27 SCALING LAWS IN CONCRETE STRUCTURES

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Abstract
There are many experimental evidence on the existence of size effect in concrete structures. The problem has two aspects – statistical and deterministic. Although the statistical aspects are not negligible, in the present paper it is demonstrated that the scaling law is controlled by the structural energy release due to cracking. If a stable crack growth before reaching peak load is possible strong size effect may be expected. For infinitely large structures of these type scaling law based on the linear elastic fracture mechanics must be used. On the contrary, concrete structures fail at crack initiation without any size effect i.e. scaling law based on the strength criteria apply. Due to the finite size of the concrete fracture process zone, size effect for any small structure must exist. Recent numerical investigations using sophisticated numerical tools show that there are many practical examples which exhibit extensive cracking in smaller size range and almost no cracking in larger size range i.e. by increasing size failure mechanism is changing. Practical implication of this is that the size effect may disappear in the case of many large concrete structures. Although this has a strong mechanical background it can also be interpreted from the multifractal damage point of view.
Keywords: Scaling law, concrete, cracking, size effect, fracture stiffening.

1 Introduction

The size effect in quasibrittle materials such as concrete is a well known phenomenon and there are a number of experimental and theoretical studies [1–6] which confirm existence of it. There are two aspects of size effect: (1) Statistical and (2) Deterministic, based on fracture mechanics. In the past, the size effect has been mainly treated from the statistical point of view [7].

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Currently, two major completely opposite groups of deterministic scaling laws exist which in a simple close form define the size effect phenomenon for different problems.

The first group of scaling laws is based on a multifractal aspects of damage [8]. The fundamental assumption in multifractal damage concept is perfect homogeneity of the material when structure size $d \to \infty$ and scaling law is of the form:

$$\sigma_N = (A + \frac{C}{d})^{1/2} \tag{1}$$

were σ_N is nominal strength (failure load divided by characteristic area), d is a measure of structure size, A and C are two constants obtained by fitting test or calculated data. Eq. (1) is schematically plotted in Fig. 1a. As can be seen, if $d \to \infty$ the nominal strength σ_N yields to a constant value different than zero. On the contrary when $d \to 0$, $\sigma_N \to \infty$. This means that the size effect is strong only in limited size range which may be larger or smaller, depending on the problem type. As will be discussed in the present paper, these kinds of scaling laws have also a clear mechanical background.

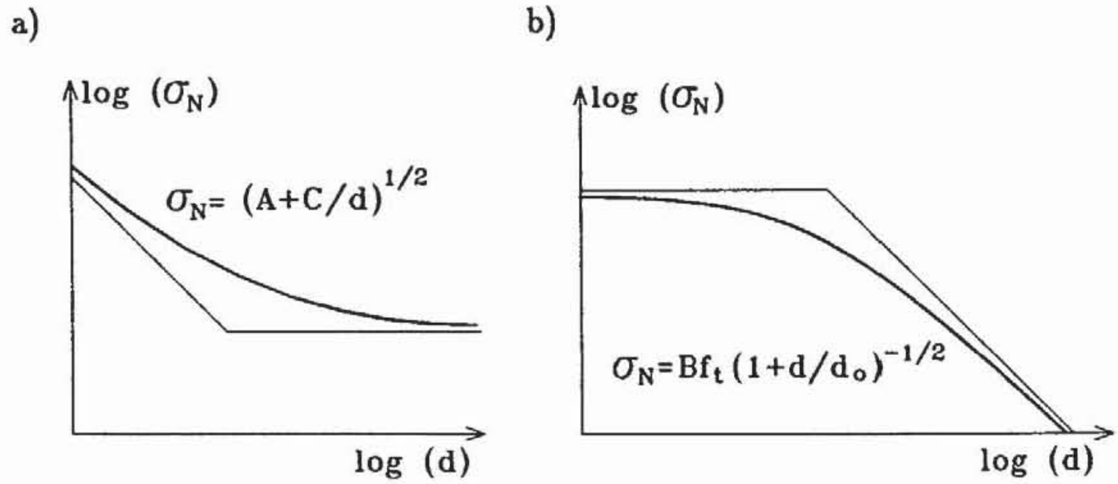

Fig. 1 Scaling laws: (a) Multifractal damage approach.
(b) Energy approach – Bažant size effect law.

