4 is divided and multiplied by the total length of the plumb line, H_A, then we obtain

$$C_A = \bar{g}H_A \tag{2.11}$$

Equation (2.11) reveals that geopotential number equals average gravity along plumb line multiplied by orthometric heigh, in other words, geopotential number connected to heights by gravity value.

The unit of geopotential numbers is given in geopotential units (g.p.u.) where 1 g.p.u. = 1 kGal-meter = 1000 gal-meter. It can be seen that the unit of geopotentail number has a unit of energy.

2.4 Height Systems

Heights that are derived from conventional differential levelling by spirit levelling are not unique, that means the results from conventional differential leveling are pathdependent so a misclosure always occurs in closed loop. To overcome this problem other unique height systems are adopted in height computations by means of gravimetric levelling. There are several height systems that have been defined based on different datum. Those height systems have advantages and disadvantages regarding applications and calculations.

2.4.1 Dynamic height

Geopotential numbers are potential difference between geoid and the point of interest. Any point on the earth has a unique geopotential number. As the unit of the geopotentail number is the unit of energy (kGal-meter) so if it is divided by some gravity value, it could be transformed into a metric unit. Normal gravity (γ_0) is selected as a scale factor in this case and the result of scaled values of geopotentail numbers by a constant normal gravity is called "dynamic height" as in equation (2.12)

$$H^D = \frac{C}{\gamma_0} \tag{2.12}$$

where C is the geopotentail numbers

γ_0 is the constant normal gravity at some latitude

The scale factor that has been chosen as nominal value in equation (2.12) is normal gravity at latitude 45° which is equal to 9.806199203 m^2/s (Moritz, 1992)

Equation (2.12) reveals that dynamic height is a scaled value of the geopotential number into a metric unit so the fundamental properties of geopotentail number are not change. Dynamic heights produce equipotential surfaces and form closed levelling loops. Although dynamic height look like a height but they are not geometric at all, they represent the physical truth, i.e. water runs from high elevation to low elevation.

In summary, dynamic heights are scaled potential numbers in units of length (m). Equipotential surfaces are defined by dynamic heights, they are unique and produce a closed loop regarding to the theory.

2.4.2 Orthometric height

Orthometric heights are the heights above sea level so to say they are heights above geoid. The orthometric height is defined by The National Geodetic Survey (1986) as the distance between the geoid and a point of interest along plumb line. The plumb line is a line perpendicular to all equipotential surfaces of the Earth's gravity field that intersect with it. The plumb line is shown in figure 2.9

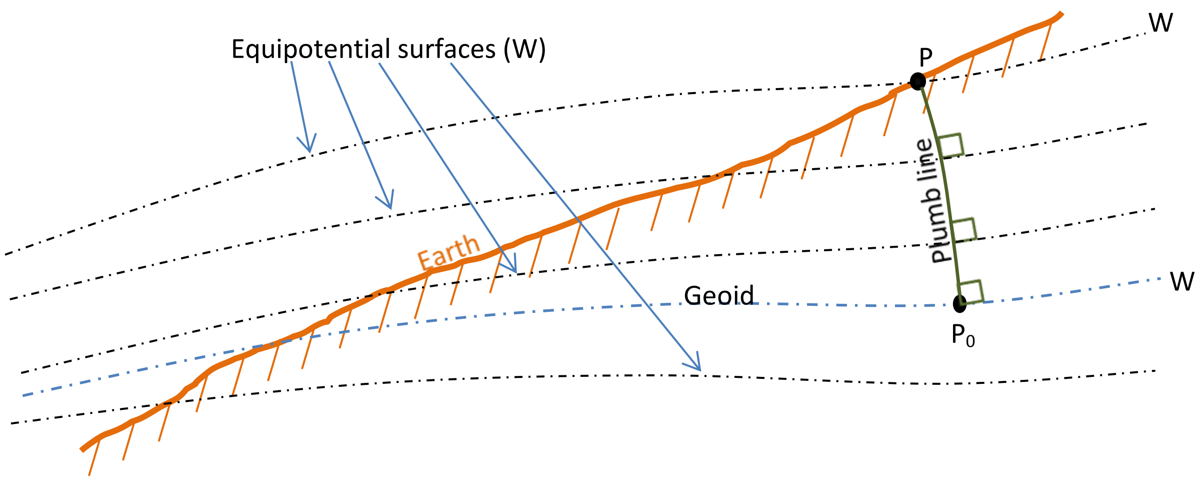

Figure 2.9 Illustration of orthomeric height and plumb line

Figure 2.8 shows the orthometric height at point P, it is clearly seen that orthometric height is measured from geoid and the relation between orthometric height and geopotential numbers shown in equation (2.5), thus, the definition of orthometric height can be written as

$$H^O = \frac{C}{\bar{g}} \tag{2.13}$$

where $\bar{g}$ is mean gravity along the plumb line between the measured point and geoid

From equation (2.8) we can see that the true orthometric height can be determined only if gravity inside the earth are known but in reality it is impossible to do so. The assumption of gravity inside the earth which is only valid for certain height differences is that the gravity value increases linearly with depth, so the average gravity is equal to the gravity at the mid-depth the average gravity along the plumb line can be obtained by adopting "Prey-reduction". The value of gravity inside the earth can be determined by first removing a Bouguer plate of constant densiy (ρ) of thickness $\frac{1}{2}H$, then a free-air reduction is applied for downward continuation and finally the Bouguer plate is restored again. The procedure of Prey-Reduction is as follows:

Density of the earth is constant, ρ = 2670 kg/m^3

Universal gravitational constant, G = 6.67 x 10^{-11} N · m^2/kg^2

$$\Delta g = 2\pi\rho g \tag{2.14}$$

Free are reduction, FA = 0.3086 mGal/m

Bouguer effect, BO = $2 \cdot \pi \cdot 2670 \cdot 6.67 \cdot 10^{-11} \cdot 10^5$ = 0.1118 mGal/m

Prey-reduction

$$\bar{g} = g_P - BO + \frac{1}{2} FA \tag{2.15}$$

Thus, equation (2.15) yields

$$\bar{g} = g_P + 0.0424H \tag{2.16}$$

The orthometric height that use mean gravity in equation (2.16) is called "Helmert height". Thus, the Helmert orhometric height is defined as follows:

$$H_P = \frac{C_P}{g_p + 0.0424 H_P} \tag{2.17}$$

where g_P is the gravity at the surface of the earth at the point of interest

The datum of Helmert heights is the Helmert geoid, it is an approximate value of the real geoid. In lowland the difference between real geoid and Helmert geoid is neglected but in the mountain area the difference could be a few centimeters.

In summary, the geoid is the datum of this height system. orthometric heights are unique as geopotential numbers, they are path independent, so closed levelling loops could be produced. They do not define equipotential surfaces due to the variable of gravity which may lead to the situation of water flowing uphill. It is not free from hypothesis on density distribution inside the Earth.

2.4.3 Normal height

In order to compute orthometric heights, the gravity inside the earth has to be known but it is impossible in reality because the distribution of density inside the Earth cannot be measured directly, therefore an assumption about the density inside the earth has to be made but it is unsatisfactory in theory especially in mountainous area.

Molodensky formulated a new height system, the height so called "normal height" which supposes that the gravity field of the Earth is normal, meaning the actual gravity potential is equal to the normal gravity potential (Molodensky, 1945). By this assumption, the physical surface of the Earth can be determined without the density of the Earth crust, only geodetic measurements are required (Heiskanen & Moritz,1967). Normal heights are defined by

$$H^N = \frac{C_P}{\bar{\gamma}} \tag{2.18}$$

where $\bar{\gamma}$ is the mean normal gravity along the plumb line between the measured point and the quasi-geoid

The datum of the normal height is the quasi-geoid. The shape of the quasi-geoid is similar to the shape of the geoid, but the quasi-geoid is not an equipotential surface of the normal or actual gravity field (Christopher Jekeli,2000).

Normal gravity can be calculated as

$$\gamma = \frac{a\gamma_{eq} \cdot \cos^2\varphi + b\gamma_{pl} \cdot \sin^2\varphi}{\sqrt{a^2 \cdot \cos^2\varphi + b^2 \cdot \sin^2\varphi}} \tag{2.19}$$

where γ is normal gravity

φ is geographic latitude

a is the semi major axis = 6378134 m

b is semi minor axis = 6356752.3141 m

γ_{eq} is normal gravity at equator = 9.7803267715 m/s^2

γ_{pl} is normal gravity at the earth pole = 9.8321863685 m/s^2

φ is average latitude in the test area = 48.485°

The assumption of the normal gravity value inside the Earth is the same as the gravity value that is it increases linearly with depth as gravity so the mean gravity along normal plumb line can be approximated by normal gravity value at the mid-depth. Mean normal gravity can be calculated as

$$\bar{\gamma} = \gamma\left(1-\left(1+f+m-2f\cdot\sin^2\varphi\right)\right)\cdot H^N \cdot a^{-1} + H^{N^2} \cdot a^{-2} \tag{2.20}$$

where $$f = \frac{a-b}{a} \tag{2.21}$$

m is the ratio between normal gravity and centrifugal acceleration at equator

m = 0.00344978600308

thus ,

$$H^N = \frac{C}{\gamma}\left[1+\left(1+f+m-2f\cdot\sin^2\varphi\right)\frac{C}{a\gamma}+\frac{C^2}{a^2\gamma^2}\right] \quad (2.22)$$

Normal heights depend on the selection of the reference ellipsoid and datum.

For summary, the normal height is the height from quasi-geoid, they are geometric distances. They are path independent as well as orthometric height so a closed loop could be produced. The assumption of Earth density and gravity measurement are not required for normal height computation.

2.4.4 Height reviews

Dynamic height is a scaled value of the geopotential number so there is no geometrical meaning. It indicates the direction of water flow, for instance, two points with the same values of dynamic height are on the same potential surface so the water will not flow from one to another point.

Orthometric and normal heights are measured from geoid and quasi-geoid, respectively, as shown in figure 2.1

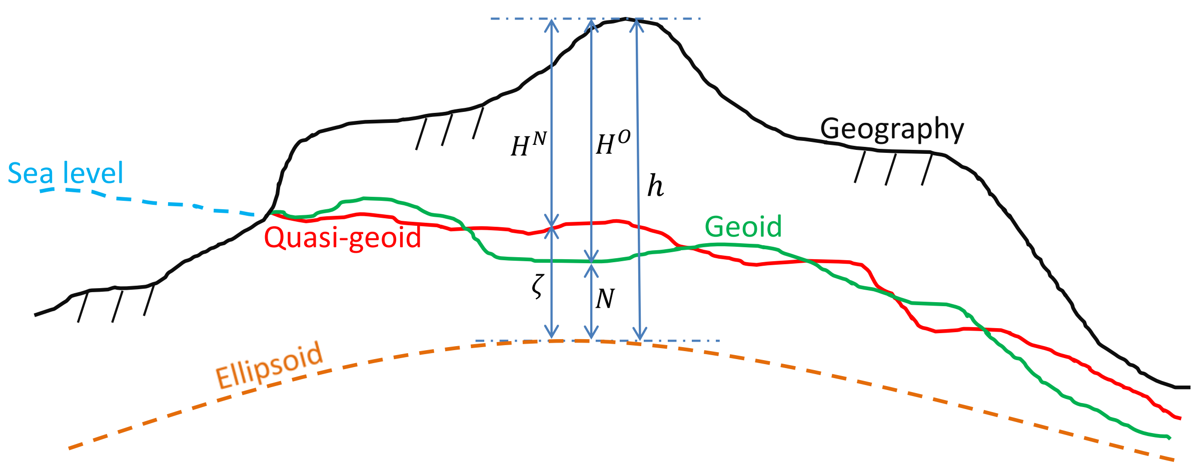

Figure 2.10 Orthometric height and normal height

$$h = H^O + N \quad (2.23)$$

$$h = H^N + \varsigma \quad (2.24)$$

where h is the ellipsoidal height

N is the geoid undulation

ς is the height anomaly

The disadvantage of orthometric and normal heights is that they are not indicate the direction of water flow although they have the same height values because they lie on different equipotenial surfaces.

If gravity is observed, dynamic heights, orthometric heights, and normal heights can be transformed from one to another because they all depend on the geopotential number as shown in equation below

$$C = \gamma_0 H^D = \bar{g} H^O = \bar{\gamma} H^N \tag{2.25}$$

2.5 Height Correction

Heights that are derived from spirit leveling are path dependent so they are not unique. Thus, a misclosure always occurs in closed surveying loops. To overcome this problem the height corrections as explained in section 2.4 have to be adopted in height calculations but computation of those height systems, except dynamic heights are quite complex and many parameters enter in the calculation. In surveying, the height differences (ΔH) are derived from spirit levelling and then the levelled height of the other points of interest can be computed as follows:

$$H_{i+1} = H_i + \Delta H_{i,i+1} \tag{2.26}$$

where H_{i+1} is the levelled height of next point

H_i is the levelled height of previous point

$\Delta H_{i,i+1}$ is the height difference between point i and $i+1$

The levelled height in (2.26) can be easily transformed into the other height systems by adding some correction, the so called “height correction” to the levelled height as follows:

$$H^{\#} = H^{lev} + Cor^{\#} \tag{2.27}$$

where $H^{\#}$ is the specified height system

H^{lev} is the levelled height

$Cor^{\#}$ is the height correction

The computation of each height correction can be determined by

- Dynamic correction

$$DC = \sum_i \frac{g_i - \gamma_0}{\gamma_0} \delta l_i \tag{2.28}$$

where g_i is the actual gravity value at the surface

γ_0 is the fixed normal gravity, in general the normal gravity at latitude 45°N

δl_i is the levelled height differences

- Orthometric correction

$$OC = \sum_i \frac{g_i - \gamma_0}{\gamma_0} \delta l_i + \frac{\bar{g}_P - \gamma_0}{\gamma_0} H_P - \frac{\bar{g}_Q - \gamma_0}{\gamma_0} H_Q \tag{2.29}$$

where $\bar{g}_P$ is the mean gravity along the plumb line at point P

H_P is the orthometric height at point P

$\bar{g}_Q$ is the mean gravity along the plumb line at point Q

H_Q is the orthometric height at point Q

- Normal correction

$$NC = \sum_i \frac{g_i - \gamma_0}{\gamma_0} \delta l_i + \frac{\bar{\gamma}_P - \gamma_0}{\gamma_0} H_P^N - \frac{\bar{\gamma}_Q - \gamma_0}{\gamma_0} H_Q^N \tag{2.30}$$

where $\bar{\gamma}_P$ is the normal gravity along the plumb line at point P

H_P^N is the normal height at point P

$\bar{\gamma}_Q$ is the normal gravity along the plumb line at point Q

H_Q^N is the normal height at point Q

2.6 Adjustment of observations

Making measurements is a fundamental part of the surveyors. The experience and skills of the surveyors as well as a good condition of the instruments are required for the observations. No matter how carefully made or how good the surveyors and instrument are, the observations are never exact and always contain errors.

By definition an error is the difference between an observed value and the true value, or

$$E = X - \bar{X} \tag{2.31}$$

where E is the error of the observation

X is the observed value from the field

$\bar{X}$ is the true value

It can be unconditionally stated that no observation is exact and always contains errors, the true value is never known which means the exact error is always unknown. The better the accurate reading of the observation, the closer the true value of the observation but it is never exact.

2.6.1 Type of errors

There are two types of errors in observations: systematic and random

- *Systematic errors* are biases of the measurement, they cause the mean of the measurements to differ significantly from the true value. The source of systematic errors are the measuring system or the conditions during the observation, which are the environment, instrument, and observer. As the system or the condition is constant, the systematic errors remain constant. On the other hand, if the conditions change, the magnitude of the systematic errors also change. Because systematic errors tend to accumulate, they are called “cumulative errors”. Systematic errors cannot be eliminated by repeating the measurements or averaging of the results. The conditions producing systematic errors conform to physical laws can be modeled mathematically (Paul R. Wolf, 2002). Thus if the conditions can be observed and are known then the correction can be made and applied to the observation values.

- *Random error*. If mistakes and systematic errors are eliminated the errors that remain in the measurement are called “random error”. They are caused by factors beyond the control of the observer. Sometimes they are called “accidental error”. It is impossible to compute or eliminate them but they can be estimated by an adjustment procedure called “least square adjustment”

2.6.2 Least squares adjustment

Since mistakes are eliminated and corrections for systematic errors are made, the random errors still remain. The presence of remaining random errors will be in the form of misclosure. For instance, sums of interior angles of a polygon is not equal to $(n-2)180°$. To account for these misclosures, adjustment has to be applied to produce mathematically perfect geometric conditions. There are so many techniques which used in the adjustment, the most widely used adjustment are made by "Least square" which is based on the laws of probability.

The advantage of least square adjustment over other arbitrary methods is that it is based upon the mathematical theory of probability. It enables all observations to be simultaneously included in the adjustment, and each observation can be weighted according to its estimated precision. Furthermore, least squares are applicable to any measurement problem regardless of its nature or geometric configuration.

The method of least square adjustment is derived from the equation of the normal distribution curve equation. It produces that unique set of residuals for a group of observations that have the highest probability of occurance (Paul R. Wolf, 2002). For observations which contain the same weight, the condition enforced by least square is that the sum of the squared residuals is minimum. For example the number of observation which has equal weight is m having residuals as $v_1, v_2, v_3, \dots, v_m$. Then the condition of unweighted least squares is

$$\sum_{i=1}^{m} v_i^2 = v_1^2 + v_2^2 + v_3^2 + \cdots + v_m^2 = minimum \qquad (2.32)$$

In some observations, a weight (w) may be assigned to the observation individually. The weight is obtained from the variance (σ^2) of the measurements if it is available. For the case of levelling, the weight can be obtained from the distances between the observation points as follows:

$$w_i = \frac{1}{\sigma_i^2} \qquad (2.33)$$

Consider the equation above, the weights are inversely proportional to the variances.

In case of the weights are assigned to the observations, then the fundamental condition of weighted least squares is

$$\sum_{i=1}^{m} w_i v_i^2 = w_1 v_1^2 + w_2 v_2^2 + w_3 v_3^2 + \cdots + w_m v_m^2 = minimum \qquad (2.34)$$

Because there are so many observations least square adjustment can be quite large so it is convenient to adopt a matrix method or linear algebra in least square adjustments so it can be solved on a computer easily. The group of observation equations can be represented in the matrix form as

$$\mathbf{Ax} = \mathbf{y} + \mathbf{e} \tag{2.35}$$

Where $\mathbf{A}$ is the design matrix or coefficient matrix of the unknowns

$\mathbf{x}$ is the unknown matrix

$\mathbf{y}$ is the observation matrix

$\mathbf{e}$ is the residual matrix

The detailed structure of these matrices are

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}, \quad \mathbf{e} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{bmatrix}$$

The normal equations for equally weighted observation equations are given in matrix form as

$$\mathbf{A^T A x} = \mathbf{A^T y} \tag{2.36}$$

If the equation (2.36) is multiplied by $(\mathbf{A^T A})^{-1}$ at both sides, then $\mathbf{x}$ can be computed as follows:

$$\hat{\mathbf{x}} = (\mathbf{A^T A})^{-1}\mathbf{A^T y} \tag{2.37}$$

Equation (2.37) is called “least square solution”

In case of weighted observations, the following equation is adopted for the computation of $\mathbf{x}$

$$\hat{\mathbf{x}} = (\mathbf{A^T A})^{-1}\mathbf{A^T y} \tag{2.38}$$

In equation (2.38) the matrices are identical to the equally weighted case, but the weight matrix $(\mathbf{W})$ is a diagonal matrix of weights defined as follows:

$$\mathbf{W} = \begin{bmatrix} w_1 & & & \\ & w_2 & & \text{zeros} \\ \text{zeros} & & \ddots & \\ & & & w_n \end{bmatrix}$$

Or in general: $\mathbf{W} = \mathbf{D}^{-1}$

with $\mathbf{D}$ dispersion matrix or variance-covariance matrix

Equation (2.38) is the general form of the least squares solution, in case the observations in an adjustment are al of equal weight, the weight matrix becomes an identity matrix ($\mathbf{I}$).

Observation equation (A model)

The adjustment of observations can be done by least square adjustment as follows:

$$\hat{\mathbf{x}} = (\mathbf{A}^{\mathbf{T}}\mathbf{W}\mathbf{A})^{-\mathbf{1}}\mathbf{A}^{\mathbf{T}}\mathbf{W}\mathbf{y} \quad (2.39)$$

$$\hat{\mathbf{y}} = \mathbf{A}\hat{\mathbf{x}} \quad (2.40)$$

where $\hat{\mathbf{x}}$ is the estimated vector of unknowns

$\hat{\mathbf{y}}$ is the vector of adjusted observations

Condition equation (B model)

In the condition equation, the vector of residual ($\mathbf{e}$) is unknown. The aim of the adjustment by condition equation is to find the smallest possible value of residual ($\hat{\mathbf{e}}$) that satisfy the condition below

$$\mathbf{w} := \mathbf{B}^{\mathbf{T}}\mathbf{y} = \mathbf{B}^{\mathbf{T}}\hat{\mathbf{e}} \quad (2.41)$$

where $\mathbf{w}$ is the misclosure vector

$\mathbf{B}^{\mathbf{T}}$ is the coefficient matrix of condition

$\hat{\mathbf{e}}$ is the possible value of the residual vector

According to equation (2.41), the vector of unknown ($\mathbf{x}$) is not involved in the condition equation.

