

## Nonlinear Analysis of Strain-Softening Damage under Monotonic and Cyclic Loading

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### Abstract

The paper reviews the modeling of damage by the microplane model and presents extensions required for cyclic loading. The general microplane constitutive model is implemented in a three-dimensional finite element code and used in damage and bifurcation analysis of various structures. The results of the analysis are compared with test results and some of the comparisons are shown in the present paper.

### Introduction

To simulate brittle failures including the size effect, the finite element code must be endowed with what has been called a localization limiter – a mathematical device that prevents localization of damage into a zone of zero volume. Alternatively, correct prediction can be obtained by a finite element code with discrete cracks characterized by a softening crack-bridging law. Such a code, however, appears to be less versatile in general situations than a nonlocal code.

In nonlocal codes, it is important that the stress-strain relation describes quite realistically not only the tensile cracking and fracture, but also the nonlinear triaxial behavior under various stress or strain histories, for both tensile and compressive stress states. A very powerful and general description of such behavior is provided by the microplane model. Its advanced form, agreeing with all the basic multiaxial test data for concrete, was presented in Bažant and Prat (1988). In order to model unloading, reloading and arbitrary cyclic loading, for arbitrary triaxial stress states, more complex rules on the microplane levels, including the rate sensitivity, have been recently introduced (Ožbolt and Bažant, 1992).

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### Nonlocal Microplane Model

In the microplane model the material properties are characterized separately on planes of various orientations within the material, called microplanes, on which there are only a few stress and strain components and no tensorial invariance requirements need to be observed. The tensorial invariance restrictions are satisfied automatically, because of integrating over all spatial directions. The microplanes may be imagined to simulate the response on various weak planes in the material (interparticle contact planes, interfaces, planes of microcracks, etc.). The constitutive properties are entirely characterized by a relation between the stress and strain components on each microplane, both normal and shear directions. The model and its implementation in a 3D finite element code is in more detail described by Bažant and Ožbolt (1990) and Ožbolt and Bažant (1992).

In the last version of the general microplane model for concrete (Ožbolt and Bažant, 1992) microplane shear stress-strain relations are the same and mutually independent for positive and negative shear strains. This is sufficient in the case of monotonic loading. However, from shear cyclic experimental evidence (for example cyclic bond-slip behavior) it is well known that a loading in one direction causing damage introduces a stiffness and strength degradation in the opposite loading direction. Due to this, the model has been further improved such that the microplane shear stress-strain law is modified in a way that the microplane shear modulus is multiplied by a damage factor which is a function of the accumulated microplane shear energy dissipation.

To avoid spurious localization, bifurcation and instabilities due to the strain-softening damage and corresponding mesh-sensitivity problems in finite element analysis, the nonlocal continuum concept based on nonlocal damage with local strains is adopted. The governing parameter in the nonlocal concept is the characteristic length  $l$  over which the strains are averaged, and it has a significant influence on the results of the analysis. Initially it was assumed (Pijaudier-Cabot and Bažant, 1987) that this length is a material parameter which can be correlated with fracture energy and approximately taken as  $3d_a$  ( $d_a$  = maximum aggregate size). However, in general 3D stress-strain situations  $l$  is difficult to interpret as a material parameter depending on the concrete mix only; rather, it may be influenced by other parameters as well. Therefore, further studies are needed to clarify the governing law between the concrete mix and stress-strain state on one side and the characteristic length on the other.

### Numerical Examples

To demonstrate the power of the nonlocal microplane code, the failure loads as well as the behavior of some test specimens under cyclic loading have been calculated.

The results of the calculated failure loads for four different cases, each of a different size, are plotted in Figs. 1a-d in order to demonstrate that the nonlocal microplane model is able to correctly predict the size effect. For comparison the experimental results are also plotted. Except those for the pull-out of headed

anchors, the size effect law giving the best fit of the experimental data is plotted as a solid curve. The agreement between experimental and numerical results is seen to be generally quite acceptable. For the pull-out of headed anchors (Fig. 1c) only calculated data are given because the assumed geometry of the specimens differs from the geometry of the test specimens. However, the calculated size effect agrees very well with test results.

