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Walking Without Impacts as a Motion/Force Control Problem

The paper deals with the synthesis of control for impactless bipedal walking. In order to avoid impacts, both the specified motion of the biped and its ground reactions are controlled, yielding a combined motion and force control problem. A method for modeling and solving such problems is proposed, and then illustrated by the example of an impactless planar walk of a seven-link bipedal robot. Some numerical results of the motion simulation are reported.

1 Introduction

Recently a number of research projects have been devoted to simulation and realization of legged locomotion, see e.g., Todd (1985), Raibert (1986), and Vukobratović et al. (1990) for a review. Usually it is distinguished between "static" and "dynamic" machines. The former maintain static equilibrium throughout their motion, they are usually represented by quadrupedal/hexapedal robots, and their speed must be low to assure stability. "Dynamic" machines may have fewer legs, are potentially faster, and attempt to execute dynamically stable motions. Bipedal robots belong to the "dynamic" machines.

The control principles used for generating legged locomotion are of broad variety in the literature. At one end of the spectrum there are passive walking mechanisms, as shown by Mochon and McMahon (1980) and McGeer (1990). The beauty of these mechanisms lies in their simplicity, in their claimed efficiency and in their consistency with the gaits of humans and legged animals, see Alexander (1984). It was shown that gravity and inertia alone may generate the locomotion pattern, and, by adding a small input of active energy, a steady walking can be produced. Raibert (1986) and Thompson and Raibert (1989) have developed similar concepts for passive hopping and running machines, also without large energy expenditure and requiring only simplified control. On the contrary, most of the bipedal robots are actively controlled. Often the walk is generated by linear feedback control, designed either to close the gap between the start and end positions of each step (Mita et al., 1984; Raibert, 1986) or to track fully specified link trajectories (Furusho and Masubuchi, 1986; Furusho and Sano, 1990; Takanishi et al., 1990). The bipeds of Miura and Shimoyama (1984), however, generate their gaits by feedforward control, and small feedback corrections are then added to maintain the walking cycle. Despite the predominant role of active control in producing the gaits of the bipeds, often not all of their degrees of freedom are managed by the control. For many bipeds the rotation about the foot joint is a kind of partly unpowered "inverted pendulum" motion. Then, the bipeds are partly passive walking machines. Moreover, though plane motions are usually the chosen locomotion targets, many bipeds balance actively to pick up the feet as known from the human walking on stilts. Again, the stilt bipeds walk in a partly passive manner (Miura and Shimoyama, 1984; Raibert, 1986; Furusho and Sano, 1990).

In dynamic walking, each time a leg hits the ground, impacts on the system may occur. Large forces may also accompany the end of the double-support phase, due to the pushing off with the trailing leg. Moreover, impacts are observed in systems that balance during walking, and in running and hopping machines. Finally, a wide range of locomotion patterns of humans and animals are also featured by impacts of some kind (Alexander, 1984). Thus, impacts play a role in both machine and natural walking. On the other hand, the engineering and natural bipeds and quadrupeds differ essentially. The living creatures have partially elastic bodies, superior mobility and agility, and perfectly developed systems for sensing and controlling their movements. Damping abilities and dexterose balancing with the whole body allow them to produce comparatively smooth locomotions. Machines lack many of these features. Moreover, the legged robots should not only walk but additionally perform some engineering tasks. Impacts due to the collisions of legs with the ground may destabilize the walking cycle of bipeds and deteriorate the functional capabilities of legged vehicles, and should thus be avoided (Daberkow et al., 1992). One of the possibilities for minimizing these phenomena is to design a control scheme for walking without impacts. This paper is a contribution to such an approach.

The reported method for control synthesis is based on robot dynamics and the condition of impactless walk. The latter is specified by a motion assuring, throughout the whole walking cycle, that the feet are placed on the ground with zero velocity. Moreover, the support is shifted gradually from the trailing to the leading leg. The number of control inputs is time-invariant, specified by the number of motion and force requirements. Thus the control problem considered turns out as a mixed motion and force control problem.

The proposed approach is very far from the passive walking; all degrees of freedom of the robot are definitely controlled.

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Contributed by the Dynamic Systems and Control Division for publication in the JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL. Manuscript received by the Dynamic Systems and Control Division March 1991; revised manuscript received April 1992. Associate Technical Editor: H. Kazerooni.

