Rotation capacity of plastic hinges and allowable degree of moment redistribution

by

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1 Introduction

According to MC 78 the maximum moments of continous beams calculated according to the theory of elasticity can be redistributed without check of the ductility requirements, if the condition of equilibrium is fulfilled and if the reduction is not less than given by eqn. (1)

$$\begin{array}{ll} \text{beton C 12 to C 35} & \delta = 0.44 + 1.25\ x/d \geq 0.75 \\ \text{beton C 40 to C 50} & \delta = 0.56 + 1.25\ x/d \geq 0.75 \end{array} \qquad (1)$$

Eqn. (1) is valid for beams with a slenderness $l/d \leq 20$. It was deduced by Macchi /1/, using the allowable plastic rotation capacity according to MC 78, Fig. 8.2.

The structural analysis can also be performed by non-linear methods. In this case it must be verified, that the necessary plastic rotation is not larger than the allowable value according to Fig. 8.2 of MC 78.

This figure was proposed by Siviero /2/. He plotted the rotation capacity measured in about 350 tests on beams performed approximately between 1960 and 1970 as a function of the ratio x/d (Fig. 1). The allowable plastic rotation - shown in Fig. 1 as dotted line - was evaluated by a statistical analysis of the data and shall represent the 5%-fractile of the test results. The rotation capacity increases significantly with decreasing ratio x/d. This indicates that in the tests very ductile bars were used.

In Table 1 some characteristic values of the tests evaluated by Siviero are summarised. It can be seen that the used reinforcing steel had a rather large relationship rupture strength f_t to yield stress f_y ($f_t/f_y \sim 1.4$ to 1.8). The corresponding unit elongation A_G must have been rather large as well ($A_G \gtrsim 10$ %).

Furthermore in many tests bars with rather poor bond behavior (smooth bars or deformed bars with a related rib area less than required by MC 78) were used. In addition, the height of the test specimens was relatively small. All these factors increase the rotation capacity.

In Fig. 2 the stress-strain relationship of reinforcing bars used today in Germany are shown. These curves were found by evaluating a large number of quality control tests from several steel mills /16/. The upper and lower lines are approximate limits of the scatter expected in practice. It can be seen that modern reinforcing steel is less ductile than the steel employed in the tests. This is especially true for cold-worked bars and for welded wire mesh produced from cold worked wires. Furthermore modern deformed bars have a very good bond behavior. Also the height of real beams will often be larger than the height of the test beams.

For these reasons the plastic rotation capacity given in MC 78 may not always be reached when using modern - especially cold-worked - reinforcement. This is indicated in Fig. 3 which shows the rotation capacity (elastic and plastic components) of beams on two supports reinforced with cold-worked deformed bars as a function of the percentage of reinforcement $\mu = A_s/b \cdot d$ according to tests /17/ and calculation (see Section 2).

The typically roof-shaped curve has a maximum rotation capacity at a critical percentage of reinforcement μ_{crit}. μ_{crit} depends on the dimensions of the section, the material behavior and the confinement. In the present case μ_{crit} amounts to about 0.42 %. For $\mu < \mu_{crit}$ the beam fails due to rupture of the reinforcement, i.e. the ductility of the reinforcing bars is fully utilized.

The rotation capacity decreases rapidly with decreasing percentage of reinforcement, because only few cracks are formed and the contribution of concrete between cracks is significant. For $\mu \geq \mu_{crit}$ the failure of the beam is due to crushing of the concrete in the compression zone. The steel strains are smaller than the values which can be sustained by the bars.

These tests demonstrate that for small percentages of reinforcement the rotation capacity of plastic hinges is governed by the ductility of the reinforcement.

The analytical models of plastic hinges proposed so far (e.g./18,19/) predict the behavior of these hinges qualitatively rather well. However, the quantitative agreement between test results and calculation is not too well. Therefore an analytical model for plastic hinges was developed which is based on the work in /18,19/. In the following only a brief description of the study is given, for details see /20/.

