


CRACKING BEHAVIOUR UNDER NON-MONOTONIC LOADING – Literature Review –

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ABSTRACT

Repeated and long term loads may produce a significant increase of crack widths owing to the increase of concrete deformations resulting in an increase of steel stress in the crack and the increase of slip along the reinforcing bar which causes a decrease of tension stiffening. Crack widths predicted on the basis of short term static tests do not provide a satisfactory guide to crack widths in service.

The present study reviews the available data concerning the influence of cyclic and long term loading on the cracking behaviour and discusses the basic influencing parameters together with the proposed methods of predictions. Only a small part of the approaches on carck control are developed to consider the influence of repeated or sustained loads as well. A comprehensive study is required to develop an appropriate prediction based on all significant influences.

Key-words:

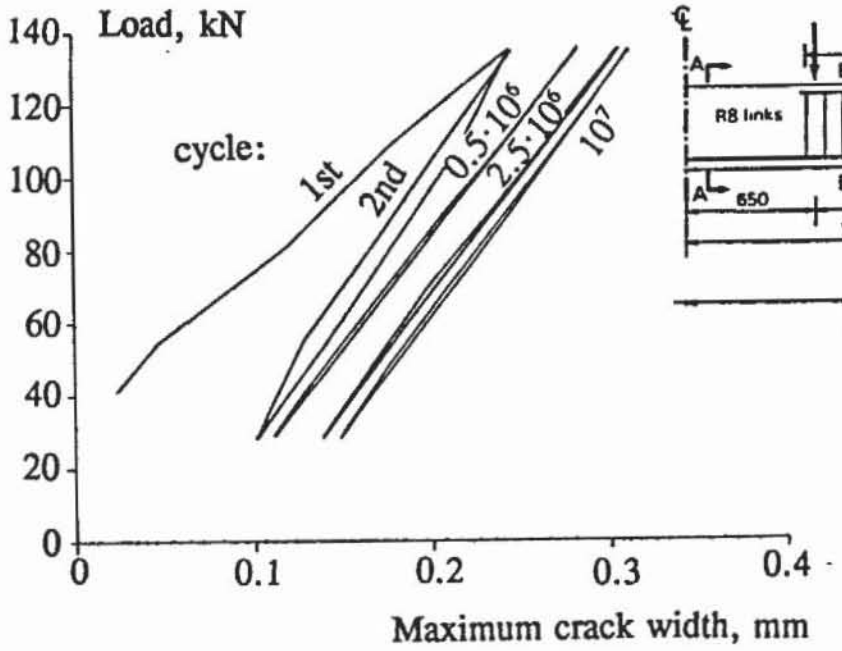
cracking, crack width, crack spacing, **cyclic load, sustained load,** bond (concrete to reinforcement), slip, tension stiffening, steel stress, steel ratio

1. INTRODUCTION

Crack widths predicted on the basis of short term static tests do not provide a satisfactory guide to crack widths in service. Repeated and long term loads may produce a significant increase of crack widths owing to the increase of concrete deformations resulting in an increase of steel stress in the crack and the increase of slip along the reinforcing bar which causes a decrease of tension stiffening.

The present study intends to review the available data concerning the influence of cyclic and long term loading on the cracking behaviour and discusses the basic influencing parameters together with the proposed methods of predictions. **Fig.1** indicates a typical example for the development of the maximum crack width in a beam subjected to repeated loading.

The crack widths increase with increasing number of load cycles or with time under sustained loading /1/ to /16/, however, the **rate of development** is decreasing. In case of repeated loading (Fig.1), the highest values of crack widths are observed under the maximum value of repeated load. **Unloading** induces decrease in crack widths without



