ANALYSIS OF HEADED ANCHORS EMBEDDED IN CONCRETE USING A NONLINEAR FRACTURE MODEL

GOTTFRIED SAWADE[1], ROLF ELIGEHAUSEN[2]
[1]FMPA BW Otto-Graf-Institut, [2] IWB, Universität Stuttgart
Pfaffenwaldring 4, D-7000 Stuttgart 80, Germany

ABSTRACT

This paper represents an energetical model of the fracture behaviour of concrete where crack opening is considered as time dependent dissipative process. States of mechanical equilibrium can be obtained by simulation of a relaxation process. Application of this model to calculations of the bearing capacity of anchorages confirms recent approachs based on linear fracture mechanics.

INTRODUCTION

The use of fastenings systems such as headed -,expansion- or undercut anchors in the building construction industry is on the increase. At present, the design of fastenings is mainly based on empirical equations/1/.In order to get a better understanding of the anchor behaviour, fracture mechanics must be used. However, due to the complicated fracture process (mixed mode crack propagation) few theoretical investigations are available only /2,3/.Therefore numerical studies were performed to investigate the behaviour of headed anchors embedded in a large concrete block. In the investigations a newly developed energetical model based on the linear irreversible thermodynamics was employed.

THERMODYNAMICAL MODEL

In this model /4/ damage due to tension stresses are considered as discontinuities of the field of displacements on the crack surface S_{Re}. Therefore in a finite element V_e an additional strain $\underline{\epsilon}_{fr}$, depending on the average crack opening $\underline{w}_e$ follows as:

$$\underline{\epsilon}_{fr} = \frac{S_{Re}}{V_e} \underline{N}(\Omega_e) \cdot \underline{w}_e \qquad (1)$$

with: $\underline{w}_e^T = [w_n, w_t]$ - crack opening normal and tangential to crack surface,

$\underline{N}$ describes the transformation from the global to the local coordinates of the crack surface. The entropy production L_e in the element V_e due to crack opening is:

$$L_e = S_{Re} \cdot \underline{w}^{\cdot T} \cdot \underline{K} \cdot \tag{2}$$

$\underline{K}$ is the thermodynamical force of crack opening with:

$$\underline{K} = \{\underline{N}^T [\underline{D} \cdot \underline{B} \cdot \underline{l} - \underline{\epsilon}_{fr}] - \underline{G}_{,w}\} \tag{3}$$

$\underline{D}$ is tensor of elasticity, $\underline{l}$ is the vector of nodal displacements. $\underline{G}_{,we}$ is the partial derivation of crack surface energy:

$$\underline{G}_{,w}^T = [\delta G/\delta w_n, \delta G/\delta w_t] \tag{3a}$$

Equ. (4) contains a simple ad hoc approach for G, with G_f as specific surface energy, β_z and β_t as tension-strength and shear-strength of concrete respectively:

$$G = G_f \cdot \{1 - \exp(-[\frac{\beta_z}{G_f} \cdot w_n + \frac{\beta_t}{G_f} \cdot |w_t|])\} \quad , w_n \geq 0 \tag{4}$$

Assuming linear irreversible thermodynamics, the following relation for the crack-opening rate is valid:

$$\underline{w}^{\cdot} = \underline{R} \cdot \underline{K} \tag{5}$$

with:

$$\underline{R} = \begin{vmatrix} r_n & 0 \\ 0 & r_t \end{vmatrix} \quad ; \; r_s = \begin{cases} 0 & \text{for } K_s < 0 \\ r_{os} & \text{for } K_s \geq 0 \quad s=n,t \end{cases} \tag{6}$$

The phenomenological parameters r_{on} and r_{ot} are assumed to be equal for reasons of simplicity. In case of previously unknown crack orientation, the direction of a newly generated crack increment follows from the principle of maximum entropy production. The necessary condition is therefore:

$$\delta \{\underline{K}^T \cdot \underline{R} \cdot \underline{K}\}_{w=0} / \delta \Omega_e = 0 \tag{7}$$