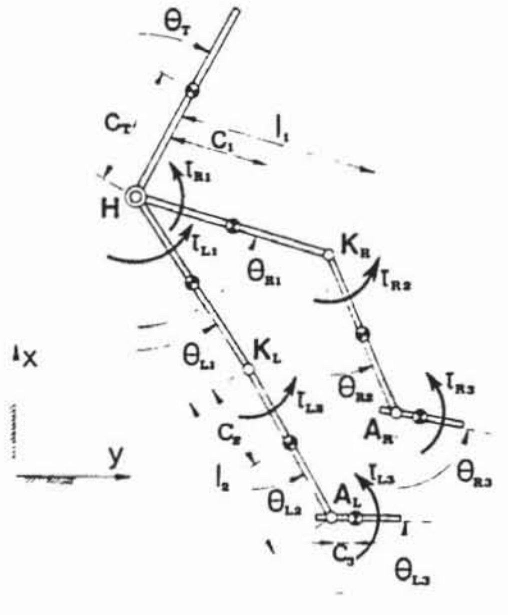

Fig. 1 Seven-link biped model

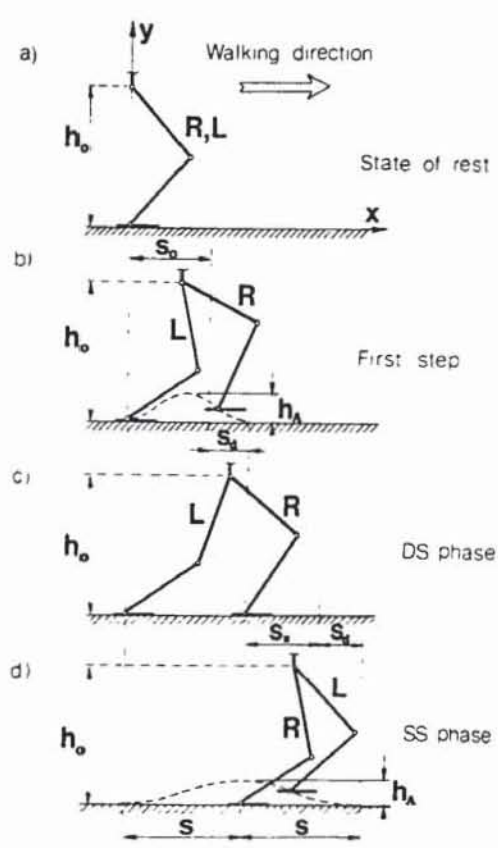

Fig. 2 Model of the biped impactless locomotion

While the passive walking covers a wide range of natural walking patterns of humans and animals, the impactless walking may refer to a "careful walk of a human who carries a full cup of hot tea." The approach presented allows the synthesis of the nominal control for impactless walking by means of continuous time-histories of reference motion. The resulting control torques/forces serve for the feedforward control of the robot. In addition, feedback corrections are introduced to assure asymptotic stability. The paper illustrates the impactless planar walk of a seven-link biped as shown in Figs. 1 and 2. However, the approach is general and it may be adopted for other more sophisticated cases of walking as well as for multilegged robots.

2 Biped Dynamics and Constraints on the System

As shown by Vukobratović et al. (1990), the dynamic equations of a biped are derived usually for the single-support (SS) phase, and the biped is treated as an open-loop multibody system with tree structure. Then, in the double-support (DS) phase, constraints due to contacting the ground with the other leg are additionally imposed on the system. In this paper, first a "flying" biped is introduced. Such a biped has tree topology and the **n** dynamic equations of motion (Schiehlen, 1991) read as

$$M(y)\ddot{y}=h(\dot{y},y)+B^T\tau, \tag{1}$$

where $y = [y_1, \ldots, y_n]^T$ is the n-position vector; M is the $n \times n$ symmetric positive-definite mass matrix; the n-vector h represents the applied and centrifugal forces; $\tau = [\tau_1, \ldots, \tau_k]^T$ are the control torques/forces; B is the $k \times n$ control input matrix; $k = n - m$ is the number of degrees of freedom (DOF) in the SS phase; and m is the number of contraints due to contacting the ground with one leg.

During walking, the biped is constrained by contact with the ground, and the number of ground constraints ("material constraints") varies in turns from m (SS phase) to $2m$ (DS phase), with corresponding variations in the biped's DOF from $k = n - m$ to $l = n - 2m$. In order to model a demanded walking pattern, k or l motion specifications ("motion program constraints") have to be set, respectively. In the SS phase the number of control actuators and motion program constraints are the same. In the DS phase, however, m control actuators are redundant. Thus, in the DS phase, m requirements ("force program constraints") can be imposed additionally on the ground reactions of the system. For the case at hand, the m reactions on the trailing leg are supposed to change smoothly from the values encountered at the beginning of DS phase to zero at the end of this phase, so that the leg will lift off the ground without an impact. The impactless walking will require to model the motion specifications appropriately, too. The motion program constraints must ensure the link time-trajectories of class C^2, and assure that the feet will touch the ground with zero velocity. The motion and force program constraints constitute a program of motion, and the objective of this paper is to determine a control strategy ensuring the realization of that program. As seen, the control problem considered is a kind of a mixed motion and force control problem (Raibert and Craig, 1981).

Summarizing, the following three groups of constraints specifying a demanded walking pattern have been found:

(*a*) $l = n - 2m$ program constraints specifying the motion of the upper part of the "body" (hip, trunk, ...).

(*b*) Alternatively:

1) (SS phase) m program constraints on the motion of the lifted leg (ankle trajectory, foot configuration).
2) (DS phase) m material constraints on the trailing leg due to its placement on the ground. Reactions of these constraints are controlled (appropriate force program constraints are added).

