NONLOCAL FRACTURE ANALYSIS – IDENTIFICATION OF MATERIAL MODEL PARAMETERS

J. Ožbolt, M. Petrangeli and R. Eligehausen
Institut für Werkstoffe im Bauwesen, Stuttgart University, Germany

Abstract

In the present paper a short description of the nonlocal microcrack interaction approach is presented. The physical meaning of this approach when used in the smeared finite element code is discussed. The present nonlocal "averaging" consists of a long-range interaction (crack interaction function) and local averaging. The first controls the crack propagation and the second is responsible for the correct energy consumption due to cracking. It is demonstrated that the nonlocal material model parameters may be identified by usual concrete fracture properties such as tensile strength, concrete fracture energy and maximum aggregate size. The nonlocal microcrack interaction approach is compared with the nonlocal strain approach and a possible simplification is considered.

1. Introduction

To simulate brittle failures, the finite element code based on the smeared crack approach must contain a mathematical device which prevents localization of damage into a zone of zero volume – called localization limiters. Currently in engineering practice most of the finite element codes are based on the local continuum approach. However, there are many evidences that these codes cannot always correctly simulate brittle failure of concrete structures (Rots, 1988; de Borst, 1991; Ožbolt and Eligehausen, 1991). Therefore, the nonlocal continuum concept has recently been introduced as a promising general concept for macroscopic modeling of fracture process in quasibrittle materials such as concrete.

Recently a few forms of the nonlocal concept, in which all variables that are associated with strain softening are nonlocal and all other variables are local, has been introduced (Pijaudier-Cabot and Bažant, 1987; De

Borst, 1991; De Borst and Muhlhaus, 1992). The governing parameter in these approaches is the characteristic length l over which the strains are averaged. However, l is difficult to interpret as a material parameter depending on the concrete mix only, but may be influenced by other physical mechanisms as well. Recently, it has been demonstrated that the nonlocality in the fracture process zone can be expressed in terms of microcrack interactions (Bažant, 1991). Therefore, the new nonlocal microcrack interaction approach is developed and implemented into the finite element code (Bažant, 1992; Ožbolt, 1993). However, in spite of the fact that the new nonlocal approach finds its physical ground in the microcrack interaction, the main problem remains the identification of the nonlocal material model parameters with usual macroscopical material properties such as tensile strength and fracture energy.

In the present paper a short description of the new nonlocal concept is presented and the possible simplifications are discussed. Attention is focused on the identification of the nonlocal material model properties with usual concrete properties such as tensile strength, fracture energy and maximum aggregate size.

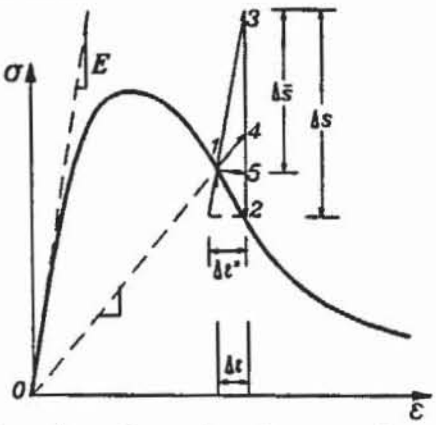

Fig. 1 Local and nonlocal stress increments

2. Nonlocality due to the microcrack interaction

During the softening process in the quasibrittle materials a number of microcracks develop. These microcracks interact up to the final failure when one or more dominant macrocracks form. Therefore, the nonlocality may be interpreted by the fact that the microcrack opening at place (μ) can stabilize or destabilize the microcrack development in the neighborhood point (ν). Using the principle of superposition (Kachanov, 1987) each loading step can be decomposed in two substeps: (1) In the first substep for a strain increase $\Delta\epsilon$ (see Fig. 1) one can imagine that all the cracks are temporarily closed and the stress increase $\Delta\sigma$ is found as the corresponding elastic stress increment i.e. the cracks are able to transfer the stresses. (2) In the second substep the stresses over the microcrack surfaces are relaxed (unfrozen). This is equivalent to applying the pressure over the crack surface. If during the load step no crack growth occurs unfrozening (substep 2) would result in a stress drop ΔS_{34} (see Fig. 1). However if the cracks grow, the larger stress drop ΔS_{32} takes place. This stress drop is defined by the local microcrack stress-strain law. Crack opening at one place cause crack opening or closing at another position i.e. microcracks are interacting. Due to this, the actual stress drop is defined by the ΔS_{35} – nonlocal stress drop.