The second group of scaling laws are in the form of the Bažant size effect law [3] which finds its physical background in balance between released and consumed fracture energy and has a form:

$$\sigma_N = Bf_t(1 + \beta)^{-1/2}; \quad \beta = d/d_0 \tag{2}$$

were f_t = tensile strength of concrete, B and d_0 are two constants, to be determined either experimentally or by a more sophisticated analysis. Eq. (2) is schematically plotted if Fig. 1b. According to Eq. (2) for $d \to \infty$ nominal strength $\sigma_N \to 0$. However, for small structures ($d \to 0$) size effect disappear and scaling law based on the strength criteria must be used. Derivation of the Bažant size effect law is based on a hypotheses that the concrete fracture energy (G_F) is constant and size independent and that the stable crack propagation before reaching peak load is possible i.e. the critical crack length (peak load) must be proportional with a structure size.

In a number of recent experimental and theoretical studies there are clear indications that both of the above two approaches have only a limited range of applicability. The reason is due to the fact that the fundamental assumptions of these approaches are not always fulfilled. For example, the homogeneity conditions, assumed in multifractal size effect law (MFSL), can not be fulfilled in a cases where a nature of the problem is such that even for infinity small load singularity of stresses must exist i.e. localization of strains in a small volume of material. On the other hand, the Bažant size effect law relay on the assumption of critical crack length proportionality. It is easy to demonstrate that this assumption is not fulfilled in many practical applications. Therefore, in the present study some major deterministic aspects which control crack growth will be considered and supported with a couple of numerical examples.

2 Crack propagation and size effect in concrete structures

It has been generally agreed that the main reason for size effect lie in concrete cracking. In elasticity or plasticity, were no cracking takes place, scaling law is based on strength criteria i.e. the nominal strength (σ_N) must be proportional to the structure size. Therefore, let us consider what are a major criteria for crack initiation and its propagation under the assumptions that the tensile strength (f_t) and the concrete fracture energy (G_F) are material constants.

In any concrete structure crack in a critical cross section starts when the tensile stress become larger than tensile strength ($\sigma > f_t$). This is necessary condition for crack initiation. Once the crack initiated, its further propagation is controlled by energy balance between structural energy release rate (dU/da) and concrete energy consumption limit (G_F), with U= energy accumulated in the structure and a= crack length. Only two crack propagation possibilities exist: (1) $dU/da \geq G_F$ — unstable crack propagation and (2) $dU/da < G_F$ — stable crack propagation. If unstable crack propagation is taking place, the energy which is released at unit crack propagation can not be consumed by concrete. Therefore, the load must decrease after crack initiation. This means that the maximal load is reached when the concrete tensile strength is reached i.e. there is no size effect. On the contrary, if stable crack propagation is possible, the structural energy release rate caused by cracking can be consumed by concrete. As a consequence, after crack initiation the load increases and peak load is controlled by cracking process rather than by tensile strength. Therefore, the size effect must be strong.

The structural energy release rate, and therefore cracking, is generally a function of geometry, loading type (problem type) and size. With this respect, two typical structure geometries exist: (1) Positive geometries, where after crack initiation unstable crack propagation is taking place and (2) Negative geometries, where the crack after initiation grows in a stable manner. In the case of positive geometry no crack propagation is possible and, therefore, there is no reason for size effect i.e. scaling law based on strength criteria must be employed. However, in the case of negative geometry stable crack propagation before reaching peak load is possible. As a consequence, size effect must be strong.

The above mentioned geometries are two extreme cases, however, one has to account for two additional aspects: (1) The size of the concrete fracture process zone (FPZ) is a function of aggregate size and it has a finite dimension. Therefore, if the size of the FPZ is relatively to the structure size large ($d \to 0$), the lower scaling law boundary conditions must account for this effect. (2) Due to the nonlinearity, the failure mode may be changed i.e. in certain size range structure may act as a structure of positive geometry and in another as a structure of negative geometry.

3 Unstable crack propagation – positive concrete geometry

According to definition, positive geometries in a seance of LEFM are those for which the stress intensity factor (K_I) is increasing when the crack grows at constant nominal stress. This means that the load decreases immediately after crack initiation with no size effect. However, since the concrete FPZ has a finite size, different then zero, for relatively small structure sizes formation of a stable softening zone or, in a sense of LEFM, formation of a stable equivalent crack must be possible.