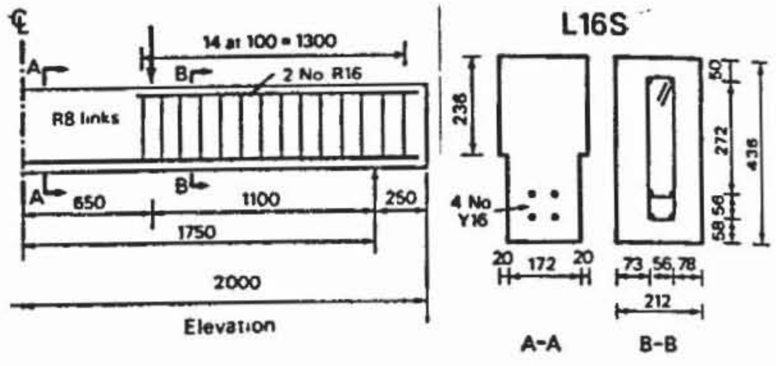
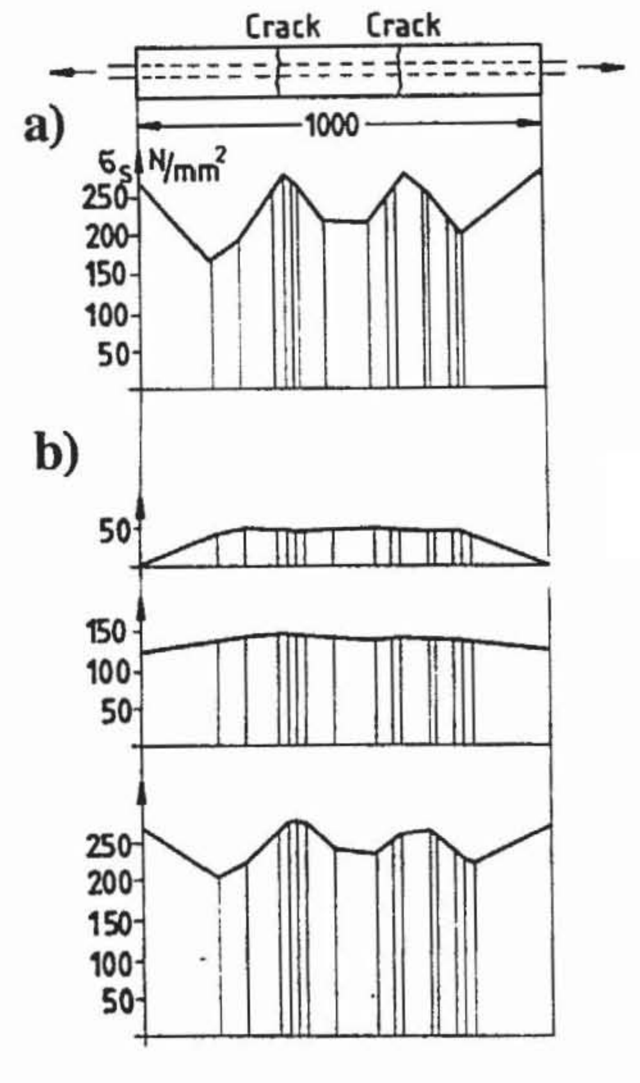


***Fig.1** Increase of crack width under repeated loading; Lovegrove and Din /1/*

complete closing, because bond stresses between steel and concrete do not withdraw by unloading, but a self-equilibrium of bond stresses remains between the cracks. This can be seen from the steel stress distributions based on strain measurements which are presented in **Fig.2.a** for initial loading and for the tenth load cycle, respectively.



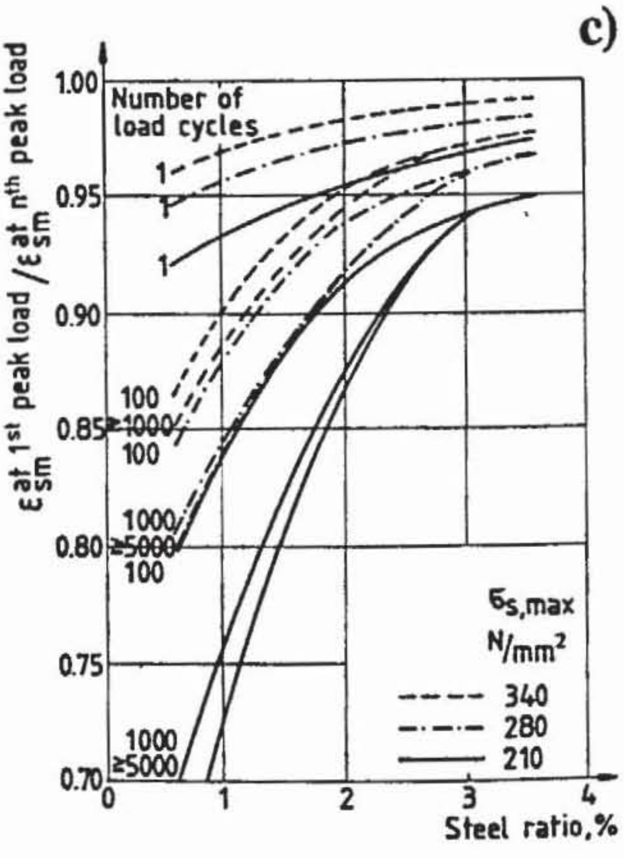
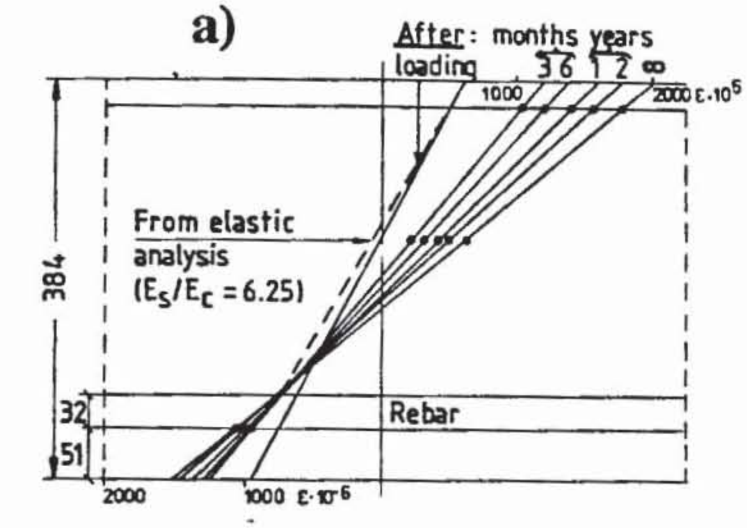
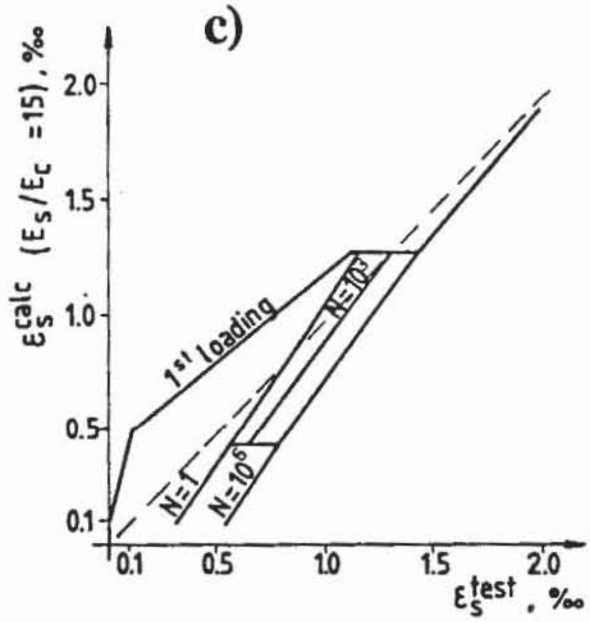