The leg subject to these constraints will be called "managed leg."

(*c*) m material constraints on the supporting leg (SS phase) or on the leading leg (DS phase). Reactions of these constraints are not controlled. The corresponding leg will be called "standing leg."

At the end of SS phase the constraints (*b*1) on the managed leg, which is just placing on the ground, turn into constraints (*c*), the leg becomes the standing leg, and the DS phase begins. Simultaneously, the constraints (*c*) on the previous standing leg turn into constraints (*b*2), the leg becomes the managed leg and the encountered ground reactions on it will vanish by control at the end of DS phase. At the instant of transition from DS to SS phase there is no change in standing/managed leg, and the constraints ($b2$) on the managed leg turn only into constraints ($b1$). An individual approach is usually needed to model the first step, when the biped starts from the state of rest and achieves the state of steady walking (see Fig. 2); and the same concerns the modelling of the last step to stop the biped. These aspects will be considered in Section 4.

The constraints (a)-(c) are assumed to be holonomic, i.e.,

$$\phi(y,t)=0, \tag{2}$$

where $\phi = [\phi_a^T\ \phi_b^T\ \phi_c^T]^T$ are at least twice differentiable functions; ϕ_a, ϕ_b, and ϕ_c are vectors of dimensions l, m, and m, respectively, representing the equations of constraints (a)-(c); and t is the time. Equation (2) is assumed to be independent, $\text{rank}(\partial\phi/\partial y) = n$. Then, the governing equations of the constrained motion of the biped can be written in the following form:

$$M\ddot{y}=h+B^T\tau+C_b^T\lambda_b+C_c^T\lambda_c, \tag{3}$$

$$C_c\ddot{y}+c_{c0}=0, \tag{4}$$

where $\lambda_b = [\lambda_{b1}, \ldots, \lambda_{bm}]^T$ are the specified reactions of constraints ($b2$); $C_c^T\lambda_c$ represents the total of reaction forces of constraints (c); $C_i(y, t) = \partial\phi_i/\partial y$ for $i = b$ or c; and $c_{c0}(\dot{y}, y, t) = (\partial\phi_c/\partial y)^{\cdot}\dot{y} + (\partial\phi_c/\partial t)^{\cdot}$.

By introducing an orthogonal complement matrix $D_c(y, t)$ to the matrix C_c in the n-space of biped's configuration determined by the n-position vector y, the n dynamic equations (3) can be reduced to k canonical equations,

$$D_cM\ddot{y}=D_c(h+B^T\tau+C_b^T\lambda_b). \tag{5}$$

The $k \times n$ full-rank matrix D_c is related to the matrix C_c such that $D_cC_c^T = 0$ (Kamman and Huston, 1984), which means a matrix formulation of the principle of virtual work.

The governing equations of the biped's constrained motion consist now of Eqs. (5) and (4), and the reactions of constraints (c) are available from the reaction equation (Schiehlen, 1991; Blajer, 1991) as

$$\lambda_c(\dot{y},y,t)=-(C_c^TM^{-1}C_c)^{-1}[c_{c0}+C_cM^{-1}(h+B^T\tau+C_b^T\lambda_b)]. \tag{6}$$

The reactions of constraints ($b2$) in Eqs. (5) and (6), λ_b, vanish in the SS phase, and take in the DS phase the specified values according to the force program constraints. The latter are assumed as

$$\lambda_b-\lambda_{bn}(t)=0, \tag{7}$$

where the subscript "n" refers to specified or nominal values, respectively. The functions $\lambda_{bn}(t)$ must be continuous and satisfy the mentioned boundary conditions.

The benefit of dealing with Eqs. (5) and (4) is that they govern both the gait phases, irrespective of which leg is the standing one. At the instant of standing/managed leg transition (at the end of each SS phase), C_c, D_c, and C_b have only to be rearranged by appropriate column pivoting. This problem is illustrated in more detail in Section 4.

3 Control Synthesis

The constraints (2) prescribe implicitly the biped's position in space. Thus, the nominal trajectories $y_n(t)$ can be precomputed as solutions to these equations. Then, by differentiation one can find the nominal time-histories $\dot{y}_n(t)$ and $\ddot{y}_n(t)$. For the numerical calculations, the differentiated forms of the constraint equations are helpful,

$$\dot{\phi}=C\dot{y}+\partial\phi/\partial t=0,$$
$$\ddot{\phi}=C\ddot{y}+c_0=0, \tag{8}$$

where $C(y, t) = \partial\phi/\partial y$, and $c_0(\dot{y}, y, t) = (\partial\phi/\partial y)^{\cdot}\dot{y} + (\partial\phi/\partial t)^{\cdot}$.

