THE ROTATION CAPACITY OF PLASTIC HINGES IN REINFORCED CONCRETE BEAMS AND SLABS

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SUMMARY

A mathematical model for calculating the rotation capacity of plastic hinges in reinforced concrete beams and slabs is presented. The model is based on the integration of the section curvature along the beam, taking into account the contribution of concrete between cracks (tension stiffening) and the shifting of the tensile force by shear cracks. The material behavior of reinforcing bars, concrete and bond is described as realistically as possible. The analytically predicted rotation capacities of about 70 beams compare favourably with the experimental results. The parameter studies demonstrate that the plastic rotation capacity of hinges given by the CEB-FIP Model Code is unconservative for cold worked deformed reinforcing bars with a low ratio tensile strength to yield stress and a low value of the uniform elongation.

INTRODUCTION

The actual load-bearing capacity of reinforced concrete structures can be utilized in the design by using the theory of plasticity or by admitting redistribution of moments. In this case it is presupposed that the plastic hinges forming in the highly stressed areas have a large enough rotation capacity. However, this capacity is not arbitrarily large, but narrowly limited. Knowledge of the rotation capacity of reinforced concrete hinges is therefore a prerequisite for the reliable application of the theory of plasticity in reinforced concrete design.

STATE OF THE ART

The current methods for determining the rotation capacity of plastic hinges are based either on the statistical evaluation of tests (Ref. 1) or on theoretical approaches (Refs. 2,3).

In the CEB-FIP Model Code (Ref. 4) the plastic rotation capacity is given as a function of the related height of the compression zone. The curve was obtained by a statistical evaluation of 350 tests (Ref. 1). The scatter of the experimental results is very large (Fig. 1) indicating that basic influencing factors are neglected by the approach. In this case a statistical evaluation is relatively meaningless. Moreover, in many tests very ductile reinforcing bars (ratio tensile strength R_m to yield stress $R_e \geq 1.4$, elongation at maximum load (uniform elongation) $A_G \geq 12$ %) with a relatively poor bond behavior were employed. These parameters increase the rotation capacity significantly. In contrast to that modern cold worked reinforcement is less ductile ($R_m/R_e \lesssim 1{,}1$, $A_G \lessapprox 4$ to 6 %) and has a very good bond behavior. Therefore the plastic rotation capacity given in Ref. 4 may not always be reached when using cold worked reinforcement.

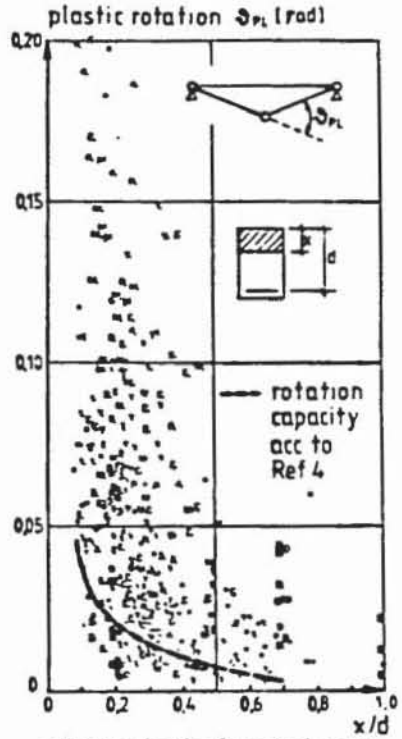


Fig. 1: Plastic rotational capacity of concrete hinges (after Siviero, Ref. 1)

