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Nonlinear Control of High Purity Distillation Columns

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Abstract: Two simple models of distillation columns are studied to investigate their suitability for the practical use with exact I/O-linearization. An extension of exact I/O-linearization, the asymptotically exact I/O-linearization is applied to the control of a high purity distillation column, using one of these models to derive the static state feedback law. Simulation studies demonstrate the advantage of asymptotically exact I/O-linearization versus classical exact I/O-linearization techniques. Experimental results show the excellent performance of asymptotically exact I/O-linearization using a simple distillation model.

Keywords: Exact I/O-linearization, asymptotically exact I/O-linearization, nonlinear control, distillation control, nonlinear observer, dynamic simulation, experimental studies.

1 Introduction

High purity distillation columns show an extremely, set point dependent nonlinear behavior with respect to manipulated variables and disturbances. Thus, the use of nonlinear control, e.g. exact I/O-linearization, instead of linear control has some advantages. The most important one is the good performance in a wide range of the nominal operating point. This is necessary if product specifications vary (that means set points have to be changed) or feed flow and feed concentrations alter, because of differing plant capacities or upsets in upstream units.
The idea of exact I/O-linearization using static state feedback laws was first introduced systematically by (Kravaris & Chung, 1987) and (Isidori, 1989). Several investigations on applications based on simulation studies are published (e.g. (Allgöwer et al., 1989) and (Castro et al., 1990)). Only a few studies show experimental results (Levine & Rouchon, 1991).

The derivation of feedback laws for exact I/O-linearization is based on dynamic process models. Many models with differing complexity for distillation columns are available. The more detailled a model is the more difficulties arise to determine the plenty of necessary parameters (e.g. parameters for physical properties like mass transfer coefficients). Therefore, it is obvious, that control laws for exact I/O-linearization should be derived from the simplest model which produces satisfying results. Another reason is, that using a less detailled model results in less complex feedback laws.
For this reason, we first describe two different models of a distillation column: a very simple model 1 (CMO-model with relative volatility) and a more detailled but still simple model 2 (CMO-model including real vapour-liquid-equilibria). Static state feedback laws for exact I/O-linearization are derived from these two models and investigated in simulation studies with a very detailed model used as the real process.
It is shown in section 2, that the feedback law derived on the basis of model 1 leads to setpoint dependent instability in a wide range around the operating point. The control law based on model 2 extends the stable region about the set point.
Exact I/O-linearization usually demands many state variables of the process which cannot be measured completely in practice or measurement instruments are very expensive and/or produces deadtimes, e.g. liquid compositions or internal flows. For practical applications an observer for a distillation column is introduced and its use with exact I/O-linearization is demonstrated.
Implementing an observer in the control structure and exactly linearizing the I/O-behavior of this observer by using corrector terms in the feedback law leads to the asymptotically exact I/O-linearization which is explained in a nutshell for systems with a relative degree of 1 in section 3.
Simulation studies of asymptotically exact I/O-linearization with the feedback law based on the equilibrium model demonstrate the excellent control performance.
In section 4, experimental studies using a distillation column on a pilot plant scale produce almost perfect results and show the practicability of asymptotically exact I/O-linearization .

1.1 Models of Distillation Columns

The determination of the feedback laws, regarded in this paper, are based on two different models of a distillation column.
Both models consist of dynamic component balances for liquid composition on each tray, constant molar holdup, neglecting energy balance and vapour holdup (Doherty & Perkins, 1982). To simplify the model equations only one liquid feedstream is considered.
Component balance for the upper section of the column (rectifying section) leads to:

$$n_k \dot{x}'_{i,k} = \varepsilon D(x'_{i,k-1} - x'_{i,k}) + V(x''_{i,k+1} - x''_{i,k}) \tag{1}$$

and the lower section of the column (stripping section):

$$n_k \dot{x}'_{i,k} = (\varepsilon D + F)(x'_{i,k-1} - x'_{i,k}) + V(x''_{i,k+1} - x''_{i,k}). \tag{2}$$

Where n_k is the molar liquid holdup on tray k, $x'_{i,k}$ and $x''_{i,k}$ are the liquid and vapour compositions of component i on tray k, ε is the external reflux ratio, D is the distillat stream, F is the feed stream and V is the vapour stream inside the column.
Vapour compositions $x''_{i,k}$ of component i on tray k are calculated with the constant relative volatility α (Henley & Seader, 1981):

$$x''_{i,k} = \frac{\alpha x'_{i,k}}{1 + (\alpha - 1)x'_{i,k}} \tag{3}$$

The so resulting model 1 is a nonlinear ODE-system.

The more detailled model (model 2) also consists of dynamic component balances (Eq.(1) and (2)) but a different calculation of vapour compositions $x''_{i,k}$ is used:

$$\begin{aligned} x''_{i,k} &= x''_{i,k+1} + EMG_k(K_{i,k}x'_{i,k} - x''_{i,k+1}) \\ &\quad \text{with } K_{i,k} = K_{i,k}(p_k, T_k, x'_{i,k}). \end{aligned} \tag{4}$$

Where p_k is the pressure and T_k is the temperature on tray k. The different behavior between a real and a theoretical plate of a distillation column is taken into account by the use of constant Murphree Efficiency EMG_k (Murphree, 1925).
Additional equations are necessary for the temperatures on each tray, calculated from the boiling point condition:

$$0 = 1 - \sum_{i=1}^{n} K_{i,k}x'_{i,k}, \tag{5}$$

and those for the calculation of the pressure profile of the column:

$$p_k = p_{k-1} + \Delta p_{stat} + \Delta p_{dyn}V^2. \tag{6}$$

(With Δp_{stat} as the static and $\Delta p_{dyn}V^2$ as the dynamic pressure loss of one tray).
This leads to a strongly nonlinear DAE-system with implicit algebraic equations (liquid compositions as dynamic and temperatures as algebraic states).