Using the nominal motion and forces, the time-history of the nominal control of the biped for the impactless walk can be synthesized according to the inverse dynamic technique as

$$\tau_n(t)=(\hat{D}_c\hat{B}^T)^{-1}\hat{D}_c(\hat{M}\ddot{y}_n(t)-\hat{h}-\hat{C}_b^T\lambda_{bn}(t)), \tag{9}$$

where the superscript "$\hat{\ }$" denotes that $y_n(t)$ and $\dot{y}_n(t)$ are substituted for y and $\dot{y}$ in the terms indicated. Note that the nominal values of ground reactions on the standing leg at the end of SS phase are needed to formulate $\lambda_{bn}(t)$. They are available from Eq. (6) after substitution of $\dot{y}_n(t)$ and $y_n(t)$ for $\dot{y}$ and y. In fact, the nominal values of reactions of constraints (c) can be obtained throughout the motion.

The nominal time-histories $\tau_n(t)$ for control of impactless walking can be precomputed and stored. On this basis a feedforward control scheme is designed. Obviously, a feedback controller is needed in addition to maintain the nominal motion. Such a controller is proposed in the following, assumed that the measured values of y, $\dot{y}$, and λ_b are available.

In the SS phase the motion is preprogrammed by constraints (a) and ($b1$), and in the DS phase the nominal motion is due to constraints (a) only. Violations of these constraints will be reduced by a motion feedback control. By analogy to the constraint stabilization method (Baumgarte, 1972; Ostermeyer, 1990), $\ddot{y}$ can be expressed as

$$\ddot{y}=-C^{-1}(c_0+K_v\dot{\phi}^*+K_p\phi^*), \tag{10}$$

where C and c_0 are as defined in Eq. (8); $\phi^* = [\phi_a^T\ \phi_{b1}^T\ 0^T]^T$ or $\phi^* = [\phi_a^T\ 0^T\ 0^T]^T$ represents the motion program constraints in the SS or DS phase, respectively, determined for the measured values of y and $\dot{y}$; 0 denotes the m-null vector; and K_v and K_p are diagonal matrices of feedback gains. Note that violations of neither constraints (c) nor, in the DS phase, constraints ($b2$) are included in (10). The latter is important since correcting violations of constraints ($b2$) could be in conflict with force control of the ground reactions.

On the assumption that the actual and nominal states of biped motion are close enough, the feedback control scheme can be proposed as follows:

$$\Delta\tau=-G_p[K_v\dot{\phi}^*+K_p\phi^*)+G_f\left(K_f\Delta\lambda_b+K_{f0}\int\Delta\lambda_b dt\right), \tag{11}$$

$$G_p(t)=(\hat{D}_c\hat{B}^T)^{-1}\hat{D}\hat{M}\hat{C}^{-1}, \tag{12}$$

$$G_f(t)=(\hat{D}_c\hat{B}^T)^{-1}\hat{D}\hat{C}_b^T, \tag{13}$$

where $\dot{\phi}^*(\dot{y}_s, y_s, t)$, $\phi^*(y_s, t)$, and $\Delta\lambda_b = \lambda_{bs} - \lambda_{bn}(t)$; $\dot{y}_s$, y_s, and λ_{bs} are the measured values; and K_f and K_{f0} are the diagonal matrices of force feedback gains. As shown by Ostermeyer (1990), the integrator term $K_{f0}\int\Delta\lambda_b dt$ is of importance for stable realization of the force program constraints. In the SS phase the dimension of $G_p(t)$ is $k \times n$ and $G_f(t)$ vanishes, and in the DS phase the dimensions are $l \times n$ and $m \times n$, respectively. The time-variant matrices G_p and G_f are assumed to be precomputed and stored. Thus, the feedback control of Eq. (11) reads symbolically as $\Delta\tau = \Delta\tau(\dot{y}_s, y_s, \lambda_{bs}, t)$.

A different strategy of feedback corrections have been used by Miura and Shimoyama (1984). At the end of each step, the feedforward inputs for the subsequent steps can be modified according to the feedback information, and so over several steps to achieve a steady walking. The continuous control proposed in this paper seems to be preferable since disturbances can be more adequately accommodated.

4 Seven-Link Bipedal Model

An application of the proposed control concept is illustrated for the seven-link biped shown in Fig. 1. The modelled walking pattern is demonstrated in Fig. 2. The biped starts from the state of rest; both feet contact the ground but the whole weight is supported by the left (L) leg. The first step starts with the right (R) leg. At the beginning of the first DS phase the biped achieves a steady walking cycle.

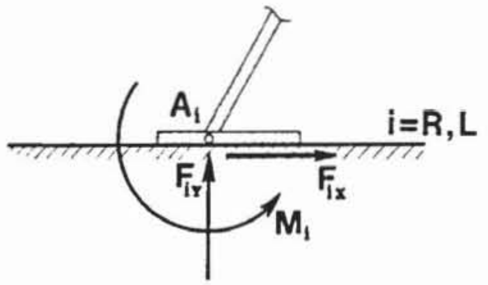

Fig. 3 Ground reactions on the foot

The "flying" biped has nine degrees of freedom, $n = 9$, its position coordinates are $y = [x_H, y_H, \theta_{R1}, \theta_{R2}, \theta_{R3}, \theta_{L1}, \theta_{L2}, \theta_{L3}, \theta_T]^T$, control inputs are $\tau = [\tau_{R1}, \tau_{R2}, \tau_{R3}, \tau_{L1}, \tau_{L2}, \tau_{L3}]^T$, and $k = 6$, $m = 3$, and $l = 3$. The dynamic equations in the form of Eq. (1) are derived in the Appendix.