The possible value of the residual vector can be calculated as follows:

$$\hat{\mathbf{e}} = \mathbf{W}^{-\mathbf{1}}\mathbf{B}^{\mathbf{T}}(\mathbf{B}\mathbf{W}^{-\mathbf{1}}\mathbf{B}^{\mathbf{T}})^{-\mathbf{1}}\mathbf{B}\mathbf{y} \quad (2.42)$$

Since $\hat{e}$ is known, then the adjusted observations can be determined by

$$\hat{\mathbf{y}} = \mathbf{y} - \hat{\mathbf{e}} \quad (2.43)$$

2.6.3 Setting up of observation and condition equation

The observation and condition equation in the example as shown in figure 2.11, table 2.4a, 2.4b, and 2.4c could be done as follows:

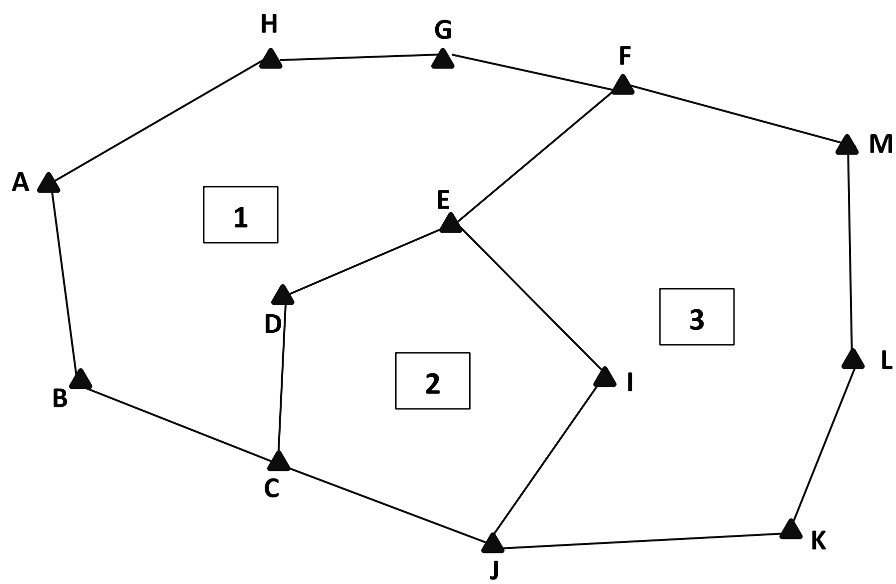

Figure 2.11 Example of closed loop

Point ID	Geopotential differences :ΔC (m^2/s^2)
A	
B	10.125
C	5.139
D	-7.249
E	5.142
F	8.479
G	-15.147
H	6.459
A	-12.945

Table 2.4a Geopotential differences of loop 1

Point ID	Geopotential differences :ΔC (m^2/s^2)
C	
D	-7.249
E	5.142
I	10.159
J	-25.137
C	17.083

Table 2.4b Geopotential differences of loop 2

Point ID	Geopotential differences :ΔC (m^2/s^2)
E	
I	10.159
J	-25.137
K	20.159
L	5.478
M	-15.128
F	12.943
E	-8.479

Table 2.4c Geopotential differences of loop 3

According the data in table 2.4a to 2.4c, the observation and condition equation could be set up as follows:

The unknown matrix

$$\mathbf{x} = \left[D_{AB} \quad D_{BC} \quad D_{CD} \quad D_{DE} \quad D_{EF} \quad D_{FG} \quad D_{GH} \quad D_{HA} \quad D_{CJ} \quad D_{JI} \quad D_{IE} \quad D_{JK} \quad D_{KL} \quad D_{LM} \quad D_{MF} \right]^T$$

where D_{ij} is the geopotential difference between point i and j

The observation matrix

$$\mathbf{y} = [10.125 \quad 5.193 \quad -7.249 \quad 5.124 \quad 8.479 \quad -15.147 \quad 6.495 \quad -12.945 \\ -17.083 \quad 25.137 \quad -10.159 \quad 20.159 \quad 5.478 \quad -15.128 \quad 12.943]^T$$

Setting up of condition equation (B matrix)

$$\mathbf{B} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Setting up of observation equation (A matrix)

$$\mathbf{A} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

After the observation and condition equation were set up, then the observations could be adjusted by the least squares adjustment as explained in section 2.6.2.

Chapter 3

Methodology

3.1 Introduction

The integrated fieldwork has been done since 1996, so many data have been collected. Those data are available in hard copies and soft copies and there are also several points that have been located in the field since that time. At the first step, those data have to be checked and it has to be made sure that the points that have the same point names in one year are the same points measured in the other years. Then the data collection as well as the correction of the data will be done.

3.2 Workflow of the project

The workflow of this thesis is shown in the diagram below.

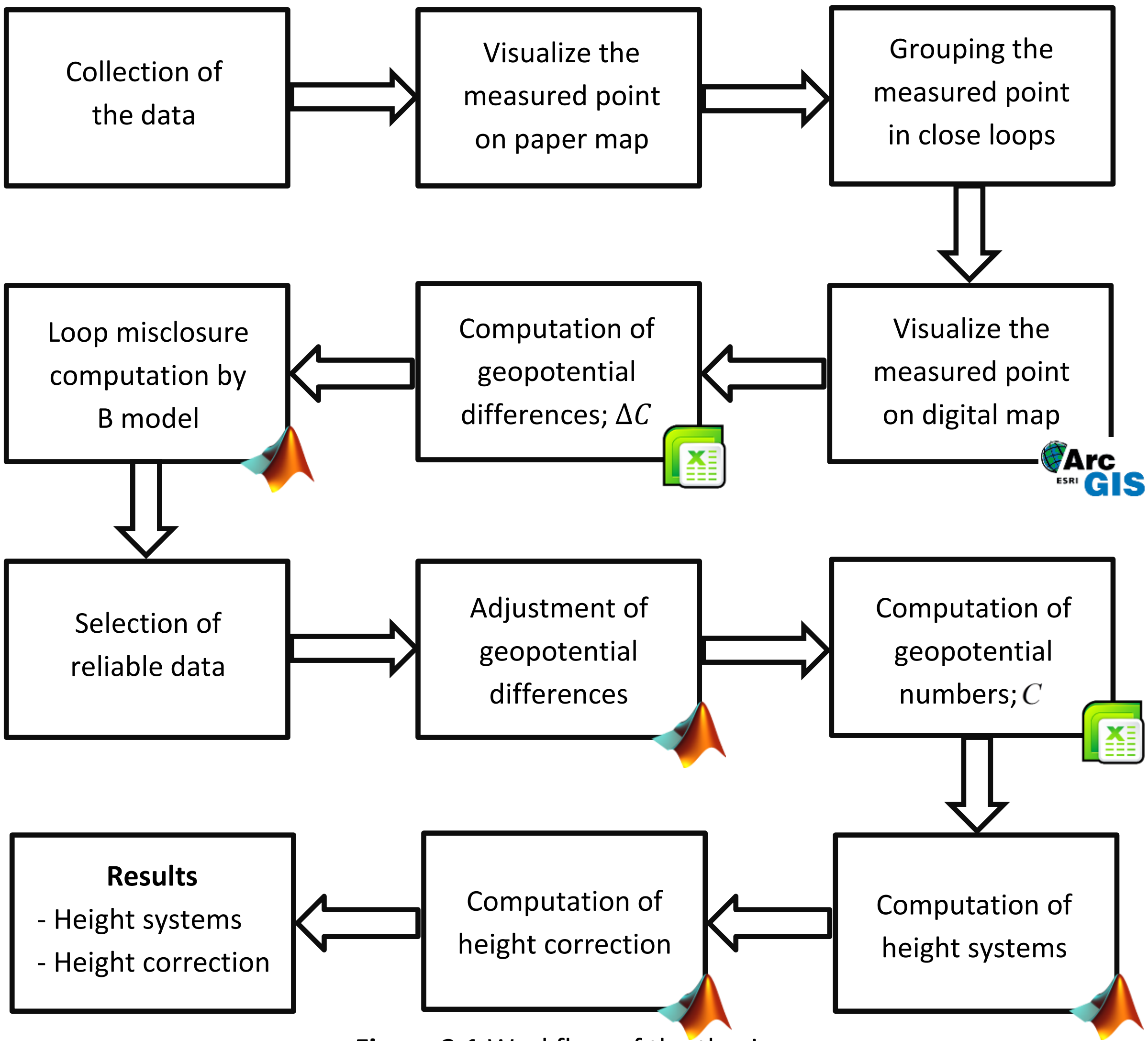

Figure 3.1 Workflow of the thesis

Softwares that have been used for data collection, data adjustment, height computations, and height correction are shown as follows:

Symbol	Software name	Application
	Microsoft Excel 2010	Data collection and computations
	Matlab 2012	Least squares adjustment and data plotting
	ArcGIS10.1	Visualization of the measured points on digital map

Table 3.1 Explanation of software symbols

According to figure 3.1, the workflow can be explained as follows;

(1) Collection of the data that have been observed at Swabian Alb test area from intergrated fieldworks since 1996 to 2013. The observed points have to be checked to make sure that the points which have the same point id. are the same points that have been measured in the other years. In some cases, the point id. were changed in the other years so they have to be checked as well, by consideration of their height differences and gravity values.

(2) After the data have been identified, the visualization of those points will be made by drawing them on a paper map as shown in figure 3.2, then the surveying paths of each years will be drawn on transparency paper as shown in figure 3.3.

(3) The surveying paths of each year can be seen on the paper map in (2), then the closed loops can be identified.

(4) Visualization of the observed points and surveying paths of each year as well as closed loops from (2) and (3) in digital form, software called "ArcGis" version 10 is adopted for the visualization.

(5) Geopotential differences (ΔC) are computed on Microsoft excel.

(6) Computation of each loop misclosure by observation equation (B-Model) on Matlab software.

(7) Selection of reliable data by comparing the loop misclosures in (6) with allowable loop misclosures. The data which make the loop misclosures exceed the allowable values have to be neglected and remove from the adjustment.

(8) Adjustment of geopotential differences by observation equation (A-Model) and condition equation (B-Model). Unweighted and weighted adjustments have to be done for both equations. The results from both models, unweighted and weighted have to be similar to make sure that the adjustment is quite correct. The adjustment is done on Matlab software. Then the sum of adjusted potential differences in each loops has to be checked, it has to be equal to zero.

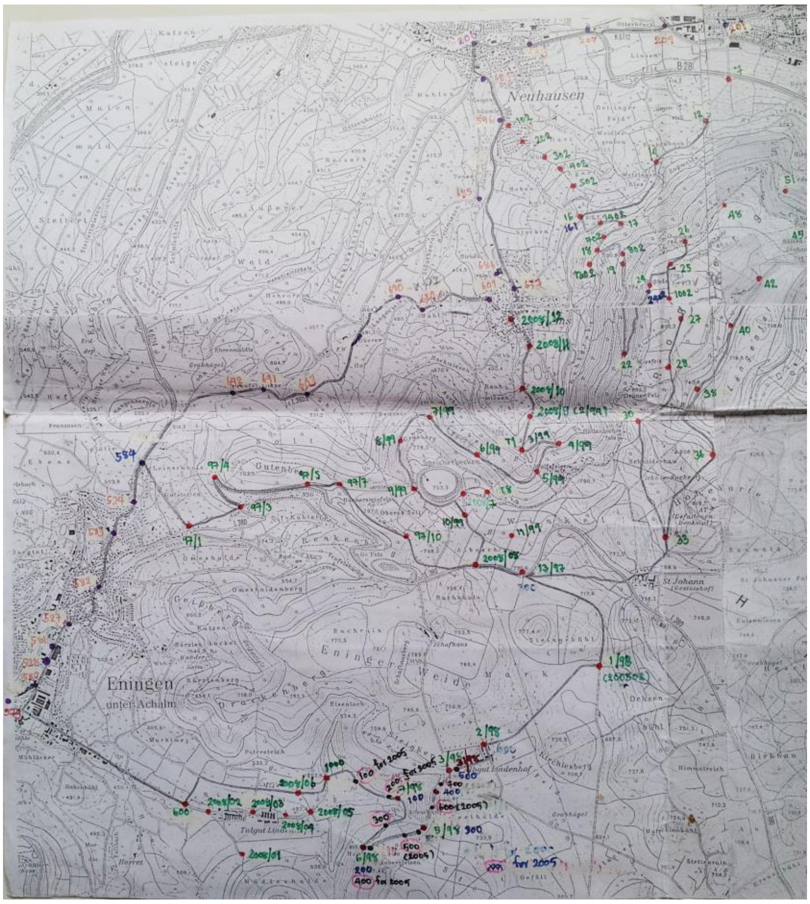

Figure 3.2 Visualization of observed points on paper map

(9) Computation of potential numbers (C) at each measured points from adjusted geopotential differences on Microsoft Excel.

(10) Computation of height systems; dynamic heights, orthometric heights, and normal heights as explained in section 2.4.

(11) Computation of height corrections; dynamic correction, orthometric correction, and normal correction as described in section 2.5.

(12) Summary of the results; height systems and height corrections.

3.3 Weights of the data

The errors of the measurement depend on the distances, it means the longer the distance, the higher the error so the weight matrix has to be adopted in the adjustment computation. The weight is inverse proportional to variances as explained in section 2.6. The variances of the observation can be computed from the distance between two points as follows;

$$\sigma^2 = \sqrt{K} \tag{3.1}$$

where σ^2 is the variances of the observations

K is the distance between two points (km)

Then, the weight of the observation is

$$w_i = \frac{1}{\sigma_i^2} \tag{3.2}$$

3.4 Computation of the geopotential difference

Geopotential difference can be computed by

$$\Delta C = \frac{1}{2}(g_{i+1} + g_i)\Delta H_{i+1,i} \tag{3.3}$$

where g_{i+1} is the gravity value at the next point

g_i is the gravity value at start point

$\Delta H_{i+1,i}$ is the height difference by spirit levelling between start point and the next point

3.5 Loop misclosures

Loop misclosures of geopotential difference of each loops can be computed by the condition equation (B-Model) by adjustment theory as

$$\mathbf{v} = \mathbf{B}^{\mathbf{T}}\mathbf{y} \tag{3.4}$$

where $\mathbf{e}$ is the loop misclosure

$\mathbf{B}^{\mathbf{T}}$ is the condition equation matrix

$\mathbf{y}$ is the observation data (geopotential differences)

3.6 Allowable loop misclosure

Since the loop misclosures $(\mathbf{e})$ are produced from non-adjusted levelled height differences the loop misclosures have to be compared allowable misclosure (e_{all}). The allowable loop misclosure is based on the order of leveling, in this case, precise order of leveling is adopted as follows,

$$e_{\text{all}} = 1\text{mm}\sqrt{K} \tag{3.5}$$

where e_{all} is the allowable loop misclosure (mm)

K is the total distance of the close loop (km)

Then the observed loop misclosure has to be compared to the allowable loop misclosure, in case the observed misclosure is larger than the allowable loop misclosure $(e > e_{\text{all}})$, the data in the loop have to be rejected. On the other hand, if the observed misclosure is smaller than the allowable loop misclosure $(e < e_{\text{all}})$, those data are considered as reliable data and the adjustment will be done.

Due to the height differences acquired by spirit levelling are not unique, they are path dependent unlike geopotential differences so the latter will be used to compute the loop misclosure instead of height differences but the unit of geopotentail differences are not in metric so they have to be divided by the normal gravity value at the latitude the test area which is 48.485°. The value of mentioned normal gravity value is 9.806199203 m/s^2, finally geopotential differences are in the unit of meter and then they could be compared with the allowable loop misclosure.

3.7 Geopotential number

Computation of height systems requires geopotential numbers, in case the surface gravity value at the point of interest is known, the geopotential numbers can be calculated from Helmert orthometric height, thus one specified Helmert orthometric height has to be known.

Point 580 is the reference point for orthometric height, the height value is given by the State Mapping Agency of Baden-Wurttemberg, then geopotential numbers can be determined.

Helmert orthometric height

$$H_P = \frac{C_P}{g_P + 0.0424 \cdot 10^{-5} H_P} \tag{3.6a}$$

then C_P can be computed by arranging the equation above

$$C_P = H_P \left(g_P + 0.0424 \cdot 10^{-5} H_P \right) \tag{3.6b}$$

$$C_P = 0.0424 \cdot 10^{-5} H_P^2 + H_P g_P \tag{3.6c}$$

where H_P is the orthometric height of point 580 = 435.17856 m

g_P is the gravity at the surface of the earth at point 580 = 9.80814029 m^2/s

According to equation (3.4b), the geopotential number at point 580 is 4268.3004 m^2/s^2

Since the geopotential number at point 580 is the start point, the geopotential numbers at every point in each loop can be determined by adding the geopotential number with adjusted geopotential differences as follows;

$$C_{n+1} = C_n + \Delta C_{n,n+1} \tag{3.7}$$

where C_{n+1} is the geopotential number of the next point

C_n is the geopotential number at the start point

$\Delta C_{n,n+1}$ is the geopotential difference between start point and the next point

3.8 Data collection

Several points have been observed in the test field since 1996 to 2013. These observed points and their paths are illustrated in a paper map as shown in figure 3.3

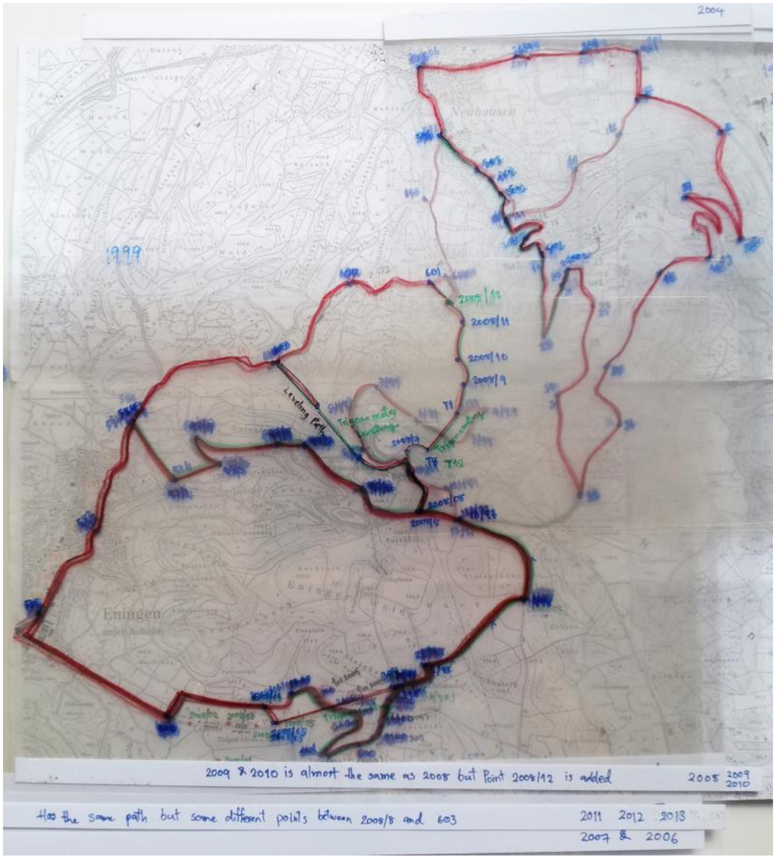

Figure 3.3 Surveying paths since 1996 to 2013 on a paper map

Then the observed points and their paths were illustrated in a digital map via ArcGis10.1 as shown in figure 3.4