If $\Delta \boldsymbol{S}_\mu$ represents the local nonelastic stress increment tensor, then the normal crack surface traction at place μ can be calculated as:

$$\Delta p_\mu = \boldsymbol{n}_\mu \Delta \boldsymbol{S}_\mu \boldsymbol{n}_\mu \tag{1}$$

with $\boldsymbol{n}_\mu$ to be a unit normal to the crack surface at place μ. Since Δp_μ represents only a local stress drop at place μ, subsequent stress drop due to the unfrozening of the microcracks at places ν should be added. Using superposition principle and two important simplifications introduced by Bažant (1992): (1) Constant stress Δp_μ along the crack surface and (2) Considerations of Mode I only, the total nonlocal inelastic stress increment $\Delta \overline{p}_\mu$ is obtained as:

$$\Delta \overline{p}_\mu = \langle \Delta p_\mu \rangle + \sum_{\nu \neq \mu} \Lambda_{\mu\nu} \Delta \overline{p}_\nu \tag{2}$$

in which $\langle .. \rangle$ is the averaging operator over the crack length, μ= 1,...N, ν= 1,...N with N= total number of microcracks and $\Lambda_{\mu\nu}$ are the crack influence coefficients representing the average pressure at the frozen crack μ caused by a unit uniform pressure applied on unfrozen crack ν with all the other cracks being frozen.

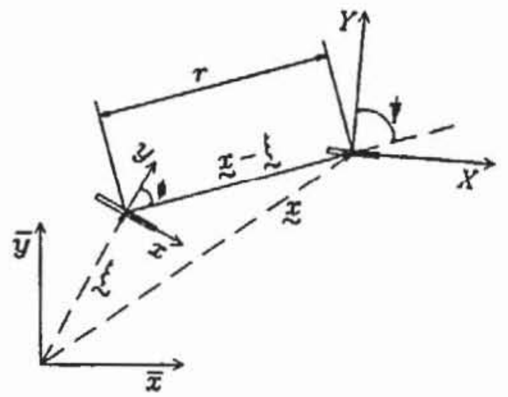

Fig. 2 Microcrack interaction

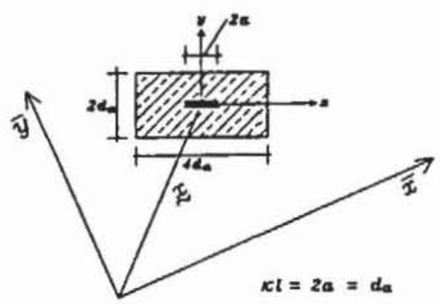

Fig. 3 Local averaging volume

For the use in the continuum fracture finite element analysis, Eq. 2 has to be modified following two assumptions: (1) The dominant microcrack direction, at each integration point, is normal to the direction of the total maximum principal strain $\epsilon^{(1)}$. (2) Instead of working with tractions perpendicular to the microcrack (Δp) one has to work with the projection of the nonelastic stress increment tensor in direction perpendicular to the microcrack ($\Delta S^{(1)}$). Accepting this Eq. 2 can be rewritten as:

$$\Delta \overline{S}_{\mu}^{(1)} = \langle \Delta S_{\mu}^{(1)} \rangle + \sum_{\nu=1}^{N} \frac{1}{V_{c\mu}} \Lambda_{\mu\nu} \Delta \overline{S}_{\nu}^{(1)} \Delta V_{\nu} \tag{3}$$

where $V_{c\mu} = \sum_{\mu=1}^{N} \Delta V_{\mu}$ = the variable which can be interpreted as an average representative crack volume and it must be introduced in order to smooth-out (average) the contribution of the randomly distributed microcracks caused by concrete inhomogeneity.