The constraints (a) represent the trajectories of hip position and trunk orientation,

$$\phi_a = \begin{bmatrix} x_H - x_{Hn}(t) \\ y_H - h_0 \\ \theta_T \end{bmatrix} = 0, \tag{14}$$

whereas constraints (b) and (c) are

$$\phi_{b,c} = \begin{bmatrix} x_H + l_1\sin\theta_{i1} + l_2\sin\theta_{i2} - x_{Ai} \\ y_H - l_1\cos\theta_{i1} - l_2\cos\theta_{i2} - y_{Ai} \\ \theta_{i3} - \pi/2 \end{bmatrix} = 0, \tag{15}$$

where i = R or L, and x_{Ai} and y_{Ai} are ankle coordinates. During the first step it yields $x_{Hn}(t) = at^5 + bt^4 + ct^3$, and the coefficients a, b, and c are chosen so that $x_{Hn}(0) = 0$, $\dot{x}_{Hn}(0) = 0$, $\ddot{x}_{Hn}(0) = 0$, $x_{Hn}(T_0) = s_0$, $\dot{x}_{Hn}(T_0) = v_0$, and $\ddot{x}_{Hn}(T_0) = 0$, where T_0 and s_0 are the period and the length of the first step, and v_0 is the velocity of steady walking; then $x_{Hn}(t) = s_0 + v_0(t - T_0)$ so that $s_d = v_0T_d$ and $s_s = v_0T_s$, where T_d and T_s are the periods of DS and SS phases, respectively. Note that $s = s_d + s_s$ must be assured, see Fig. 2. For constraints ($b2$) and (c), x_{Ai} are constant distances and $y_{Ai} = 0$, and for constraints ($b1$) they are set as appropriate trajectories $x_{Ain}(t)$ and $y_{Ain}(t)$ so that $\dot{x}_{Ain} = 0$ and $\dot{y}_{Ain} = 0$ at the beginning and at the end of SS phase, which assures impact avoidance. The trajectories $y_{Ai}(t)$ are shown in Figs. 4 and 5. The nine constraints (14) and (15) prescribe implicitly the biped position and the equations can be solved for $y_n(t)$. The nominal time-histories $\dot{y}_n(t)$ and $\ddot{y}_n(t)$ can then be obtained recursively from the differentiated forms of the constraint equations.

For the case at hand the matrices C_c and D_c are:

$$C_c = \begin{bmatrix} 1 & 0 & l_1\cos\theta_{R1} & l_2\cos\theta_{R2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & l_1\sin\theta_{R1} & l_2\sin\theta_{R2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{16}$$

$$D_c = \begin{bmatrix} -l_1\cos\theta_{R1} & -l_1\sin\theta_{R1} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -l_2\cos\theta_{R2} & -l_2\sin\theta_{R2} & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{17}$$

Equations (16) and (17) are valid when constraints (c) are imposed on the R leg, i.e., when the R leg is the standing one (DS + SS phase cycle). For the other case, i.e., when the L leg becomes the standing one (successive DS + SS phase cycle), columns 3–5 and 6–8 in Eqs. (16) and (17) should replace each other, and R ↔ L. Also, it is easy to find that, for the case of constraints ($b2$) imposed on the L leg,

$$C_b = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & l_1\cos\theta_{L1} & l_2\cos\theta_{L2} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & l_1\sin\theta_{L1} & l_2\sin\theta_{L2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \tag{18}$$

and for the other case, Eq. (18) has to be rearranged appropriately.

Table 1 Biped parameters used for simulation

Parameter	$j = 1$	$j = 2$	$j = 3$
m_j	5.3 kg	2.25 kg	0.5 kg
J_j	0.08 kg m²	0.09 kg m²	0.006 kg m²
l_j	0.3 m	0.35 m	—
c_j	0.07 m	0.145 m	0.05 m
m_T	14.8 kg		
J_T	0.9 k m²		
c_T	0.2 m		

The force program constraints are specified as

$$\lambda_{bn}(t) = (\lambda_{b0}/2)(1 + \cos(\pi t^*/T_d)), \tag{19}$$

where the components of $\lambda_b = [F_{bix}, F_{biy}, M_{bi}]^T$, i = R or L, are shown in Fig. 3; λ_{b0} denotes the values encountered at the beginning of DS phase; t^* is the actual time of the current DS phase; and T_d is the DS phase period.