In case of isotropic behaviour of G and $\underline{R}$, the crack orientation calculated with respect to equ. (7) is in accordance with the usual normal-tension criterium.
The loading of the specimen results from the vector $\underline{d}_o(t)$ of prescribed displacements in some selected nodal points. The remaining unknown displacements $\underline{d}(t)$ are obtained from:

$$\underline{K}_{red} \cdot \underline{d}(t) = \underline{f}[\underline{d}_o(t)] + \Sigma \, \underline{g}_e[\underline{w}_e(t)] \tag{8}$$

$\underline{K}_{red}$ is the reduced stiffness matrix, $\underline{f}$ and $\underline{g}$ are linear functions of $\underline{d}_o(t)$ and $\underline{w}_e(t)$ respectively. Solving of the differential equation system (5,7,8) is done by means of discrete time steps $t_k=k\cdot dt, k=0,1,.M$. For k=0 the crack opening is assumed to be zero, meaning no damage of the specimen at the beginning of loading. The calculation-scheme is as follows:

1. nodal displacement at time t_k:
$$\underline{d}(t_k) = \underline{K}_{red}^{-1}\{\underline{f}[\underline{d}_o(t_k)]+\Sigma \underline{g}_e[\underline{w}_e(t_k)]\} \tag{9}$$

2. orientation of cracks in at present uncracked elements :
$$\delta\{\underline{K}[\underline{l}(t_k)]\cdot\underline{R}\cdot\underline{K}[\underline{l}(t_k)]\}/\delta\Omega = 0 \tag{10}$$

3. crack opening displacement at time t_{k+1}:
$$\underline{w}_e(t_{k+1}) = dt\cdot\underline{R}\cdot\underline{K}[\underline{l}(t_k),\underline{w}_e(t_k)] + \underline{w}_e(t_k) \tag{11}$$

This scheme is simple as at any time step the vector of nodal displacements and the crack opening can be calculated from the values of the previous time step. It is especially advantagous that $\underline{K}_{red}$ remains unchanged during the whole process. To obtain thermodynamical equilibrium ($\underline{K}=0$), the loading function $\underline{d}_o(t_k)$ has to be expressed by means of a step-function:

$$\underline{d}_o(t_k) = p(t_k)\cdot\underline{d}_\infty \tag{12}$$

$$p=p_m \quad ; \quad t_m \le t_k < t_m + dt\cdot M$$

where $\underline{d}_\infty$ is a unit displacement, and M an integer.
Loading is kept constant during the relaxation interval M·dt. At sufficiently high relaxation times, the thermodynamical force $\underline{K}$ tends to zero. The stress vector at the crack surface depends only on the crack-opening in aggreement with the Hillerborg-model.

MODELLING OF ANCHORAGES

ASSUMPTIONS

For the calculation of the bearing capacity of axially loaded headed anchors, the following assumptions are made:

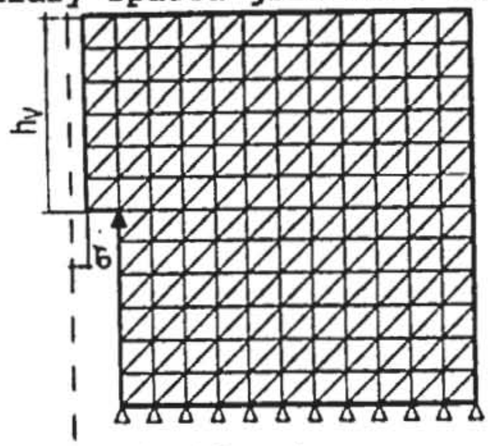
1. Loading is axial-symmetrical
2. The headed stud is assumed to be rigid.
3. Crack growth takes place only in the r-z-plane (circumferential cracking)
4. The ultimate load results from the load-displacement curve, in each loading-step the thermodynamic equilibrium is reached because the loading process is sufficient slow.