The nonlocal nonelastic stress increment ($\Delta \overline{S}_{\mu}^{(1)}$) consists of an average local nonelastic stress increment ($\langle \Delta S_{\mu}^{(1)} \rangle$) and the nonelastic stress increment due to the microcrack interaction. The "averaging" applies only if both interacting microcracks are opening i.e. for positive strain increment the nonelastic stress increment must be negative (increase of damage). In Eq. 3, $\Lambda_{\mu\nu}$ are the coefficients of the crack interaction function Λ. For the purpose of the smeared macrocontinuous analysis some properties of this function, which control microcrack interaction over a distance, must be preserved and another have to be simplified (Bažant, 1992). Preserved must be the long-range asymptotic property. However, close-range properties of this function must be simplified since on the macroscale it is impossible to deal with microcracks of the finite size. Therefore, the singularities near the crack tip have to be smoothed out. Generally, the crack interaction function is of the form:

$$\Lambda(\boldsymbol{x}, \boldsymbol{\xi}) = k(r) f(\phi) \tag{4}$$

where r and ϕ are the polar coordinates with the origin in the center of the crack and measured from the crack direction (see Fig. 2), $\boldsymbol{x}$ is the position where the unit stress ($\sigma = 1$) perpendicular to the crack length is applied (unfrozen crack) and $\boldsymbol{\xi}$ is the position of the frozen microcrack at which the stress is calculated. In order to avoid singularity at the crack tip and preserve asymptotic behavior it is reasonable to assume $k(r)$ function as (Bažant, 1992):

$$k(r) = \left(\frac{\kappa l r}{r^2 + l^2} \right) \tag{5}$$

where l is an empirical constant which can be identified with what has been called the *characteristic length* of the nonlocal continuum (Pijaudier-Cabot and Bažant, 1987) and κ is an empirical parameter such that κl roughly represents the average of effective microcrack length $2a$ and may be approximately taken as a maximum aggregate size d_a.

An important property of the Λ function is its positive value in the direction of microcrack and its negative value in the direction perpendicular to the microcrack (shielding). In this way, the function controls the crack propagation i.e. it controls the "averaging" in the softening continuum in which the local analysis has no meaning. This is of great importance for the correct representation of cracking in the case when a number of microcracks form one open macrocrack as well as in the case of complicated stress-strain situations. Λ has a nonnegligeble value inside a volume of a diameter $D \approx 8\kappa l \approx 8d_a$ and, therefore, it is responsible for the long range interaction.

The discrete microcrack system operator $\langle .. \rangle$ in Eq. 2 is used in order to average the stress along the microcrack length. In the smeared crack analysis, however, one must introduce a local averaging volume. Namely, due to concrete inhomogeneity, the microcracks are randomly distributed and the energy consumption required for a unit macrocrack propagation is taking place inside the material volume whose size and shape is dependent on the aggregate size and shape as well as on the stress-strain state. Therefore, it is reasonable to assume that the local averaging volume depends on the aggregate size and the current stress-strain state.

Another reason why one needs the local averaging is due to the fact that the nonelastic stress increments are calculated using the local stress-strain law which is not related to any length, such as for example the stress-strain law employed in the crack band approach. Therefore, the local averaging in Eq. 3 is calculated as (Ožbolt, 1993):

$$\langle \Delta S_\mu^{(1)} \rangle = \frac{1}{V_\mu} \sum_{\nu=1}^{n} \Delta S_\nu^{(1)} \alpha_{\mu\nu} \Delta V_\nu \tag{6}$$

where $V_\mu = \sum_{\nu=1}^{n} \alpha_{\mu\nu} \Delta V_\nu =$ local representative volume, $n=$ number of integration points which define this volume and $\alpha_{\mu\nu}$ is a weighting function which assures continuity on the boundaries and accounts for the stronger contribution of the points which are closer to each other. Similar as in the nonlocal strain approach the weighting function may be taken as a bell shape function or as an exponential function. Based on experience, the local representative volume at the integration point μ is defined by the volume of a prism which has a length of $4d_a$, in the microcrack direction, and shorter length of $2d_a$ in the perpendicular directions (i.e. in the principle strain direction), with $d_a =$ max. aggregate size (see Fig. 3).