Using the nominal motion characteristics $y_n(t)$, $\dot{y}_n(t)$, and $\ddot{y}_n(t)$, the nominal control $\tau_n(t)$ can be synthesized from Eq. (9), whereas the reaction values of constraints (c), $\lambda_{cn}(t) = [F_{cix}, F_{ciy}, M_{ci}]^T$, i = R or L, can be determined from Eq. (6). For the inertial and geometrical parameters of the biped as in Table 1, and for the gait parameters: $s = 0.25$ m, $s_0 = s_s = 0.15$ m, $s_d = 0.1$ m, $v_0 = 0.5$ m/s, $h_0 = 0.55$ m and $h_A = 0.1$ m, the simulation results are shown in Figs. 4 and 5. As demonstrated in the graphs, the introduced program of motion assures the absence of impacts both in the ground reactions on the biped and in the control strategy. The simulation results for a perturbed motion stabilized according to the proposed method are not reported in this paper.

5 Discussion

The paper contributes to the inverse dynamic technique for determining the control inputs of a system that has to produce desired system outputs. Here, a compact description was given for modelling of an impactless bipedal walk and for synthesizing the required control law. The desired motion was fully specified. In contrary to the standard problems of this type, however, a system with a variable number of degrees of freedom had to be modelled and analyzed, and mixed motion and force control was synthesized.

Commonly feedback control uses the measured deviations $\Delta y = y_s - y_n(t)$ and $\Delta\dot{y} = \dot{y}_s - \dot{y}_n(t)$, and forces the system so that $y(t) \to y_n(t)$. In the concept of this paper, $\dot{\phi}^*(\dot{y}_s, y_s, t)$, $\phi^*(y_s, t)$ and $\Delta\lambda_b = \lambda_{bs} - \lambda_{bn}(t)$ are the deviations for the feedback control resulting in $\phi^*(y(t), t) \to 0$ and $\lambda_b(t) \to \lambda_{bn}(t)$. There are at least two reasons for doing so. Firstly, the nominal motion and forces are controlled directly, as opposed to the indirect control by tracking $y_n(t)$ available as solutions

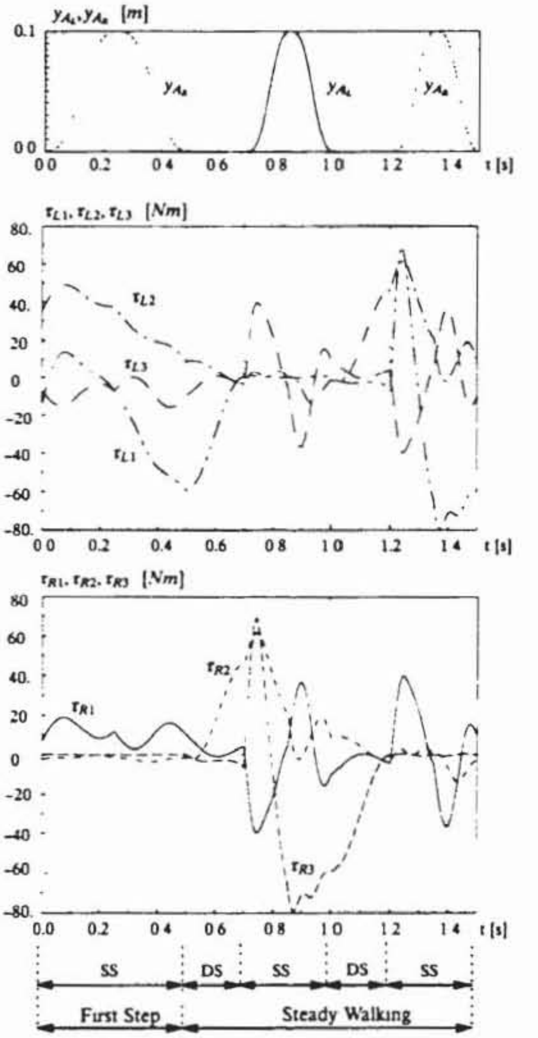

Fig. 4 Control strategy for the modeled impactless walking

to the nominal motion and constraints of the system. Note also that $\dim(\tau) < \dim(y)$. Secondly, forcing the system so that $y(t) \rightarrow y_n(t)$ might be in conflict with the force control, which manages $\lambda_b(t) \rightarrow \lambda_{bn}(t)$ and in principle allows some small violation of constraints ($b2$). Violation of these constraints cannot be corrected by the motion control.

References

Alexander, R.McN, 1984, "The Gaits of Bipedal and Quadrupedal Animals," *The International Journal of Robotics Research*, Vol. 3, No. 2, pp. 49-59.

Baumgarte, J., 1972, "Stabilization of Constraints and Integrals of Motion in Dynamical Systems," *Computer Methods in Applied Mechanics and Engineering*, Vol. 1, pp. 1-16.

Blajer, W., 1991, "Contribution to the Projection Method of Obtaining Equations of Motion," *Mechanics Research Communications*, Vol. 18, pp. 293-301.

Daberkow, A., Gao, J., and Schiehlen, W., 1992, "Walking Without Impacts," *Proc. of the 8th CISM-IFToMM Symp. on Theory and Practice of Robots and Manipulators*, Cracow, 1990 (in press).

Furusho, J., and Masubuchi, M., 1986, "Control of a Dynamical Biped Locomotion System for Steady Walking," ASME JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL, Vol. 108, pp. 111-118.

