# DETERMINATION OF AVERAGE ROUGHNESS AND PROFILE AUTOCORRELATION WIDTH OF METALLIC SURFACES WITH A WHITE LIGHT SENSOR

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A new procedure to determine simultaneously a horizontal descriptor of the surface – the autocorrelation width – and the most important vertical descriptor – the root-mean-square roughness – is presented. It is based on the inversion of an analytic contrast formula. After a short introduction to white light random phase contrast measurement we describe the elimination process and show first experimental verifications.

## 1. Introduction

For roughness measurement of metallic surfaces a noncontacting, fast, and compact device is desirable. White light speckle contrast measurements [1–5,7] offer interesting possibilities but have been restricted so far to a relatively small measuring range [3,6]. They also suffered from the fact that speckle contrast does not only depend on the surface parameter to be measured (average roughness $R_q$, $R_a$, $R_z$, see e.g. [11]) but also on other unknown statistical characteristics of the surface. Ambiguities introduced in this way are, however, not as critical as in methods using highly coherent light. A strictly monotonic depence of the contrast as a function of roughness over a wide range ($0.06\ \mu m \leqslant R_a \leqslant 10\ \mu m$) was obtained [4,5] by an incoherent superposition of a uniform intensity to the random intensity pattern.

In refs. [4,5] this was supported by low temporal and spatial coherence of the light, relative large aperture, and defocussing of the imaging lens. The resulting random phase contrast structure was scanned with a detector aperture larger than the nominal speckle diameter. Therefore this low contrast method allows for a reliable, compact, and comparatively cheap sensor with important instrumental advantages (non-laser illumination, simple detectors, insensitivity to vibrations, alignment and focussing).

In this paper we describe a further step towards an unambiguous determination of the average roughness or some other vertical descriptor ($R_a$, $R_q$, $R_z$, ...). We calculate from the measured data an estimation of the autocorrelation width $w_a$ (horizontal descriptor) and use it to invert an analytic contrast formula $C = C(R_q, w_a)$, which thus yields an estimation of $R_q$.

## 2. Stochastic phase contrast with incoherent superposition of a uniform intensity

In this section we summarize the measuring process given in detail in ref. [4]. A rough surface is illuminated (fig. 1) by a white light tungsten lamp via beamsplitter BS. The surface is imaged into the image plane IP and generates a random intensity structure in a volume around IP. This structure (cf. fig. 5 in ref. [4]) either resembles the well-known laser speckle patterns nor the white light speckle structures shown in ref. [3]. Therefore we speak of random white light phase contrast by defocussing.

The intensity is scanned in the detector plane defocussed by $\Delta z$. The contrast is defined as in the case of speckle contrast as

$$C = \sigma_I / \langle I \rangle \quad (1)$$

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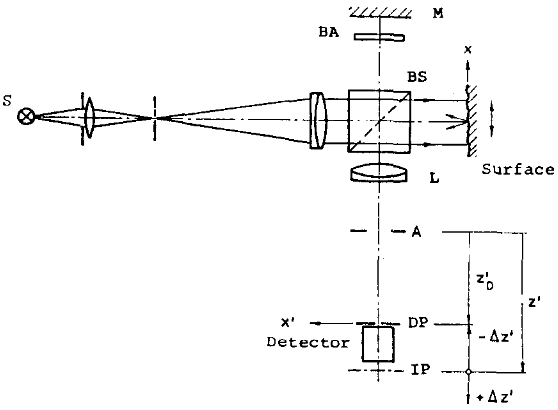


Fig. 1. Arrangement for the measurement of the contrast with superposed incoherent intensity. BS beamsplitter, BA beam attenuator, M mirror, L imaging lens, A imaging aperture, IP image plane, DP detector plane, S white light source.