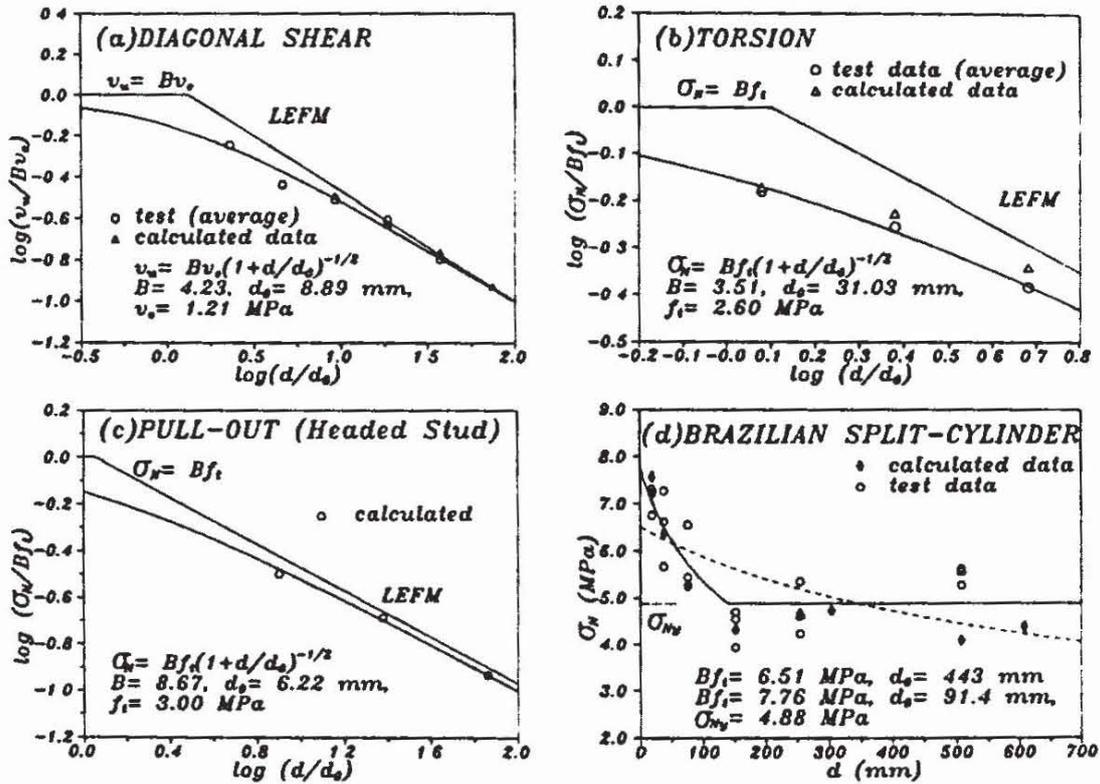


Fig. 1 Size effect - comparison between calculated and test data.

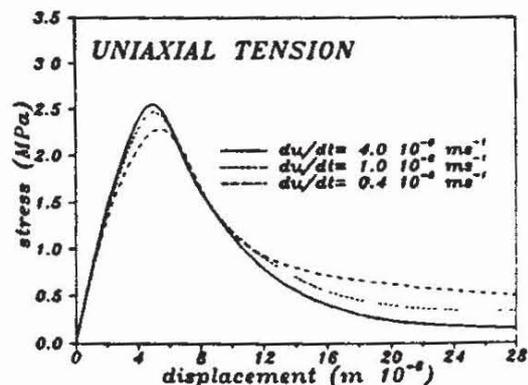
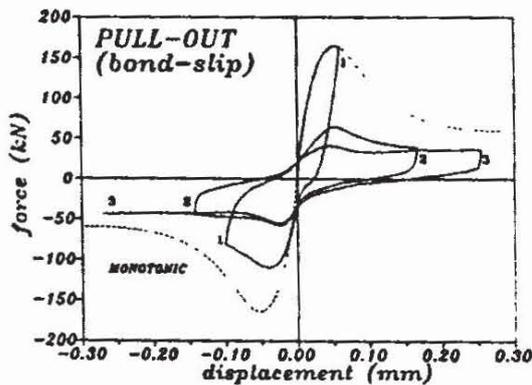


Fig. 2 Calculated bond-slip curve. Fig. 3 Uniaxial tension - rate dependency.

Fig. 2 shows the calculated load-slip curves for a deformed steel bar embedded in a concrete cylinder and pulled out by monotonic and cyclic loading. The analysis is performed by the use of axisymmetric finite elements employing spatial discretization with modeling of the ribs of a deformed steel bar. The comparison between numerical results and test results (Eligehausen et al., 1983) indicates good agreement. The present approach is able to correctly predict the

monotonic as well as cyclic behavior including friction and degradation of pull-out resistance due to the previous damage. However, the calculated slip is much smaller than observed in experiments, which can be explained by several reasons.

Fig. 3 shows the load-displacement curves for a specimen loaded in monotonic tension using three different rates of loading. Analysis is performed using 3D finite elements. Again, comparison between the calculated results and the experimental evidence (Reinhardt and Cornelissen, 1984) indicates qualitatively good agreement.

### Conclusions

1. The nonlocal microplane model implemented in a finite element code is capable to correctly predict the size effect and the fracture process in complicated 3D situations such as monotonic and cyclic pull-out of a deformed steel bar from a concrete block, as well as the influence of the loading rate on the tensile strength of concrete. Since such a code can also model quite well all the phenomena associated with nonlinear triaxial response of concrete in compressive states, this particular type of numerical modeling is shown to be of very broad applicability.
2. Better and more fundamental understanding of the characteristic length  $l$  as a function of the relevant influencing parameters is needed, otherwise, the value of  $l$  would have to be based on empirical rules.

### Appendix.- References

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