Furusho, J., and Sano, A., 1990, "Sensor-Based Control of a Nine-Link Biped," *The International Journal of Robotics Research*, Vol. 9, No. 2, pp. 83-98.

Kamman, J. W., and Huston, R. L., 1984, "Dynamics of Constrained Dynamic Systems," ASME *Journal of Applied Mechanics*, Vol. 51, pp. 899-903.

Kreuzer, E., and Leister, G., 1991, "Programmsystem NEWEUL '90," Anleitung AN-23, Stuttgart University, Institute B of Mechanics.

McGeer, T., 1990, "Passive Dynamic Walking," *The International Journal of Robotics Research*, Vol. 9, No. 2, pp. 62-82.

Mita, T., Yamagushi, T., Kashiwase, T., and Kawase, T., 1984, "Realization of a High Speed Biped Using Modern Control Theory," *International Journal of Control*, Vol. 40, pp. 107-119.

Miura, H., and Shimoyama, I., 1984, "Dynamic Walking of a Biped," *The International Journal of Robotics Research*, Vol. 3, No. 2, pp. 60-74

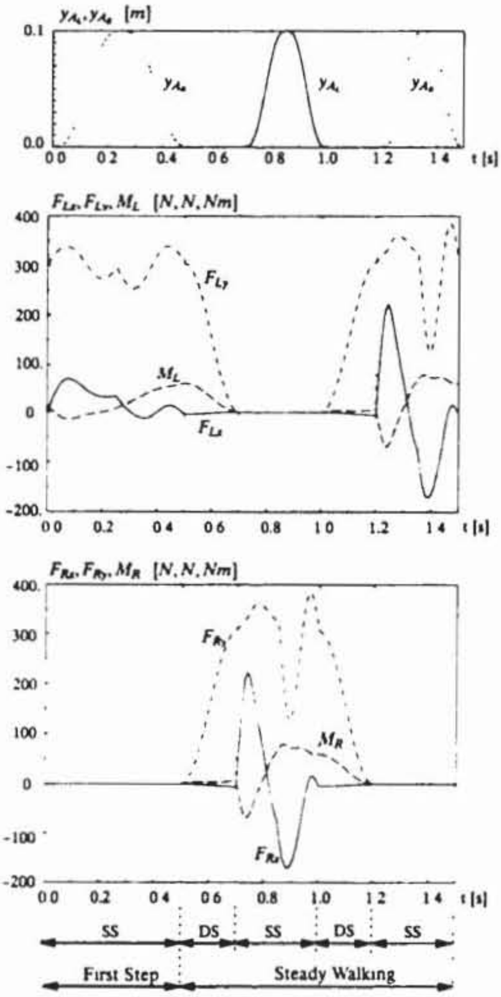

Fig. 5 Variations in the ground reactions

Mochon, S., and McMahon, T. A., 1981, "Ballistic Walking: An Improved Model," *Mathematical Biosciences*, Vol. 52, pp. 241-260.

Ostermeyer, G.-P., 1990, "On Baumgarte Stabilization for Differential Algebraic Equations," *Real Time Integration Methods for Mechanical System Simulation*, Haug, E. J. and Deyo, R. C., eds., NATO ASI Series, Vol. F69, Springer-Verlag, Berlin, pp. 193-207.

Raibert, M. H., 1986, *Legged Robots that Balance*, MIT Press, Cambridge, Mass.

Raibert, M. H., and Craig, J. J., 1981, "Hybrid Position/Force Control of Manipulators," ASME JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL, Vol. 103, pp. 126-133.

Schiehlen, W., 1990, *Multibody Systems Handbook*, Springer-Verlag, Berlin.

Schiehlen, W., 1991, "Computational Aspects in Multibody System Dynamics," *Computer Methods in Applied Mechanics and Engineering*, Vol 90, pp. 569-582.

Takanishi, A., Egusa, Y., Tochizawa, M., Takeya, T., and Kato, I., 1990, "Realization of Dynamic Biped Walking Stabilized with Trunk Motion," *Proc of the VIIth CISM-IFToMM Symp. on Theory and Practice of Robots and Manipulators*, RoManSy '88, Hermes, Paris, pp 68-79

Thompson, C. M., and Raibert, M. H., 1989, "Passive Dynamic Running," Report AI-TR-1179, LL-6, MIT Artificial Intelligence Lab., Cambridge, CA, pp. 135-146.

Todd, D. J., 1985, *Walking Machines: An Introduction to Legged Robots*, Kogan Page, London.

Vukobratović, M., Borovac, B., Surla, D., and Stokic, D., 1990, *Biped Locomotion: Dynamics, Stability, Control and Application*, Springer-Verlag, Berlin-Heidelberg.