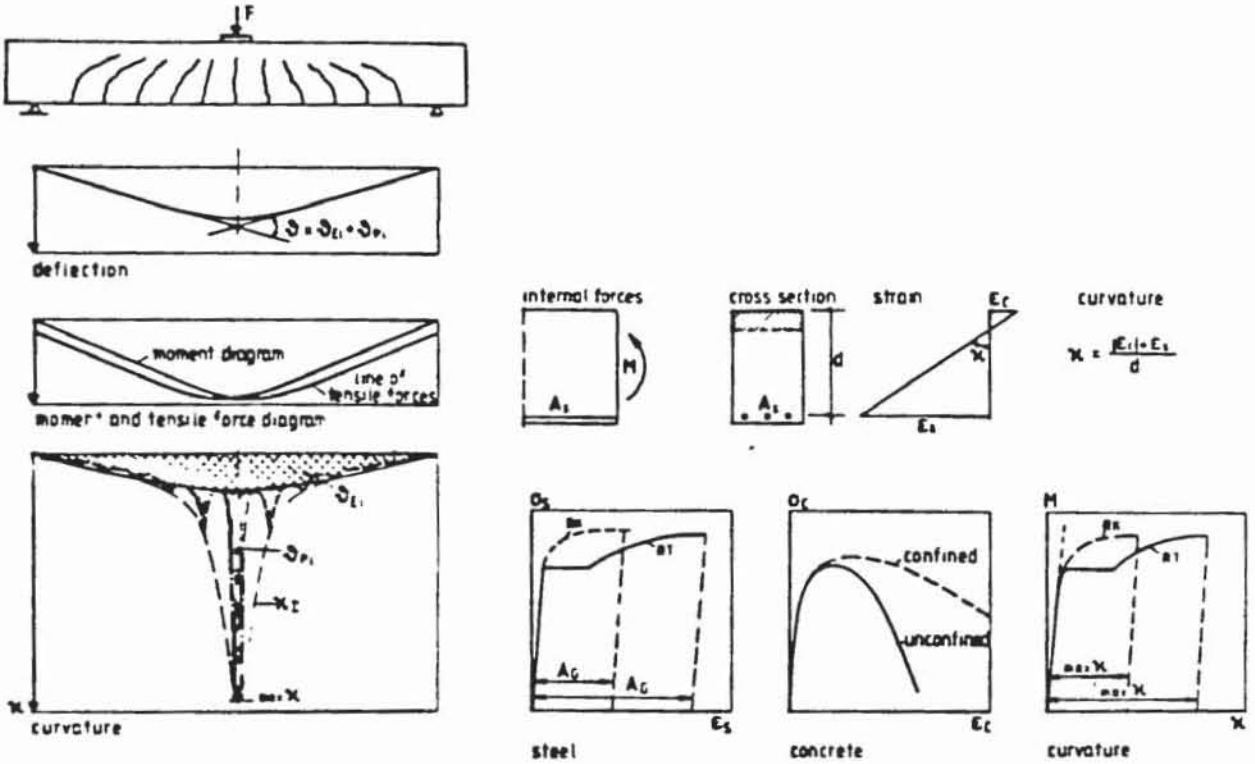
The basic work on analytical models of plastic hinges was performed by Dilger (Ref. 2) and Bachmann (Ref. 3). While Dilger disregards the important influence of the contribution of concrete between cracks (tension stiffening) on the rotation capacity, Bachmann assumes constant bond stresses between cracks thus neglecting the influence of displacements on the bond behavior. Both authors assume simplified relationships to describe the behavior of the bars in the inelastic range. Because the rotation capacity significantly depends on the shape of the stress-strain curve of the steel and on the bond behavior, these results cannot be transferred to reinforcement commonly used today.

Because of this unsatisfactory situation an analytical model for plastic hinges was developed, which is based on the work in Refs. 2 and 3. In this paper only a brief description of the study is given, for details see Ref. 5.

ANALYTICAL MODEL

Based on the given dimensions of the cross section (concrete and reinforcement) and the assumed stress-strain relationships of steel and concrete, the moment-curvature relationship or the tensile force-curvature relationship, respectively, are calculated (Fig. 2b), assuming plaine sections remain plaine. The distribution of moments along the beam is calculated taking into account the width of the loading plate. The load is increased until the ultimate moment previously calculated is reached. In statically indeterminate structures an statically determinate beam with a length equal to the distance between two adjacent points of zero moment is cut out of the real system. If shear cracks must be expected, the shifting of the tensile force compared to the M/z-line (M = Moment, z = lever arm)(truss analogy) is taken into account assuming an angle of the inclined compression struts according to Ref. 2. From the tensile force distribution and the tensile force-curvature relationship the curvature in the cracks is reached (Fig. 2a). The crack distance is calculated according to Ref. 6.