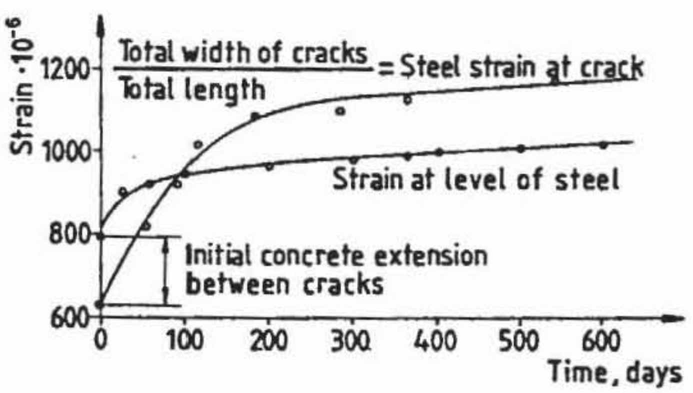
Fig.2 Tension test; Günther /13/
***a)** After the appearance of the 2^{nd} crack*
***b)** After the 10^{th} load cycle at loading*
***c)** Average steel strain related to that at first maximum load vs. steel ratio*

2. INFLUENCE OF REPEATED AND LONG TERM LOADS ON CRACKING

Both long term and repeated loads produces an increase in crack width, however, it is considerably faster under repeated loading. The test results by repeated and long term loads are discussed on a common basis owing to their similar influence.







a) Modification of strain distribution under long term loading; Stevens /5/
b) Incerase of steel strain in crack under long term loading; Illson and Stevens /6/
c) Calculated vs. measured steel stresses in crack (average of 3 results on splices under repeated load); Rehm and Eligehausen /8/

***Fig.3** Modification of strains and crack widths under long term and repeated loads*

2.1 Steel strain in the cracks

The shrinkage and the creep or cyclic creep of concrete in the compression zone of a beam, in addition to loss of tensile stress of concrete between cracks results in a redistribution of tresses and a drop in the neutral axis which causes an **increase in the steel stress in the cracks (Fig.3.a)**.The strain in the cracks (obtained as a ratio of the total widths of cracks to the total length) increases more rapidly than the average concrete strain at the level of the tension steel **(Fig.3.b)** which indicates the loss of concrete tension between cracks.

The steel strain in the cracks of the element subjected to bending may exceed the theoretical value obtained by elastic analysis of the cracked section **(Figs.3.a and c)**. *Rehm and Eligehausen /9/* observed a 10 to 15 % increase above the theoretical value. In overreinforced sections it may be even more /8/.

2.2 Average steel strain

A part of the steel stress in cracks is transformed to the tensioned concrete by bond producing a reduction of steel stress between the cracks. Owing to this contribution of tensioned concrete, the phenomenon is generally refered to as tension stiffening effect. With increasing number of load cycles or increasing time under sustained load, however, the tension stiffening effect decreases and yields to an increase in the average steel strain. The smaller the relative rib area of the reinforcing bar, the faster the increase of average steel strain. The increase is explained in /3/ as a result of new microcracks surrounding the bar.