For simulation studies, a rate based distillation column model, including energybalances and constant volume holdup on each tray (Lang, 1991) is used as the real process.

1.2 An Example Column

As an example we consider an atmospheric, 40 tray, high purity, binary distillation column on a pilot plant scale (Fig.1). The real plant is 10m of height and consists of the distillation tower, a total condensor and a reboiler with electrical heating. The whole equipment is of industrial standard. A mixture of Methanol and 1-Propanol is fed on the 21st tray. Automation

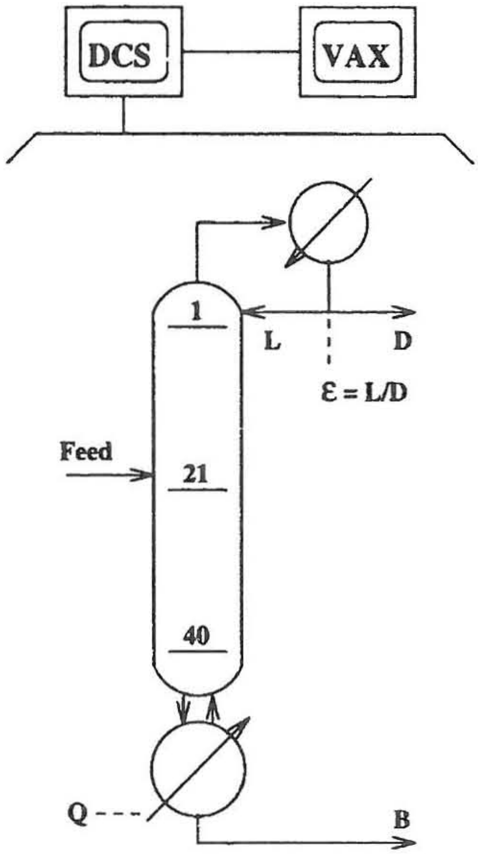

Figure 1: Structure of the controlled plant

is done via a distributed control system (DCS) and a VAX-computer connected to the DCS. The control strategy is an inferential control scheme. Earlier investigations have shown that the temperatures (respectively concentrations) on the 14th and 28th tray should be chosen as controlled variables (Allgöwer & Raisch, 1992). External reflux ratio ε and electrical heating Q of the reboiler are chosen as manipulated variables.

2 Exact I/O-Linearization of Distillation Columns

The idea of exact I/O-linearization based on static state feedback is to transform a nonlinear interacting I/O-behavior into a linear decoupled one. Differentiating the outputvector $\mathbf{w}$ until the input $\mathbf{u}$ appears (that means determining the relative degree of the system) and using the right sides of those differential equations to form a desired linear system with the new input $\mathbf{v}$ leads to the static state feedback law which compensates nonlinearities in I/O-behavior and decouples I/O-channels. Additional terms can be used to produce a desired linear I/O-behavior. The use of this linearization technique requires a minimum-phase-system (that means stable zero dynamics). The application of exact I/O-linearization technique to distillation model 1 (Eq.(1), (2) and (3)) is considered in the following. This leads to a MIMO-2x2-system with input $\mathbf{u} = [\varepsilon, Q]^T$ and output $\mathbf{w} = [x'_{14}, x'_{28}]^T$. To determine the relative degree of the system, the outputvector $\mathbf{w}$ has to be differentiated. The appearance of both input variables in the first derivation of the output vector shows the systems relative degree to be $\mathbf{r} = [1, 1]^T$. Thus it is possible to formulate the feedback law in such a way, that the desired I/O-behavior is a first order lag. With respect to the formulation of a linear system with first order lag and gain $K_i = 1$ for output w_i

$$\tau_i \dot{w}_i + (w_i - w_i^0) = v_i, \tag{7}$$

the result is the following control law (using the differential equations of $\dot{w}_1 = \dot{x}'_{1,14}$ and $\dot{w}_2 = \dot{x}'_{1,28}$) to calculate the original inputs $\mathbf{u}$ from the new inputs $\mathbf{v}$:

$$\begin{aligned} \tfrac{\tau_{14}}{n_{14}} \left[\varepsilon D(x'_{1,13} - x'_{1,14}) + V(x''_{1,15} - x''_{1,14})\right] + (x'_{1,14} - x'^0_{1,14}) &= v_1 \\ \tfrac{\tau_{28}}{n_{28}} \left[(\varepsilon D + F)(x'_{1,27} - x'_{1,28}) + V(x''_{1,29} - x''_{1,28})\right] + (x'_{1,28} - x'^0_{1,28}) &= v_2. \end{aligned} \tag{8}$$

$x''_{1,k}$ are calculated with Eq.(3).
τ_{14} and τ_{28} are the desired time constants of the linearized system, whereas $x'^0_{1,14}$ and $x'^0_{1,28}$ are the set points of the nominal operating point.
The states to be measured are the liquid concentrations on trays 13,14,15 and 27,28,29 as well as the distillat stream D. Disturbance F (feed flow) is also needed.
To implement the control law it is necessary to determine electrical heating Q instead of the vapour stream V. Q is the real input variable of the plant. The I/O-behavior of input Q and the real electrical heating of the reboiler is a first oder lag with gain 1. Because of its small time constant (about 10 sec.), with regard to the time constants of the plant (several hours), this I/O-behavior can be neglected. With respect to high purity distillation columns the following assumption can be made: changes in reboiler composition (and therefore temperature too) can be neglected. Assuming a good level control, a steady state energy balance of the reboiler can be formulated with the reboiler inletstream εD and its enthalpy h'_{col}, the reboiler leaving vapourstream V with its enthalpy h''_{reb}, the liquid enthalpy of the leaving liquidstream h'_{reb} and the electrical heating Q as follows:

$$0 = \varepsilon D h'_{col} - V h''_{reb} - (\varepsilon D - V) h'_{reb} + Q. \tag{9}$$