3. Identification of the nonlocal material model properties

The relation between the nonlocal input parameters and usual concrete properties, such as tensile strength and concrete fracture energy, is not possible to define in close form. Therefore, based on the present nonlocal physical background as well as on the current experience with the nonlocal analysis, it has been concluded: (1) The uniaxial tensile strength is mainly controlled by the local tensile strength and (2) The maximum aggregate size (the characteristic length) together with the area under the local stress-strain curve control the concrete fracture energy (G_F).

Under the assumption that the characteristic length in the fracture process zone is larger or at least equal to the minimum size of the finite element, the average length L_c in two dimensional analysis (area in 3D case) in which the fracture energy is consumed, is approximately equal to the area (volume in 3D case) in which the local averaging is performed divided by the average microcrack length (area in 3D case):

$$L_c = (2\kappa l)(4\kappa l)/(\kappa l) = 8\kappa l \tag{7}$$

or with $\kappa l = d_a$

$$L_c = (2d_a)(4d_a)/(d_a) = 8d_a \tag{8}$$

In the above equations it is assumed that the local averaging is performed inside the rectangle of $2\kappa l$ times $4\kappa l$ (see Fig. 3). Concrete fracture energy (G_F) can be now calculated as:

$$G_F = L_c g_f \tag{9}$$

with g_f= area under the local tensile stress-strain curve i.e. local microcrack fracture energy. Eq. 9 tells us that the concrete fracture energy (G_F), in terms of the smeared crack band fracture analysis, is consumed in a band of width L_c. Of course, one may argue that the size of the local averaging zone may be taken smaller or larger than assumed above. However, using the standard microplane model local stress-strain curve and specifying a realistic max. aggregate size for concrete, resulting G_F is approximately equal to a value which one would expect for that kind of concrete. Note, however, that this value can be increased or decreased simply by changing the area under the local stress-strain curve.

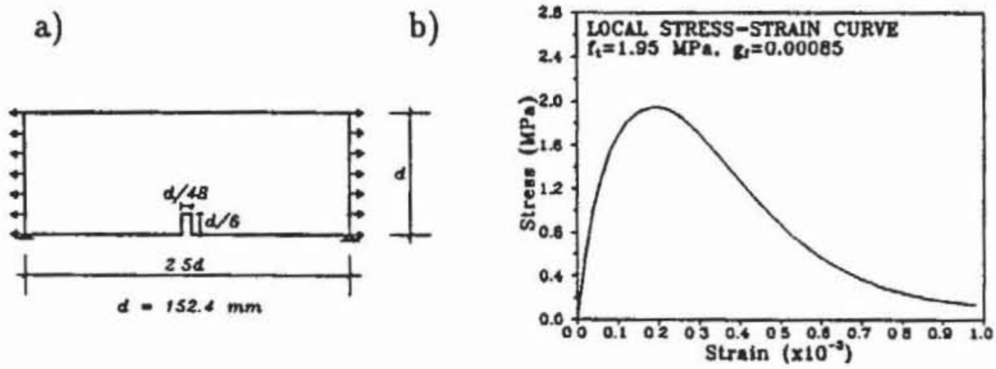

Fig. 4 Test specimen a) Geometry b) Local tensile stress-strain curve

The above assumptions have been checked on a simple specimen loaded in eccentric tension. In Figs. 4a geometry of the specimen, boundary conditions and applied loading are plotted. The specimen was loaded up to failure such that the symmetric failure mode in eccentric tension was enforced by prescribing horizontal displacement at the vertical sides of the specimen. The local material properties are taken as follows: concrete tensile strength $f_{t,loc}$ = 1.95 MPa, Young's modulus E= 27500 MPa, local concrete fracture energy g_f= 0.00085 MPa and the maximum aggregate size d_a= 9 mm. The local tensile stress-strain curve, obtained using microplane material model, is plotted in Fig. 4b. According to Eq. 9, the resulting concrete fracture energy is found to be $G_F = L_c g_f = 0.061$ N/mm.