The increase of average steel strain in a tensioned element as a function of the number of load cycles, the steel ratio and the maximum cyclic load is presented in **Fig.2.c**. The influence of repeated loading on the average steel strain decreases with increasing steel ratio.

a) **b)**

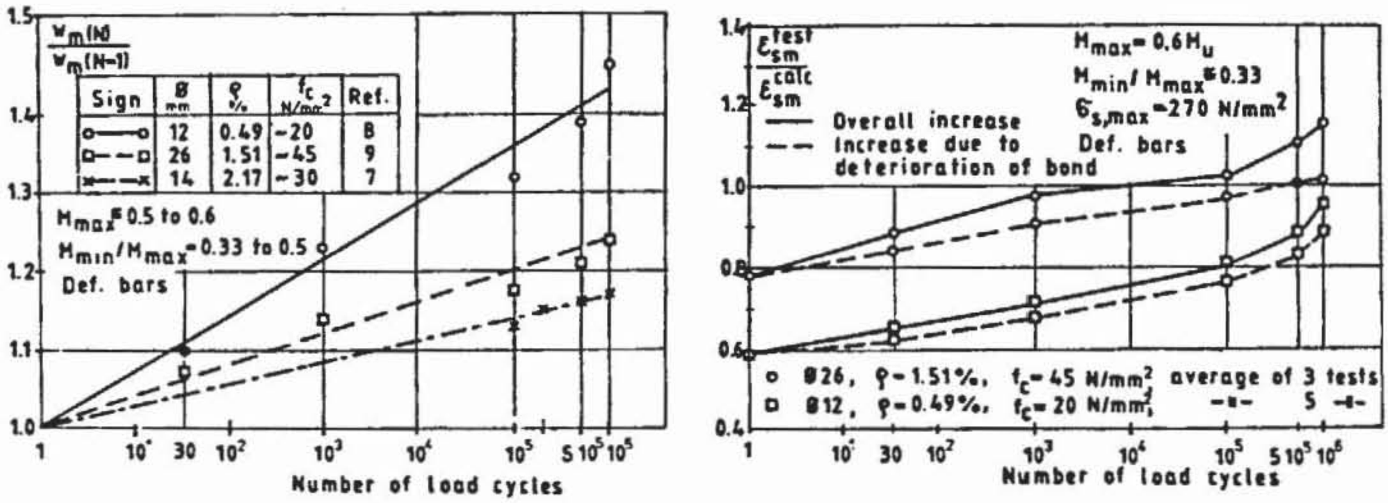


Fig.4 Increases of average steel strain and average crack width; Rehm & Eligehausen /8/
***a)** Measured to calculated average steel strains versus number of load cycles*
***b)** Average crack width after N cycles to that after the 1st cycle versus N*

Fig.4.a indicates the increase of the average steel strain (obtained on splices of plates and beams with two different steel ratios subjected to repeated loading by a maximum steel stress of 270 N/mm²) related to the calculated average steel strain. The average steel strain increases approximately linearly with the logarithm of the number of load cycles. The continuous lines of Fig.4.a indicate the overall increase of the average steel strain and the dotted lines show the average strain increase due to the deterioration of the interaction as a difference of the overall increase reduced by the measured strain increase in cracks.

2.3 Crack spacing

The decrease of average crack spacing in elements subjected to long term or repeated loads depends on the actual state of cracking at the beginning of the non-monotonic loading /8/.

Long term or repeated loads applied on an element being in the so called initial cracking (i.e. the steel stress is only slighly above that just after cracking), the decrease of average crack spacing may be significant. *Holmberg /7/* observed a decrease by more than 50% .

Long term or repeated loads applied on an element wich have already reached the so called stabilized cracking state, the modification of average crack spacing is minor. A 10 % decrease of the average crack spacing was observed by *Soretz /2/* after 2 years sustained loading and by *Rehm and Eligehausen /9/* after 10^6 load cycles. The decrease of average crack spacing, however, supports the opinions /17/ to query the existence of a stabilized crack state.

Stevens, Bryden-Smith and Hunt /4/ observed that under sustained load a few new cracks usually form between initial cracks which happened to be spaced widely, probably due to shrinkage, but the primary crack spacing remains predominant. Usually such later developments seldom widened or extended in length as much as those initially formed.

Bodó and Balogh /14/ reported that the number of cracks in prestressed concrete slabs and beams subjected to sustained loading increased by 6 % and 34 %, respectively, in 9 years.

2.4 Crack width

The range of the increase of crack width observed in elements under sustained or constant amplitude loads is rather large. This may be explained by the differences in steel ratio, rib pattern of reinforcing bar, shrinkage of concrete or state of cracking by applying the sustained or repeated loads. **Fig.4.b** shows the relative increase of crack width by three series of test results with various steel ratios. The higher the steel ratio, the smaller the relative increase of average crack width.