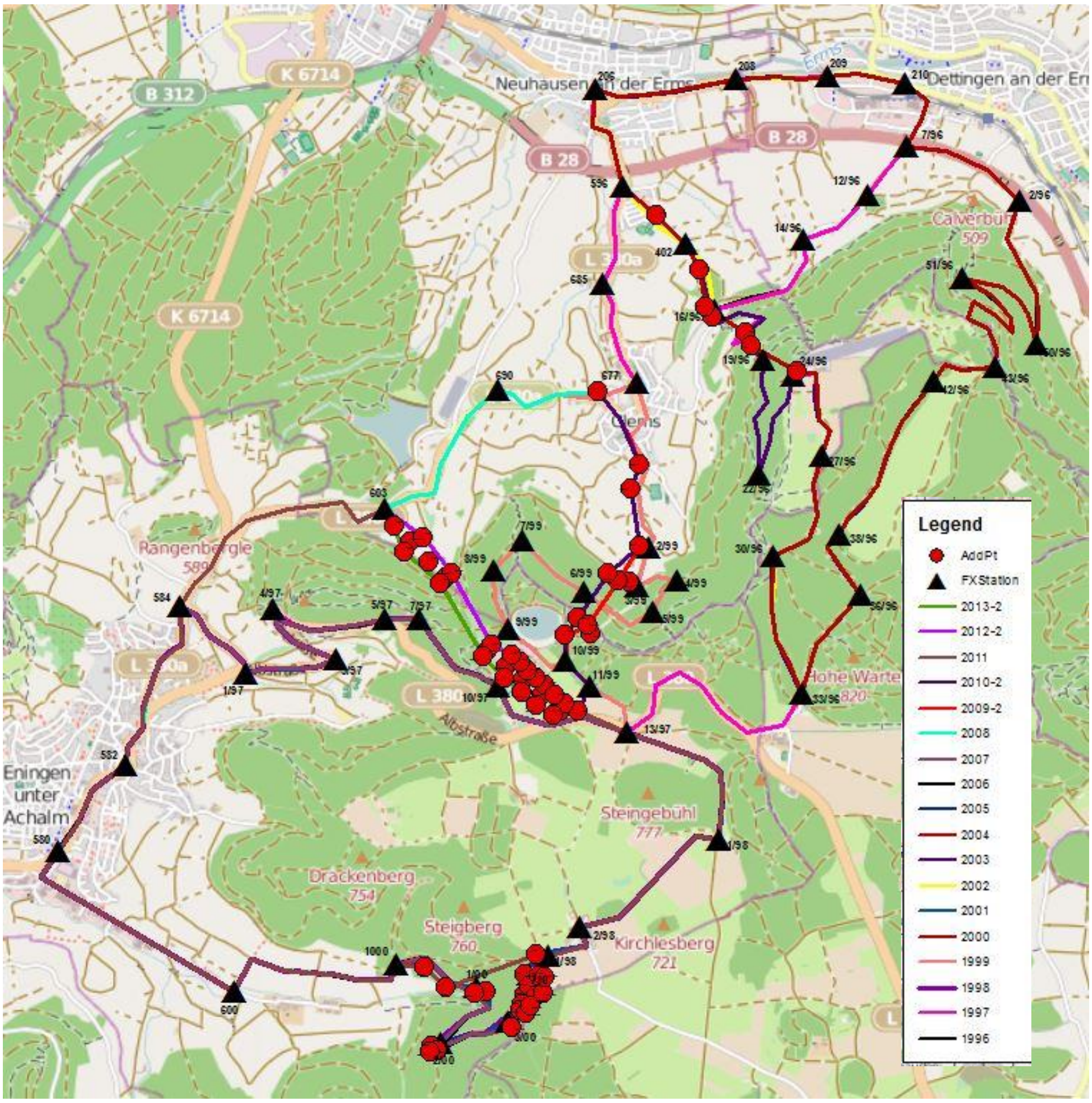

Figure 3.4 Illustration of surveying paths since 1996 to 2013 in the digital map

Figure 3.4 illustrates the observed points and the surveying paths that had been observed since 1996 to 2013. There are 121 points and 18 paths that had been observed. In some years, the same points were measured to complete a closed loop and in some years, new points were located. After the data checking and evaluating, the black triangles in figure 2.3 indicate the points that were selected for adjustment and height systems computation, the red circles indicate the points that were not taken into account in the calculation because they were not related to the other years as shown in figure 3.5

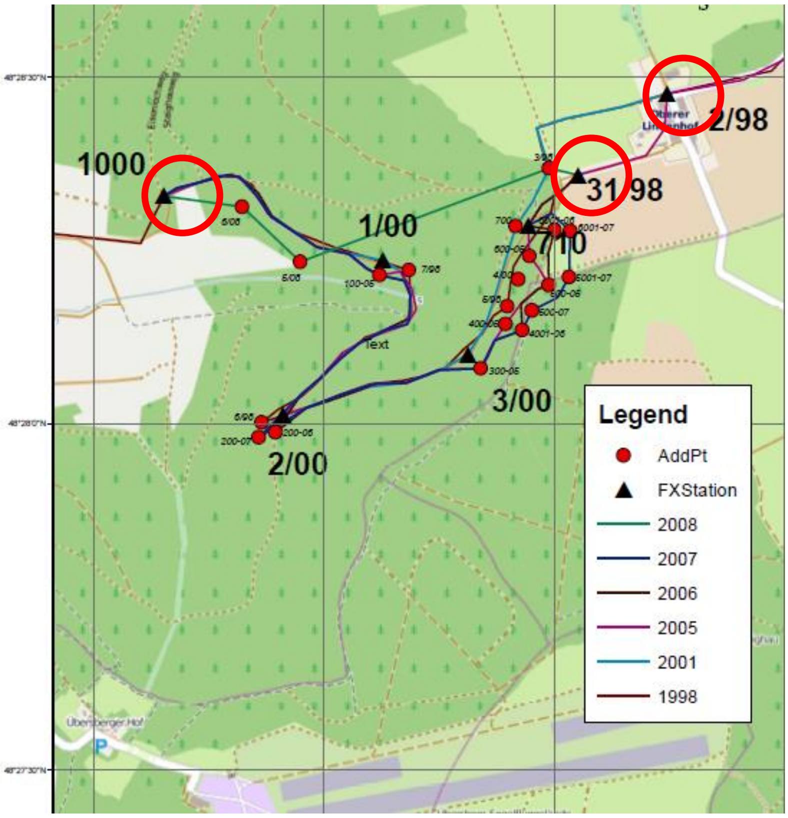

Figure 3.5 Uncorrelated points

Figure 3.5 shows that the surveying paths in 1998, 2001, 2005, 2006, 2007, and 2008 were done in the same paths but the points were located in different places. To combine those observations into the adjustment and height computation, the related points have to be specified. The raw data show that the surveying paths start from point 1000 and end at point 31/98 or 2/98 so these points are considered as junction points for combining the observation data (height differences) in 1998, 2001, 2005, 2006, 2007, and 2008 to the other years by determining the height differences between start point and end point as illustrated in figure 3.5

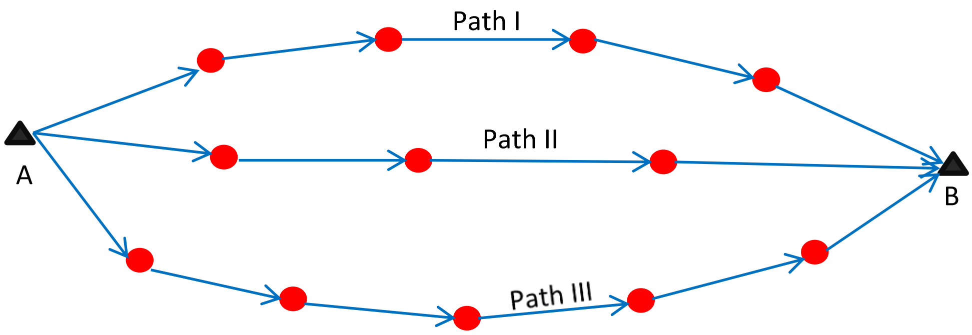

Figure 3.6 Combination of observation data

Figure 3.6 shows that the levelled height differences between point A and B can be computed from the path I, II, or III. Although the height difference from spirit leveling are path dependent but precise leveling was adopted in the data acquisition so the differences between each paths are in the range of millimeters. Moreover, the difference height differences give different geopotential differences which causes the inconsistency case in the adjustment.

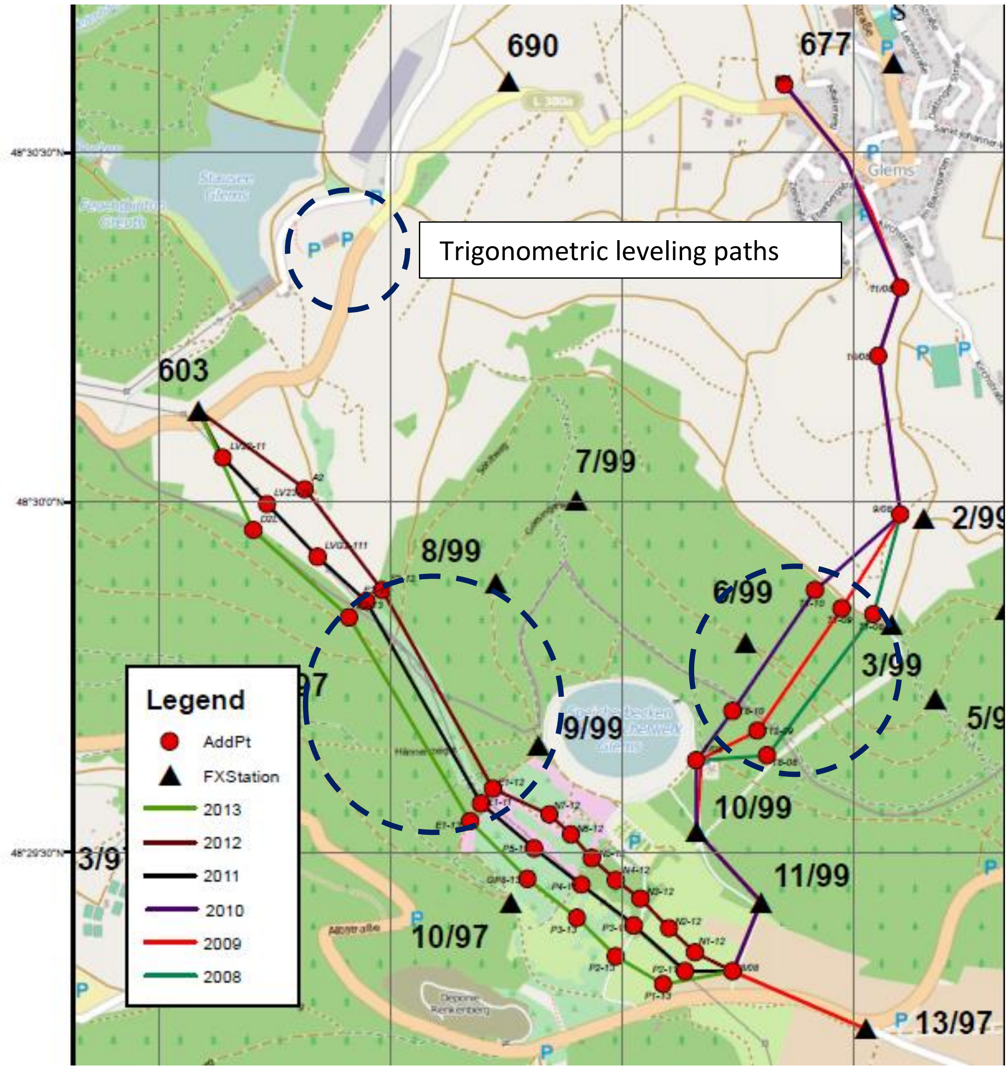

Figure 3.7 The years that trigonometric levelling was included

Because the height differences determined by trigonometric leveling are not accurate enough, this data have to be neglected. The years that trigonometric leveling were included are 2008 to 2013 as shown in figure 3.7, thus, the data in 2008 to 2013 were not take into account in height system computation.

3.9 Generation of close loops

According to figure 3.3, the surveying paths since 1996 to 2013 could generate several closed loops but since the data in 2008 to 2013 were neglected, five closed loops could be drawn as shown in figure 3.8 and figure 3.9

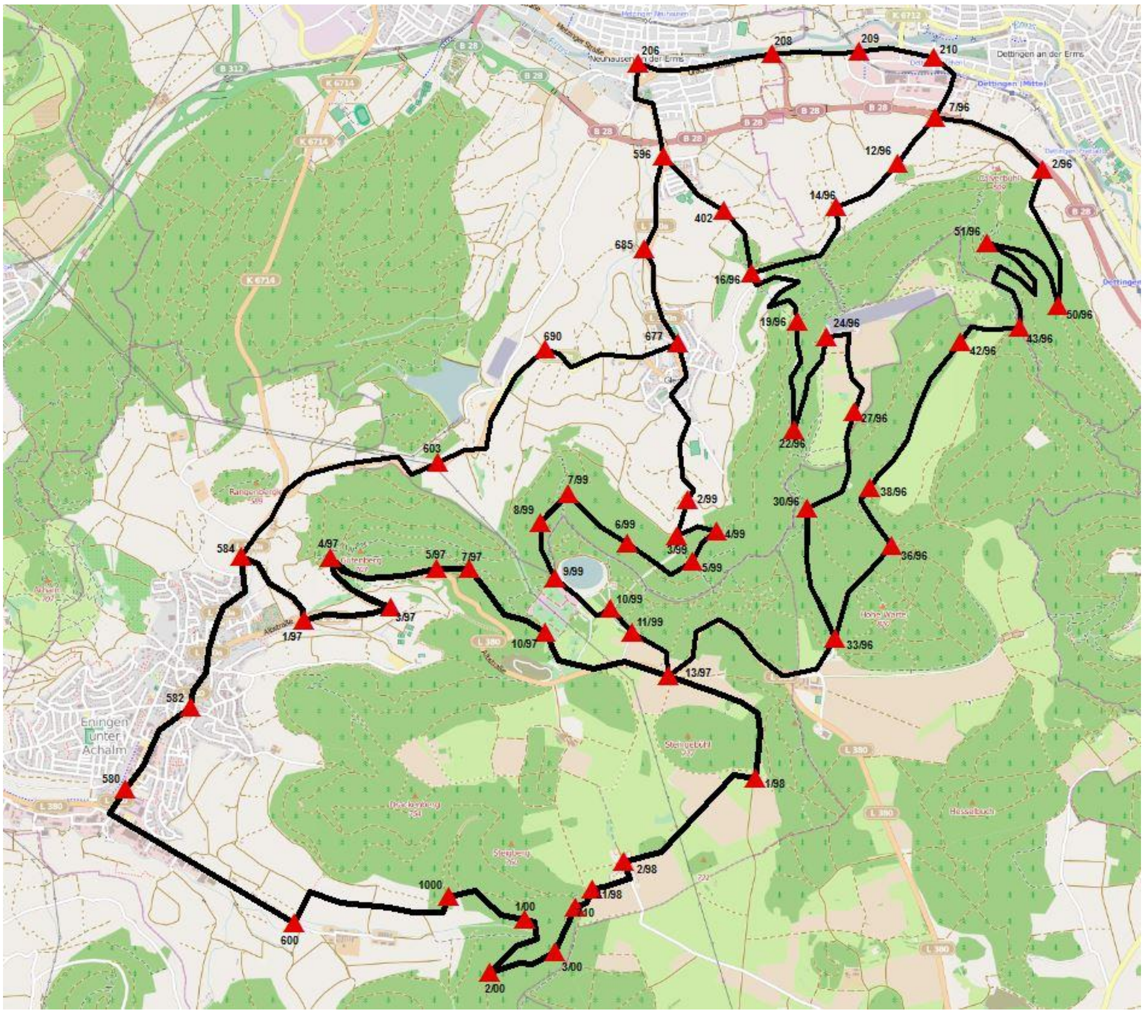

Figure 3.8 Close loops that generated from surveying paths

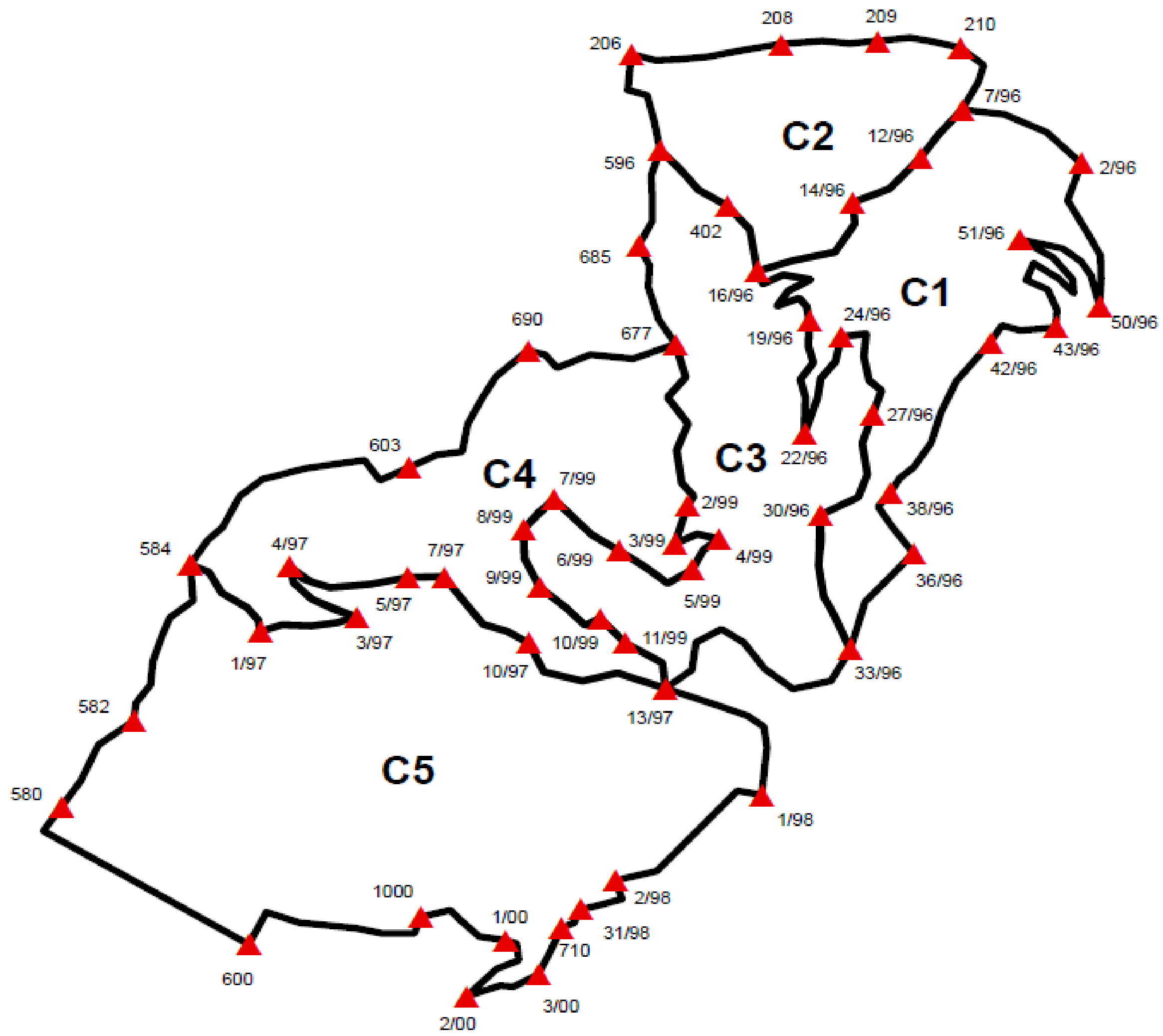

Figure 3.9 Point id in each loops

As can be seen from figure 3.9, there are 5 loops of surveying paths, 56 points were selected as fixed points in the adjustments. The summary of the loops and point id are shown in table 3.2

Loop Number	Number of point id
Loop 1	17
Loop 2	10
Loop 3	22
Loop 4	21
Loop 5	19

Table 3.2 Number of point id in each loop

The geopotential difference between the adjacent points can be determined as explained in section 3.4, then loop misclosures can also be determined by condition equation (B-model) or by the summation of geopotential differences of each loops.