The contribution of concrete between cracks is calculated for every beam section between two cracks by means of an iterative solution of the differential equation of bond, using a modified version of the program described in Ref. 7. On the basis of the calculated steel strain distribution, the distribution of curvature between the cracks is derived by using the distance of the tensile reinforcement to the neutral axis (Fig. 2a). Integration of these curvatures over the beam length yields the rotation capacity of the beam. The plastic rotation is defined as the difference between the rotation at ultimate load and at a load causing yielding of the reinforcement at the point of maximum moment (see Fig. 2a). The mathematical model can only yield reliable results if the actual material is described very accurately. Therefore the stress-strain relationship of the reinforcing steel is described by a polygon (with up to 30 points, which allows a very close representation of the real behavior). The stress-strain relationship of the concrete is formulated as proposed in Ref. 8. This model which consists of a parabola and a trilinear continuation (Fig. 4) takes into account the influence of confinement by stirrups on the strength σ_1 and corresponding strain ε_1, the descending branch of the stress-strain relationship (defined by σ_1/ε_2 and σ_3/ε_3) and on the residual strength σ_4. The values for these characteristic parameters are chosen according to Ref. 8 for the problem on hand. The bond behavior is described by the bond stress-slip relationships shown in Fig. 5, which are based on the model proposed in Ref. 9 taking into account the test results given in Ref. 10. Fig. 5 is valid for a concrete compression strength $f_c' = 30\ \mathrm{N/mm^2}$. For other values of f_c' the bond stress-slip relationships are varied according to Ref. 9.



a) Integration of the curvature

b) Moment-curvature relationship

Fig. 2: Mathematical model

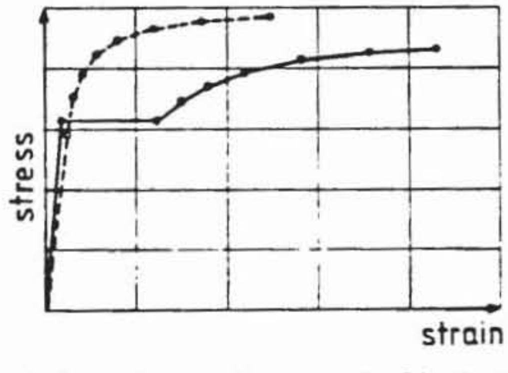


Fig. 3: Polygon defining the stress-strain relationship of the reinforcing steel

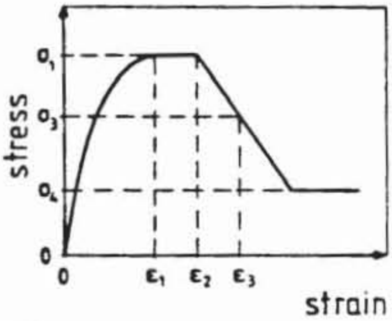


Fig. 4: Stress-strain relationship of concrete (after Ref. 8)

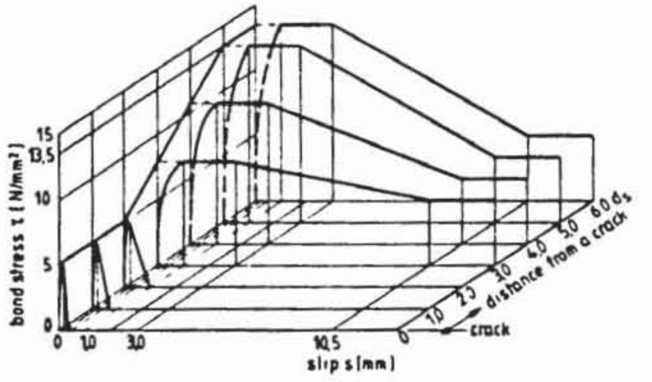


Fig. 5: Bond stress-slip relationships ($f_c{}' = 30\ N/mm^2$)

To check the validity of the assumptions, the predicted response of beams was compared with available test results. In Fig. 6 the calculated and measured distribution of the residual steel elongation after unloading from maximum load is plotted. Note, that in the experiment (Ref. 11) the crack spacing varies, while in the calculation a constant value was assumed. Fig. 7 shows the predicted rotation capacity of 70 beams as a function of the measured value. The data points scatter around the 45-degree line for perfect agreement. The coefficient of variation is only 17 %. In Fig. 8 the predicted and measured rotation capacities of otherwise identical beams are plotted as a function of the percentage of reinforcement. The typical behavior found in the tests is captured quite well by the calculation.