Most of the test results show that the increase of characteristic crack width (w_k) agrees with the increase of the average crack width rather well /8/. The characteristic crack width denotes a 95 % fractile value of crack widths. The maximum crack width (w_{max}) is often used in the literature instead of the characteristic width with a similar meaning. While the probality level of w_{max} was not always given, both notations w_k and w_{max} were kept.

Fig.5 gives a comparison of the increase in crack width produced by sustained or repeated loads. A 35 % increase was reached in 21 months or in $5 \cdot 10^5$ load cycles (giving a much shorter tesing time), respectively under 270 N/mm² sustained or maximum cyclic load.

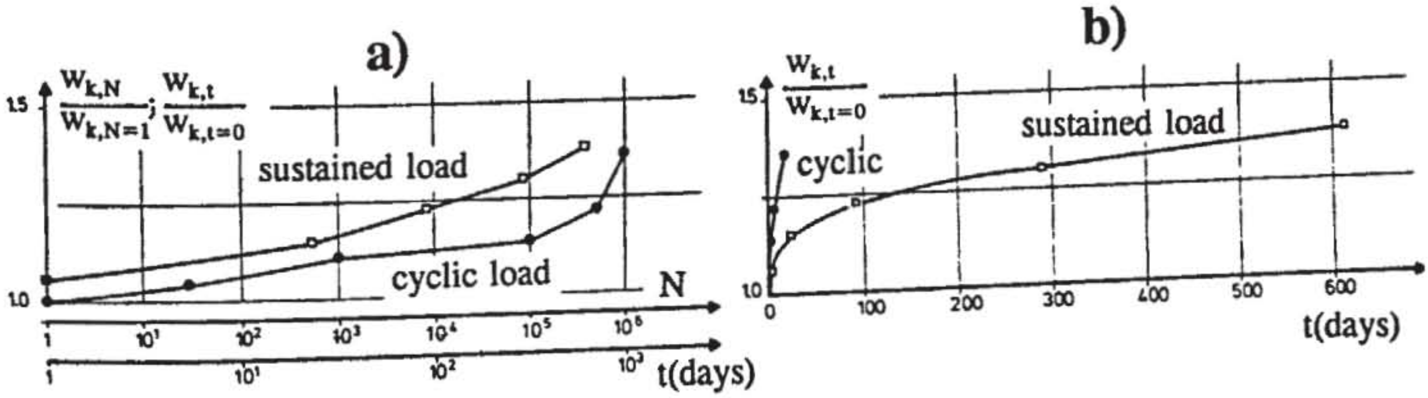


Fig.5 Increase in crack width under sustained or repeated loads; Rehm and Eligehausen /9/
• Results with repeated load   *□ Results with sustained load*

Lovegrove and Din /1/ shown that the cyclic maximum crack width increases almost linearly with the logarithm of the number of load cycles and reached a 60 % increase in 10^7 load cycles (**Fig.6**).