This equation is added to the control law and implemented in the dynamic simulator DIVA (Holl et al., 1987). An explicit formulation is not necessary. The feedback law is not explicitly solved for the input $\mathbf{u} = [\varepsilon, Q]^T$ that need to be determined. Eq.(8) and (9) are treated as an algebraic system and solved simultaneously during simulation. Necessary state variables of the process ($x'_{1,i}(i = 13, 14, 15, 27, 28, 29)$ and D) are easily available in simulation.
Simulating the plant with the complex and the very simple model shows, that in steady state different values of the reflux ratio ε and electrical heating Q are necessary to reach the same operating point. Therefore, the feedback law derived from model 1 cannot produce the

values of ε and Q, necessary to keep the complex model at its nominal point. An extension of the classical feedback structure has to be realized in such a way, that constant offsets $\delta\mathbf{u}$ are added to the calculated values $\mathbf{u}$ as can be seen in Fig.2.
Investigation of stability was done as follows: linearizing the so controlled plant (including

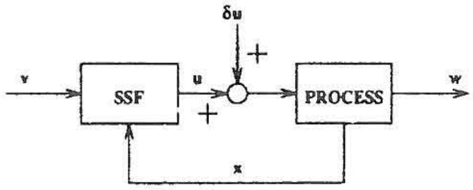

Figure 2: Extended structure of exact I/O-linearization via static state feedback (SSF) with constant offset-compensator

feedback law) at several setpoints and analyzing the poles of the resulting linear system. It was found that only in a small region of setpoints, far away from the nominal operating points, stability is given. That means the new system is setpoint dependent unstable. In addition, simulation studies around the stable operating points produce nonlinear and interacting behavior without any disturbance decoupling. Obviously, the feedback law derived from model 1 is an insufficient basis for a successful application of exact I/O-linearization.
For this reason, no guarantee for stability etc. can be given.
The possible step in order to solve this problem seems to be the use of the more detailled model 2. Deriving the static state feedback law in the same manner as above leads to the same control law like above (Eq.(8)) but using Eq.(4) to calculate vapour compositions $x''_{1,k}$. Although the equilibrium model is a DAE-system, it can be treated as an ODE-system, because values of the algebraic states (temperatures) can be get from the process or Eq.(5) can be solved simultaneously.
It it is easy to see, that disturbances in feed temperature and feed composition do not appear explicitly in the feedback law. The relative degree of these disturbances is greater than 1 and therefore they are decoupled from the outputs. Changes in feed flow can be easily measured (even in practice), so that a compensation of this disturbance can be realized.
This nonlinear control law was tested with the complex process model. Stability analysis mentioned above shows a stable behavior in a region around the nominal operating point. The offset problem is still present. The so controlled system shows neither linear nor decoupled I/O-behavior (Fig.3 and 4).

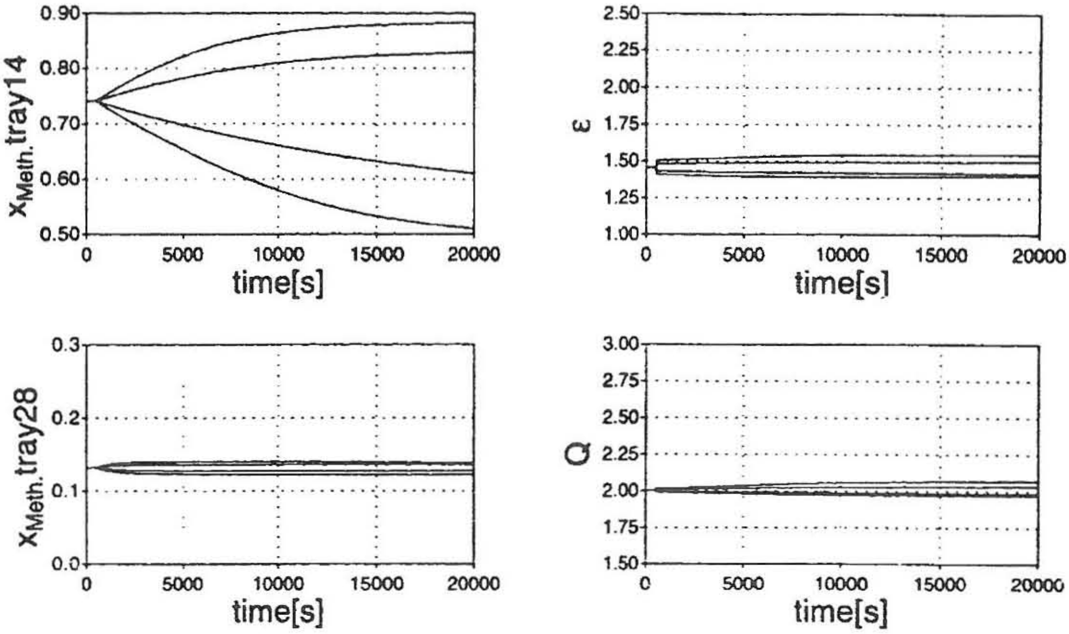

Figure 3: Classical I/O-linearization structure with constant offset-compensator, using model 2, after stepchanges in v_1 of +/- 0.1 and +/- 0.05