From Figs. 6 to 8 it can be concluded that the proposed analytical model is sufficient accurate for practical purposes.

PARAMETER STUDIES

The influence of the percentage of reinforcement ($\mu = A_s/b \cdot d$) on the rotation capacity (elastic and plastic components) of beams on two supports reinforced with cold woked deformed bars is shown in Fig. 8. The typically roof-shaped curve has a maximum rotation capacity at a critical percentage of reinforcement μ_{crit}. μ_{crit} depends on the dimensions of the section, the material behavior and the confinement. In the present case μ_{crit} amounts to about 0.42 %. For $\mu < \mu_{crit}$ the beam fails due to rupture of the reinforcement, i.e. the ductility of the reinforcing bars is fully utilized. The rotation capacity decreases rapidly with decreasing percentage of reinforcement, because only few cracks are formed and the contribution of concrete between cracks is significant. For $\mu \geq \mu_{crit}$ the failure of the beam is due to crushing of the concrete in the compression zone. The steel strains are smaller than the values which can be sustained by the bars.

In general it was assumed in literature that the rotation capacity of plastic hinges and hence the possible degree of moment distribution mainly depends on the deformation capacity of the compressed concrete. By contrast, theoretical consideration and test results show that at small percentages of reinforcement the rotation capacity is governed by the behavior of the steel and may be very small. For this reason, in the following studies small reinforcement percentages ($\mu \leq \mu_{crit}$) were assumed.

The influence of the stress-strain relationship of steel on the rotation capacity was studied on a single-span beam loaded in midspan (Fig. 9a). The assumed stress-strain relationships (Fig. 9b) cover approximately the range valid for welded wire fabric produced in Germany (no failure in the welds). As anticipated the ultimate moment increases with increasing tensile strength, but because of the decreasing ratio R_m/R_e the length of the plastic hinge decreases (Fig. 9d). Furthermore the decreasing ductility (characterized by the uniform elongation A_G) the maximum curvature decreases as well (Fig. 9c). As result of these effects, the rotation (elastic and plastic components) of the beam reinforced with the more ductile steel 3 amounts to about 1.6-times the value valid for steel 1. The difference in the plastic rotation of the hinge is even larger. On the whole,

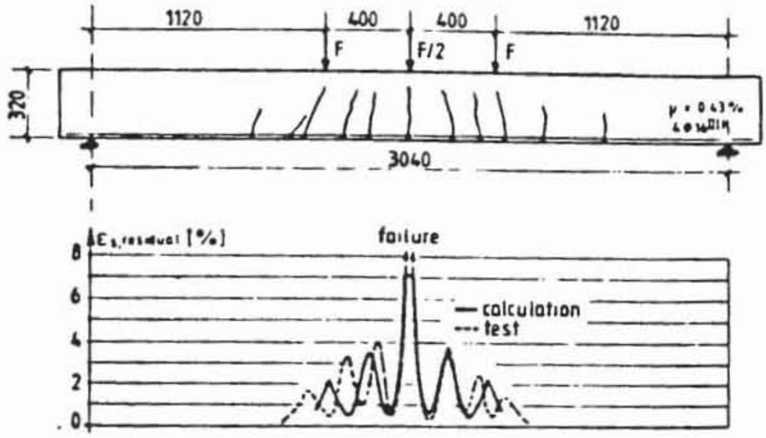
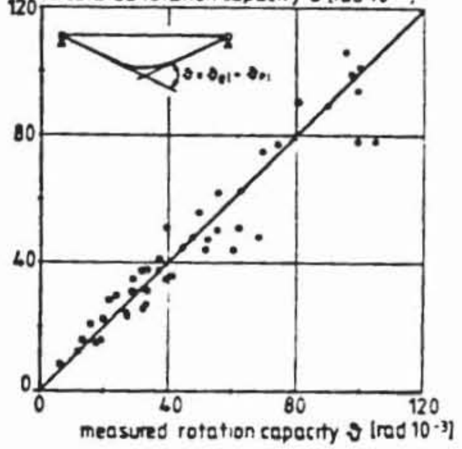