2 Analytical Model

Based on the given dimensions of the cross section (concrete and reinforcement) and the assumed stress-strain relationships of steel and concrete, the moment-curvature relationship or the tensile force-curvature relationship, respectively, are calculated (Fig. 4b), assuming plaine sections remain plaine. The distribution of moments along the beam is calculated taking into account the width of the loading plate. The load is increased until the ultimate moment previously calculated is reached. In statically indeterminate structures an statically determinate beam with a length equal to the distance between two adjacent points of zero moment is cut out of the real system. If shear cracks must be expected, the shifting of the tensile force compared to the M/z-line (M = Moment, z = lever arm) (truss analogy) is taken into account assuming an angle of the inclined compression struts according to Ref. /18/. From the tensile force distribution and the tensile force-curvature relationship the curvature in the cracks is reached (Fig. 4a). The crack distance is calculated according to /21/.

The contribution of concrete between cracks is calculated for every beam section between two cracks by means of an iterative solution of the differential equation of bond, using a modified version of the program described in Ref. /22/. On the basis of the calculated steel strain distribution, the distribution of curvature between the cracks is derived by using the distance of the tensile reinforcement to the neutral axis (Fig. 4a). Integration of these curvatures over the beam length yields the rotation capacity of the beam. The plastic rotation is defined as the difference between the rotation at ultimate load and at a load causing yielding of the reinforcement at the point of maximum moment (see Fig. 4a).

The mathematical model can only yield reliable results if the behaviour of the material is described very accurately. Therefore the stress-strain relationship of the reinforcing steel is described by a polygon (with up to 30 points, which allows a very close representation of the real behaviour(Fig. 5)). The stress-strain relationship of the concrete is formulated as proposed in Ref. /23/. This model which consists of a parabola and a trilinear continuation (Fig. 6) takes into account the influences of confinement by stirrups on the strength σ_1 and corresponding strain ε_1, the descending branch of the stress-strain relationship (defined by σ_1/ε_2 and σ_3/ε_3) and on the residual strength σ_4. The values for these characteristic parameters are chosen according to Ref. /23/ for the problem on hand. The bond behavior is described by the bond stress-slip relationships shown in Fig. 7 which are based on the model proposed in Ref. /24/ taking into account the test results given in Ref. /25/. Fig. 7 is valid for a concrete compression strength f_c = 30 N/mm². For other values of f_c the bond stress-slip relationships are varied according to Ref. /24/.

To check the validity of the assumptions, the predicted response of beams was compared with available test results. In Fig. 8 the calculated and measured distribution of the residual steel elongation after unloading from maximum load is plotted. Note that in the experiment (Ref. /17/) the crack spacing varies, while in the calculation a constant value was assumed. Fig. 9 shows the predicted rotation capacity of 70 beams as a function of the measured value. The data points scatter around the 45-degree line for perfect agreement. The coefficient of variation is only 17 %. In Fig. 3 the predicted and measured rotation capacities of otherwise identical beams are plotted as a function of the percentage of reinforcement. The typical behavior found in the tests is captured quite well by the calculation.

From these figures it can be concluded that the proposed analytical model is sufficiently accurate for practical purposes.

3 Parameter Studies

3.1 Influence of stress-strain relationship of steel

Fig. 10 shows schematically the influence of the stress strain curve on the rotation capacity of a single span reinforced concrete beam loaded in midspan (Fig. 10a).

In Fig. 10b, which shows the moment distribution along the beam, the influence of the ratio f_t/f_y on the rotation capacity is investigated. It is assumed that the strength of the steel and the unit elongation are constant. Under these conditions the maximum section curvature and the ultimate moment are almost constant. The plastic rotation capacity is approximately proportional to the length of the plastic zone. Because this length increases with increasing ratio f_t/f_y, the rotation capacity also increases considerably with increasing ratio f_t/f_y.

In Fig. 10c, which shows the distribution of section curvature along the beam length, a constant ratio f_t/f_y, but different values for the unit elongation are assumed. In this case the length of the plastic zone is almost constant. However, with increasing unit elongation the maximum section curvature increases, resulting in increasing rotation capacity.

The reinforcement percentage of the beam investigated in Fig. 10 is relatively small and therefore the beam will fail by rupture of the steel. For higher reinforcement ratios leading to a concrete failure the rotation capacity will also increase with increasing ratio f_t/f_y, but will almost be independent of the unit elongation.