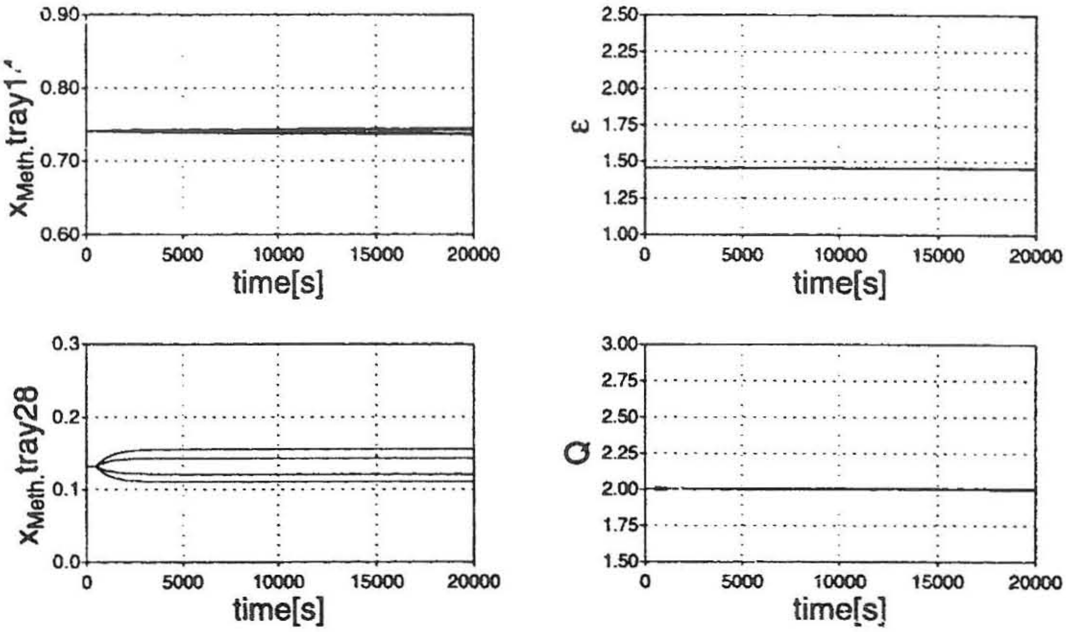

Figure 4: Classical I/O-linearization structure with constant offset-compensator, using model 2, after stepchanges in v_2 of +/- 0.1 and +/- 0.05

Also disturbance decoupling could not be realized. Fig.5 shows the system behavior after a step change in feedflow of +20% (solid line) and a step change in feedcomposition of +10% Methanol (dashed line). Both disturbances force the outputs to differ from their nominal set-

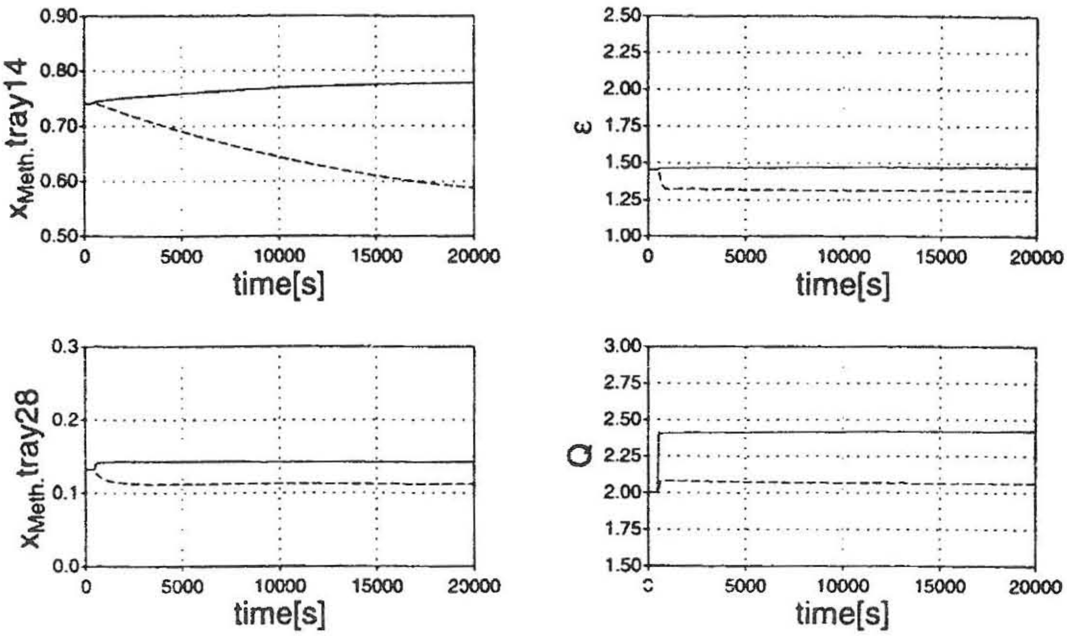

Figure 5: Classical I/O-linearization structure with constant offset-compensator, using model 2, after a step change in feedflow of +20% (solid line) and a step change in feedcomposition of +10% Methanol (dashed line).

points, although a change in feedflow as a measurable disturbance should be compensated and a disturbance in feedcomposition should be decoupled from the outputs.

Using an observer as a model based measurement leads to a control structure as shown in Fig.6. Observer corrector terms are formulated with measurable ouputs $\mathbf{y}$ and estimated outputs $\hat{\mathbf{y}}$, in contrast to the controlled outputs $\mathbf{w}$. The model of the state estimator is based on model 2. Eq.(1) and (2) are extended by adding the corrector terms on the right side:

$$n_k \dot{\hat{x}}'_{1,k} = \epsilon D(\hat{x}'_{1,k-1} - \hat{x}'_{1,k}) + V(\hat{x}''_{1,k+1} - \hat{x}''_{1,k}) + Corr_{14}, \tag{10}$$

$$n_k \dot{\hat{x}}'_{1,k} = (\epsilon D + F)(\hat{x}'_{1,k-1} - \hat{x}'_{1,k}) + V(\hat{x}''_{1,k+1} - \hat{x}''_{1,k}) + Corr_{28}. \tag{11}$$