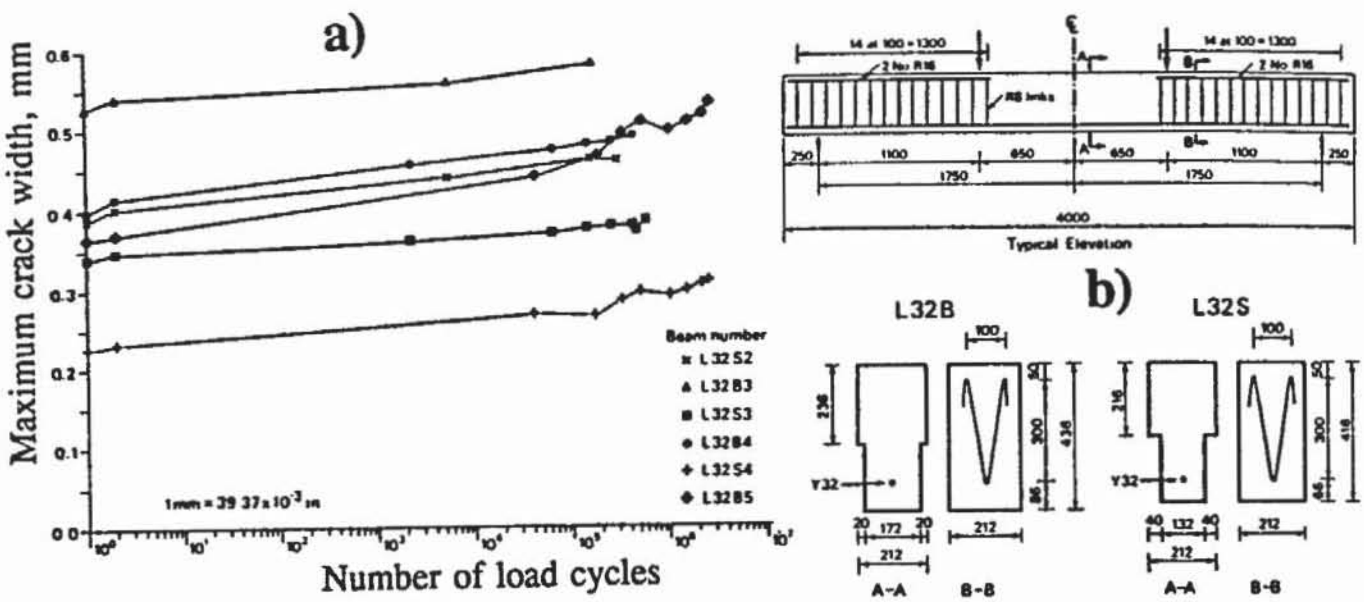


***Fig.6** Crack width at maximum load versus number of load cycles; Lovegrove and Din /1/*
***a)** Beams with main reinforcing bars of 32 mm*
***b)** Details of test beams (28 days cube strength of concrete was 58 N/mm², average proof stresses of 16 mm and 32 mm bars were 467 N/mm² and 435 N/mm², resp.)*

Test results by *Bhuvasorakul* /15/ show a 50 to 140 % increase of crack width in 10^5 load cycles for the various simply supported beams (**Fig.7**). The load amplitude was set to produce steel stresses between 69 and 276 N/mm² with a frequency of 2 s^{-1}. The crack widths were recorded photographically using a camera equipped with a close-up lens mounted to a special positioning jig.

Jagdish /16/ reported an increase of 80 % in beams after $5 \cdot 10^4$ load cycles.

Stevens, Bryden-Smith and Hunt /4/ observed that the ultimate crack width of reinforced concrete beams under sustained load was about twice the initial, although the increase was sligthly greater for lower amounts of concrete cover.

Bodó and Balogh /14/ reported an increase in crack width by about 80 % in 9 years on concrete beams and slabs prestressed with wires and loaded permanently by concrete blocks. The elements were stored outdoor on a riverside exposed to environmental conditions which produced a fluctuation of crack measurements concerning summer and winter periods superimposed to the overall increasing tendency.

The addition of fibres to the conventionally reinforced concrete reduces the crack width. The increase in crack width due to repeated loading also decreases with increasing fiber content /12/. Paddled fibres experienced better than the hooked or straight fibres. The benefitial influence of fibre addition decreases with increasing steel ratio of conventional reinforcement.

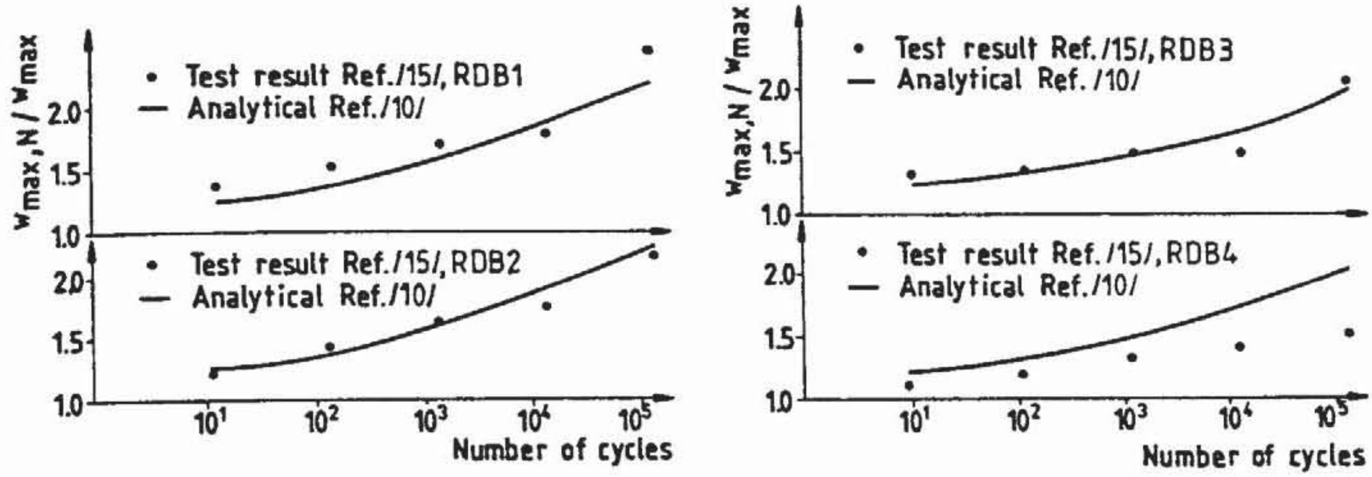


Fig.7 Crack width ratio vs number of load cycles; Bhuvasorakul /15/ (quoted after Ref. /10/) Details of specimens marked RDB1 to RDB4: A_s=400 mm², 568 mm², 200 mm² and 284 mm², resp.; effective depth=262 mm, 373 mm, 130 mm and 186 mm, resp.; concrete cover=50.8 mm, width of beams=203 mm

4. PREDICTION OF LONG TERM AND CYCLES DEPENDENT CRACK WIDTH

A large number of proposals are already available to predict crack width under monotonic loading, but only a few of them are developed to considere repeated or sustained loading as well.