Figure 3.9 to figure 3.13 illustrate the closed loops individually as well as the direction of the surveying used in the least squares adjustment

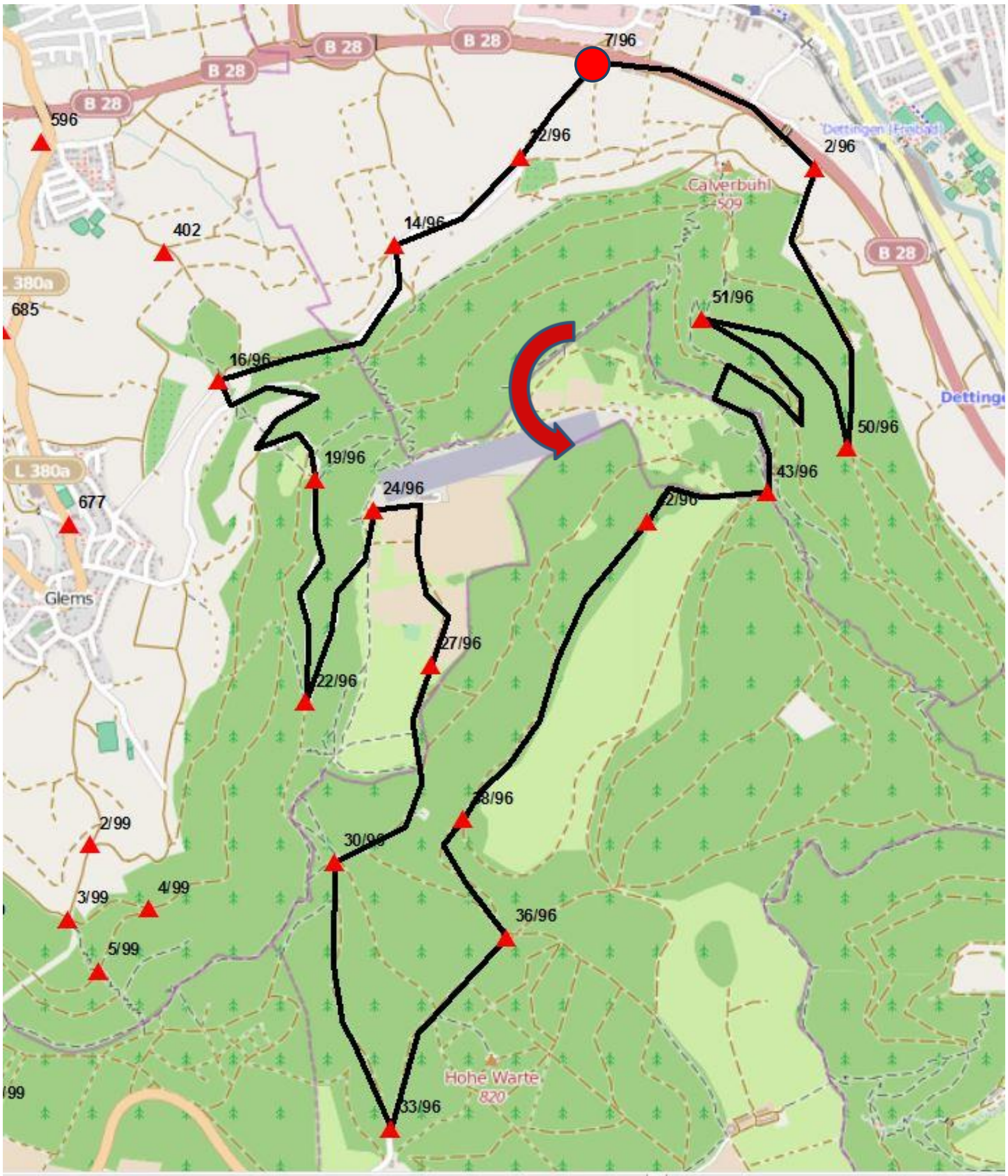

Figure 3.10 Loop 1

There are 17 observed points in loop 1. The red solid circle indicates the start point of the surveying path and the curved arrow indicates the direction of the surveying path, thus, in figure 3.10, the loop started at point 7/96 and ran in anti-clockwise direction. So the surveying path ran from point 7/96 - 12/96 - 14/96 - 16/96 - 19/96 - 22/96 - 24/96 - 27/96 - 30/96 - 33/96 - 36/96 - 38/96 - 42/96 - 43/96 - 51/96 - 50/96 - 2/96 back to 7/96

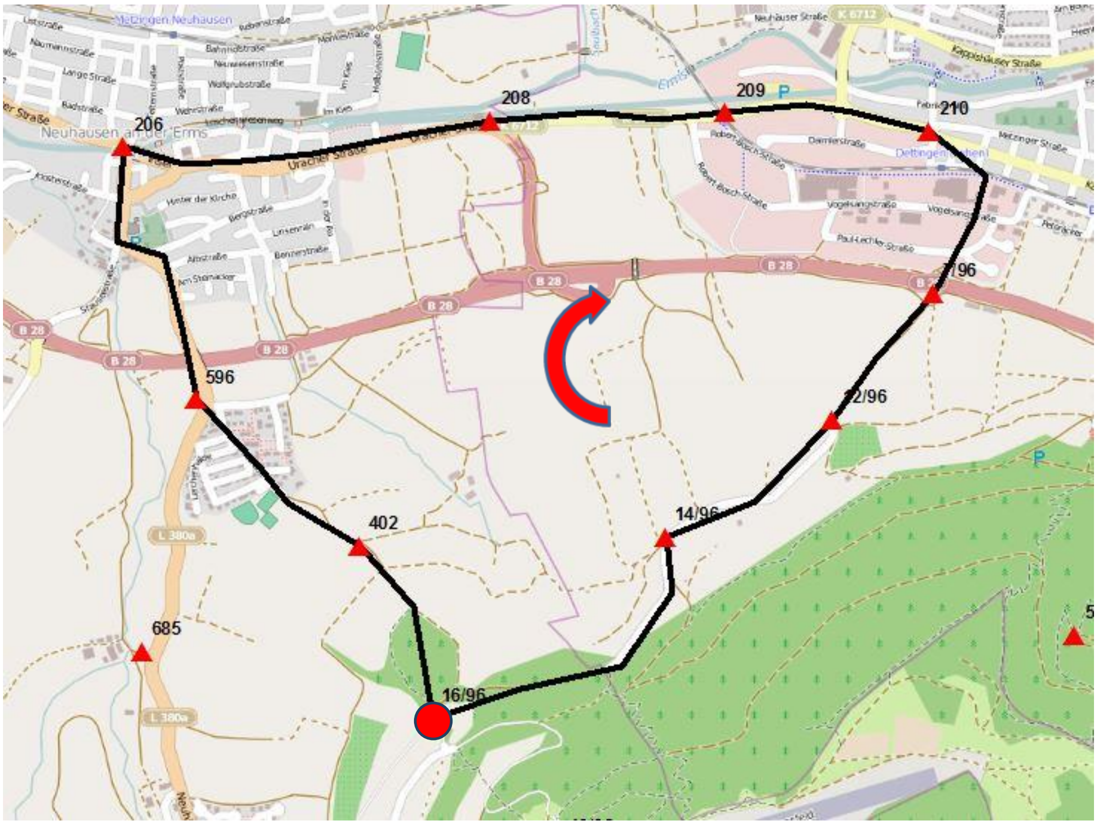

Figure 3.11 Loop 2

There are 10 observed points in loop 2. The start point of this loop is point number 16/96 (point number 16 in year 1996). The direction of the surveying path is counter clockwise. The surveying path started at point 16/96 - 402 - 596 - 206 - 208 - 209 - 21 - 7/96 - 12/96 - 14/96 back to 16/96

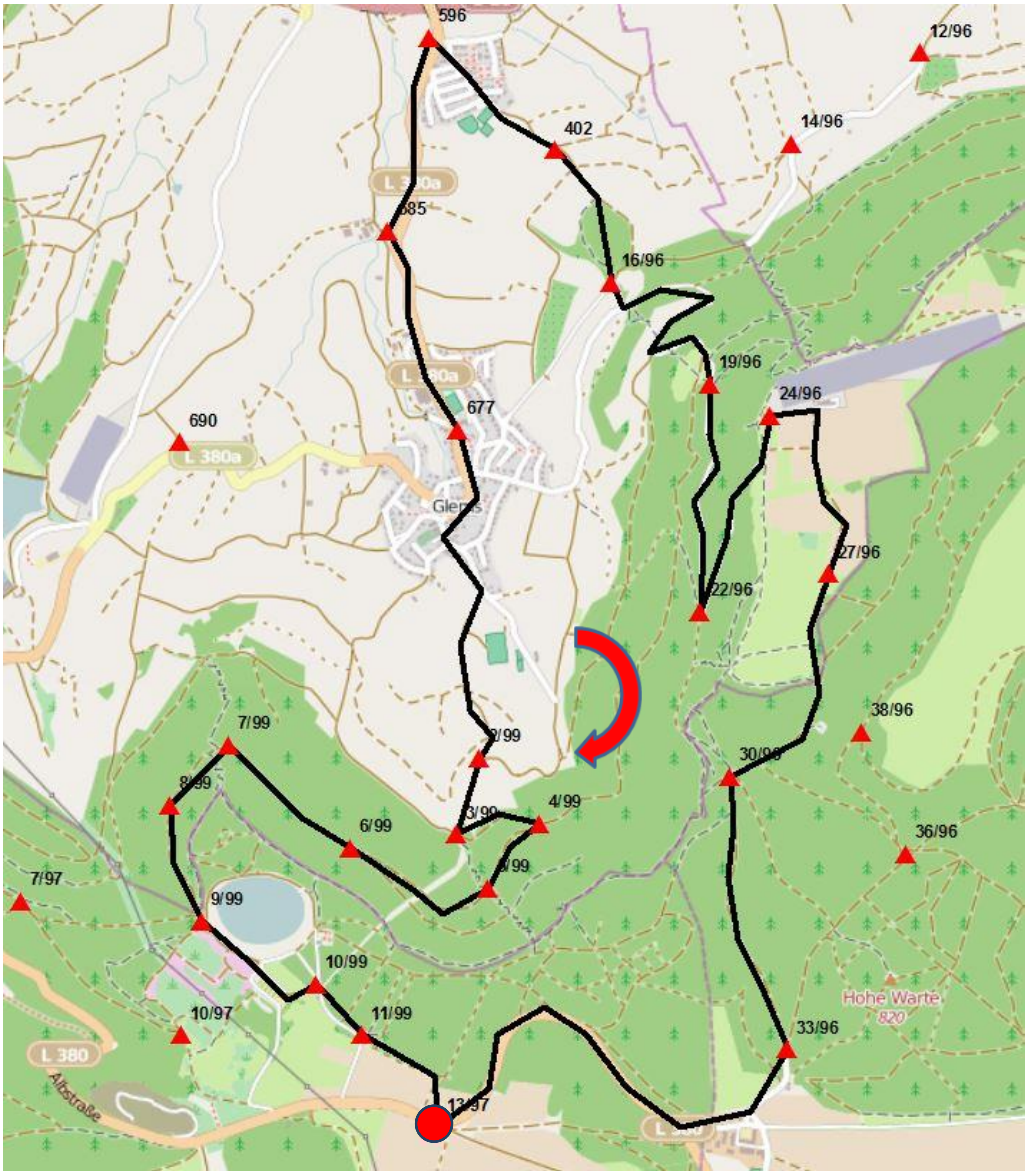

Figure 3.12 Loop 3

There are 22 points that had been observed in loop 3. The observation started from point 33/96 and ran counter clockwise from 33/96 - 13/97 - 11/99 - 10/99 - 9/99 - 8/99 - 7/99 - 6/99 - 5/99 - 4/99 - 3/99 - 2/99 - 677 - 685 - 596 - 402 - 16/96 - 19/96 - 22/96 - 24/96 - 27/96 - 30/96 back to 33/96

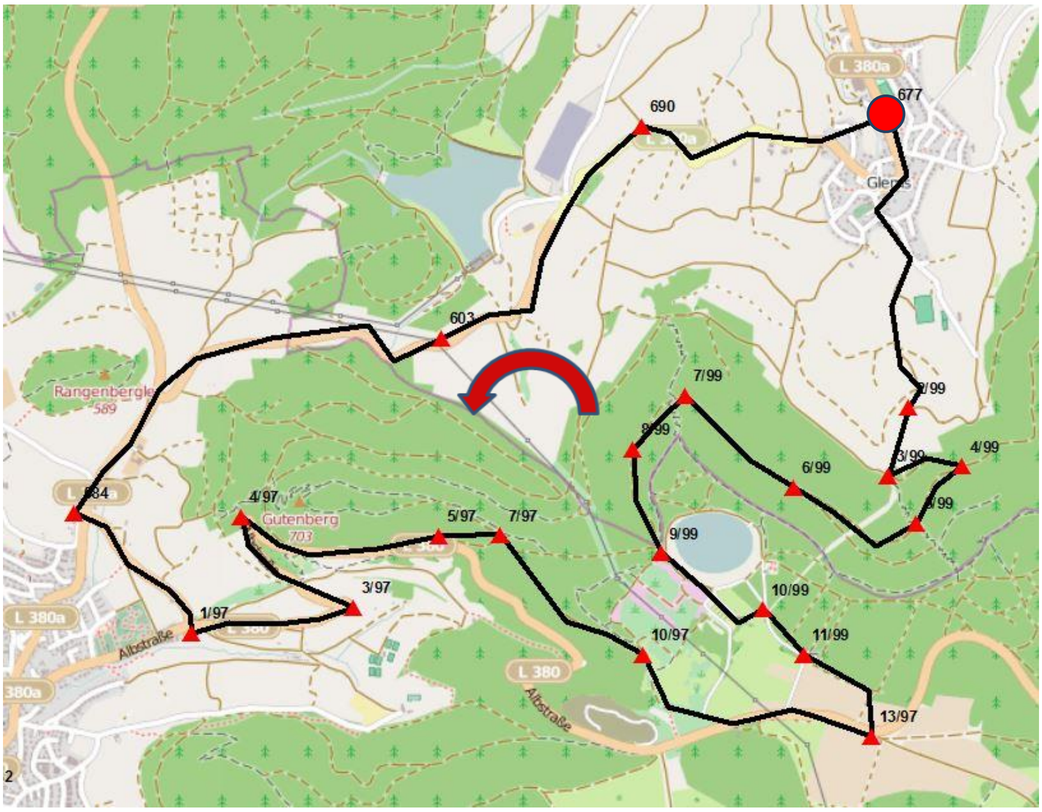

Figure 3.13 Loop 4

There are 21 observed points in loop 4. The surveying path started at point 677 and ran anti clockwise from 677 - 2/99 - 3/99 - 4/99 - 5/99 - 6/99 - 7/99 - 8/99 - 9/99 - 10/99 - 11/99 - 13/97 - 10/97 - 7/97 - 5/97 - 4/97 - 3/97 - 1/97 – 584 – 603 – 690 back to 677.

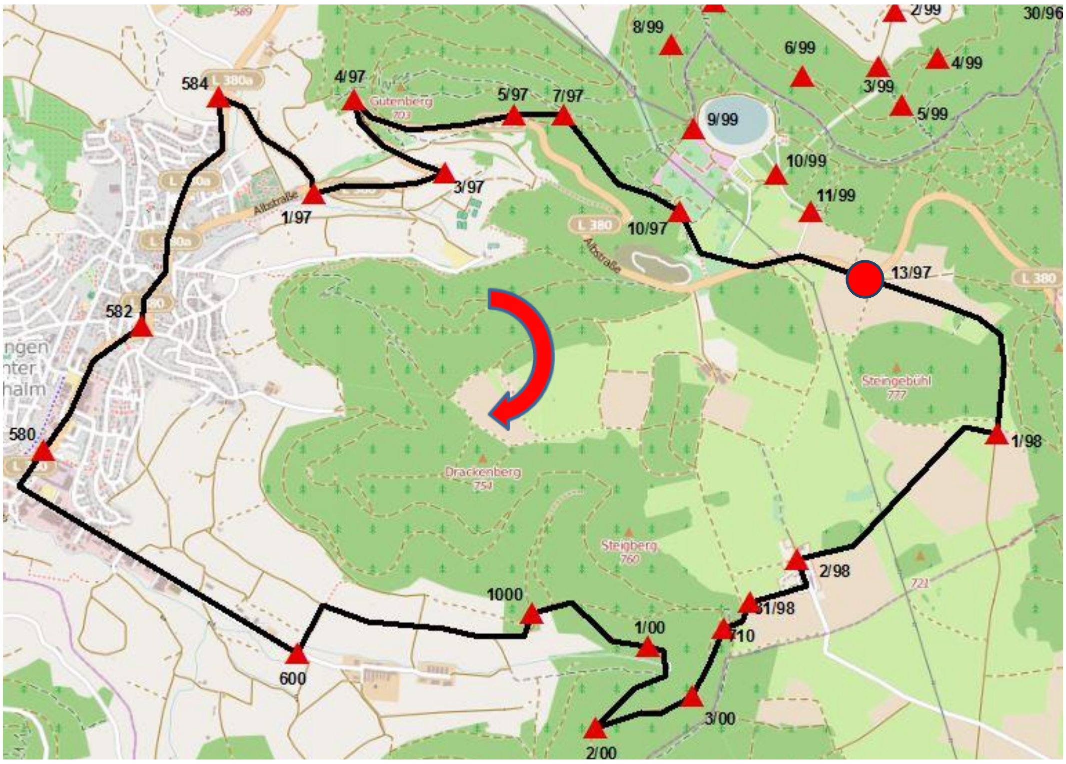

Figure 3.14 Loop 5

There are 19 observed points in loop 5. The surveying path started at point 13/97 and the direction of surveying is counter clockwise from point 13/97 - 1/98 - 2/98 - 31/98 - 710 - 3/00 - 2/00 - 1/00 - 1000 - 580 - 582 - 584 - 1/97 - 3/97 - 4/97 - 5/97 - 7/97 - 10/97 back to 13/97.

3.10 Grouping of the data

Since the close loops were generated by tracing the surveying paths since 1996 to 2013, the data (height differences, gravity values, and distances between observed points will be put into groups according to their loops for geopotential differences.

Point id	Height differences; ΔH (m)	Distance (m)	Gravity (mGal)	Geopotential difference; $\Delta C;\left(\mathrm{m}^2/\mathrm{s}^2\right)$	Year
96/7			980822,120		1996
96/12	57,65436	560,4	980809,492	565,483	
96/14	13,70818	545,7	980806,305	134,451	
96/16	21,60818	964,5	980801,338	211,934	
96/19	113,75889	1141,6	980775,949	1115,734	
96/22	83,29436	984,35	980758,320	816,924	
96/24	65,74404	703,8	980745,001	644,786	
96/27	19,83665	788,9	980743,191	194,547	
96/30	-9,07374	1130,9	980744,941	-88,990	
96/33	-13,28528	1056,7	980749,168	-130,295	
96/36	6,5708	845,1	980748,034	64,443	
96/38	-37,01157	652,9	980755,075	-362,992	
96/42	-0,82561	1250,3	980755,142	-8,097	
96/43	-8,39046	561,2	980755,223	-82,290	
96/51	-126,62073	1420	980780,658	-1241,856	
96/50	-83,65924	800	980797,177	-820,521	
96/2	-84,89477	1102,6	980818,241	-832,654	
96/7	-18,41345	1062,1	980822,120	-180,603	

Table 3.3a Data of loop 1 from year 1996

Point id	Height differences; ΔH (m)	Distance (m)	Gravity (mGal)	Geopotential difference; $\Delta C;\left(\mathrm{m}^2/\mathrm{s}^2\right)$	Year
96/16			980801,338		2002
96/19	113,77940	1102,5	980775,949	1115,935	
96/24			980745,001		
96/30	10,76030	1919,9	980744,941	105,531	

Table 3.3b Data of loop 1 from year 2002

Point id	Height differences; ΔH (m)	Distance (m)	Gravity (mGal)	Geopotential difference; $\Delta C; (m^2/s^2)$	Year
96/16			980801,338		2003
96/19	113,7965	1190,5	980775,949	1116,103	

Table 3.3c Data of loop 1 from year 2003

Point id	Height differences; ΔH (m)	Distance (m)	Gravity (mGal)	Geopotential difference; $\Delta C; (m^2/s^2)$	Year
96/7			980822,120		2004
96/30	366,51298	17445,9	980744,941	3594,699	

Table 3.3d Data of loop 1 from year 2004

Point id	Height differences; ΔH (m)	Distance (m)	Gravity (mGal)	Geopotential difference; $\Delta C; (m^2/s^2)$	Year
96/7			980822,120		1996
96/12	57,65436	560,4	980809,492	565,483	
96/14	13,70818	545,7	980806,305	134,451	
96/16	21,60818	964,5	980801,338	211,934	
402	-70,04010	601,4	980861,727	-686,975	2002
596	-42,97250	704,5	980825,808	-421,493	
206	-32,30664	800	980832,725	-316,873	
208	5,83917	2030	980832,030	57,272	
209	7,17297	710	980830,764	70,355	
201	9,29160	610	980829,209	91,135	
96/7	30,06630	610	980822,120	294,898	

Table 3.4a Data of loop 2 from year 1996 and 2002

Point id	Height differences; ΔH (m)	Distance (m)	Gravity (mGal)	Geopotential difference; $\Delta C; (m^2/s^2)$	Year
402			980861,727		2003
596	-42,9707	614,2	980825,808	-421,475	

Table 3.4b Data of loop 2 from year 2003

Point id	Height differences; ΔH (m)	Distance (m)	Gravity (mGal)	Geopotential difference; $\Delta C; (\mathrm{m}^2/\mathrm{s}^2)$	Year
677			980816,409		1999
2/99	30,00080	1060	980808,758	294,252	
3/99	55,31310	570	980795,662	542,512	
4/99	45,96310	390	980785,900	450,802	
5/99	64,31360	570	980774,372	630,775	
6/99	40,54780	390	980765,779	397,681	
7/99	9,67560	500	980763,609	94,895	
8/99	31,29900	520	980756,230	306,968	
9/99	26,53920	350	980753,104	260,284	
10/99	2,33610	530	980753,666	22,911	
11/99	-14,24060	500	980757,378	-139,665	
13/97	-8,00050	490	980759,198	-78,466	
96/33	51,89472	1240	980749,168	508,960	1997
96/30	13,28528	1060	980744,941	130,295	1996
96/27	9,07374	1130	980743,191	88,990	
96/24	-19,83665	790	980745,001	-194,547	
96/22	-65,74404	700	980758,320	-644,786	
96/19	-83,29436	980	980775,949	-816,924	
96/16	-113,7589	1140	980801,338	-1115,734	
402	-70,0401	601,4	980861,727	-686,975	2002
596	-42,9725	704,5	980825,808	-421,493	
685	-1,23553	860	980825,350	-12,118	1997
677	38,89953	990	980816,409	381,535	