To determine vapour composition, Eq.(4) is used. The algebraic relation between V and Q is given in Eq.(5). Corrector terms used in the observer are formulated as the difference of the estimated temperatures on tray 14 and 28 and the one measured from the process:

$$\begin{aligned} Corr_i &= \eta_i(T_i - \hat{T}_i) \\ & \quad i = 14, 28 \end{aligned} \tag{12}$$

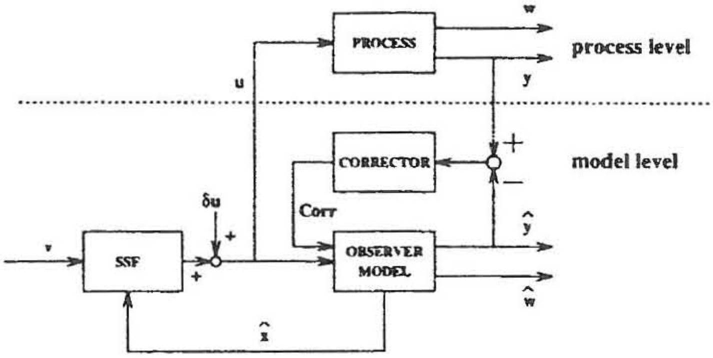

Figure 6: Extended structure of exact I/O-linearization via static state feedback (SSF) with offset-compensator and observer

The corrector gains η_i are determined by physical insight: if the estimated concentration of the lower boiling component $\hat{x}'_{1,k}$ is too small (that means $\hat{T}_i > T_i$) then the sign of η_i must be negative to compensate this lag. The value of η can be chosen in a way, that the values of corrector terms $Corr_i$ are in a bandwith of the other terms on the right side of the component balances. This leads to a good convergence performance of the observer.
The necessity of an offset is still present, but the offset values of $\delta\mathbf{u}$ are about 20% of the offsets used in simulations above.
The new control system leads to a "more linear" I/O-behavior of the process as can be seen if Fig.7 and 8.
This might be a result of the observer, based on the same model like the static state feedback law. Here, the observer corrector terms include an offset $\delta\mathbf{u}^*$ in the control structure, but a total compensation is not realized. For that reason the smaller $\delta\mathbf{u}$ in the use with an observer produces smaller set point dependent offset errors (using a constant offset $\delta\mathbf{u}$) and results in a better I/O-performance than before. But still disturbance decoupling cannot be found (Fig.9) and may lead to an unstable behavior (after a step change in feedcomposition). The reason is that because of no disturbance decoupling, the stability radius is left if the process is too far away from its nominal operating point.

In the next section we will show, that by the use of asymptotically exact I/O-linearization these problems can be overcome.

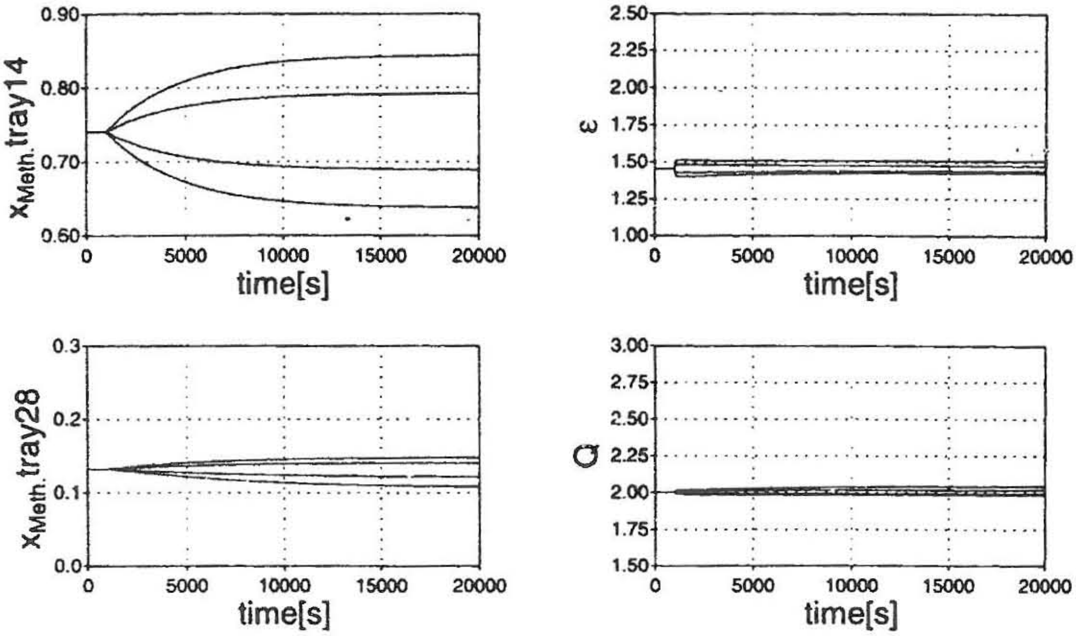

Figure 7: Classical I/O-linearization structure with constant offset-compensator and observer, using model 2, after stepchanges in v_1 of +/- 0.1 and +/- 0.05