Let us define a critical cross section as a section in which a possibility for a small defect in material exist (statistical aspect) or, because of the nature of the problem, a nonuniform strain distribution (deterministic aspect) is possible. It can be shown that for $d \to 0$ strain gradients ($d\epsilon/dx$) in the critical cross section will tend to infinity i.e. strong strain localization is possible. In the same time $U \to 0$ and $dU/da \to 0$. Assuming G_F to be a constant different than zero $G_F/(dU/da) \to \infty$ and therefore, theoretically, $\sigma_N \to \infty$. Practically, however, $d = 0$ has no physical meaning and therefore $\sigma_{Nd\to 0} = \sigma_{N plasticity}$. The phenomena may be interpreted as a fracture stiffening effect [9]. This nonlinear effect disappear when size of the fracture process zone become negligible in comparison to the structure size ($d \to \infty$). For such a case in critical cross section $d\epsilon/dx \to 0$ and, therefore, at crack initiation $G_F/(dU/da) \to 0$. This means failure at crack initiation and no size effect.

Having in mind both limit cases, scaling law for positive concrete geometry may be approximately of a form:

$$\sigma_N = Bf_t(1+\alpha)^{1/2}, \quad \alpha = d_0/d \tag{3}$$

where B and d_0 are two constants which depend on the problem type and material fracture properties, similar as in the case of Bažant size effect law. Note, that in contrast to the Bažant size effect law for $d > d_0$ size effect is small and for $d < d_0$ it is maximal. For each particular problem B and d_0 must be obtained from experiments or sophisticated analysis. General shape of a curve from Eq. (3) is essentially the same as predicted by MFSL (see Fig. 1a).

Comparing Eq. (3) with Eq. (1) (MSFL) the same shape of a scaling law, although with different physical background, can be observed. According to multifractal damage concept the size effect is strong in limited size range, however, when $d \to \infty$ it disappear. For positive concrete geometries nonmechanical arguments, exploited in multifractal damage theory, coincide with mechanical arguments discussed above. In contrast to this, Eq. (3) is in contradiction with Bažant size effect law which relay on the assumption that the crack length at peak load growth proportionally with the structure size. This basic hypothesis can not hold for positive concrete geometries were no stable crack propagation before reaching peak load is possible.

In the above discussion statistical aspects play important role in a sense that they may influence the position of the critical cross section, when for example uniform strain field exists (uniaxial tension), or in a seance that they influence concrete tensile strength or concrete fracture energy. However, essentially, the scaling law is controlled by equilibrium between energy release rate and concrete fracture energy.