In practice for a certain type of reinforcement (e.g. cold worked bars) the unit elongation increases with increasing ratio f_t/f_y (compare Fig. 2). The combined effect is investigated in Fig. 11 using the same beam as in Fig. 10. Plotted is the plastic rotation capacity as a function of the shape of the stress-strain diagram. The assumed stress-strain relationships (Fig. 11a) cover approximately the range valid for welded wire fabric produced in Germany (no failure in the welds). The plastic rotation capacity of the beam reinforced with the more ductile steel 1 is about 3 times larger than for steel 3 (Fig. 11b).

From this figure the significant influence of the bond between bars and concrete on the plastic rotation can also be seen. While the dotted line is valid for the so-called naked state (no bond between cracks), the full line takes the influence of bond (tension stiffening due to contribution of concrete between cracks) into account. Under otherwise constant conditions, the plastic rotation capacity is reduced to about 40 % due to bond compared to the "naked state". The influence of the bond is smaller for higher percentages of reinforcement, but still significant. This can be explained by the fact, that for steel strains $\varepsilon > \varepsilon_y$ small differences in forces caused by the bond action result in large changes of steel strain. The influence of bond is especially pronounced for steel with a low ratio f_t/f_y.

The importance of the steel behavior on the rotation capacity has also been pointed out in /18,19/.

In Fig. 12 the plastic rotation capacity according to CEB is compared with the theoretical values using two different types of reinforcement: heat treated bars and welded wire mesh produced from cold-worked wires. The average stress-strain relationships given in Fig. 2 (heat treated steel f_t/f_y = 1,18, ε_g = 8 %, welded wire mesh: f_t/f_y = 1,06, ε_g = 3 %) were assumed. The calculated values are valid for the following parameters:

- Reinforced concrete beam or slab (h = 30 cm, l/h = 6) with tension and shear reinforcement, but without compression reinforcement. It was assumed, that sufficient shear reinforcement was provided to prevent a shear failure.

- Concrete strength C 40. A parabolic-rectangular stress-strain diagram according to CEB, but with a maximum strain of 5 ‰ was assumed.

- Bond behavior according to /24/, however, the bond strength was reduced to 50 % of the values given in /24/ to model the situation over the support (bad bond condition).

It can be seen that for heat treated steel and values $x/d \gtrsim 0.15$ the calculated plastic rotation capacities agree fairly well with the CEB-curve; however, for values $x/d \lesssim 0.15$, the calculated values decrease due to rupture of the steel. For welded wire mesh the calculated rotation capacities are smaller over the full range of x/d than the CEB curve. This is caused by the small ratio f_t/f_y and the small unit elongation of the employed reinforcement.

In Fig. 12, the results of three tests on slabs reinforced with welded wire mesh /26/ are plotted as well. The test results agree rather well with the theoretical values.

3.2 Beam or slab slenderness

If the span is kept constant, the slenderness increases with decreasing height. Because the maximum section curvature increases with decreasing height, the rotation capacity of the specimen increases with decreasing height or increasing slenderness, respectively (Fig. 13).

When increasing the span (height constant), the length of the plastic zone increases (Fig. 14) and therefore the plastic rotation capacity increases as well (Fig. 15). When doubling the slenderness, the rotation capacity increases approximately by 50 %.

In /17/ the influence of the beam slenderness on the rotation capacity was investigated in experiments by varying the span. The results are plotted in Fig. 16. The plastic rotation increases with increasing slenderness. However, the increase is less than shown in Fig. 15. This might be due to the different reinforcement employed: The calculation is valid for welded wire mesh ($f_t/f_y = 1.06$), while in the tests rather ductile bars ($f_t/f_y \sim 1.2$) were used.

4 Proposal for the revision of the CEB-FIP Model Code

As shown in Section 3, the plastic rotation capacity is significantly influenced by the stress-strain relationship of the tension reinforcement. While the allowable plastic rotation given in MC 78 is valid only for rather ductile steel, the rotation capacity may be much less for less ductile reinforcement. To take this influence into account, it is proposed to classify the reinforcement as follows:

Type 1: $f_{st}/f_{yt} \gtrsim 1.10$ (2)

$\varepsilon_g \gtrsim 6\ \%$

Type 2: $1.05 \leq f_{st}/f_{yt} < 1.10$ (3)

$2.5\ \% \leq \varepsilon_g \leq 6\ \%$

where

f_{st} = strength of the reinforcement obtained in a test

f_{yt} = yield strength or proof stress obtained in a test

ε_g = unit elongation measured in tests (plastic elongation at failure outside the failure zone)

The conditions given in eqns. (2) and (3) are defined as 5 %-fractile. If the reinforcement is delivered in coils and then straightened in the work shop, the conditions in eqns. (2) and (3) must be fulfilled for the finished product.