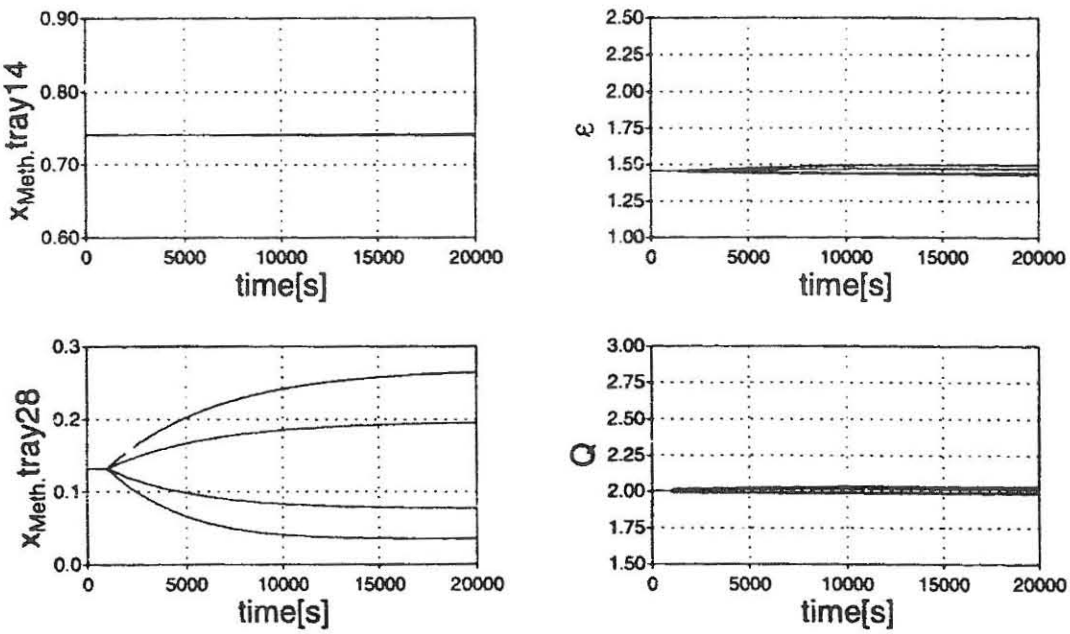

Figure 8: Classical I/O-linearization structure with constant offset-compensator and observer, using model 2, after stepchanges in v_2 of +/- 0.1 and +/- 0.05

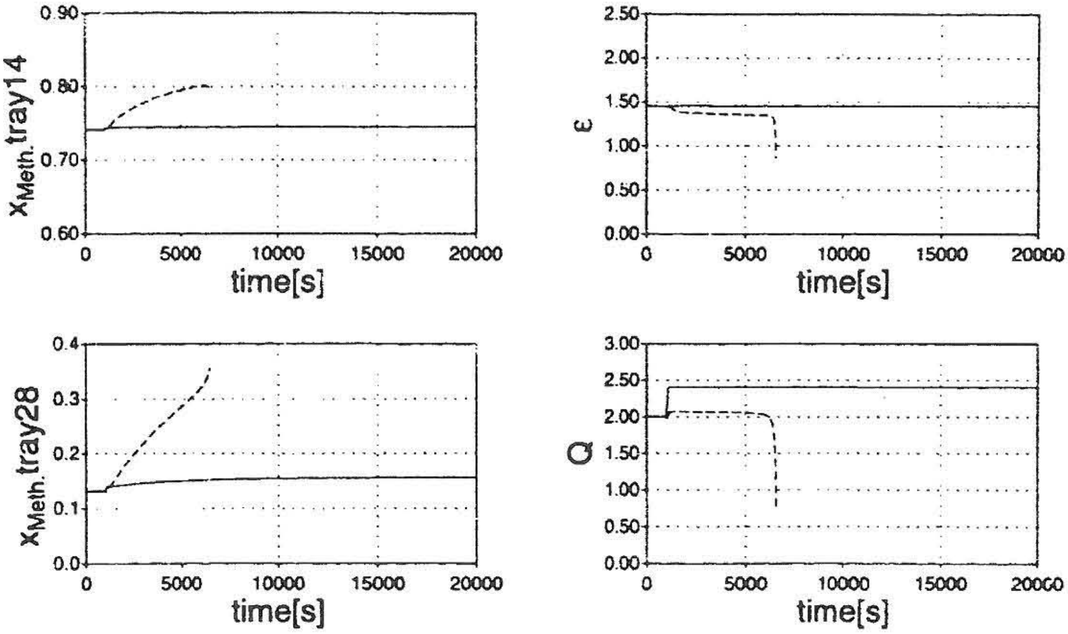

Figure 9: Classical I/O-linearization structure with constant offset-compensator and observer, using model 2, after stepchanges in feedflow of +20% (solid line) and in feedcomposition of +10% Methanol (dashed line)

3 Asymptotically Exact I/O-Linearization of Distillation Columns

The classical use of state estimators like observers leads to a structure for exact I/O-linearization as shown in Fig.10 (solid lines). Here, the observer is used as a modelbased measurement. A modification of this structure is called asymptotically exact I/O-linearization and was introduced by (Friedrich et al., 1992). Basically, the idea is to realize an exact I/O-linearization of the observer under consideration of corrector terms, interpreted as measurable disturbances Fig.10 (dashed line). Implementing the observer in such a way, this control structure is called modelbased control. The observer can be linearized exactly, since no model-plant-mismatch is present and the I/O-linearization is perfect between $\mathbf{v}$ and the observer outputs $\hat{\mathbf{w}}$. If the observer is stable and guarantees convergence between the estimated outputs $\hat{\mathbf{y}}$ and the measured outputs of the process $\mathbf{y}$, the result is an asymptotically exact I/O-linearization between the new inputs $\mathbf{v}$ and the process outputs $\mathbf{w}$. If the observer dynamic is much faster than the process, then the I/O-behavior is nearly exactly linear and decoupled. The advantage of this control structure is twofold: the model to be linearized is the observer model and therefore completely known. Furthermore, I/O-linearization requires knowledge of all states. As the state variables of the observer are naturally available, this is no restriction on the applicability. In (Friedrich et al., 1992) it is shown, that this method can only be used if the model is of relative degree 1 for all considered outputs $\hat{\mathbf{w}}$. As we