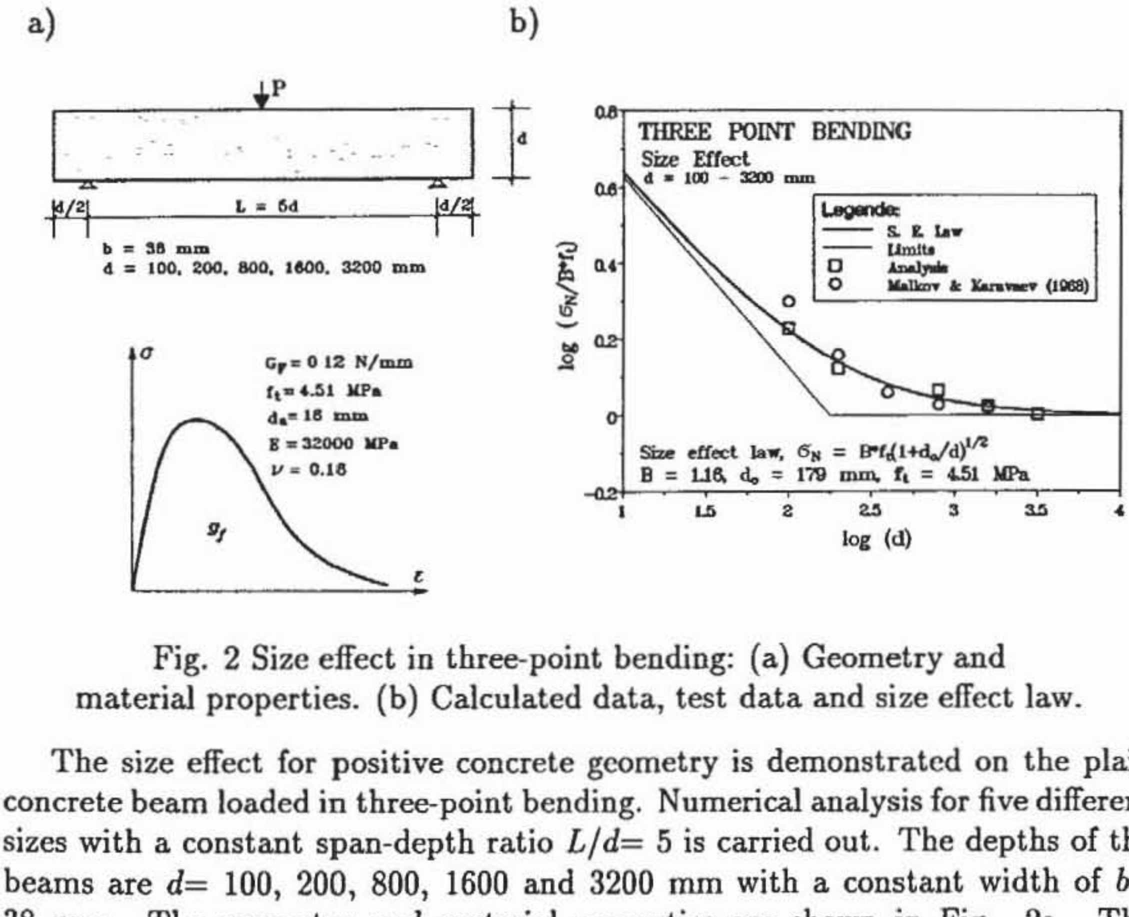

Fig. 2 Size effect in three-point bending: (a) Geometry and material properties. (b) Calculated data, test data and size effect law.

The size effect for positive concrete geometry is demonstrated on the plain concrete beam loaded in three-point bending. Numerical analysis for five different sizes with a constant span-depth ratio $L/d = 5$ is carried out. The depths of the beams are $d = 100$, 200, 800, 1600 and 3200 mm with a constant width of $b = 38$ mm. The geometry and material properties are shown in Fig. 2a. The nonlinear nonlocal finite element analysis is performed using microplane material model and nonlocal microcrack interaction approach ([10], [11]). In Fig. 2b the nominal strength is plotted versus beam depth. For comparison the experimental results are also plotted. The experimental results have been extrapolated up to a beam depth of 2 m using equation proposed by Malkov and Karavaev [12]. The numerical results agree sufficiently well with experimental observations. For beams smaller than approximately 500 mm size effect is strong, however, for larger beam sizes it disappear i.e. $\sigma_N \to f_t$.

4 Stable crack propagation – negative concrete geometry

According to definition, negative geometries are those for which stable crack growth before reaching peak load is possible, or in terms of LEFM, geometries for which the stress intensity factor (K_I) decreases when the crack length increases at constant nominal stress (σ_N). This means that because of strain localization, caused by a nature of the problem or by initial damage, after reaching tensile strength stable crack propagation is possible for any geometry size under the assumption that the failure mode is not changing when the structure size is increasing.

For extremely large structures ($d \to \infty$) relative size of the fracture process zone yields to zero. It can be demonstrated that for such a case crack length at peak load increases proportionally with structure size [13]. Therefore, the size effect must be maximal and equal to the size effect predicted by LEFM. For smaller concrete structures of these geometries, the size of the fracture process zone is relatively to the structure size large. In the limit case ($d \to 0$), following the same arguments as in the case of positive geometries, theoretically $\sigma_N \to \infty$ however, practically $\sigma_N \to \sigma_{N plasticity}$.