Table 3.5a Data of loop 3 from year 1996

Point id	Height differences; ΔH (m)	Distance (m)	Gravity (mGal)	Geopotential difference; $\Delta C; (\mathrm{m}^2/\mathrm{s}^2)$	Year
96/19			980775,949		2002
596	-226,792	2408,4	980825,808	-2224,378	

Table 3.5b Data of loop 3 from year 2002

Point id	Height differences; ΔH (m)	Distance (m)	Gravity (mGal)	Geopotential difference; $\Delta C; (\mathrm{m}^2/\mathrm{s}^2)$	Year
96/19			980775,9485		2003
596	-226,7841	2408,5	980825,808	-2224,300	

Table 3.5c Data of loop 3 from year 2003

Point id	Height differences; ΔH (m)	Distance (m)	Gravity (mGal)	Geopotential difference; $\Delta C; \left(\mathrm{m}^2/\mathrm{s}^2\right)$	Year
96/30			980744,941		2004
596	-386,5764	5565,3	980825,808	-3791,485	

Table 3.5d Data of loop 3 from year 2004

Point id	Height differences; ΔH (m)	Distance (m)	Gravity (mGal)	Geopotential difference; $\Delta C; \left(\mathrm{m}^2/\mathrm{s}^2\right)$	Year
677			980816,409		1999
2/99	30,0008	1060	980808,758	294,252	
3/99	55,3131	570	980795,662	542,512	
4/99	45,9631	390	980785,900	450,802	
5/99	64,3136	570	980774,372	630,775	
6/99	40,5478	390	980765,779	397,681	
7/99	9,6756	500	980763,609	94,895	
8/99	31,299	520	980756,230	306,968	
9/99	26,5392	350	980753,104	260,284	
10/99	2,3361	530	980753,666	22,911	
11/99	-14,2406	500	980757,378	-139,665	
13/97	-8,0005	490	980759,198	-78,466	
97/10	21,50972	1240	980753,411	210,958	1997
97/7	-61,2289	780	980764,680	-600,507	
97/5	-45,6793	350	980773,865	-448,008	
97/4	-50,8282	820	980784,412	-498,512	
97/3	-25,0099	430	980789,872	-245,294	
97/1	-54,0104	450	980800,905	-529,731	
584	3,31334	1000	980800,749	32,497	
603	-13,3748	1680	980804,047	-131,180	
690	-23,264	1590	980810,017	-228,175	
677	-35,1802	1570	980816,409	-345,052	

Table 3.6 Data of loop 4 from year 1997 and 1999

Point id	Height differences; ΔH (m)	Distance (m)	Gravity (mGal)	Geopotential difference; $\Delta C; \left(\mathrm{m}^2/\mathrm{s}^2\right)$	Year
580			980814,029		1998
600	33,1787	1890	980806,479	325,4201	
1000	54,7979	1440	980795,114	537,4582	
1/00	29,8093	750	980787,768	292,3671	2000
2/00	63,7101	590	980775,144	624,8568	

Table 3.7a Data of loop 5

Point id	Height differences; ΔH (m)	Distance (m)	Gravity (mGal)	Geopotential difference; $\Delta C; \left(m^2/s^2\right)$	Year
3/00	47,0649	539	980766,065	461,599	2000
710	1,2966	201	980766,690	12,717	2005
31/98	21,1947	144	980762,902	207,870	
2/98	18,32007	737	980761,420	179,676	
1/98	0,76511	1225	980762,274	7,504	
13/97	14,9883	1200	980759,198	146,999	
97/10	21,510	1240	980753,411	210,958	1997
97/7	-61,229	780	980764,680	-600,507	
97/5	-45,679	350	980773,865	-448,008	
97/4	-50,828	820	980784,412	-498,512	
97/3	-25,010	430	980789,872	-245,294	
97/1	-54,010	450	980800,905	-529,731	
584	3,313	1000	980800,749	32,497	1998
582	-42,13185	1490	980804,191	-413,230	
580	-31,05288	1140	980814,029	-304,569	

Table 3.7a Data of loop 5 (con't)

Point id	Height differences; ΔH (m)	Distance (m)	Gravity (mGal)	Geopotential difference; $\Delta C; \left(m^2/s^2\right)$	Year
600			980806,479		1998
1/98	236,9551	5660	980762,274	2324,019	

Table 3.7b Data of loop 5 from year 1998

Point id	Height differences; ΔH (m)	Distance (m)	Gravity (mGal)	Geopotential difference; $\Delta C; \left(m^2/s^2\right)$	Year
1000			980795,114		2001
1/98	182,1559	4140	980762,274	1786,546	

Table 3.7c Data of loop 5 from year 2001

Point id	Height differences; ΔH (m)	Distance (m)	Gravity (mGal)	Geopotential difference; $\Delta C; \left(m^2/s^2\right)$	Year
1000			980795,114		2007
2/98	181,38477	2938	980761,420	1778,982	

Table 3.7d Data of loop 5 from year 2007

Point id	Height differences; ΔH (m)	Distance (m)	Gravity (mGal)	Geopotential difference; $\Delta C;\left(\mathrm{m}^2/\mathrm{s}^2\right)$	Year
1000			980795,114		2005
1/00	29,80276	711	980787,768	292,3029	
2/00	63,71299	633	980775,144	624,8852	
3/00	47,0561	538	980766,065	461,5124	

Table 3.7e Data of loop 5 from year 2005

Point id	Height differences; ΔH (m)	Distance (m)	Gravity (mGal)	Geopotential difference; $\Delta C;\left(\mathrm{m}^2/\mathrm{s}^2\right)$	Year
1000			980795,114		2006
1/98	182,1559	4140	980762,274	1786,546	

Table 3.7f Data of loop 5 from year 2006

3.11 Computation of loop misclosures

The Condition equation (B-Model) is created for loop misclosures of all 5 loops, geopotential differences $\left(\Delta C\right)$ from table 3.2a to table 3.6e are considered as the observation values to be put in the observation matrix ($\mathbf{y}$). The loop misclosures can be computed by $\mathbf{e} = \mathbf{B}^{\mathbf{T}}\mathbf{y}$.

There are 22 conditions and 77 observations for the computation the size of the matrices as shown below

$$\mathbf{e}_{[22\times1]} = \mathrm{B}^{\mathrm{T}}_{[22\times7]}\mathbf{y}_{[77\times1]} \tag{3.8}$$

It reveals that there are 22 loop misclosures from least square adjustment, the mentioned loop misclosures will be compared to the allowable loop misclosure as explained in section 3.6.

Chapter 4

Results and Discussions

4.1 Loop misclosures

Since the condition equation was done based on the geopotential differences, the loop misclosures of all 22 conditions were determined. The loop misclosures are in the unit of energy per mass (m^2/s^2) so they have to be converted to unit of length (m) by dividing them by normal gravity at latitude 48.485° which equals to 9.8061992 m/s^2. The results of the miosclosures were shown in table 4.1.

Loop No.	Misclosure (m)	Allowable misclosure (m)	Total Distance (m)
1	0.000	0.004	15571.05
	0.021		
	-0.002		
	0.038		
	-0.017		
2	0.020	0.003	8136.50
	0.020		
3	-0.018	0.004	16065.90
	0.002		
	-0.005		
	-0.014		
4	0.004	0.004	15780.00
5	-0.007	0.004	16416.00
	-0.004		
	-0.003		
	0.002		
	-0.010		
	-0.001		
	-0.002		
	-0.001		
	0.003		
	-0.026		

Table 4.1 Results of loop misclosures

According to table 4.1, the second column are the misclosures derived from B-matrix ($\mathbf{e}$) and the third column are the allowable loop misclosures (e_{all}). They have to be compared for the reliability check of the data. In case the observed misclosure is larger than the allowable misclosure that means the data in that loop is not accurate enough for the height computation because some of the observed data is deviated from the others so they have to be eliminated and not taken into account for the data adjustment and height calculation. The red numbers in column two of table 4.1 show the loops that have to be neglected.

4.2 Adjustment of geopotential differences

After the reliability of the data has been checked, the adjustment of the data was done. From the beginning there were 22 conditions for 5 loops but since the unreliable data were eliminated, 9 conditions were left for the adjustment. So the size of A the matrix was reduced from $\left[77\times 56\right]$ to $\left[64\times 56\right]$ and the size of B matrix was reduced from $\left[22\times 77\right]$ to $\left[9\times 64\right]$ as well as the size of the observation matrix that reduced from $\left[77\times 1\right]$ to $\left[64\times 1\right]$.

Unweighted and weighted least squares adjustment were adopted for the computation. The weight matrix was assigned regarding the distances from one point to another point as described in section 3.3. Observation equation (A-matrix) and condition equation (B-matrix) were both adopted in the adjustment for comparing of the results. If the results from both equations were equal that means the adjustment was quite correct. The adjusted geopotential differences results were shown in table 4.2

Observed Points		Potential differences; ΔC $\left[\mathrm{m^2/s^2}\right]$			Diff. UW and W $\left[\mathrm{m^2/s^2}\right]$
From	To	Observed	Unweighted Adj. (UW)	Weighted Adj. (W)	
96/7	96/12	565.4831	565.4670	565.4678	0.0009
96/12	96/14	134.4509	134.4348	134.4359	0.0011
96/14	96/16	211.9339	211.9178	211.9139	0.0039
96/16	96/19	1115.7343	1,115.7828	1,115.7866	0.0038
96/19	96/22	816.9237	816.9315	816.9316	0.0001
96/22	96/24	644.7858	644.7935	644.7924	0.0011
96/24	96/27	194.5468	194.5409	194.5429	0.0021
96/27	96/30	-88.9902	-88.9961	-88.9948	0.0013
96/30	96/33	-130.2950	-130.2872	-130.2869	0.0004
96/33	96/36	64.4430	64.4410	64.4406	0.0004
96/36	96/38	-362.9915	-362.9935	-362.9937	0.0001
96/38	96/42	-8.0972	-8.0992	-8.1001	0.0010
96/42	96/43	-82.2899	-82.2919	-82.2918	0.0000

Table 4.2 Adjusted geopotential references

Observed Points		Potential differences; $\Delta C\left[\mathrm{m}^2/\mathrm{s}^2\right]$			Diff. UW and W $\left[\mathrm{m}^2/\mathrm{s}^2\right]$
From	To	Observed	Unweighted Adj. (UW)	Weighted Adj. (W)	
96/43	96/51	-1241.8555	-1,241.8575	-1,241.8587	0.0011
96/51	96/50	-820.5206	-820.5225	-820.5229	0.0004
96/50	96/2	-832.6544	-832.6564	-832.6572	0.0008
96/2	96/7	-180.6028	-180.6048	-180.6055	0.0007
96/24	96/30	105.5311	105.5448	105.5481	0.0033
96/16	402	-686.9754	-687.0243	-687.0134	0.0109
402	596	-421.4931	-421.5117	-421.5074	0.0044
596	206	-316.8730	-316.8871	-316.8888	0.0018
206	208	57.2725	57.2584	57.2472	0.0112
208	209	70.3547	70.3406	70.3398	0.0008
209	210	91.1348	91.1207	91.1209	0.0003
210	96/7	294.8980	294.8839	294.8841	0.0003
402	596	-421.4754	-421.5117	-421.5074	0.0044
96/33	13/97	-508.9596	-508.9499	-508.9479	0.0020
13/97	11/99	78.4656	78.4651	78.4656	0.0005
11/99	10/99	139.6655	139.6650	139.6655	0.0005
10/99	9/99	-22.9114	-22.9119	-22.9113	0.0005
9/99	8/99	-260.2844	-260.2849	-260.2844	0.0005
8/99	7/99	-306.9680	-306.9686	-306.9680	0.0005
7/99	6/99	-94.8949	-94.8954	-94.8948	0.0005
6/99	5/99	-397.6807	-397.6812	-397.6807	0.0005
5/99	4/99	-630.7750	-630.7755	-630.7750	0.0005
4/99	3/99	-450.8018	-450.8024	-450.8018	0.0005
3/99	2/99	-542.5121	-542.5126	-542.5121	0.0005
2/99	677	-294.2516	-294.2521	-294.2516	0.0005
677	685	-381.5347	-381.5250	-381.5242	0.0008
685	596	12.1184	12.1281	12.1282	0.0000
96/19	596	-2224.3779	-2,224.3188	-2,224.3073	0.0115
96/19	596	-2224.3004	-2,224.3188	-2,224.3073	0.0115
677	690	345.0520	345.0418	345.0389	0.0029
690	603	228.1746	228.1643	228.1613	0.0030
603	584	131.1801	131.1698	131.1665	0.0034
584	97/1	-32.4973	-32.5001	-32.5001	0.0000
97/1	97/3	529.7310	529.7282	529.7291	0.0009
97/3	97/4	245.2943	245.2915	245.2924	0.0010
97/4	97/5	498.5122	498.5094	498.5097	0.0003
97/5	97/7	448.0084	448.0056	448.0068	0.0012
97/7	97/10	600.5075	600.5047	600.5050	0.0003
97/10	13/97	-210.9579	-210.9607	-210.9610	0.0003
13/97	1/98	-146.9994	-146.9919	-146.9909	0.0010
1/98	2/98	-7.5039	-7.4965	-7.4954	0.0011

Table 4.2 Adjusted geopotential references (con't)

Observed Points		Potential differences; ΔC $\left[\mathrm{m^2/s^2}\right]$			Diff. UW and W $\left[\mathrm{m^2/s^2}\right]$
From	To	Observed	Unweighted Adj. (UW)	Weighted Adj. (W)	
2/98	31/98	-179.6763	-179.6689	-179.6697	0.0008
31/98	710	-207.8702	-207.8627	-207.8672	0.0045
710	3/00	-12.7166	-12.7092	-12.7132	0.0040
3/00	2/00	-461.5987	-461.5913	-461.5930	0.0018
2/00	1/00	-624.8568	-624.8494	-624.8509	0.0015
1/00	1000	-292.3671	-292.3596	-292.3604	0.0008
1000	600	-537.4582	-537.4508	-537.4490	0.0018
600	580	-325.4201	-325.4127	-325.4095	0.0032
580	582	304.5695	304.5769	304.5777	0.0008
582	584	413.2302	413.2376	413.2396	0.0020

Table 4.2 Adjusted geopotential references (con't)

The results from the adjustment shown that the differences between unweighted and weighted adjustment is up to 0.0115 m^2/s^2 or 0.0012 mm in metric unit. The results from observation equation and condition equation are similar. The squared sum of the residuals shown in table 4.3

Type of adjustment	Squared sum of the residuals
Unweighted observation equation (Unweighted A-matrix)	0.01604
Weighted observation equation (Weighted A-matrix)	0.01675
Unweighted condition equation (Unweighted B-matrix)	0.01604
Weighted condition equation (Weighted B-matrix)	0.01675

Table 4.3 Squared sum of the residuals

The squared sums of the residuals from both adjustment types are equal which is a proof that the result of the adjustment by observation equation is equal to the one from the condition equation. Then the adjustment is checked by summing up the adjusted potential differences in each loop, if the summation was zero that means there was no loop misclosure, thus, the adjustment is successful. After that the geopotential numbers could be calculated as well as the height computation can be made.

The adjusted potential differences from table 4.2 were grouped in their own loops and the summations of the potential differences of each loop have to be computed to check that the summation of the potential differences of each loop is zero. The results of the summation and potential numbers are shown in table 4.4 to table 4.8

Point numbers	Unweighted Potential diff. $\Delta C\left[\mathrm{m}^2/\mathrm{s}^2\right]$	Weighted Potential diff. $\Delta C\left[\mathrm{m}^2/\mathrm{s}^2\right]$
96/7		
96/12	565.4670	565.4678
96/14	134.4348	134.4359
96/16	211.9178	211.9139
96/19	1115.7828	1115.7866
96/22	816.9315	816.9316
96/24	644.7935	644.7924
96/27	194.5409	194.5429
96/30	-88.9961	-88.9948
96/33	-130.2872	-130.2869
96/36	64.4410	64.4406
96/38	-362.9935	-362.9937
96/42	-8.0992	-8.1001
96/43	-82.2919	-82.2918
96/51	-1241.8575	-1241.8587
96/50	-820.5225	-820.5229
96/2	-832.6564	-832.6572
96/7	-180.6048	-180.6055
Sum	0.0000	0.0000

Table 4.4 Summation of potential differences of loop 1

Point numbers	Unweighted Potential diff. $\Delta C\left[\mathrm{m}^2/\mathrm{s}^2\right]$	Weighted Potential diff. $\Delta C\left[\mathrm{m}^2/\mathrm{s}^2\right]$
96/16		
402	-687.0243	-687.0134
596	-421.5117	-421.5074
206	-316.8871	-316.8888
208	57.2584	57.2472
209	70.3406	70.3398
210	91.1207	91.1209
96/7	294.8839	294.8841
96/12	565.4670	565.4678
96/14	134.4348	134.4359
96/16	211.9178	211.9139
Sum	0,0000	0,0000

Table 4.5 Summation of potential differences of loop 2

Point numbers	Unweighted Potential diff. $\Delta C\left[\mathrm{m}^2/\mathrm{s}^2\right]$	Weighted Potential diff. $\Delta C\left[\mathrm{m}^2/\mathrm{s}^2\right]$
96/33		
13/97	-508.9499	-508.9479
11/99	78.4651	78.4656
10/99	139.6650	139.6655
9/99	-22.9119	-22.9113
8/99	-260.2849	-260.2844
7/99	-306.9686	-306.9680
6/99	-94.8954	-94.8948
5/99	-397.6812	-397.6807
4/99	-630.7755	-630.7750
3/99	-450.8024	-450.8018
2/99	-542.5126	-542.5121
677	-294.2521	-294.2516
685	-381.5250	-381.5242
596	12.1281	12.1282
402	421.5117	421.5074
96/16	687.0243	687.0134
96/19	1115.7828	1115.7866
96/22	816.9315	816.9316
96/24	644.7935	644.7924
96/27	194.5409	194.5429
96/30	-88.9961	-88.9948
96/33	-130.2872	-130.2869
Sum	0.0000	0.0000

Table 4.6 Summation of potential differences of loop 3

Point numbers	Unweighted Potential diff. $\Delta C\left[\mathrm{m}^2/\mathrm{s}^2\right]$	Weighted Potential diff. $\Delta C\left[\mathrm{m}^2/\mathrm{s}^2\right]$
677		
690	345.0418	345.0389
603	228.1643	228.1613
584	131.1698	131.1665
97/1	-32.5001	-32.5001
97/3	529.7282	529.7291
97/4	245.2915	245.2924
97/5	498.5094	498.5097
97/7	448.0056	448.0068
97/10	600.5047	600.5050

Table 4.7 Summation of potential differences of loop 4

Point numbers	Unweighted Potential diff. $\Delta C\left[\mathrm{m}^2/\mathrm{s}^2\right]$	Weighted Potential diff. $\Delta C\left[\mathrm{m}^2/\mathrm{s}^2\right]$
13/97	-210.9607	-210.9610
11/99	78.4651	78.4656
10/99	139.6650	139.6655
9/99	-22.9119	-22.9113
8/99	-260.2849	-260.2844
7/99	-306.9686	-306.9680
6/99	-94.8954	-94.8948
5/99	-397.6812	-397.6807
4/99	-630.7755	-630.7750
3/99	-450.8024	-450.8018
2/99	-542.5126	-542.5121
677	-294.2521	-294.2516
Sum	0.0000	0.0000

Table 4.7 Summation of potential differences of loop 4 (con't)

Point numbers	Unweighted Potential diff. $\Delta C\left[\mathrm{m}^2/\mathrm{s}^2\right]$	Weighted Potential diff. $\Delta C\left[\mathrm{m}^2/\mathrm{s}^2\right]$
13/97		
1/98	-146.9919	-146.9909
2/98	-7.4965	-7.4954
31/98	-179.6689	-179.6697
710	-207.8627	-207.8672
3/00	-12.7092	-12.7132
2/00	-461.5913	-461.5930
1/00	-624.8494	-624.8509
1000	-292.3596	-292.3604
600	-537.4508	-537.4490
580	-325.4127	-325.4095
582	304.5769	304.5777
584	413.2376	413.2396
97/1	-32.5001	-32.5001
97/3	529.7282	529.7291
97/4	245.2915	245.2924
97/5	498.5094	498.5097
97/7	448.0056	448.0068
97/10	600.5047	600.5050
13/97	-210.9607	-210.9610
Sum	0.0000	0.0000

Table 4.8 Summation of potential differences of loop 5

According to table 4.4 to table 4.8, the summation of potential differences of all 5 loops are equal to zero, that means the loop misclosures do not exist anymore and the adjustment of the data were successful. Then the geopotential numbers would be computed.