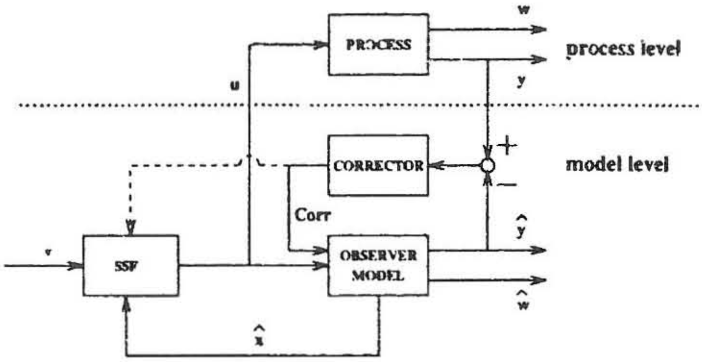

Figure 10: Classical structure of I/O-linearization via static state feedback (SSF) using an observer as modelbased measurement (only solid lines); modified structure for asymptotically exact I/O-linearization (including dashed line)

will see in the following, the observer model used in this paper satisfies these requirements. Additionally a third advantage of this modified structure is, that no offsets on the input $\mathbf{u}$ has to be considered, because this is automatically done by implementing the observer corrector terms in the feedback law.

A method to use asymptotically exact I/O-linearization with models of relative degree greater than 1 is described in (Gilles et al., 1994).

Using the same observer as introduced above it is easy to see, that manipulated variables ε and Q appear in the first derivative of outputvector $\hat{\mathbf{w}} = [\hat{x}'_{1,14}, \hat{x}'_{1,28}]^T$. Thus the relative degree is $\hat{\mathbf{r}} = [1, 1]^T$. Therefore, asymptotically exact I/O-linearization can be applied in the above manner. F as a measurable disturbance is decoupled as well as disturbances in feed concentration which are left in zerodynamics. The resulting feedback law for a first order lag I/O-behavior with timeconstants τ_i is analogue to Eq.(8):

$$\begin{aligned} \frac{\tau_{14}}{n_{14}}\left[\epsilon D(\hat{x}'_{1,13}-\hat{x}'_{1,14})+V(\hat{x}''_{1,15}-\hat{x}''_{1,14}+Corr_{14})\right]+(\hat{x}'_{1,14}-\hat{x}'^{0}_{1,14}) &= v_1 \\ \frac{\tau_{28}}{n_{28}}\left[(\epsilon D+F)(\hat{x}'_{1,27}-\hat{x}'_{1,28})+V(\hat{x}''_{1,29}-\hat{x}''_{1,28}+Corr_{28})\right]+(\hat{x}'_{1,28}-\hat{x}'^{0}_{1,28}) &= v_2 \end{aligned} \tag{13}$$

For the reasons mentioned above, an explicit formulation for V and ϵ is not necessary. Again, energy balance of the reboiler to calculate electrical heating Q can be used (Eq.(9). The application of this control law to the linearization of the complex process model produces almost perfect results in case of the distillation column considered. Fig.11 and 12 show the good linear and approximately decoupled I/O-behavior after step changes in the new inputs $\mathbf{v}$. Changing the desired time constants produce similar results.

Also disturbance decoupling with respect to step changes in feed flow and feed composition is given, as can be seen in Fig.13.

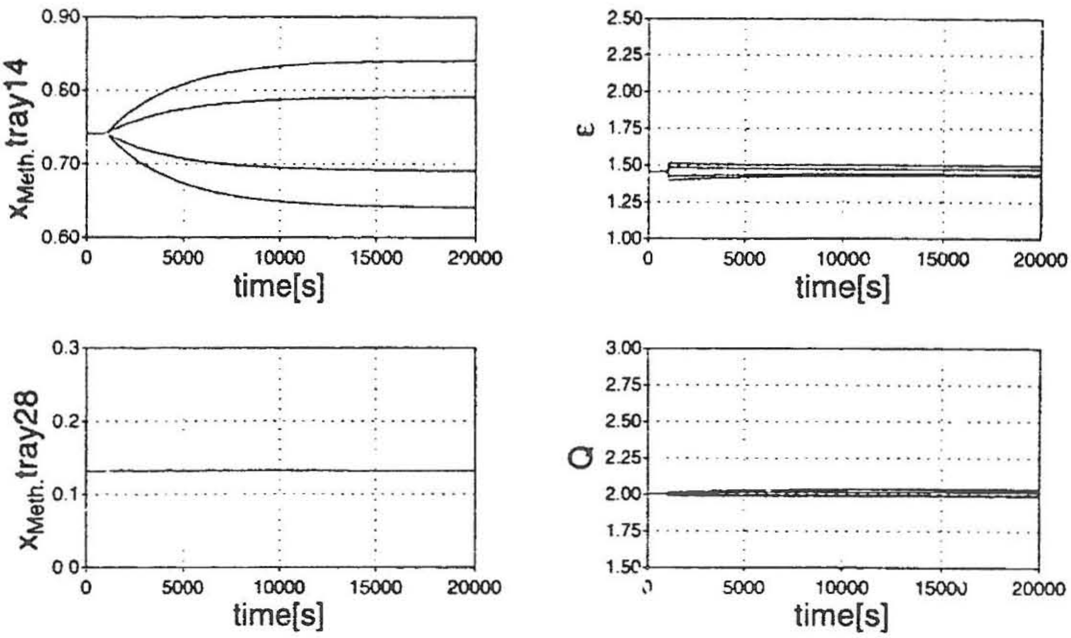

Figure 11: Asymptotically exact I/O-linearized distillation column (using model 2) after stepchanges in v_1 of +/-0.1 and +/-0.05