Fig. 6: Residual steel strain in the area of a plastic hinge according to calculation and experiment



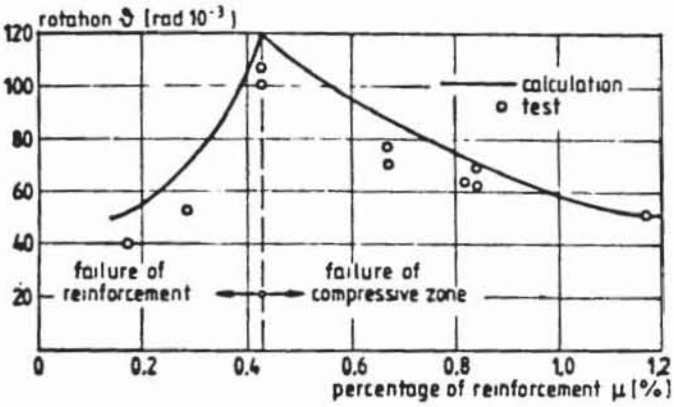


Fig. 7: Rotation capacity of reinforced concrete hinges according to calculation and experiment

Fig. 8: Influence of the percentage of reinforcement on the rotation capacity of reinforced concrete beams

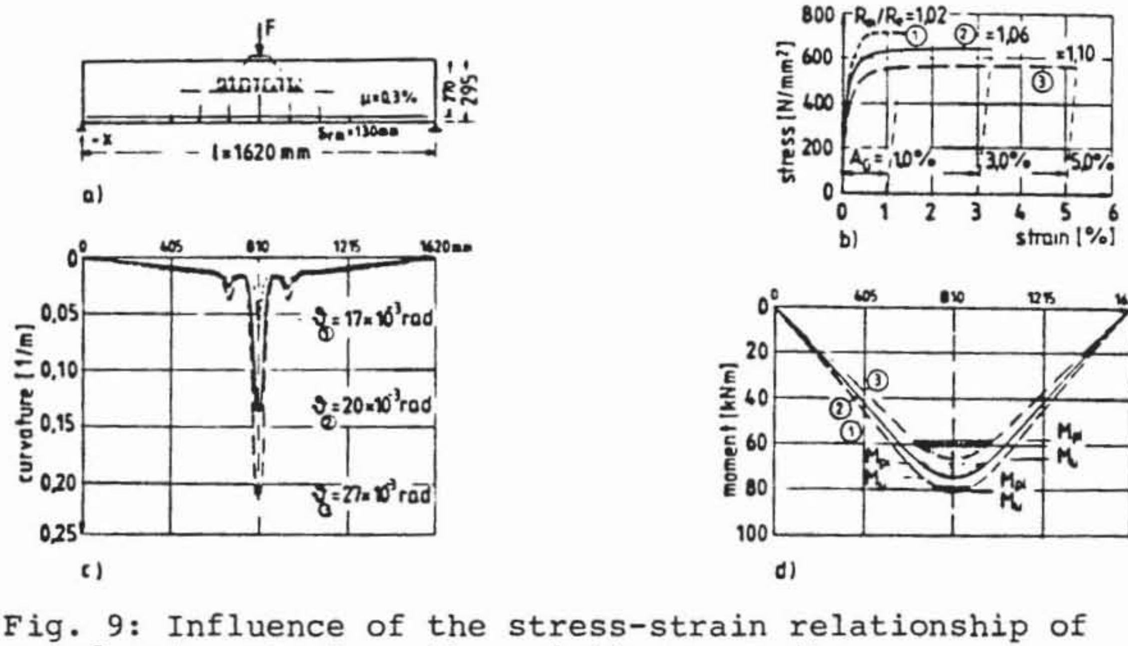


Fig. 9: Influence of the stress-strain relationship of reinforcing steel on the rotation capacity