APPENDIX

The motion Eqs. (1) for the biped shown in Fig. 1 can easily be obtained by a symbolic multibody system formalism like NEWEUL (Kreuzer and Leister, 1991; Schiehlen, 1990);

$$\begin{bmatrix} M_{HH} & M_{HR} & M_{HL} & M_{HT} \\ M_{HR}^T & M_{RR} & 0 & 0 \\ M_{HL}^T & 0 & M_{LL} & 0 \\ M_{HT}^T & 0 & 0 & M_{TT} \end{bmatrix} \begin{bmatrix} \ddot{p}_H \\ \ddot{\theta}_R \\ \ddot{\theta}_L \\ \ddot{\theta}_T \end{bmatrix} = \begin{bmatrix} h_H \\ h_R \\ h_L \\ h_T \end{bmatrix} = B^T\tau,$$

where $p_H = [x_H, y_H]^T$, $\theta_i = [\theta_{i1}, \theta_{i2}, \theta_{i3}]^T$ (i = R or L). After introducing

$$m = 2(m_1 + m_2 + m_3) + m_T,$$
$$\mu_1 = m_1 c_1 + (m_2 + m_3) l_1,$$
$$\mu_2 = m_2 c_2 + m_3 l_2,$$
$$\mu_3 = m_3 c_3,$$
$$\mu_T = m_T c_T,$$

the submatrices and subvectors M_{HH}, M_{HR}, M_{HL}, M_{HT}, M_{RR}, M_{LL}, h_H, h_R, h_L, and h_T read as follows:

$$M_{HH} = \mathrm{diag}(m, m),$$

$$M_{Hi} = \begin{bmatrix} \mu_1 \cos\theta_{i1} & \mu_2 \cos\theta_{i2} & \mu_3 \cos\theta_{i3} \\ \mu_1 \sin\theta_{i1} & \mu_2 \sin\theta_{i2} & \mu_3 \sin\theta_{i3} \end{bmatrix},$$

$$M_{HT} = [\mu_T \cos\theta_T, \quad -\mu_T \sin\theta_T]^T,$$

$$M_{ii} = \begin{bmatrix} J_1 + (m_2 + m_3) l_1^2 & \mu_2 l_1 \cos(\theta_{i2} - \theta_{i1}) & \mu_3 l_1 \cos(\theta_{i3} - \theta_{i1}) \\ \bullet & J_2 + m_3 l_2^2 & \mu_3 l_2 \cos(\theta_{i3} - \theta_{i2}) \\ \bullet & \bullet & J_3 \end{bmatrix},$$

$$M_{TT} = [J_T],$$

$$h_i = \begin{bmatrix} \mu_1 \dot\theta_{i1}(\dot x_H \sin\theta_{i1} - \dot y_H \cos\theta_{i1}) + \mu_2 l_1 \dot\theta_{i2}(\dot\theta_{i2} - \dot\theta_{i1}) \sin(\theta_{i2} - \theta_{i1}) \\ + \mu_3 l_1 \dot\theta_{i3}(\dot\theta_{i3} - \dot\theta_{i1}) \sin(\theta_{i3} - \theta_{i1}) - g\mu_1 \sin\theta_{i1} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \mu_2 \dot\theta_{i2}(\dot x_H \sin\theta_{i2} - \dot y_H \cos\theta_{i2}) + \mu_2 l_1 \dot\theta_{i1}(\dot\theta_{i2} - \dot\theta_{i1}) \sin(\theta_{i2} - \theta_{i1}) \\ + \mu_3 l_2 \dot\theta_{i3}(\dot\theta_{i3} - \dot\theta_{i1}) \sin(\theta_{i3} - \theta_{i1}) - g\mu_2 \sin\theta_{i2} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \mu_3 \dot\theta_{i3}(\dot x_H \sin\theta_{i3} - \dot y_H \cos\theta_{i3}) + \mu_3 l_1 \dot\theta_{i1}(\dot\theta_{i3} - \dot\theta_{i1}) \sin(\theta_{i3} - \theta_{i1}) \\ + \mu_3 l_2 \dot\theta_{i2}(\dot\theta_{i3} - \dot\theta_{i2}) \sin(\theta_{i3} - \theta_{i2}) - g\mu_3 \sin\theta_{i3} \end{bmatrix}$$

$$h_T = [\mu_T \dot\theta_T (\dot x_H \sin\theta_T + \dot y_H \cos\theta_T) + \mu_T g \sin\theta_T],$$

where J_1, J_2, J_3, J_T denote the moments of inertia relative to the joints, and i = R or L, accordingly. The matrix B is obtained as

$$B = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix}.$$

$$h_H = \begin{bmatrix} \mu_T \dot\theta_T^2 \sin\theta_T + \sum_{i=R,L} (\mu_1 \dot\theta_{i1}^2 \sin\theta_{i1} + \mu_2 \dot\theta_{i2}^2 \sin\theta_{i2} + \mu_3 \dot\theta_{i3}^2 \sin\theta_{i3}) \\ \\ \mu_T \dot\theta_T^2 \cos\theta_T - \sum_{i=R,L} (\mu_1 \dot\theta_{i1}^2 \cos\theta_{i1} + \mu_2 \dot\theta_{i2}^2 \cos\theta_{i2} + \mu_3 \dot\theta_{i3}^2 \cos\theta_{i3}) - mg \end{bmatrix}.$$

(Contents continued)