The elasticity tensor is assumed to be orthotropic with respect to circumferential stresses:

$$\underline{D} = \frac{E(1-\mu)}{(1-2\mu)(1+\mu)} \begin{vmatrix} 1-\mu & \mu & c\mu & 0 \\ \mu & 1-\mu & c\mu & 0 \\ c\mu & c\mu & c(1-\mu) & 0 \\ 0 & 0 & 0 & 2(1+\mu)^{-1} \end{vmatrix} \tag{13}$$

If circumferential stress is to be excluded (e.g. in case of anchors in intersecting cracks) c has to be set to zero.
In the finite element discretization the nodal points form a

regulary spaced grid with the coordinates (r_i, z_j) (see Fig.1)



$$r_i = r_0 + i \cdot \Delta z \qquad 0 \le i \le N_i$$
$$z_j = j \cdot \Delta z : \qquad 0 \le j \le N_j$$

$$\Delta z = h_v / 6$$
$$r_0 = 1 \text{ mm}$$

Fig.1 FE-Idealization

The quotient S_{Re}/V_e in equ. (1) was set approximately to $1/\Delta z$. The time interval dt was normalized to $dt = 0.2 \cdot \Delta z / (E \cdot r_{on})$. In the following described calculations the influence of embedment depth h_v, the concrete stengths β_z, β_t and circumferential stresses (c) was investigated. The Youngs-modulus E, the specific surface energy G_f and the Poisson-modulus μ was set to:

- $E = 30000$ N/mm², $\mu = 0$, $G_f = 0.1$ N/mm

For each of the load-displacement curves, 150 loading steps were considered, the relative duration of one relaxation interval being M=100.

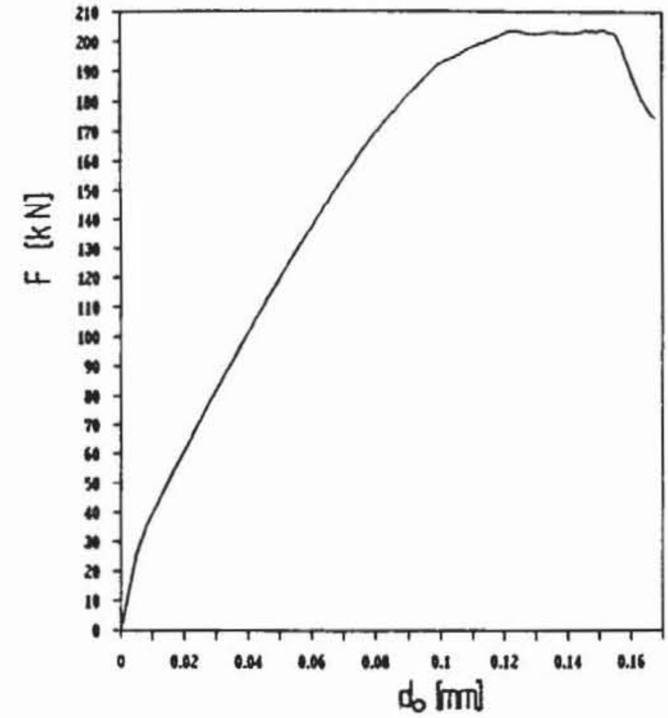
RESULTS

Table 1 contains the cases considered and the results obtained.

Table 1

No.	h_v mm	β_z N/mm²	β_t N/mm²	c -	F_{max} kN	$\frac{F_{max}}{\sqrt{EG_f} \cdot h_v^{1.5}}$	emp. equ.(15) kN
1	50	3	0	1	36	1.85	41
2	150	3	0	1	203	2.01	212
3	450	3	0	1	1168	2.23	1103
4	150	6	3	1	236	2.34	212
5	150	3	0	0	100	1.00	-

Fig. 2 shows the load-displacement curve for case 2. The crack contour for this case at 0.95 F_{max} is shown in Fig.3. The anchorage fails due to the propagation of a single circumferential crack, growing in a stable manner. Comparison of case 2 and case 4 yields little influence of β_z and β_t on ultimate load. The failure load is dependent mainly on the embedment depth, the Youngs-modulus and the total specific crack surface energy.