The unit elongation should be calculated according to the following equation /16/:

$$\varepsilon_g = 2\,\varepsilon_{10} - \varepsilon_5 \tag{4}$$

with

$\varepsilon_5, \varepsilon_{10}$ = rupture elongation measured over a length of 5 Ø or 10 Ø, resp.

In Fig. 17 modified curves for the allowable plastic rotation are shown. For type 1 steel, the proposal coincides with the current CEB-curve, only for $x/d \lesssim 0.15$ the reduction of the rotation capacity due to rupture of the steel is taken into account. For type 2 reinforcement, the proposed allowable plastic rotations are lower than according to the current CEB curve. The latter proposal should be considered as preliminary, because the plastic rotation for values $x/d \gtrsim 0.2$ significantly depend on the assumption for the stress-strain curve of the concrete and could easily be improved by using compression reinforcement and/or by confining the compression zone.

In the lower part of the figure a proposal for the possible degreee of redistribution of moments is given. For type 1 steel, the maximum allowabble degree of redistribution could be increased to 30 % ($\delta \geq 0.7$). For type 2 reinforcement, a redistribution by not more than 15 % is proposed ($\delta \geq 0.85$). This degree of moment redistribution has been confirmed by tests /16/.

The allowable plastic rotation according to Fig. 17a is valid for a slenderness l/d = 6. For plastic zones over a support the span l is the distance between the points with M = 0. A slenderness of 6 over supports corresponds approximately to a ratio $l/d \sim 20$ (l = distance between supports). Because of the increase of the allowable rotation with increasing values l/d (see Fig. 15), it is proposed to allow the possible degree of moment redistribution according to Fig. 17b independent of the slenderness (also for values $l/d > 20$).

The possible degree of moment redistribution δ is given by eqn. (5).

$$\delta \geq 0{,}44 + 1{,}25\,\frac{x}{d} \quad \begin{cases} \geq 0{,}70 & \text{for continous beams and braced frames reinforced with steel type 1} \\ \geq 0{,}85 & \text{for continous beams and braced frames reinforced with steel type 2} \\ \geq 0{,}90 & \text{for sway frames} \end{cases} \qquad (5)$$

Eqn. (5) is valid for concrete C 12 to C 35. It must be checked, whether its validity can be extended to concrete $\leq$ C 50.

According to a proposal of Com. VII, the conditions given in eqns. (2) and (3) shall also be applied to prestressing steel. In general, the ductility of prestressing steel is rather low and therefore this steel would often be classified as type 2. In this case the possible degree of moment redistribution of prestressed beams will be limited to $\delta \geq 0{,}85$.

It seems reasonable to limit the possible degree of moment redistribution also by Fig. 17b when in the static analysis of linear elements the theory of plasticity is used.

In the region of splices by overlapping, by mechanical devices, or by welding the steel strains are rather small. Furthermore the strength and the ductility of the bars might be reduced by welding. Therefore it is proposed that splicing of the reinforcement is not allowed in the regions where plastic hinges are anticipated in the analysis.

Because the type of steel has a significant influence on the allowable degree of moment redistribution, it must be possible to distinguish type 1 and type 2 steel clearly and easily on site, for instance by different rib patterns.

5 Open problems

In the following some problems which should be investigated by a common research are listed:

a) The investigations described in Section 3 are valid for reinforced concrete. In prestressed concrete cracking might occur at rather a high fraction of the ultimate load. Furthermore the employed prestressing steel might be rather brittle. Therefore it must be checked whether Fig. 17b can also be applied to prestressed concrete.

b) Fig.17a is valid for specimen with no or only nominal confinement of the concrete in the compression zone. With increasing degree of confinement by closely spaced stirrups the rotation capacity can be improved considerably, provided at sufficiently high percentages of reinforcement the failure is caused by a failure of the compressed concrete. It should be checked how this significant influence can be taken into account in the code provisions.