Lovegrove and Din /1/ developed an empirical equation to predict the maximum crack width under cyclic loading based on the upper bound of their test results, as a function of the initial crack width (w_{max}) and the number of load cycles (N):

$$w_{max,N}/w_{max}=(0.382-0.0227\cdot \log N)\cdot \log N \qquad \text{for } 10^5 \le N \le 10^8 \tag{1}$$

Balaguru and Shah /10/ developed a semi-analytical equation which intends to take into account the modifications induced by repeated loading in the compressed concrete and in the tensioned concrete causing reductions in the tension stiffening contribution. Cyclic creep of steel is neglected assuming that the reinforcing steel is cyclically stable at service load.

The maximum crack width is considered as a multiple of the maximum crack spacing and the steel strain at crack (ϵ_{s2}) reduced by the average concrete strain between cracks (ϵ_{ctm}). The maximum crack spacing is calculated by the conventional approach taking the double length required to reach the strength of concrete in tension by evenly distributed bond stresses. The ratio of cyclic and static crack widths for a flexural member may be expressed as:

$$\frac{w_{max,N}}{w_{max}}=\frac{c_2(e_{s2,N}-e_{ctm,N})\frac{h-x_N}{d-x_N}}{c_1(e_{s2}-e_{ctm})\frac{h-x}{d-x}}=1.2[1+1.22(\frac{\log N}{7})^2]\frac{\frac{h-x_N}{d-x_N}e_{s2,N}}{\frac{h-x}{d-x}e_{s2}} \tag{2}$$

where coefficients c_1 and c_2 include the bond properties. The ratio c_2/c_1 was set to 1.2 based on measurements on wire-mesh reinforced mortar specimens and is assumed to be valid for reinforced concrete beams. The assumption on a constant c_2/c_1 ratio, however, may lead to overestimation of crack width for small number of load cycles. The steel strain (ϵ_{s2}) and the neutral axis (x) for static loading are calculated by performing a cracked section elastic analysis. With cyclic loading, $\epsilon_{s2,N}$ and x_N can be similarly calculated using $E_{c,N}$ rather than E_c: $E_{c,N}=\sigma_{c,max}/(\sigma_{c,max}/E_c+\epsilon_c^{creep})$.

ϵ_{ctm} and $\epsilon_{ctm,N}$ were rewritten based on test results by Bhuvasorakul /15/ to consider the tension stiffening effect and its modification during cycling. A comparison of test results and predictions by this approach of Balaguru and Shah /10/ is indicated in Fig.7.

EUROCODE 2 on Design of Concrete Structures predicts the characteristic crack width as a multiple of the average crack spacing, the average steel strain (taking also into account tension stiffening) and a coefficient relating the average crack width to the characteristic value being 1.7 for load induced cracking. The ratio of the characteristic crack width after long term or repeated loading ($w_{k,N}$) to its initial value (w_k) may be expressed as:

$$\frac{w_{k,N}}{w_k}=\frac{1-\beta_1\beta_{2,N}(\frac{\sigma_{sr}}{\sigma_{s2}})^2}{1-\beta_1\beta_2(\frac{\sigma_{sr}}{\sigma_{s2}})^2}=\frac{1-0.5\beta_1(\frac{\sigma_{sr}}{\sigma_{s2}})^2}{1-\beta_1(\frac{\sigma_{sr}}{\sigma_{s2}})^2}$$

where β_2 is the coefficient which considers the deterioration of tension stiffening for long term and repeated loading
=1.0 at first loading and
=0.5 for long term or repeated loads independently on their duration.

Cyclic creep strain of concrete is not accounted. The crack spacing is assumed to be constant independently on the duration of load or the number of load cycles.

CEB-FIP Model Code 1990 predicts the characteristic crack width as a multiple of the maximum crack spacing and the average steel strain reduced by the average concrete tensile strain and the concrete shrinkage strain. The ratio of the characteristic crack width after long term or repeated loads to its initial value may be expressed for stabilised cracking as:

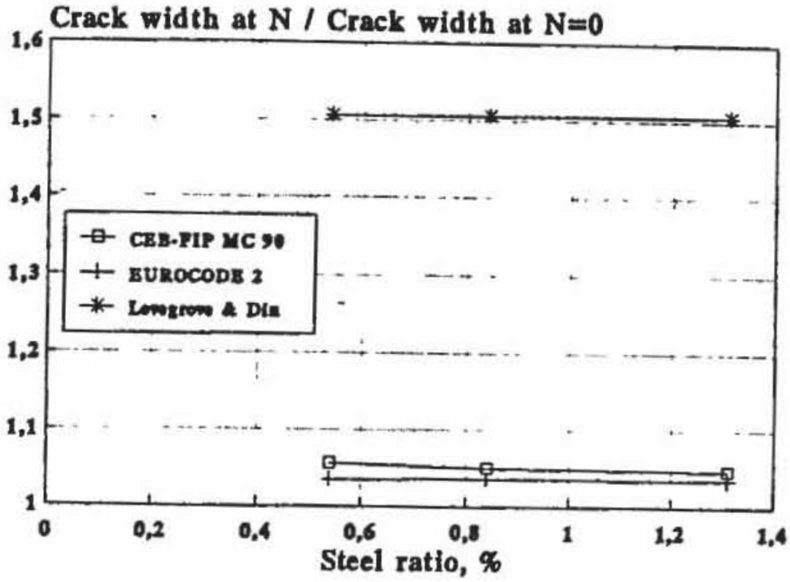
$$\frac{w_{k,N}}{w_k}=\frac{\epsilon_{s2}-\beta_N\epsilon_{sr2}-\epsilon_{cs}}{\epsilon_{s2}-\beta\epsilon_{sr2}-\epsilon_{cs}}=\frac{\epsilon_{s2}-0.38\epsilon_{sr2}-\epsilon_{cs}}{\epsilon_{s2}-0.60\epsilon_{sr2}-\epsilon_{cs}}$$

where β is an empirical factor to assess averaged strain
=0.60 for short term loading and
=0.38 for long term and repeated loading in case of stabilized cracking

ϵ_{sr2} $=\frac{f_{ctm}(t)}{\varrho_{s,ef}E_s}\left(1+\frac{E_s}{E_c}\varrho_{s,ef}\right)$ is the steel strain at crack under forces producing the mean value of the concrete tensile strength at time of cracking /$f_{ctm}(t)$/

in which $\varrho_{s,ef}$ $=A_s/A_{c,ef}$: A_s related to the effective area of concrete in tension

Fig.8 indicates a comparison of the above proposals obtained on a typical singly reinforced rectangular cross section subjected to a bending moment applied as a cyclic load up to 2 million load cycles. The predicted increase in crack width differ considerably. The Eurocode 2 and the CEB FIP Model Code 90 predict a relative increase below 10 %, while the formula by Lovegrove and Din gives an increase by about 50 %. The Formula by Balaguru and Shah predicts an increase above 150 %, however, the considered 2 million load cycles might be out of the range of applicability of Eq.(2). Nevertheless, it was not presented in Fig.8.



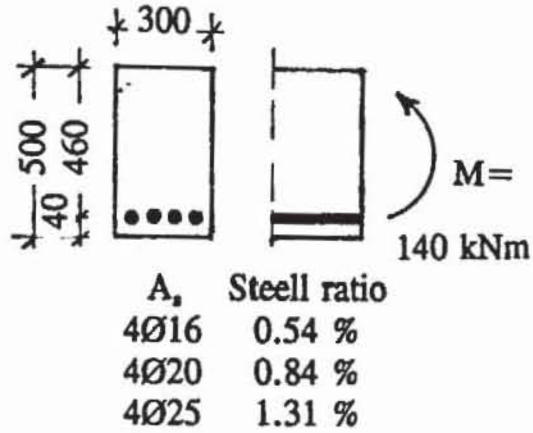


Fig.8 Relative increase of crcak width under repeated loading as a function of the steel ratio

5. CONCLUSIONS

Crack widths predicted on the basis of short term static tests do not provide a satisfactory guide to crack widths in service. Repeated and long term loads may produce significant increase in crack widths owing to the increase of concrete deformations resulting in an increase of steel stres in the crack and the increase of slip along the reinforcing bar which causes a decrease of tension stiffening. Test data with long term and repeated loading indicate that:

a) Crack widths increase with loading time or increasing number of load cycles and may even exceed the double of the initial value of crack width. Crack widths at unloading have also an increasing tendency.

b) Stell stress in the crack of an element subjected to bending may increase above the theoretical value based on cracked section analysis due to the drop of neutral axis.

c) The increase of average steel strain is a function of steel ratio and load level in addition to the number of load cycles or time elapsed under sustained load.

d) The decrease in average crack spacing may be significant if the sustained or repeated load level is only slightly above the cracking load.

Besides the stress redistribution in the vicinity of the reinforcing bar, the creep of the compressed concrete is to be considered as well in predicting the crack width for elements in bending under long term and cycles. Cyclic creep of steel seems to be neglectable in this respect. In most proposals the cyclic steel stress level is considered to be an influencing parameter besides the duration or number of load cycles. The benefitial effect of fibre

reinforcement if available in addition to the conventional reinforcement should be considered by the prediction of both the short term and the long term cracking.

A comprehensive experimental and theoretical study is required in order to predict the cracking behaviour of reinforced concrete elements in service conditions including repeated and long term loading.

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