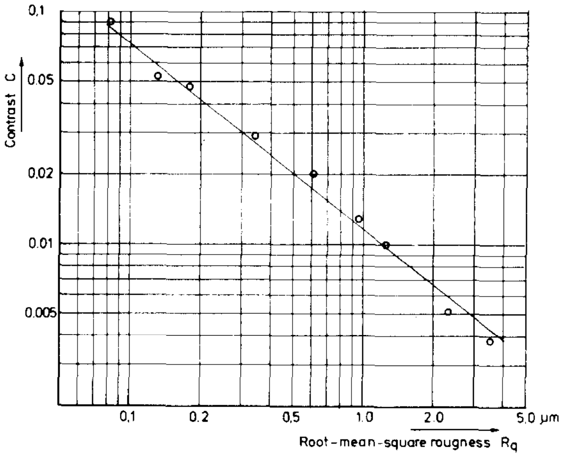


Fig. 2. Measured contrast $C$ over root-mean-square roughness $R_q$. The scanning length on ground samples was 7 mm. The contrast was averaged over 6 measurements along different scanning lines.

with variance $\sigma_I^2$ and mean value $\langle I \rangle$ of the intensity. For practical measurements $C$ is illustrated by an online microcomputer from a sufficiently large number (~2000) of discrete intensity values. The intensity ratio $t$ of the incoherent superposition via beamsplitter BS and plane mirror M is defined as

$$t = I_u / I_s \,, \tag{2}$$

where $I_u$ is the uniform intensity via mirror M and $I_s$ is the intensity via the rough sample in the limit of vanishing roughness, both measured in the detector plane.

The dependence of the contrast $C$ on $t$ as a parameter is shown in detail in refs. [4,5]. Fig. 2 shows the contrast $C$ over root mean square roughness $R_q$, which is defined as

$$R_q = \sigma_h = \left( \int_{-\infty}^{+\infty} h^2 \, w(h) \, \mathrm{d}h \right)^{1/2} = \langle h^2 \rangle^{1/2} \,, \tag{3}$$

where $\langle h \rangle$ is the zero mean stochastic process of the profile height $h$ assumed to be stationary and ergodic, $w(h)$ is the first order probability density function, and $\sigma_h$ the standard deviation. Even a small amount of incoherent superposed light (e.g. straylight or reflections from the beamsplitter ($t \approx 0.05$)) not only reduces the level of the curves $C = C(R_q)$ but also changes the slope of these curves. This is also true for the roughness parameters $R_a$ and $R_z$. In ref. [4] a nearly straight curve in log–log-representation was measured with $t = 1.4$ over a relatively large range of roughness values $R_a$ ($0.06\ \mu\mathrm{m} \leqslant R_a \leqslant 10\ \mu\mathrm{m}$). This clearly exceeds the measuring range given by [6] for conventional white light speckle methods.

## 3. Analytic expression for the stochastic contrast

White light speckle theory is reviewed in ref. [7]. Using propagation theory of the mutual coherence [8–10] and introducing interlaced facets of surface profile autocorrelation and spatial coherence we developed the following analytic expression for the contrast $C$, which is valid in the case of laser speckle as well as in the case of our low contrast white light phase structure [5]:

$$C = \left\{ \frac{\nu_1}{(1 + 16W^2\sigma_h^2)^{1/2}} \left[ 1 + \exp\left( \frac{-16k_0^2\sigma_h^2}{1 + 16W^2\sigma_h^2} \right) \right] \right.$$

$$+ \frac{2\nu_2}{[(1 + 12W^2\sigma_h^2)(1 + 4W^2\sigma_h^2)]^{1/2}}$$

$$\times \left[ \exp\left( \frac{-4k_0^2\sigma_h^2}{1 + 4W^2\sigma_h^2} \right) + \exp\left( \frac{-12k_0^2\sigma_h^2}{1 + 12W^2\sigma_h^2} \right) \right]$$

$$\left. - \frac{2(\nu_1 + 2\nu_2)}{1 + 8W^2\sigma_h^2} \exp\left( \frac{-8k_0^2\sigma_h^2}{1 + 8W^2\sigma_h^2} \right) \right\}^{1/2}$$

$$\times \left[ (\nu_1 + n)t + n + \frac{\nu_1}{(1 + 8W^2\sigma_h^2)^{1/2}} \right.$$

$$\left. \times \exp\left( \frac{-4k_0^2\sigma_h^2}{1 + 8W^2\sigma_h^2} \right) \right]^{-1} . \tag{4}$$