The crack pattern and the calculated load-displacement curve at termination of the analysis are plotted in Figs. 5a,b. The area under the dashed line (unloading) and softening line (see Fig. 5a) represents the total energy consumed by fracture. The concrete fracture energy (G_F) is approximately equal to the total consumed fracture energy, obtained by integrating the area under the load – crack width diagram, divided by the net cross-section area of the specimen. Doing this one obtains $G_F \approx 0.065$ N/mm which is approximately the same as the value that has been calculated as input data. For the tensile strength calculated as $f_t = P_U/bd_{net}$, with P_U = calculated peak load, bd_{net} = net cross-section area of the specimen, one obtains f_t= 2.14 MPa. This is approximately the same as the value given by the local tensile strength (input data).

a)

b)

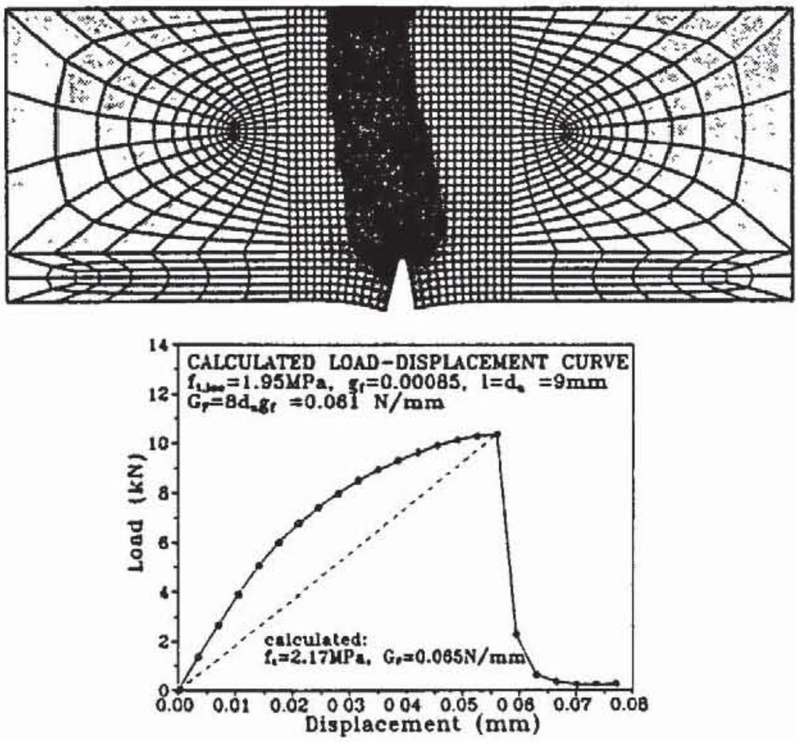

Fig. 5 Results a) Crack pattern b) Load-displacement curve

The above procedure for calculating G_F may be used under the condition that the minimum finite element size in the fracture process zone is at least equal to or smaller than the maximum aggregate size. If the minimum element size is larger than the max. size of the aggregate (what can happen when modeling a very large structure) one should increase the characteristic length such that it would be approx. equal to the min. element size ($l = d_a^*$). Doing this, one must replace the local concrete fracture energy g_f by the modified value g_f^* so that the fracture energy G_F would be equal to the value obtained when using Eq. 8 i.e.:

$$G_F = L_c(d_a)g_f = L_c(d_a^*)g_f^* \tag{10}$$

From Eq. 10 it follows that the modified local fracture energy must be equal to:

$$g_f^* = \frac{L_c(d_a)g_f}{L_c(d_a^*)} \tag{11}$$

The modified local fracture energy has to be obtained by keeping constant the hardening part of the stress-strain curve, as well as its tensile strength, and modifying the softening part of it only. The reason for this is due to the fact that the softening part of the local stress-strain curve mainly influence fracture energy G_F. If this modification is not possible, what can happen in the case when the min. element size is much larger than the aggregate size, one must refine the finite element mesh.