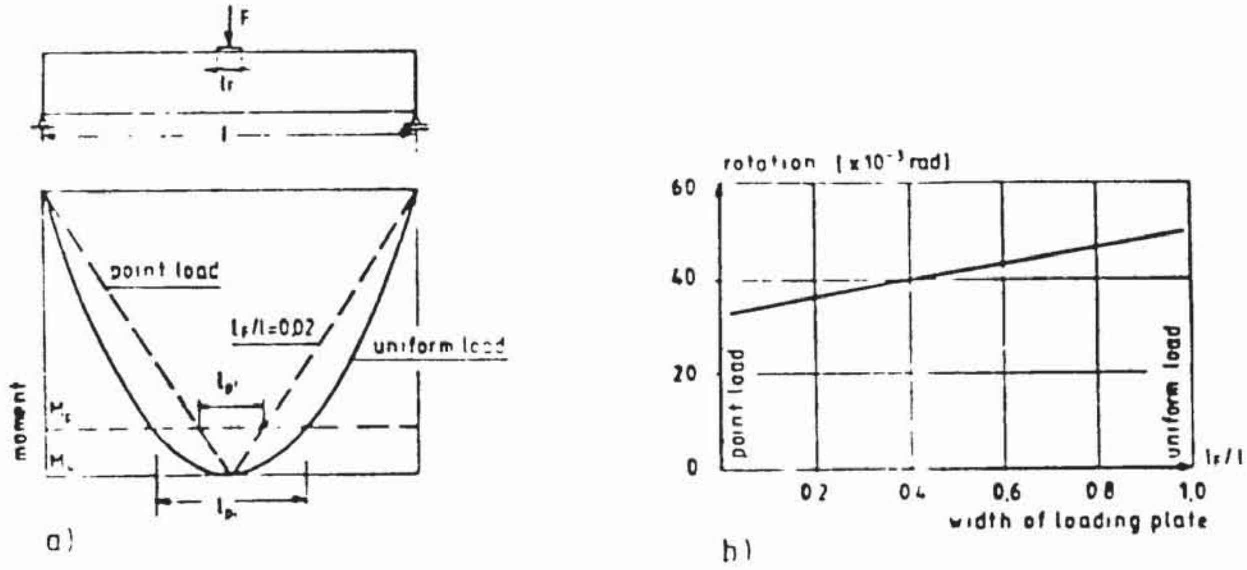


Fig. 10: Rotation capacity as a function of the width of the loading plate

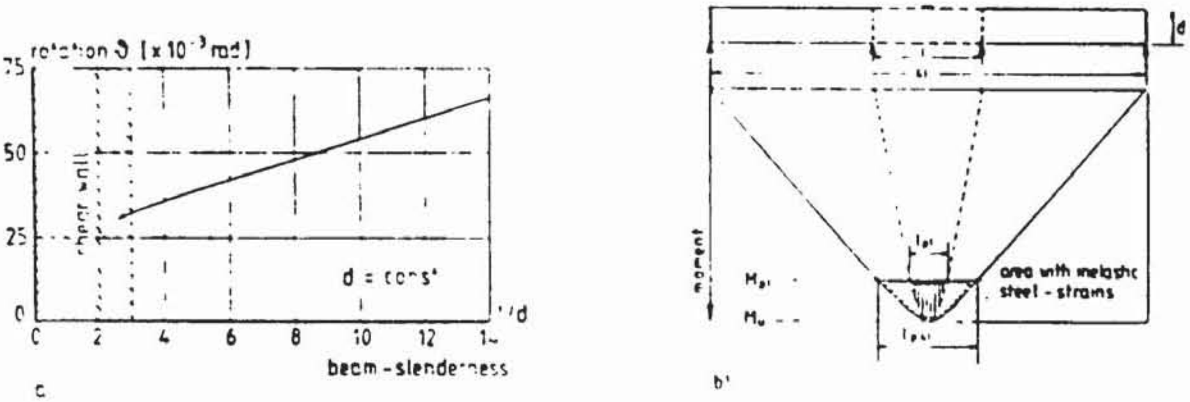


Fig. 11: Rotation capacity as a function of beam slenderness

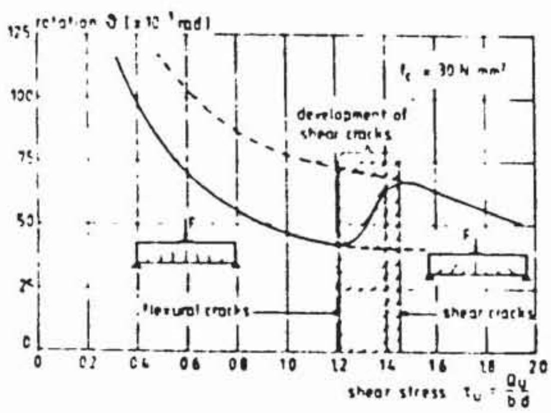


Fig. 12: Influence of shear cracks on rotation capacity

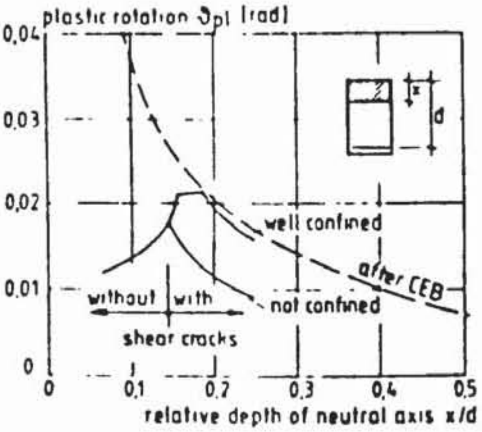


Fig. 13: Influence of confinement on plastic rotation capacity

it can be stated that the rotation capacity of reinforced concrete hinges increases with increasing values for the ratio R_m/R_e and/or the uniform elongation A_G. The importance of the steel behavior on the rotation capacity has also been pointed out in Refs. 2 and 3.

In the following studies a stress-strain relationship according to line 2 of Fig. 9b is assumed.

Under otherweise constant conditions, the rotation capacity increases with increasing width of the loading plate because of the increased length of the plastic hinge (Fig. 10). This agrees with earlier studies (Ref. 2). A similar effect has the increase of the beam length (Fig. 11).

In Fig. 12 the rotation capacity is plotted as a function of the shear stress at failure. In this study the beam cross-section and the reinforcement were kept constant and the beam length was varied, resulting in a constant ultimate moment and increasing shear stresses with decreasing span. With increasing shear stresses (decreasing slenderness) the rotation capacity decreases (compare Fig. 11). When shear cracks are formed (in the example at $\tau_u \sim 1.2 - 1.5$ N/mm²), the rotation capacity is significantly increased due to the shifting of the tensile force (truss analogy) and the corresponding increase of the length of the plastic hinge. Qualitatively the same result was found in Ref. 3. Note, that the rotation capacity may drastically be reduced if a shear failure occurs before the bending strength is reached.