In (4) $\sigma_h = R_q$ is the root-mean-square roughness of the surface. The spectral response of the light–instrument–sample–detector chain is assumed to be gaussian

$$S(k) = \text{const.} \times \exp[-(k - k_0)^2/2W^2] \tag{5}$$

with wave number $k$, mean wave number $k_0$, and wave number $W$ of the spectral width:

$$k = 2\pi/\lambda, \quad k_0 = 2\pi/\lambda_0, \quad W = 2\pi/\Delta\lambda . \tag{6}$$

$w_c$ = equivalence width of the spatial coherence , (7)

$w_a$ = equivalence width of the surface profile autocorrelation , (8)

$w_b = |1/\beta' - 1/\beta'_D|D$ = width of the point spread function , (9)

$\beta'$ = transverse magnification related to the image plane, (10)

$\beta'_D$ = transverse magnification related to the detector plane , (11)

$D$ = imaging aperture , (12)

$n = w_b/w_a$ = number of autocorrelation facets in one dimension , (13)

$d = w_c/w_a$ = number of autocorrelation facets within coherence width $(d \geqslant 2)$ . (14)

The interlacing parameters $\nu_1$ and $\nu_2$ are

$$\nu_1 = (2n - d)(d - 1) , \tag{15}$$

$$\nu_2 = n(2d - 2)(2d - 3) - \tfrac{10}{3}d^3 + 8d^2 - \tfrac{14}{3}d . \tag{16}$$

The autocorrelation width $w_a$ as a horizontal descriptor of surface roughness has the advantage that it does not depend on high frequency components of the surface power spectrum, whose influence on measurements is difficult to assess in any case. Therefore comparison with mechanically measured samples should be more reliable.

In fig. 3 the dependence of the contrast $C$ on roughness $\sigma_h$ for various autocorrelation widths according to (4) is shown. We note that from a measured value of $C$ it is not possible to find $\sigma_h$ unambiguously in the field of curves with varying $w_a$.

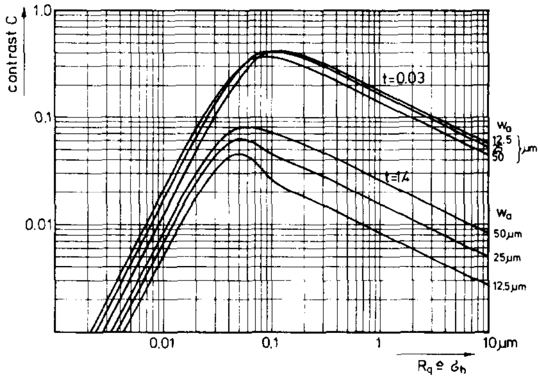


Fig. 3. Contrast $C$ calculated by eq. (4) over root-mean-square roughness $R_q$ with correlation width $w_a$ of the surface and intensity ratio $t$ of the incoherent superposition. Mean wavenumber $k_0$ = 11.5 1/μm, spectral bandwidth $w$ = 2.0 1/μm, diameter $D$ of aperture A in fig. 1. $D$ = 13 mm, coherence width $w_c$ = 108 μm, width of the point spread function, $w_b$ = 469 μm.