4.3 Geopotential numbers results

After the potential differences were adjusted and the loop misclosures of potential differences were checked to be zero, the geopotential numbers (C)were computed according to section 3.7. The point 580 was selected as the start point of the computation. The results for the potential numbers of each loops are shown in table 4.9 to table 4.13

Point numbers	**Unweighted Potential number;** $C_{UW}\left[\mathrm{m^2/s^2}\right]$	**Weighted Potential number;** $C_W\left[\mathrm{m^2/s^2}\right]$	**Differences between** C_{UW} **and** C_W $\left[\mathrm{m^2/s^2}\right]$
96/7	4109.0586	4109.0582	0.0003
96/12	4674.5255	4674.5260	-0.0005
96/14	4808.9603	4808.9619	-0.0016
96/16	5020.8781	5020.8758	0.0023
96/19	6136.6609	6136.6623	-0.0014
96/22	6953.5924	6953.5939	-0.0016
96/24	7598.3859	7598.3863	-0.0004
96/27	7792.9267	7792.9292	-0.0025
96/30	7703.9306	7703.9344	-0.0038
96/33	7573.6434	7573.6476	-0.0042
96/36	7638.0844	7638.0882	-0.0037
96/38	7275.0909	7275.0945	-0.0036
96/42	7266.9917	7266.9944	-0.0027
96/43	7184.6999	7184.7025	-0.0027
96/51	5942.8423	5942.8439	-0.0015
96/50	5122.3198	5122.3210	-0.0012
96/2	4289.6634	4289.6638	-0.0004
96/7	4109.0586	4109.0582	0.0003

Table 4.9 Geopotential numbers of loop 1

Point numbers	Unweighted Potential number; $C_{UW}\left[\mathrm{m}^2/\mathrm{s}^2\right]$	Weighted Potential number; $C_W\left[\mathrm{m}^2/\mathrm{s}^2\right]$	Differences between C_{UW} and C_W $\left[\mathrm{m}^2/\mathrm{s}^2\right]$
96/16	5020.8781	5020.8758	0.0023
402	4333.8538	4333.8624	-0.0085
596	3912.3421	3912.3550	-0.0129
206	3595.4550	3595.4662	-0.0111
208	3652.7134	3652.7133	0.0000
209	3723.0540	3723.0531	0.0009
210	3814.1747	3814.1741	0.0006
96/7	4109.0586	4109.0582	0.0003
96/12	4674.5255	4674.5260	-0.0005
96/14	4808.9603	4808.9619	-0.0016
96/16	5020.8781	5020.8758	0.0023

Table 4.10 Geopotential numbers of loop 2

Point numbers	Unweighted Potential number; $C_{UW}\left[\mathrm{m}^2/\mathrm{s}^2\right]$	Weighted Potential number; $C_W\left[\mathrm{m}^2/\mathrm{s}^2\right]$	Differences between C_{UW} and C_W $\left[\mathrm{m}^2/\mathrm{s}^2\right]$
96/33	7573.6434	7573.6476	-0.0042
13/97	7064.6935	7064.6997	-0.0062
11/99	7143.1585	7143.1653	-0.0067
10/99	7282.8235	7282.8308	-0.0073
9/99	7259.9116	7259.9194	-0.0078
8/99	6999.6267	6999.6350	-0.0083
7/99	6692.6581	6692.6670	-0.0089
6/99	6597.7628	6597.7722	-0.0094
5/99	6200.0816	6200.0915	-0.0099
4/99	5569.3061	5569.3165	-0.0105
3/99	5118.5037	5118.5147	-0.0110
2/99	4575.9911	4576.0026	-0.0115
677	4281.7390	4281.7511	-0.0121
685	3900.2140	3900.2268	-0.0128
596	3912.3421	3912.3550	-0.0129
402	4333.8538	4333.8624	-0.0085
96/16	5020.8781	5020.8758	0.0023
96/19	6136.6609	6136.6623	-0.0014
96/22	6953.5924	6953.5939	-0.0016
96/24	7598.3859	7598.3863	-0.0004

Table 4.11 Geopotential numbers of loop 3

Point numbers	Unweighted Potential number; $C_{UW}\left[\mathrm{m^2/s^2}\right]$	Weighted Potential number; $C_W\left[\mathrm{m^2/s^2}\right]$	Differences between C_{UW} and C_W $\left[\mathrm{m^2/s^2}\right]$
96/27	7792.9267	7792.9292	-0.0025
96/30	7703.9306	7703.9344	-0.0038
96/33	7573.6434	7573.6476	-0.0042

Table 4.11 Geopotential numbers of loop 3 (con't)

Point numbers	Unweighted Potential number; $C_{UW}\left[\mathrm{m^2/s^2}\right]$	Weighted Potential number; $C_W\left[\mathrm{m^2/s^2}\right]$	Differences between C_{UW} and C_W $\left[\mathrm{m^2/s^2}\right]$
677	4281.7390	4281.7511	-0.0121
690	4626.7808	4626.7899	-0.0092
603	4854.9451	4854.9513	-0.0062
584	4986.1149	4986.1177	-0.0028
97/1	4953.6149	4953.6177	-0.0028
97/3	5483.3431	5483.3468	-0.0037
97/4	5728.6345	5728.6393	-0.0047
97/5	6227.1439	6227.1489	-0.0050
97/7	6675.1495	6675.1557	-0.0061
97/10	7275.6542	7275.6607	-0.0065
13/97	7064.6935	7064.6997	-0.0062
11/99	7143.1585	7143.1653	-0.0067
10/99	7282.8235	7282.8308	-0.0073
9/99	7259.9116	7259.9194	-0.0078
8/99	6999.6267	6999.6350	-0.0083
7/99	6692.6581	6692.6670	-0.0089
6/99	6597.7628	6597.7722	-0.0094
5/99	6200.0816	6200.0915	-0.0099
4/99	5569.3061	5569.3165	-0.0105
3/99	5118.5037	5118.5147	-0.0110
2/99	4575.9911	4576.0026	-0.0115
677	4281.7390	4281.7511	-0.0121

Table 4.12 Geopotential numbers of loop 4

Point numbers	Unweighted Potential number; $C_{UW}\left[\mathrm{m}^2/\mathrm{s}^2\right]$	Weighted Potential number; $C_W\left[\mathrm{m}^2/\mathrm{s}^2\right]$	Differences between C_{UW} and C_W $\left[\mathrm{m}^2/\mathrm{s}^2\right]$
13/97	7064.6935	7064.6997	-0.0062
1/98	6917.7015	6917.7087	-0.0072
2/98	6910.2051	6910.2134	-0.0083
31/98	6730.5362	6730.5437	-0.0075
710	6522.6734	6522.6764	-0.0030
3/00	6509.9642	6509.9633	0.0010
2/00	6048.3730	6048.3702	0.0027
1/00	5423.5235	5423.5193	0.0042
1000	5131.1639	5131.1589	0.0050
600	4593.7131	4593.7099	0.0032
580	4268.3004	4268.3004	0.0000
582	4572.8773	4572.8781	-0.0008
584	4986.1149	4986.1177	-0.0028
97/1	4953.6149	4953.6177	-0.0028
97/3	5483.3431	5483.3468	-0.0037
97/4	5728.6345	5728.6393	-0.0047
97/5	6227.1439	6227.1489	-0.0050
97/7	6675.1495	6675.1557	-0.0061
97/10	7275.6542	7275.6607	-0.0065
13/97	7064.6935	7064.6997	-0.0062

Table 4.13 Geopotential numbers of loop 5

The results from table 4.9 to table 4.13 reveal that the difference between unweighted and weighted potential numbers is up to 0.0129 $\mathrm{m}^2/\mathrm{s}^2$. That means the geopotential numbers are not very much affected by the weights that had been assigned in the adjustment because the maximum weight in the adjustment is 2.635 and the minimum weight is 0.644, the average of the weights is 1.185. The weights larger than 2 appeared twice in the computation as well as the very low weights did not occur frequently in the adjustment so the weights did not affect the results significantly.

The results also show that the potential numbers derived from the unweighted adjustment are lower than those from the weighted adjustment.

4.4 Height computation results

After the geopotential numbers were computed, the height systems have to be calculated according to the section 2.4. The results of the height computations are shown in the following:

4.4.1 Heights system results

Point No.	Unweighted (m)			Weighted (m)		
	Dynamic	Orthometric	Normal	Dynamic	Orthometric	Normal
96/7	419.0266	418.9395	418.9198	419.0266	418.9394	418.9197
96/12	476.6909	476.5977	476.5736	476.6909	476.5978	476.5737
96/14	490.4000	490.3058	490.2805	490.4002	490.3060	490.2806
96/16	512.0106	511.9148	511.8875	512.0104	511.9145	511.8873
96/19	625.7940	625.6928	625.6548	625.7942	625.6929	625.6549
96/22	709.1017	708.9994	708.9532	709.1018	708.9996	708.9534
96/24	774.8553	774.7539	774.7011	774.8554	774.7540	774.7011
96/27	794.6939	794.5913	794.5382	794.6942	794.5915	794.5384
96/30	785.6184	785.5156	785.4634	785.6188	785.5160	785.4637
96/33	772.3322	772.2278	772.1781	772.3326	772.2283	772.1786
96/36	778.9037	778.7993	778.7491	778.9040	778.7997	778.7495
96/38	741.8869	741.7823	741.7354	741.8873	741.7827	741.7358
96/42	741.0610	740.9564	740.9095	741.0613	740.9567	740.9098
96/43	732.6692	732.5658	732.5185	732.6694	732.5660	732.5187
96/51	606.0291	605.9282	605.8924	606.0293	605.9284	605.8926
96/50	522.3553	522.2597	522.2306	522.3554	522.2598	522.2307
96/2	437.4440	437.3548	437.3338	437.4441	437.3548	437.3338
96/7	419.0266	418.9395	418.9198	419.0266	418.9394	418.9197

Table 4.14 Height system results of loop 1

Point No.	Unweighted (m)			Weighted (m)		
	Dynamic	Orthometric	Normal	Dynamic	Orthometric	Normal
96/16	512.0106	511.9148	511.8875	512.0104	511.9145	511.8873
402	441.9504	441.8609	441.8393	441.9513	441.8618	441.8402
596	398.9662	398.8818	398.8632	398.9675	398.8831	398.8645
206	366.6512	366.5711	366.5547	366.6524	366.5722	366.5559
208	372.4902	372.4091	372.3925	372.4902	372.4091	372.3925
209	379.6633	379.5811	379.5641	379.6632	379.5810	379.5641
210	388.9555	388.8718	388.8544	388.9554	388.8717	388.8544
96/7	419.02662	418.93948	418.91977	419.02659	418.93945	418.91974
96/12	476.69086	476.59774	476.57363	476.69091	476.59780	476.57368
96/14	490.40003	490.30580	490.28048	490.40019	490.30596	490.28064
96/16	512.01062	511.91478	511.88754	512.01038	511.91454	511.88730

Table 4.15 Height systems results of loop 2

Point No.	Unweighted (m)			Weighted (m)		
	Dynamic	Orthometric	Normal	Dynamic	Orthometric	Normal
96/33	772.3322	772.2278	772.1781	772.3326	772.2283	772.1786
13/97	720.4314	720.3268	720.2818	720.4320	720.3274	720.2824
11/99	728.4329	728.3286	728.2826	728.4336	728.3292	728.2833
10/99	742.6755	742.5718	742.5239	742.6762	742.5725	742.5246
9/99	740.3390	740.2361	740.1876	740.3398	740.2369	740.1884
8/99	713.7961	713.6947	713.6471	713.7969	713.6955	713.6480
7/99	682.4926	682.3906	682.3468	682.4935	682.3915	682.3477
6/99	672.8155	672.7135	672.6708	672.8165	672.7144	672.6717
5/99	632.2614	632.1601	632.1214	632.2624	632.1612	632.1224
4/99	567.9373	567.8398	567.8057	567.9383	567.8408	567.8068
3/99	521.9661	521.8714	521.8415	521.9672	521.8725	521.8426
2/99	466.6427	466.5519	466.5272	466.6439	466.5531	466.5284
677	436.6359	436.5476	436.5258	436.6372	436.5489	436.5270
685	397.7294	397.6454	397.6267	397.7307	397.6467	397.6280
596	398.9662	398.8818	398.8632	398.9675	398.8831	398.8645
402	441.9504	441.8609	441.8393	441.9513	441.8618	441.8402
96/16	512.0106	511.9148	511.8875	512.0104	511.9145	511.8873
96/19	625.7940	625.6928	625.6548	625.7942	625.6929	625.6549
96/22	709.1017	708.9994	708.9532	709.1018	708.9996	708.9534
96/24	774.8553	774.7539	774.7011	774.8554	774.7540	774.7011
96/27	794.6939	794.5913	794.5382	794.6942	794.5915	794.5384
96/30	785.6184	785.5156	785.4634	785.6188	785.5160	785.4637
96/33	772.3322	772.2278	772.1781	772.3326	772.2283	772.1786

Table 4.16 Height systems results of loop 3

Point No.	Unweighted (m)			Weighted (m)		
	Dynamic	Orthometric	Normal	Dynamic	Orthometric	Normal
677	436.63594	436.54764	436.52580	436.63717	436.54887	436.52704
690	471.8220	471.7296	471.7056	471.8230	471.7306	471.7066
603	495.0894	494.9954	494.9691	495.0900	494.9960	494.9697
584	508.4656	508.3707	508.3431	508.4659	508.3710	508.3434
97/1	505.1514	505.0570	505.0294	505.1516	505.0573	505.0297
97/3	559.1711	559.0728	559.0408	559.1715	559.0732	559.0412
97/4	584.1850	584.0856	584.0512	584.1855	584.0860	584.0517
97/5	635.0212	634.9197	634.8808	635.0217	634.9202	634.8813
97/7	680.7071	680.6046	680.5615	680.7077	680.6053	680.5622
97/10	741.9444	741.8410	741.7928	741.9450	741.8417	741.7935
13/97	720.43137	720.32681	720.28179	720.43200	720.32744	720.28242
11/99	728.43294	728.32856	728.28262	728.43363	728.32924	728.28331
10/99	742.67546	742.57180	742.52386	742.67620	742.57254	742.52460
9/99	740.33899	740.23609	740.18760	740.33979	740.23688	740.18839

Table 4.17 Height systems results of loop 4

Point No.	Unweighted (m)			Weighted (m)		
	Dynamic	Orthometric	Normal	Dynamic	Orthometric	Normal
8/99	713.79609	713.69468	713.64715	713.79694	713.69553	713.64800
7/99	682.49257	682.39057	682.34680	682.49348	682.39147	682.34771
6/99	672.81549	672.71348	672.67077	672.81645	672.71444	672.67173
5/99	632.26143	632.16014	632.12140	632.26245	632.16115	632.12241
4/99	567.93727	567.83977	567.80575	567.93834	567.84084	567.80681
3/99	521.96612	521.87141	521.84146	521.96724	521.87253	521.84258
2/99	466.64268	466.55190	466.52718	466.64386	466.55307	466.52836
677	436.63594	436.54764	436.52580	436.63717	436.54887	436.52704

Table 4.17 Height systems results of loop 4 (con't)

Point No.	Unweighted (m)			Weighted (m)		
	Dynamic	Orthometric	Normal	Dynamic	Orthometric	Normal
13/97	720.4314	720.3268	720.2818	720.4320	720.3274	720.2824
1/98	705.4417	705.3371	705.2935	705.4424	705.3379	705.2943
2/98	704.6772	704.5734	704.5292	704.6781	704.5742	704.5300
31/98	686.3552	686.2531	686.2091	686.3560	686.2539	686.2098
710	665.1582	665.0567	665.0143	665.1585	665.0570	665.0146
3/00	663.8621	663.7613	663.7184	663.8620	663.7612	663.7183
2/00	616.7907	616.6915	616.6526	616.7905	616.6912	616.6524
1/00	553.0709	552.9749	552.9415	553.0705	552.9745	552.9411
1000	523.2572	523.1625	523.1323	523.2566	523.1620	523.1318
600	468.4499	468.3599	468.3341	468.4496	468.3595	468.3338
580	435.2655	435.1786	435.1556	435.2655	435.1786	435.1556
582	466.3251	466.2366	466.2097	466.3252	466.2367	466.2098
584	508.4656	508.3707	508.3431	508.4659	508.3710	508.3434
97/1	505.1514	505.0570	505.0294	505.1516	505.0573	505.0297
97/3	559.1711	559.0728	559.0408	559.1715	559.0732	559.0412
97/4	584.1850	584.0856	584.0512	584.1855	584.0860	584.0517
97/5	635.0212	634.9197	634.8808	635.0217	634.9202	634.8813
97/7	680.7071	680.6046	680.5615	680.7077	680.6053	680.5622
97/10	741.9444	741.8410	741.7928	741.9450	741.8417	741.7935
13/97	720.4314	720.3268	720.2818	720.4320	720.3274	720.2824

Table 4.18 Height systems results of loop 5

According to the results from table 4.14 to table 4.18, they reveal that the height systems calculation from unweighted potential numbers adjustment and weighted potential numbers adjustment are quite the same. The difference between the unweighted and weighted result is up to 0.0013 m or 1.3 mm, so the weights do not significantly affect the height system computation.

4.4.2 Visualization of height profiles and gravity values

The height systems in table 4.14 to table 4.18 were plotted for illustrating the height profile of each loop as well as the gravity values. The plots are shown in the following:

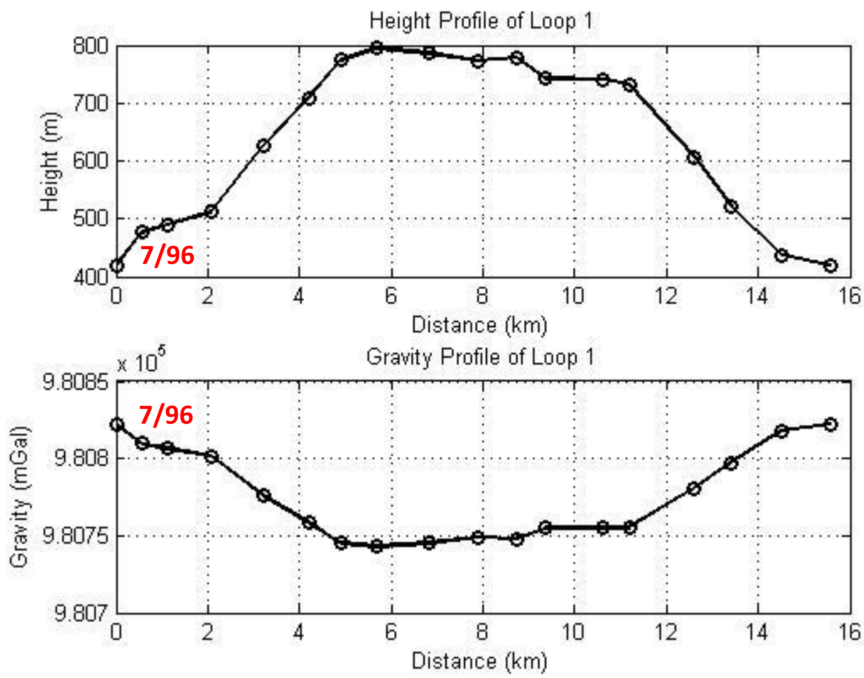

Figure 4.1 Height and gravity profile of loop 1

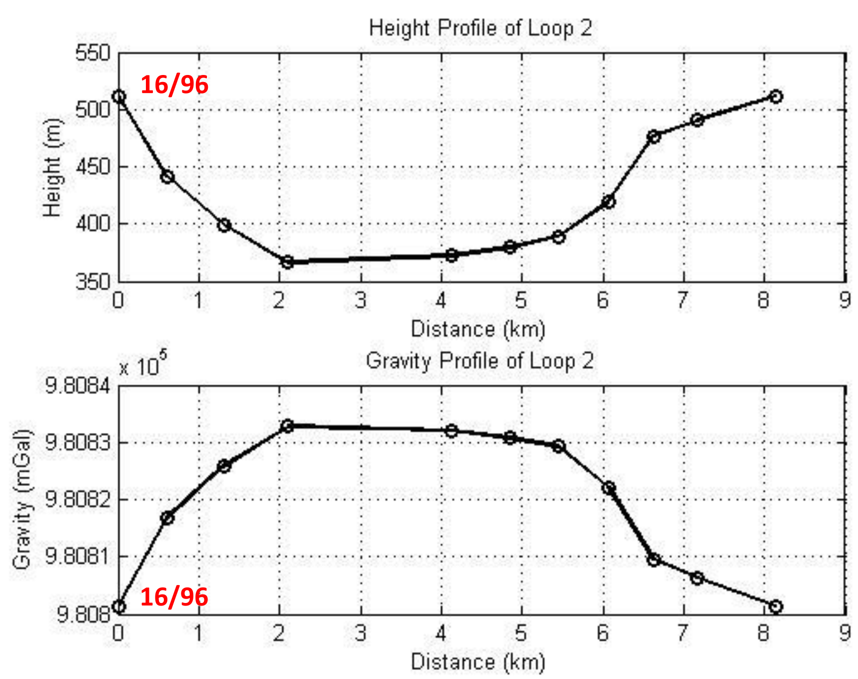

Figure 4.2 Height and gravity profile of loop 2

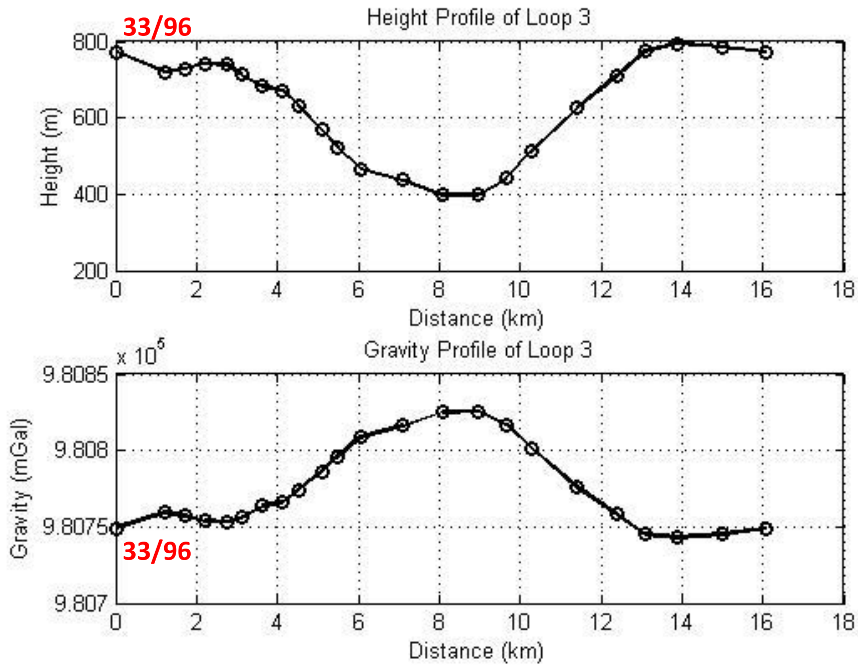

Figure 4.3 Height and gravity profile of loop 3

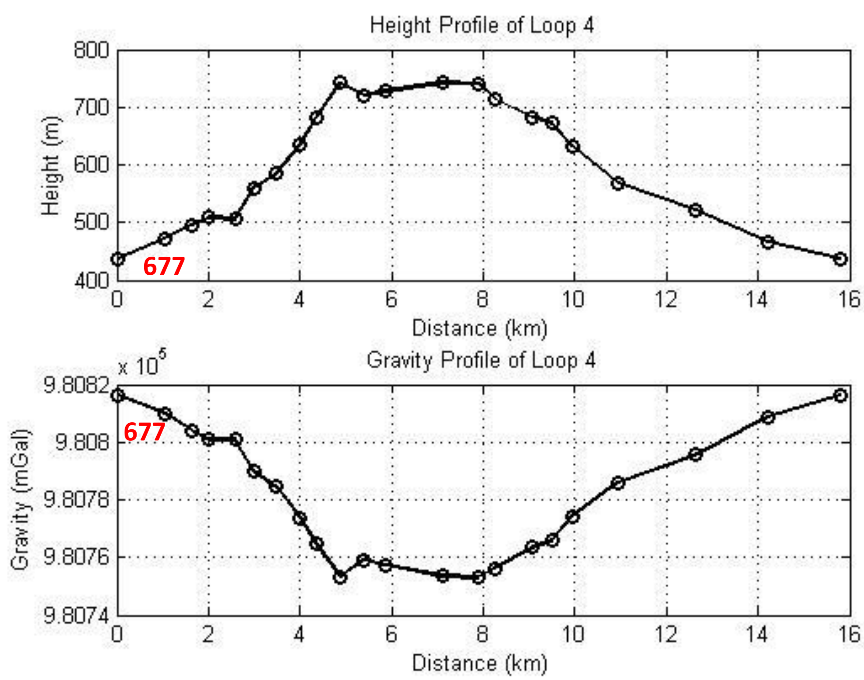

Figure 4.4 Height and gravity profile of loop 4

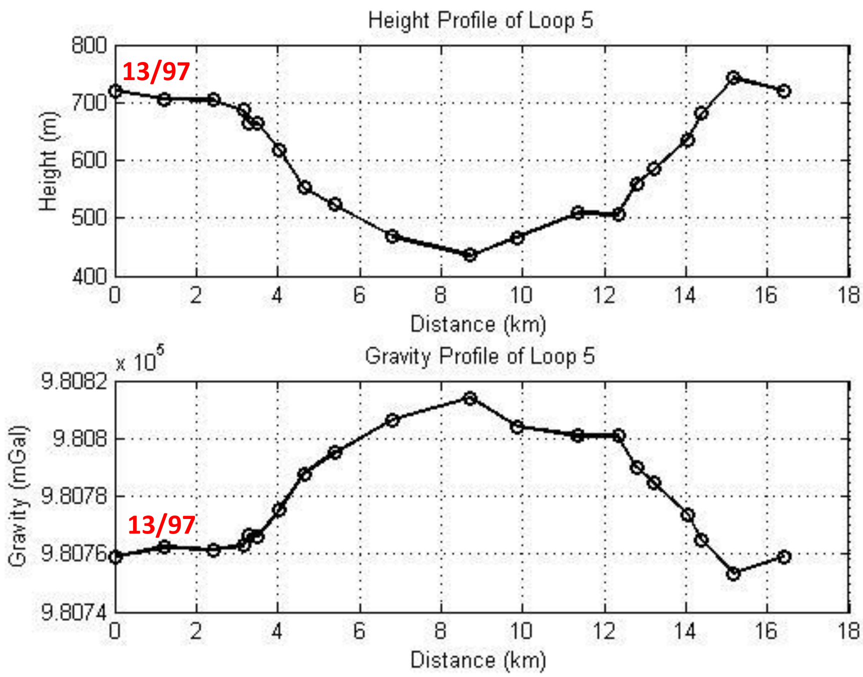

Figure 4.5 Height and gravity profile of loop 5

In figure 4.1 to figure 4.5, the start point of each loop is indicated by red point number in the image. For instance, in figure 4.1 point 7/96 is the start point of the loop and it is also the end point so the height and gravity value at the start point and the last point in the figure are equal. The figures also show that the gravity profile is a vertical flipped image of the height profile. This is because the higher the elevation from the earth surface, the lower the gravity value and on the other hand, the closer to the earth core, the higher the gravity values.

The average gravity values along the plumb line and the average normal gravity along the plumbline that were computed from unweighted and weighted adjustment have differences less than sub-mGal so the differencesof the results are not significant.

Figure 4.6 shows the strange gravity value of point 402 which is hilighted in the red circle. The gravity value at the mentioned point is very much higher than the adjacent points (point 16/96 and 596). It is clearly shown that the gravity value at point 402 is incorrect because the gravity profile has to be a vertical flipped image of height profile. So the gravity values at point 402 which occured in loop 2 and loop 3 has to be is 980816.727 mGal instead of 980861.727 mGal because the point 16/96 which is higher than point 402 has gravity value of 980801.338 mGal and also point 596 which is lower than point 402 has gravity value of 980825.828 mGal so the gravity value at point 402 has to be in between the mentioned adjacent points. Although the incorrect gravity value at point 402 was used in height computation, the difference of height computations between the correct and incorrect gravity value is in the range of millimeter.

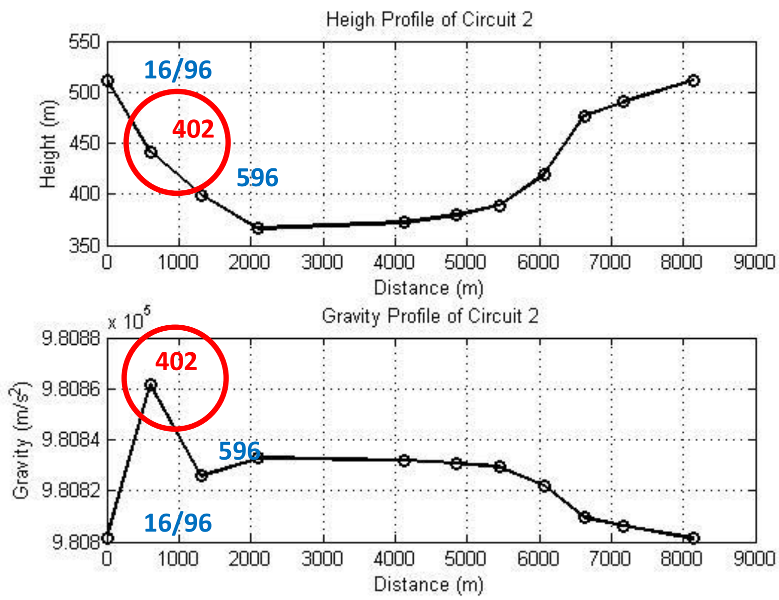

Figure 4.6 The incorrect gravity value at point 402 at loop 2

4.5 Height correction results

The results of height correction computation are shown in table 4.19 to table 4.23 and figure 4.7 to figure 4.11

Point No.	Unweighted (m)			Weighted (m)		
	DC	OC	NC	DC	OC	NC
96/7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
96/12	0.0111	0.0052	0.0008	0.0111	0.0052	0.0008
96/14	0.0138	0.0067	0.0011	0.0138	0.0067	0.0011
96/16	0.0177	0.0091	0.0015	0.0177	0.0091	0.0015
96/19	0.0358	0.0217	0.0035	0.0358	0.0217	0.0035
96/22	0.0476	0.0325	0.0060	0.0476	0.0325	0.0060
96/24	0.0560	0.0417	0.0086	0.0560	0.0417	0.0086
96/27	0.0585	0.0430	0.0096	0.0585	0.0430	0.0096
96/30	0.0573	0.0417	0.0091	0.0573	0.0417	0.0091
96/33	0.0556	0.0384	0.0084	0.0556	0.0384	0.0084
96/36	0.0564	0.0392	0.0087	0.0564	0.0392	0.0087
96/38	0.0513	0.0339	0.0067	0.0513	0.0339	0.0067
96/42	0.0512	0.0338	0.0066	0.0512	0.0338	0.0066
96/43	0.0501	0.0338	0.0062	0.0501	0.0338	0.0062
96/51	0.0293	0.0155	-0.0006	0.0293	0.0155	-0.0006
96/50	0.0142	0.0057	-0.0037	0.0142	0.0057	-0.0037
96/2	-0.0030	-0.0051	-0.0064	-0.0030	-0.0051	-0.0064
96/7	-0.0068	-0.0068	-0.0068	-0.0068	-0.0068	-0.0068

Table 4.19 Height corrections of loop 1

Point No.	Unweighted (m)			Weighted (m)		
	DC	OC	NC	DC	OC	NC
96/16	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
402	-0.0141	-0.0077	-0.0021	-0.0141	-0.0077	-0.0021
596	-0.0231	-0.0117	-0.0030	-0.0231	-0.0117	-0.0030
206	-0.0301	-0.0144	-0.0035	-0.0301	-0.0144	-0.0035
208	-0.0288	-0.0141	-0.0035	-0.0288	-0.0141	-0.0035
209	-0.0273	-0.0137	-0.0034	-0.0273	-0.0137	-0.0034
210	-0.0253	-0.0131	-0.0032	-0.0253	-0.0131	-0.0032
96/7	-0.0191	-0.0104	-0.0029	-0.0191	-0.0104	-0.0029
96/12	-0.0080	-0.0052	-0.0021	-0.0080	-0.0052	-0.0021
96/14	-0.0054	-0.0037	-0.0018	-0.0054	-0.0037	-0.0018
96/16	-0.0014	-0.0014	-0.0014	-0.0014	-0.0014	-0.0014

Table 4.20 Height corrections of loop 2

Point No.	Unweighted (m)			Weighted (m)		
	DC	OC	NC	DC	OC	NC
96/33	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
13/97	-0.0074	-0.0076	-0.0029	-0.0074	-0.0076	-0.0029
11/99	-0.0062	-0.0063	-0.0025	-0.0062	-0.0063	-0.0025
10/99	-0.0043	-0.0036	-0.0019	-0.0043	-0.0036	-0.0019
9/99	-0.0046	-0.0032	-0.0020	-0.0046	-0.0032	-0.0020
8/99	-0.0083	-0.0054	-0.0032	-0.0083	-0.0054	-0.0032
7/99	-0.0129	-0.0105	-0.0046	-0.0129	-0.0105	-0.0046
6/99	-0.0143	-0.0120	-0.0050	-0.0143	-0.0120	-0.0050
5/99	-0.0207	-0.0177	-0.0067	-0.0207	-0.0177	-0.0067
4/99	-0.0316	-0.0248	-0.0091	-0.0316	-0.0248	-0.0091
3/99	-0.0398	-0.0302	-0.0105	-0.0398	-0.0302	-0.0105
2/99	-0.0505	-0.0369	-0.0119	-0.0505	-0.0369	-0.0119
677	-0.0565	-0.0404	-0.0126	-0.0565	-0.0404	-0.0126
685	-0.0647	-0.0443	-0.0134	-0.0647	-0.0443	-0.0134
596	-0.0644	-0.0445	-0.0133	-0.0644	-0.0445	-0.0133
402	-0.0558	-0.0409	-0.0128	-0.0558	-0.0409	-0.0128
96/16	-0.0428	-0.0343	-0.0118	-0.0428	-0.0343	-0.0118
96/19	-0.0247	-0.0216	-0.0099	-0.0247	-0.0216	-0.0099
96/22	-0.0130	-0.0108	-0.0074	-0.0130	-0.0108	-0.0074
96/24	-0.0046	-0.0016	-0.0048	-0.0046	-0.0016	-0.0048
96/27	-0.0021	-0.0003	-0.0037	-0.0021	-0.0003	-0.0037
96/30	-0.0032	-0.0017	-0.0042	-0.0032	-0.0017	-0.0042
96/33	-0.0050	-0.0050	-0.0050	-0.0050	-0.0050	-0.0050

Table 4.21 Height corrections of loop 3

Point No.	Unweighted (m)			Weighted (m)		
	DC	OC	NC	DC	OC	NC
677	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
690	0.0068	0.0027	0.0006	0.0068	0.0027	0.0006
603	0.0112	0.0055	0.0010	0.0112	0.0055	0.0010
584	0.0137	0.0071	0.0013	0.0137	0.0071	0.0013
97/1	0.0130	0.0070	0.0012	0.0130	0.0070	0.0012
97/3	0.0224	0.0125	0.0023	0.0224	0.0125	0.0023
97/4	0.0266	0.0154	0.0029	0.0266	0.0154	0.0029
97/5	0.0346	0.0215	0.0043	0.0346	0.0215	0.0043
97/7	0.0413	0.0271	0.0059	0.0413	0.0271	0.0059
97/10	0.0497	0.0346	0.0083	0.0497	0.0346	0.0083
13/97	0.0466	0.0303	0.0072	0.0466	0.0303	0.0072
11/99	0.0477	0.0316	0.0075	0.0477	0.0316	0.0075
10/99	0.0497	0.0343	0.0082	0.0497	0.0343	0.0082
9/99	0.0493	0.0347	0.0081	0.0493	0.0347	0.0081
8/99	0.0457	0.0325	0.0068	0.0457	0.0325	0.0068
7/99	0.0411	0.0274	0.0054	0.0411	0.0274	0.0054
6/99	0.0396	0.0259	0.0050	0.0396	0.0259	0.0050
5/99	0.0332	0.0202	0.0033	0.0332	0.0202	0.0033
4/99	0.0224	0.0131	0.0010	0.0224	0.0131	0.0010
3/99	0.0141	0.0077	-0.0004	0.0141	0.0077	-0.0004
2/99	0.0035	0.0010	-0.0019	0.0035	0.0010	-0.0019
677	-0.0025	-0.0025	-0.0025	-0.0025	-0.0025	-0.0025

Table 4.22 Height corrections of loop 4

Point No.	Unweighted (m)			Weighted (m)		
	DC	OC	NC	DC	OC	NC
13/97	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1/98	-0.0022	-0.0022	-0.0007	-0.0022	-0.0022	-0.0007
2/98	-0.0023	-0.0015	-0.0008	-0.0023	-0.0015	-0.0008
31/98	-0.0050	-0.0025	-0.0016	-0.0050	-0.0025	-0.0016
710	-0.0081	-0.0050	-0.0024	-0.0081	-0.0050	-0.0024
3/00	-0.0083	-0.0046	-0.0025	-0.0083	-0.0046	-0.0025
2/00	-0.0158	-0.0105	-0.0043	-0.0158	-0.0105	-0.0043
1/00	-0.0267	-0.0181	-0.0065	-0.0267	-0.0181	-0.0065
1000	-0.0320	-0.0221	-0.0073	-0.0320	-0.0221	-0.0073
600	-0.0424	-0.0279	-0.0087	-0.0424	-0.0279	-0.0087
580	-0.0490	-0.0314	-0.0093	-0.0490	-0.0314	-0.0093
582	-0.0432	-0.0272	-0.0090	-0.0432	-0.0272	-0.0090
584	-0.0354	-0.0257	-0.0083	-0.0354	-0.0257	-0.0083
97/1	-0.0360	-0.0258	-0.0084	-0.0360	-0.0258	-0.0084
97/3	-0.0266	-0.0203	-0.0073	-0.0266	-0.0203	-0.0073

Table 4.23 Height corrections of loop 5

Point No.	Unweighted (m)			Weighted (m)		
	DC	OC	NC	DC	OC	NC
97/4	-0.0224	-0.0173	-0.0067	-0.0224	-0.0173	-0.0067
97/5	-0.0145	-0.0113	-0.0053	-0.0145	-0.0113	-0.0053
97/7	-0.0077	-0.0056	-0.0037	-0.0077	-0.0056	-0.0037
97/10	0.0006	0.0018	-0.0013	0.0006	0.0018	-0.0013
13/97	-0.0024	-0.0024	-0.0024	-0.0024	-0.0024	-0.0024

Table 4.23 Height corrections of loop 5 (con't)

The graphics of height corrections are illustrated in figure 4.7 to figure 4.11 in the following:

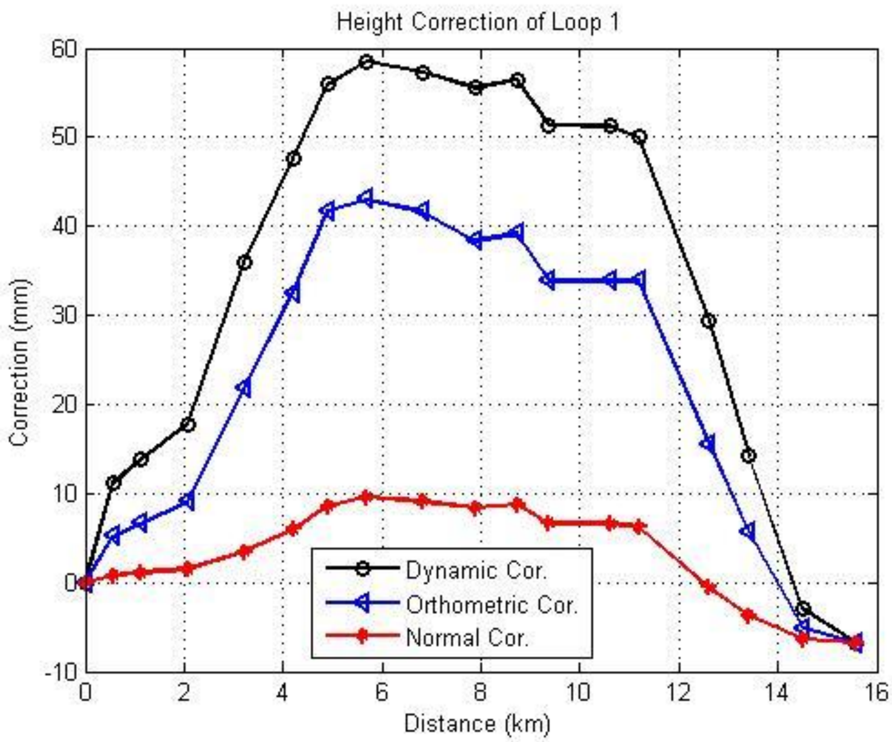

Figure 4.7 Height corrections of loop 1

The figure above shows that height corrections start at zero at the start point because there is always no correction at the beginning and they have the same values at the end point. The figure also reveals that dynamic height corrections have the highest values while normal height corrections give the lowest correction values and the orthometric corrections are in between those two corrections. Dynamic corrections have the highest values because the test area is at latitude 48.485° but normal gravity at latitude 45° is applied in dynamic height computation. The further from the latitude of 45°, the higher the dynamic correction. In some areas the dynamic correction is up to several centimeters. The results of loop 2 to loop 5 also show the same figure and the explanation as in loop 1. The results of the other loops are illustrated in figure 4.8 to figure 4.11.