This procedure has been checked on a number of different numerical examples (Ožbolt, 1993) and satisfactory results have been obtained.

4. Nonlocal strain approach comparison and possible simplifications

Presently an extensive experience in the nonlocal strain approach exists. In this approach strains are averaged independently on the stress-strain state in order to represent some average material properties in the fracture process zone i.e. the characteristic length of the nonlocal continuum is assumed as a material constant related to the max. aggregate size only. As a consequence, in numerical simulations for different problems (tension, shear, pull-out) it has been observed that the characteristic length must be adopted for a each different problem, independent of the max. aggregate size, in order to match experimental data (Bažant, Ožbolt and Eligehausen, 1994).

In microcrack interaction approach the representative volume of the material, expressed in the sense of the nonlocal strain approach, is stress-strain dependent and it is changing during the analysis as a function of damage (fracture). Since the heterogeneity of the concrete strongly depends on the aggregate size and shape, it seems reasonable to assume that the length of the microcracks κl is related to the maximum aggregate size d_a which together with the current stress-strain state in the material define microcrack interaction zone. To check if in the nonlocal analysis based on the microcrack interaction approach the nonlocal continuum properties may be related to the maximum aggregate size, size effect studies for four different problems (tensile, three-point bending, eccentric compression, diagonal shear and pull-out of headed stud) have been carried out and compared with known test evidence (Ožbolt, 1993). The fact that the predicted failure loads seems to be independent of the problem type, when the input data for the nonlocal material parameters are given by the standard concrete properties, it is a significant improvement compared to the nonlocal strain approach in which the characteristic length must be adopted for each different problem.

One possible simplification of the nonlocal approach based on the microcrack interactions would be simply dropping the contribution of the crack influence function Λ from Eq. 3 i.e. ignoring the interaction of the microcracks over a larger distance and taking into account only local averaging. This should be better than nonlocal strain approach since the local representative volume remain a function of a current stress-strain state. For simple cases this may be sufficient, however, for more complicated stress-strain histories, such as for example diagonal shear failure, this would generally lead to similar problems as in the case of the nonlocal strain approach since the interaction over a larger distance in the softening (damaged) part of the material is missing and it can not be simply replaced by satisfying equilibrium and compatibility conditions using local, elastic or nonelastic, finite element analysis.

5. Conclusions

1. Numerical studies for different problems confirmed that the nonlocal analysis based on the microcrack interaction approach is able to realistically predict the fracture process in quasibrittle materials, such as concrete. In the microcrack interaction approach the crack influence function controls the crack propagation and it is important for the correct representation of fracture when one or more macrocracks form. The local averaging inside the local representative volume controls the energy consumption during the fracture process. The averaging inside the local representative volume in the smeared fracture analysis is approximately equivalent to the stress averaging along a single microcrack of the discrete microcrack continuum.

2. It is demonstrated that the nonlocal material properties can be related to the standard concrete fracture properties of concrete. The input value for the tensile strength is approx. equal to the tensile strength calculated from the local tensile stress-strain law. The input fracture energy can be calculated as the total energy consumed in the local averaging volume divided by the microcrack length. Due to the concrete inhomogeneity, it is reasonable to assume that the local averaging volume as well as the microcrack length is related to the maximum aggregate size. Therefore, the fracture energy is a function of the max. aggregate size and the area under the local stress-strain curve.

3. The maximum size of finite elements in the fracture process zone should be approximately equal or smaller then the maximum aggregate size. If this is not the case the size of the local averaging volume (the characteristic length) must be increased such that the local tensile strength and concrete fracture energy are kept constant i.e. the area under the local softening tensile stress-strain curve has to be adopted.

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