## 4. Inversion of the contrast expression for $w_a$ and $\sigma_h$

In the range $\sigma_h \geqslant 0.15$ µm we can neglect the exponentials in (4) and obtain a simpler expression

$$C_t = \frac{\nu_1^{1/2}}{(1+16W^2\sigma_h^2)^{1/4}[(\nu_1+n)t+n]} . \qquad (17)$$

From (17) we find

$$C_0/C_t = [(\nu_1+n)t+n]/n , \qquad (18)$$

i.e. the ratio of the contrast $C_t$ for $t \neq 0$ and $C_0$ for $t = 0$ is independent of $\sigma_h$; in logarithmic scales (fig. 3) therefore the corresponding curves have constant vertical distance.

From (15), (14), and (13) we calculate

$$\nu_1 = (2w_b/w_a - w_c/w_a)(w_c/w_a - 1) . \qquad (19)$$

From (18) and (19) we get the autocorrelation width as

$$w_a = w_c \frac{2w_b - w_c}{(C_0/C_t - 1)\, w_b/t + w_b - w_c} , \qquad (20)$$

where the quantities $t$, $w_c$, $w_b$ on the right side are known and $C_0/C_t$ is the ratio of the mean intensities (cf. (4)), which are easily measured.

With $w_a$ known, the uncertainty of the determination of $\sigma_h$ from the field of curves $C = f(\sigma_h, x_a, ...)$ is resolved: (17) implies

$$\sigma_h = (1/4W)(\nu_1^2/C_0^4 n^4 - 1)^{1/2} , \qquad (21)$$

where the terms on the right side are determined by (13), (19), and (20).

The root mean square roughness $R_q = \sigma_h$ can thus be calculated by (21) in quasi real time. As the accuracy of formula (4) on which our procedure is based, is difficult to estimate, practical measurements on surfaces with known roughness values have to confirm the practability of the method.

## 5. Experimental verification

Fig. 4 shows preliminary measurements on ground surfaces using the new procedure. $\sigma_{h\ mech}$ is the known value of roughness measured mechanically with the stylus instrument, and $\sigma_{h\ opt}$ is the value calculated from (21), however with correction constants $\chi_1$, $\chi_2$ introduced in order to improve the approximation:

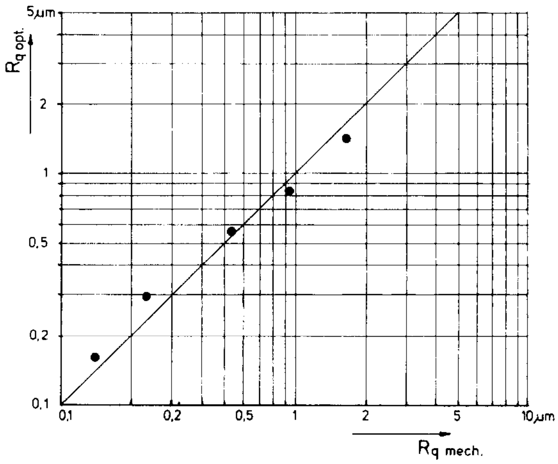


Fig. 4. Root-mean-square roughness $R_{q\ opt}$ calculated from eq. (21) over $R_{q\ mech}$ measured mechanically by a stylus instrument.

$$\sigma_h = (\chi_1/4W)(\nu_1^2/C_0^4 n^4 - \chi_2)^{1/2} ; \qquad (22)$$

they were given the values $\chi_1 = 1.05$ and $\chi_2 = 1.1$ (instead of $\chi_1 = \chi_2 = 1$ in (21)). Fig. 4 shows that the accuracy of (4) is already good enough for a starting point. No systematic fitting of the $\chi$-values was attempted so far.

The autocorrelation width $w_a$ calculated by (20) decreased monotonically from 70 µm for $\sigma_{h\ mech} = 0.14$ µm to 23 µm for $\sigma_{h\ mech} = 1.7$ µm.

## 6. Conclusion

The method of incoherent superposed stochastic phase contrast allows for a simple procedure to measure simultaneously a horizontal descriptor of the surface – the autocorrelation width – and with improved uniqueness the most important vertical descriptor – the average roughness.

This is a further step towards an optical sensor which measures standardized roughness parameters relevant for functional behaviour of surfaces.

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