In Fig. 13 the plastic rotations given by Ref. 4 are compared with the calculated values. The assumed beam slenderness is $l/d = 20$ with $d = 300$ mm. The influence of the confinement of the compression zone ($f'_c = 35$ N/mm²) was also studied. It can be seen that for small values of the ratio x/d (i.e. small percentages of reinforcement) the calculated plastic rotation is much smaller than the values given by the CEB-FIP Model Code. For higher values of x/d the CEB line is reached only if the compression zone is well confined by closely spaced stirrups. In general the plastic rotation capacity of beams reinforced with heat treated bars (which is relatively ductile) will reach or even surpass the CEB line. Deformed bars cold worked by twisting are situated between these extremes (Ref. 4).

CONCLUSIONS

Based on the present study, the following conclusions can be drawn:

1) With the presented analytical model, the rotation capacity of plastic hinges in reinforced concrete beams or slabs can be predicted with sufficient accuracy for practical purposes.

2) The rotation capacity of plastic hinges is significantly influenced by the shape of the stress-strain relationships of the reinforcement in the inelastic range. This is especially valid for low percentages of reinforcement.

3) The plastic rotation capacity given in Ref. 4 may not be reached, if cold worked reinforcement (welded wire mesh or twisted bars) and low percentages of reinforcement are employed.

4) Because of 3) the degree of moment redistribution allowed in Ref. 4 may be unconservative in some cases. This has been confirmed by tests (Ref. 12).

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