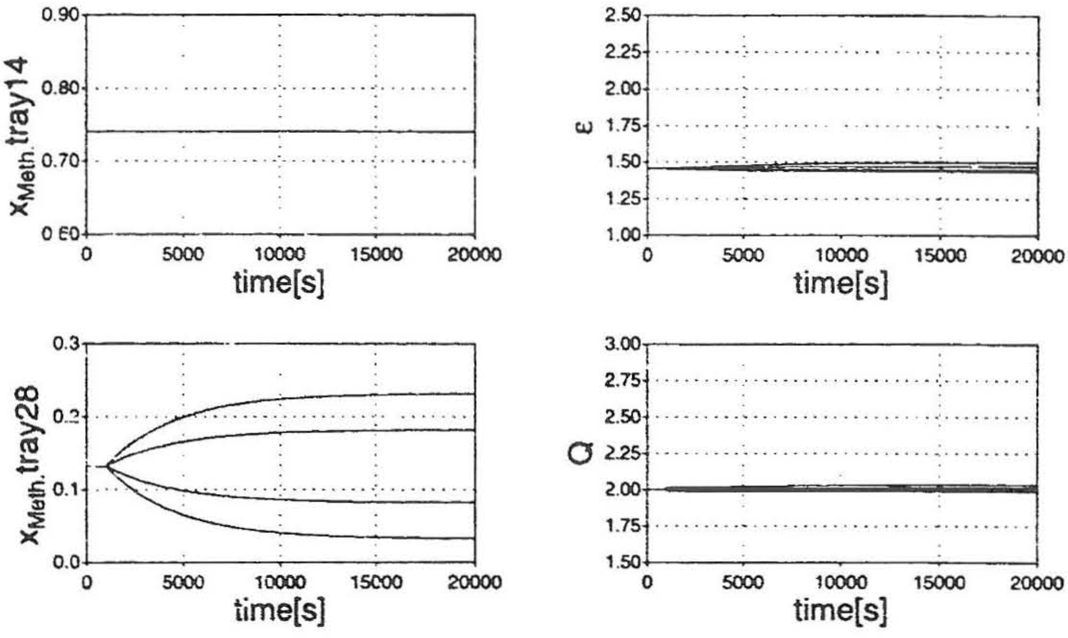

Figure 12: Asymptotically exact I/O-linearized distillation column (using model 2) after stepchanges in v_2 of +/-0.1 and +/-0.05

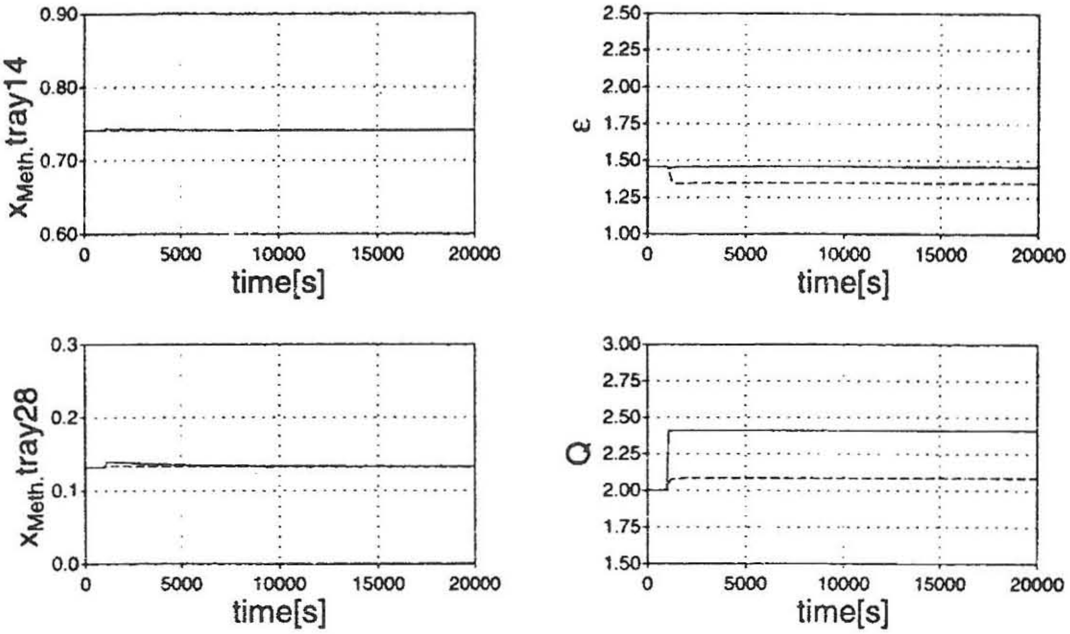

Figure 13: Asymptotically exact I/O-linearized distillation column (using model 2) after stepchanges in feed flow of +20% (solid line) and a step change in feedcomposition of +10% Methanol (dashed line).

4 Experimental Results

To realize this control structure on the real plant, the DIVA realtime environment (Kurrle et al., 1990) is used. The advantage is, that the identical control law of the simulation studies can be used by only replacing the signals of the simulated plant by those of the real distillation process.
As the composition of trays 14 and 28 could not be measured online during experiments (outputs $\mathbf{w}$), the estimated concentrations of Methanol are plotted in the following figures. (Because of the well known relationship between temperature and concentration in a Methanol/1-Propanol mixture and the fast observer dynamics, estimated and real composition on tray 14 and 28 are nearly exact.)
The excellent I/O-behavior can be seen in Fig.14 and 15. Positive as well as negative step changes of different values in the inputs $\mathbf{v}$ produce linear and decoupled answers of the outputs. Checking the so forced linearity of I/O-behavior via identification of a time response after a step change determines a time constant of 3671 sec. and a gain of 1.02 (desired time constant = 3600 sec., gain = 1).
The time constants of the uncontrolled system are about 6 hours in the upper section and about 2 hours in the lower section of the column. Applying asymptotically exact I/O-linearization, those time constants can be reduced to 1 hour in both sections with respect to the new inputvector $\mathbf{v}$. Desired time constants less than 1 hour produce insufficient linear and interacting I/O-behavior. The reason is, that hard changes in the original input variables ϵ