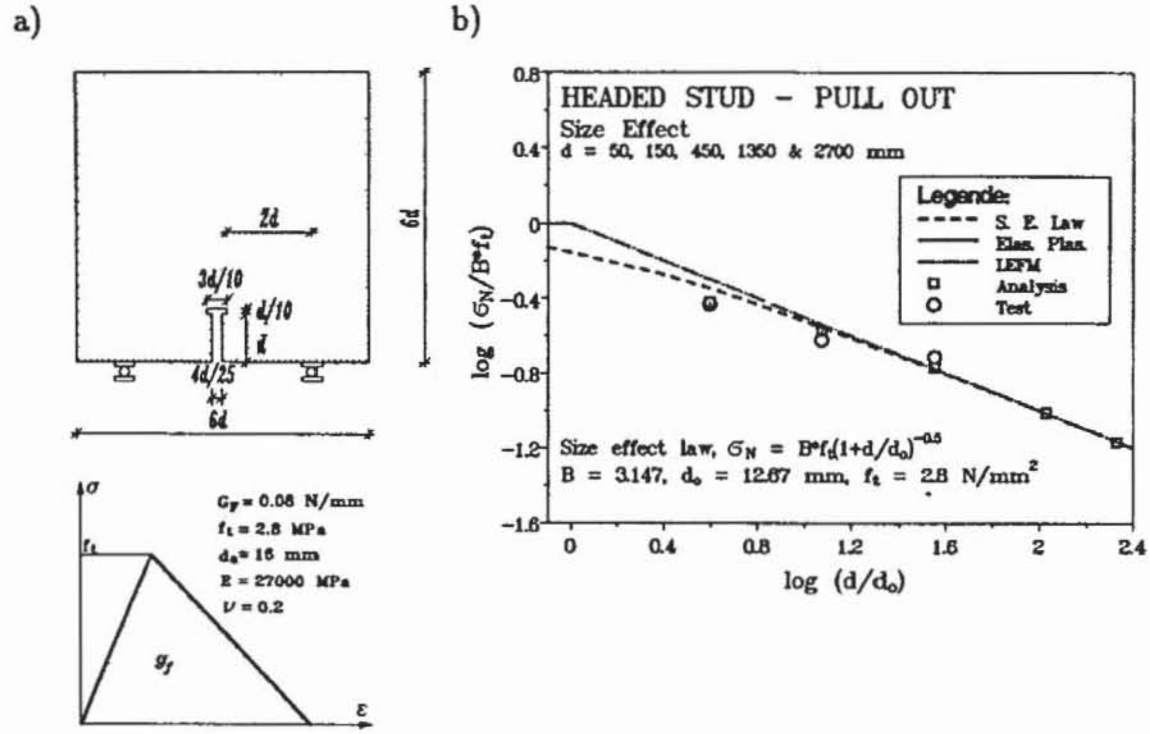

Fig. 3 Size effect in pull-out of headed stud: (a) Geometry and material properties. (b) Calculated data, test data and Bažant size effect law.

The Bažant size effect law (Eq. 2) fits above limit cases and it may be used. However, if the failure mode is changing when the structure size is increasing it does not apply. Principally, for such a cases scaling laws in a form of Eq. (3) have to be used. In order to recognize these cases, the problem is currently under intensive numerical investigation.

To demonstrate size effect for typical negative concrete geometry, numerical analysis for pull-out of headed stud from a plain concrete block has been carried out. The specimen geometry and the material properties used in the analysis are shown in Fig. 3a. The analysis has been carried out for embedment depths in range $d=$ 50 to 3200 mm using above mentioned nonlocal axisymmetric finite element code. In Fig. 3b calculated and experimentally obtained failure loads are plotted and compared with Bažant size effect law. As can be seen, numerical results are in good agreement with Bažant size effect law and with experimental results. The proportionality of the crack length at peak load is approximately fulfilled and, therefore, the size effect is strong in broad size range. The ratio between the crack length increment and the load increment (da/dP) is increasing with structure size i.e. when $d \to \infty$, $da/dP \to \infty$. Therefore, for larger structures dynamical effects become important. This partly explains sensitivity of the size effect on the loading rate.

5 Conclusions

1. Scaling law in concrete structures is controlled by cracking. If extensive cracking for any structure size before reaching peak load is possible the size effect must be strong in broad size range.
2. With respect to crack growth two typical structure geometries exist: (1) Positive, with no stable crack propagation possibility and (2) Negative, with stable crack growth before reaching peak load. Generally, when $d \to \infty$ size effect disappear in the case of positive geometries and it is maximal in the case of negative geometries.
3. For smaller concrete structures of any geometry size effect is always present. This is due to a relatively large ratio between the size of the concrete FPZ and the structure size i.e. for such cases stable crack growth before reaching peak load is always possible.
4. In practice many concrete structures exhibit extensive cracking capacity in smaller size range and limited or almost no cracking in larger size range i.e. in the smaller size range the structure acts as the structure of negative geometry and later, with increasing size, it acts as the structure with positive geometry. This is a consequence of the change in failure mechanism caused by nonlinearity and it is currently under intensive research, for each particular case, using a sophisticated numerical tools.

6 References

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