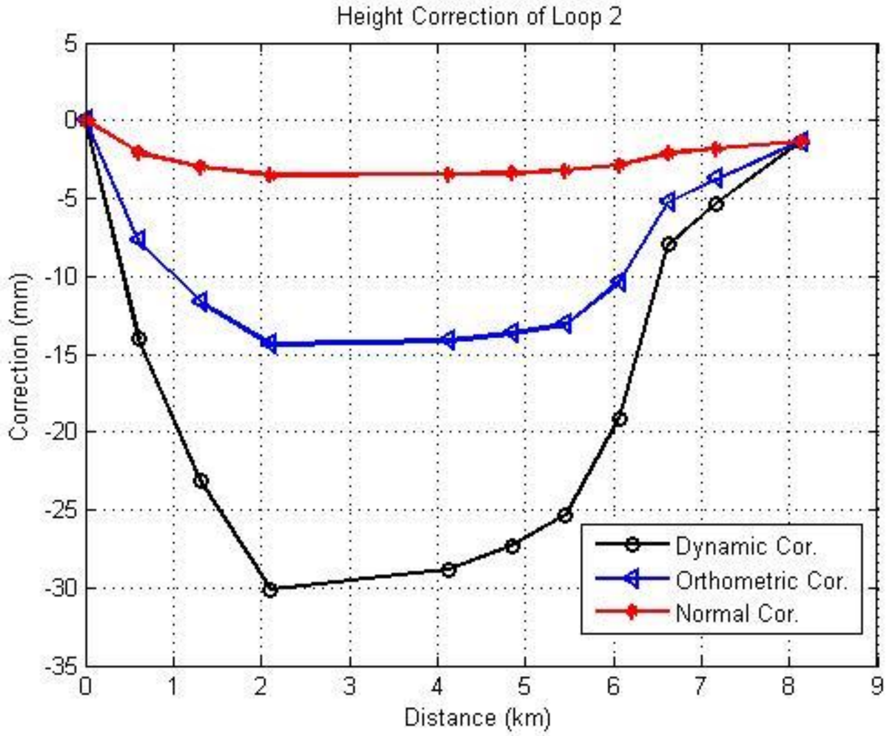

Figure 4.8 Height corrections of loop 2

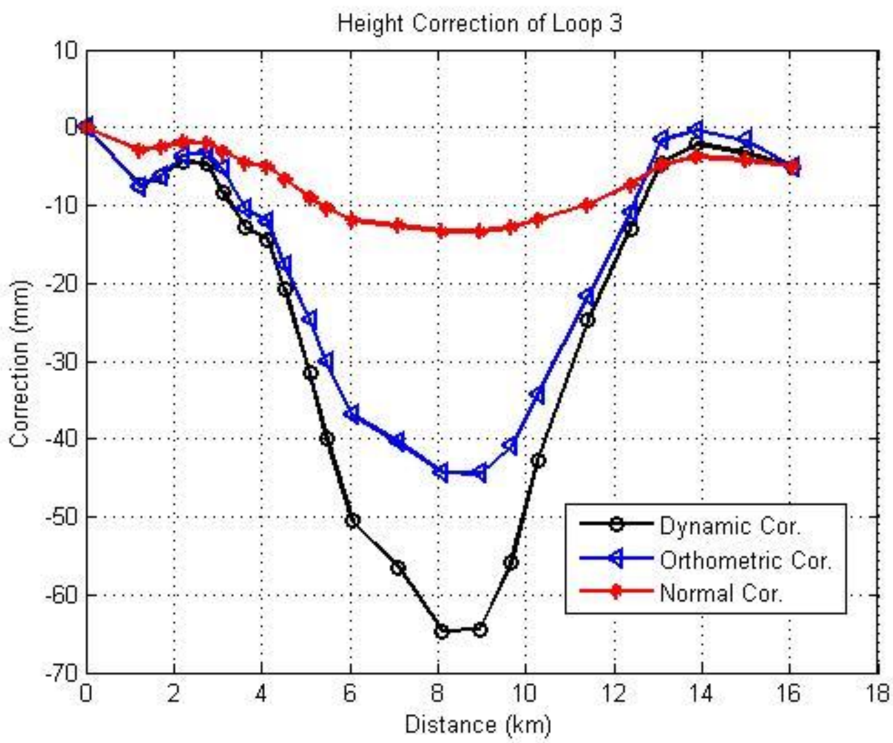

Figure 4.9 Height corrections of loop 3

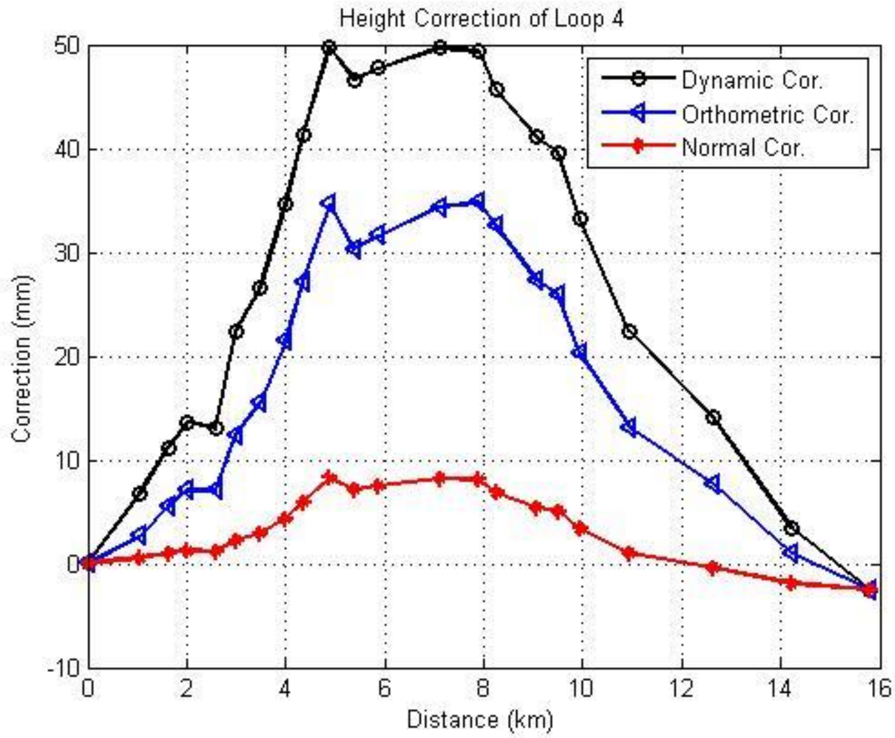

Figure 4.10 Height corrections of loop 4

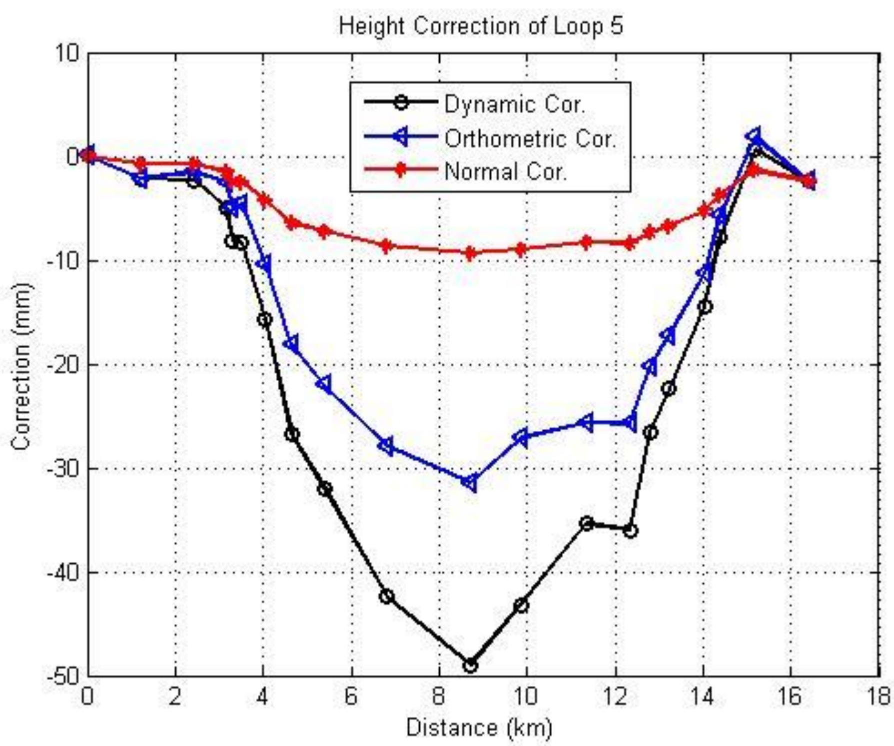

Figure 4.11 Height corrections of loop 5

Chapter 5

Summary of the Results and Conclusions

5.1 Summary of the results

After the adjustment of the geopotential difference were done, then the geopotential numbers at each points were computed. The results of the geopotential numbers which derived from unweighted and weighted adjustment are shown in table 5.1**.**

Point No.	Gravity (mGal)	Unweighted Geopotential Numbers $C_{UW}\left[\mathrm{m}^2/\mathrm{s}^2\right]$	Weighted Geopotential Numbers $C_{W}\left[\mathrm{m}^2/\mathrm{s}^2\right]$
96/7	980822.1200	4109.0586	4109.0582
96/12	980809.4920	4674.5255	4674.5260
96/14	980806.3050	4808.9603	4808.9619
96/16	980801.3377	5020.8781	5020.8758
96/19	980775.9485	6136.6609	6136.6623
96/22	980758.3200	6953.5924	6953.5939
96/24	980745.0010	7598.3859	7598.3863
96/27	980743.1910	7792.9267	7792.9292
96/30	980744.9410	7703.9306	7703.9344
96/33	980749.1680	7573.6434	7573.6476
96/36	980748.0340	7638.0844	7638.0882
96/38	980755.0750	7275.0909	7275.0945
96/42	980755.1420	7266.9917	7266.9944
96/43	980755.2230	7184.6999	7184.7025
96/51	980780.6580	5942.8423	5942.8439
96/50	980797.1770	5122.3198	5122.3210
96/2	980818.2410	4289.6634	4289.6638
402	980816.7270	4333.8538	4333.8624
596	980825.8080	3912.3421	3912.3550
206	980832.7250	3595.4550	3595.4662
208	980832.0300	3652.7134	3652.7133
209	980830.7640	3723.0540	3723.0531
210	980829.2090	3814.1747	3814.1741
13/97	980759.1979	7064.6935	7064.6997
11/99	980757.3780	7143.1585	7143.1653
10/99	980753.6660	7282.8235	7282.8308
9/99	980753.1040	7259.9116	7259.9194
8/99	980756.2300	6999.6267	6999.6350
7/99	980763.6090	6692.6581	6692.6670

Table 5.1 Gravity and geopotential numbers

Point No.	Gravity (mGal)	Unweighted Geopotential Numbers $C_{UW}\left[\mathrm{m}^2/\mathrm{s}^2\right]$	Weighted Geopotential Numbers $C_W\left[\mathrm{m}^2/\mathrm{s}^2\right]$
6/99	980765.7790	6597.7628	6597.7722
5/99	980774.3720	6200.0816	6200.0915
4/99	980785.9000	5569.3061	5569.3165
3/99	980795.6620	5118.5037	5118.5147
2/99	980808.7580	4575.9911	4576.0026
677	980816.4085	4281.7390	4281.7511
685	980825.3500	3900.2140	3900.2268
690	980810.0170	4626.7808	4626.7899
603	980804.0470	4854.9451	4854.9513
584	980800.7490	4986.1149	4986.1177
97/1	980800.9050	4953.6149	4953.6177
97/3	980789.8720	5483.3431	5483.3468
97/4	980784.4120	5728.6345	5728.6393
97/5	980773.8650	6227.1439	6227.1489
97/7	980764.6800	6675.1495	6675.1557
97/10	980753.4110	7275.6542	7275.6607
1/98	980762.2735	6917.7015	6917.7087
2/98	980761.4200	6910.2051	6910.2134
31/98	980762.9018	6730.5362	6730.5437
710	980766.6897	6522.6734	6522.6764
3/00	980766.0650	6509.9642	6509.9633
2/00	980775.1440	6048.3730	6048.3702
1/00	980787.7680	5423.5235	5423.5193
1000	980795.1141	5131.1639	5131.1589
600	980806.4790	4593.7131	4593.7099
580	980814.0290	4268.3004	4268.3004
582	980804.1910	4572.8773	4572.8781

Table 5.1 Gravity and geopotential numbers (con't)

After geopotential numbers were computed, the height systems could be determined afterward. The results of height systems of all 56 points are shown in table 5.2.

Point No.	Unweighted (m)			Weighted (m)		
	Dynamic	Orthometric	Normal	Dynamic	Orthometric	Normal
96/7	419.0266	418.9395	418.9198	419.0266	418.9394	418.9197
96/12	476.6909	476.5977	476.5736	476.6909	476.5978	476.5737
96/14	490.4000	490.3058	490.2805	490.4002	490.3060	490.2806
96/16	512.0106	511.9148	511.8875	512.0104	511.9145	511.8873
96/19	625.7940	625.6928	625.6548	625.7942	625.6929	625.6549
96/22	709.1017	708.9994	708.9532	709.1018	708.9996	708.9534
96/24	774.8553	774.7539	774.7011	774.8554	774.7540	774.7011

Table 5.2 Height systems

Point No.	Unweighted (m)			Weighted (m)		
	Dynamic	Orthometric	Normal	Dynamic	Orthometric	Normal
96/27	794.6939	794.5913	794.5382	794.6942	794.5915	794.5384
96/30	785.6184	785.5156	785.4634	785.6188	785.5160	785.4637
96/33	772.3322	772.2278	772.1781	772.3326	772.2283	772.1786
96/36	778.9037	778.7993	778.7491	778.9040	778.7997	778.7495
96/38	741.8869	741.7823	741.7354	741.8873	741.7827	741.7358
96/42	741.0610	740.9564	740.9095	741.0613	740.9567	740.9098
96/43	732.6692	732.5658	732.5185	732.6694	732.5660	732.5187
96/51	606.0291	605.9282	605.8924	606.0293	605.9284	605.8926
96/50	522.3553	522.2597	522.2306	522.3554	522.2598	522.2307
96/2	437.4440	437.3548	437.3338	437.4441	437.3548	437.3338
402	441.9504	441.8609	441.8393	441.9513	441.8618	441.8402
596	398.9662	398.8818	398.8632	398.9675	398.8831	398.8645
206	366.6512	366.5711	366.5547	366.6524	366.5722	366.5559
208	372.4902	372.4091	372.3925	372.4902	372.4091	372.3925
209	379.6633	379.5811	379.5641	379.6632	379.5810	379.5641
210	388.9555	388.8718	388.8544	388.9554	388.8717	388.8544
13/97	720.4314	720.3268	720.2818	720.4320	720.3274	720.2824
11/99	728.4329	728.3286	728.2826	728.4336	728.3292	728.2833
10/99	742.6755	742.5718	742.5239	742.6762	742.5725	742.5246
9/99	740.3390	740.2361	740.1876	740.3398	740.2369	740.1884
8/99	713.7961	713.6947	713.6471	713.7969	713.6955	713.6480
7/99	682.4926	682.3906	682.3468	682.4935	682.3915	682.3477
6/99	672.8155	672.7135	672.6708	672.8165	672.7144	672.6717
5/99	632.2614	632.1601	632.1214	632.2624	632.1612	632.1224
4/99	567.9373	567.8398	567.8057	567.9383	567.8408	567.8068
3/99	521.9661	521.8714	521.8415	521.9672	521.8725	521.8426
2/99	466.6427	466.5519	466.5272	466.6439	466.5531	466.5284
677	436.6359	436.5476	436.5258	436.6372	436.5489	436.5270
685	397.7294	397.6454	397.6267	397.7307	397.6467	397.6280
690	471.8220	471.7296	471.7056	471.8230	471.7306	471.7066
603	495.0894	494.9954	494.9691	495.0900	494.9960	494.9697
584	508.4656	508.3707	508.3431	508.4659	508.3710	508.3434
97/1	505.1514	505.0570	505.0294	505.1516	505.0573	505.0297
97/3	559.1711	559.0728	559.0408	559.1715	559.0732	559.0412
97/4	584.1850	584.0856	584.0512	584.1855	584.0860	584.0517
97/5	635.0212	634.9197	634.8808	635.0217	634.9202	634.8813
97/7	680.7071	680.6046	680.5615	680.7077	680.6053	680.5622
97/10	741.9444	741.8410	741.7928	741.9450	741.8417	741.7935
1/98	705.4417	705.3371	705.2935	705.4424	705.3379	705.2943
2/98	704.6772	704.5734	704.5292	704.6781	704.5742	704.5300
31/98	686.3552	686.2531	686.2091	686.3560	686.2539	686.2098
710	665.1582	665.0567	665.0143	665.1585	665.0570	665.0146
3/00	663.8621	663.7613	663.7184	663.8620	663.7612	663.7183
2/00	616.7907	616.6915	616.6526	616.7905	616.6912	616.6524

Table 5.2 Height systems (con't)

Point No.	Unweighted (m)			Weighted (m)		
	Dynamic	Orthometric	Normal	Dynamic	Orthometric	Normal
1/00	553.0709	552.9749	552.9415	553.0705	552.9745	552.9411
1000	523.2572	523.1625	523.1323	523.2566	523.1620	523.1318
600	468.4499	468.3599	468.3341	468.4496	468.3595	468.3338
580	435.2655	435.1786	435.1556	435.2655	435.1786	435.1556
582	466.3251	466.2366	466.2097	466.3252	466.2367	466.2098

Table 5.2 Height systems (con't)

5.2 Conclusions

The results in chapter 4 can be concluded as follows;

- There are 56 out of 121 observed points that have been selected for height computations, the data in 2008 to 2013 were neglected because trigonometric leveling was adopted for height differences computation.

- In the adjustment of various closed loops with many data, the condition equation is preferred to the observation equation because the size of the design matrix is smaller and the loop misclosures can be computed by the condition equation. The initial size of the design matrix of the observation equation (A-matrix), condition equation (B-matrix), and weight matrix in this project are $[77 \times 56]$, $[22 \times 77]$, and $[77 \times 77]$, respectively. After the unreliable data were eliminated due to the loop misclosures are exceed the allowable misclosure, the size of observation equation is reduced to $[64 \times 56]$while the size of the condition equation (B-matrix) is reduced to $[9 \times 64]$, and the size of the weight matrix is reduced to $[64 \times 64]$. It is clearly shown that the size of condition equation is significant smaller than the observation equation. Within the condition equation it is also easier to check the coefficients of the design matrix in case the results are not satisfied, e.g. the squared sum of the residuals is very high or even creating a new design matrix if the error of the coefficients could not be found, but it is time consuming to check the error of the design matrix coefficients in the observation equation. Thus, the condition equation is recommended for closed loop adjustment. The results from the condition equation and the observation equation are equal as well as the squared sum of the residuals.

- It has to be noted that the gravity value of point 402 is 980816.727 mGal instead 9980861.727 mGal as appeared in the data sheet. Point 16/96 and point 596 are the adjacent points of 402, the gravity values at point 16/96 and 596 are 980801.338 mGal and 980825.828 mGal, respectively. Point 16/96 is at the highest elevation and point 596 is at the lowest elevation so the gravity value of point 402 has to be larger than the gravity value at point 16/96 and smaller than point 596. Thus, the gravity value of 9980861.727 mGal at point 402 is unreliable, this happened because of human error during the data recording.

- The difference of the mean gravity along the plumb line $(\bar{g})$ and the mean normal gravity along the plumb line $(\bar{\gamma})$ between between unweighted and weighted adjustment are not significant.

- The difference of potential numbers (C) between unweighted and weighted adjustment are up to 0.0129 m^2/s^2 or 0.0013 m in metric unit. It can be concluded that the adjusted potential numbers are not significantly affected by the weights that had been assigned in the adjustment. The maximum weight is 2.635 and the minimum weight is 0.644, the weights that are larger than 2 appeared twice and the weights less than 0.7 also appeared twice. The distances between the measured points are around 500 meters to 1600 meters so the weights are not dramatically different.

- The difference of height systems between unweighted and weighted adjustment are up to 0.0013 meter or 1.3 millimeters, mostly they are in the sub-millimeter range, so the height systems are not significantly affected by the weight.

- The differences of height corrections $(Corr.^{\#})$ between unweighted and weighted adjustment are approximately 10^{-8} m so it can be concluded that he height corrections from unweighted adjustment are similar to the height correction from weight adjustment
- Height corrections $(Corr.^{\#})$ are proportional to height differences (ΔH), the higher the heigh differences, the higher the height corrections.

- Normal corrections (NC) have the lowest values while dynamic corrections (DC) always give the highest value and orthometric correction values are in between of those two height corrections. Swabian Alb test area is located at latitude 48.485° while the normal gravity that has been used in dynamic height was computed at latitude 45°, The further from latitude 45°, the higher the dynamic height correction.

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