c) In Fig. 17a it is assumed that failure is caused by bending and not by shear forces. In /27,28/ the influence of the shear force on the rotation capacity of plastic hinges was studied. It was shown that in cases where the shear strength was based on the strength of the shear reinforcement and a concrete contribution, in accordance with Dutch provision, the rotation capacity was rather small, because the "bending" failure occurred soon after the yield stress of the main reinforcement was reached. Similar results were found in /29/ when the shear failure occurred before reaching the full bending strength. It must be checked, whether the provisions given in the Model Code for dimensioning the shear reinforcement are suitable to ensure a sufficiently high shear strength to develop the full bending capacity of the beam.

d) In the ultimate limit state the forces caused by imposed deformations are reduced to almost zero. However, the imposed deformations concentrate in the plastic hinge and reduce the plastic rotation available for redistribution of moments caused by loads. Therefore it has to be clarified how the effect of imposed deformations on the inelastic behavior of structures shall be taken into account in the analysis.

e) Plastic hinges can occur only, if the amount of tensile reinforcement is large enough to avoid a brittle failure after cracking. It should be checked whether the value given in MC 78 ($\mu = 0.15$ %) is sufficient to guarantee the assumed plastic rotations.

f) Experimental and analytical investigations done so far are valid for normal strength concrete ($\lesssim$ C 50). The validity of the proposals has to be checked for high strength concrete (C 60 - C 90).

6 Summary

According to experimental and analytical studies, the shape of the stress-strain curve in the inelastic range has a significant influence on the rotation capacity of plastic hinges. The plastic rotation capacity given in MC 78, Fig. 8.2, is valid only for rather ductile steel which was used in the tests evaluated in /2/. The ductility of modern cold-worked reinforcing steel may be rather small, resulting in low values for the plastic rotation capacity. Therefore it is proposed to classify reinforcing steel in two groups:

Type 1: Steel with high ductility
Type 2: Steel with reduced ductility

The conditions for the classification are given in eqns. (2) and (3) of the report.

While for type 1 steel the plastic rotation capacity given in MC 78, Fig. 8.2, can be used with some small modifications for low values x/d (low percentages of reinforcement), a new curve giving smaller plastic rotation capacities is proposed for type 2 steel. Depending on the type of steel used in a structure, the maximum degree of moment redistribution is limited to

$\delta \geq 0{,}70$ steel type 1
$\delta \geq 0{,}85$ steel type 2

Figures 8.3 and 8.4 of MC 78 give the possible degree of moment redistribution as a function of the ratio x/d. These equations are valid only if the slenderness is smaller than l/d = 20. Because the rotation capacity increases with increasing slenderness, it is proposed to drop this limitation.

In the region of splices by overlapping, mechanical devices or welding, the average steel strains are rather small (often below the yield strain). Therefore it is proposed that splicing of the reinforcement by any type of splice is not allowed in the region where the formation of plastic hinges is anticipated in the analysis.

Because of its importance for the plastic rotation capacity and the possible degree of moment redistribution, it must be possible to distinguish the type of steel (type 1 or type 2) clearly and easily on site, e.g. by different rib patterns.

In Section 5 of the report some open problems are listed.

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Year	Author	Ref.	Height d mm	Slenderness l/d	f_t/f_y	μ_s %	μ_s' %	Remarks
1960-1965	CEB	/3/	180-280	10-13	MS CWS	0.4 - 6.8	0.0 - 1.7	MS=Mild Steel CWS= Cold-Worked Steel
1971	Burnett	/4/	220	6-11	1.8	1.2 - 2.4	0.2	
1965	Lenkei	/5/	280	10	1.5	0.2 - 2.4	-	1)
1966	Rao	/6/	370	4-10	1.45	1.36	-	1)
1958	Yamada	/7/	200-250	4	HT	0.9 - 2.6	0.0 - 1.0	1) HT=Hot-Rolled Steel
1962	Burns-Siess	/8/	250-450	8-14	1.5	1.1 - 2.0	0.0 - 2.0	
1954	Mc-Collister	in /8/	250	11	-	0.6 - 5.1	0.0 - 4.0	
1966	Corley	/9/	130-760	6-14	1.7	1.1 - 2.9	0.1 - 1.5	
1965	Mattock	/10/	250-510	6-22	1.7	0.7 - 2.9	0.2 - 0.4	
1969	Meschkat	/11/	280	6-13	1.5	1.2	0.15	
1970	Tadros	/12/	270	6-12	1.5	2.2	2.2	
1962	Yamashiro-Siess	/13/	250	14	1.6	0.7 - 3.3	0.7 - 3.3	
1965	Thomas-Sozen	/14/	reinforced with strands with f_t/f_y = 1584/1825 N/mm²					
1968	Nawy	/15/	tests with normal force, $x/d \geq 0.6$					