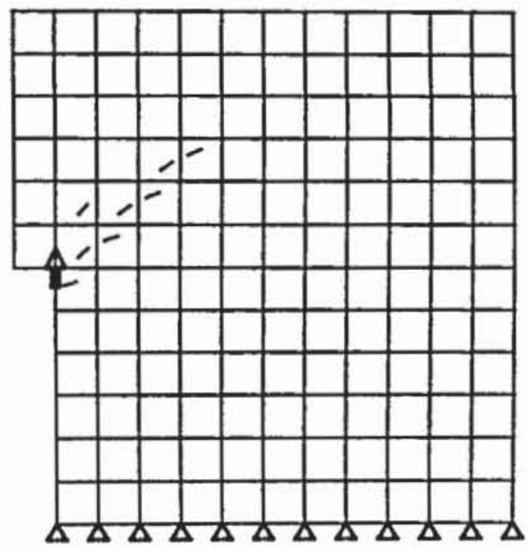


Fig. 2 Load-displacement curve case 2, (h_v=150 mm)

Fig. 3 Crack contour at $0.95 \cdot F_{max}$

Considering the size-effect, the ultimate load of case 1-4 is expressed as a function of the embedment depth:

$$F_{max} = A \cdot h_v^2/(B+h_v)^{0.5} \qquad (14)$$

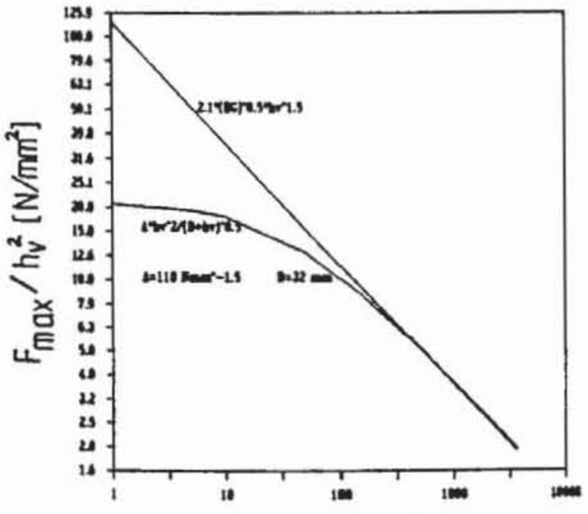


Fig. 4 Size Effect

In Fig.4 the ratios F_{max}/h_v^2 calculated from equ.(14) are plotted as a function of the embedment depth in double-logarithmic scale. For comparision the result of a linear fracture mechanics solution /5/ with:

$$F_{max} = 2.1\sqrt{EG_f} \cdot h_v^{1.5} \qquad (14a)$$

is plotted as well.

Obviously, the above nonlinear dissipative model gives ultimate loads with agree rather well with results obtained by means of linear fracture mechanics (see column 7 of table 1). This is valid for h_v> 50mm.
In column 8 of table 1 the above calculated ultimate loads are compared with results of the empirical design formula (17)/1/:

$$F_{max} = 15.5\sqrt{\beta_D}\cdot h_v^{1.5} \qquad (15)$$

The compression strength was calculated to β_D=55 N/mm² according to FIP-CEB Model-Code /6/, assuming a maximum grain diameter of 16mm. Comparision of theoretical and empirical values shows sufficient agreement.
Neglecting the circumferential stresses gives an ultimate load of only one half of the value valid for uncracked concrete (case 5). This is confirmed by empirical results /1/ obtained with anchors in crosswise cracked specimens, where the ultimate load was in range of 40-70% compared to uncracked specimens.

CONCLUSIONS

The essential features of the described energetical model are as follows:
- -The rates of crack opening displacements can be obtained as a function of the corresponding thermodynamical force.
- -The crack contour can be determined by the principle of maximum production of entropy
- -The displacements and crack-openings can be treated as a relaxation problem. This model allows the solution of a nonlinear boundary-problem by a simple step by step procedure.

The application of the case of headed anchors embedded in a large concrete block yields the following results:
- -The ultimate load is essentially influenced only by the embedment depth, the Youngs-modulus and the total specific crack-surface energy.
- -For an embedment depth h_v> 50 mm, the ultimate load follows the linear fracture mechanics.

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