1) Test-specimen with region of constant moment

<u>Table 1:</u> Main parameters of the tests evaluated by Siviero

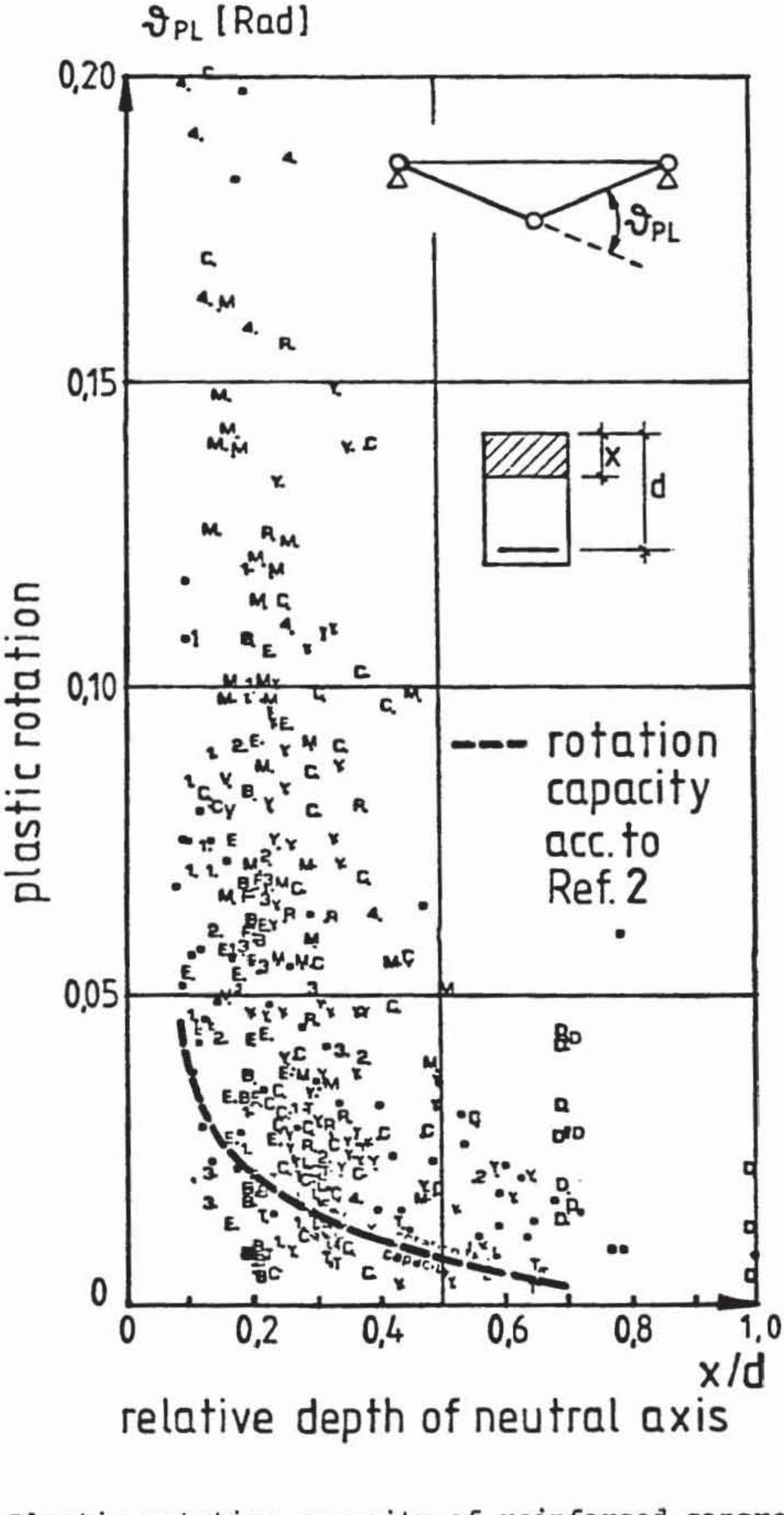


Fig. 1: Plastic rotation capacity of reinforced concrete hinges (after /2/)

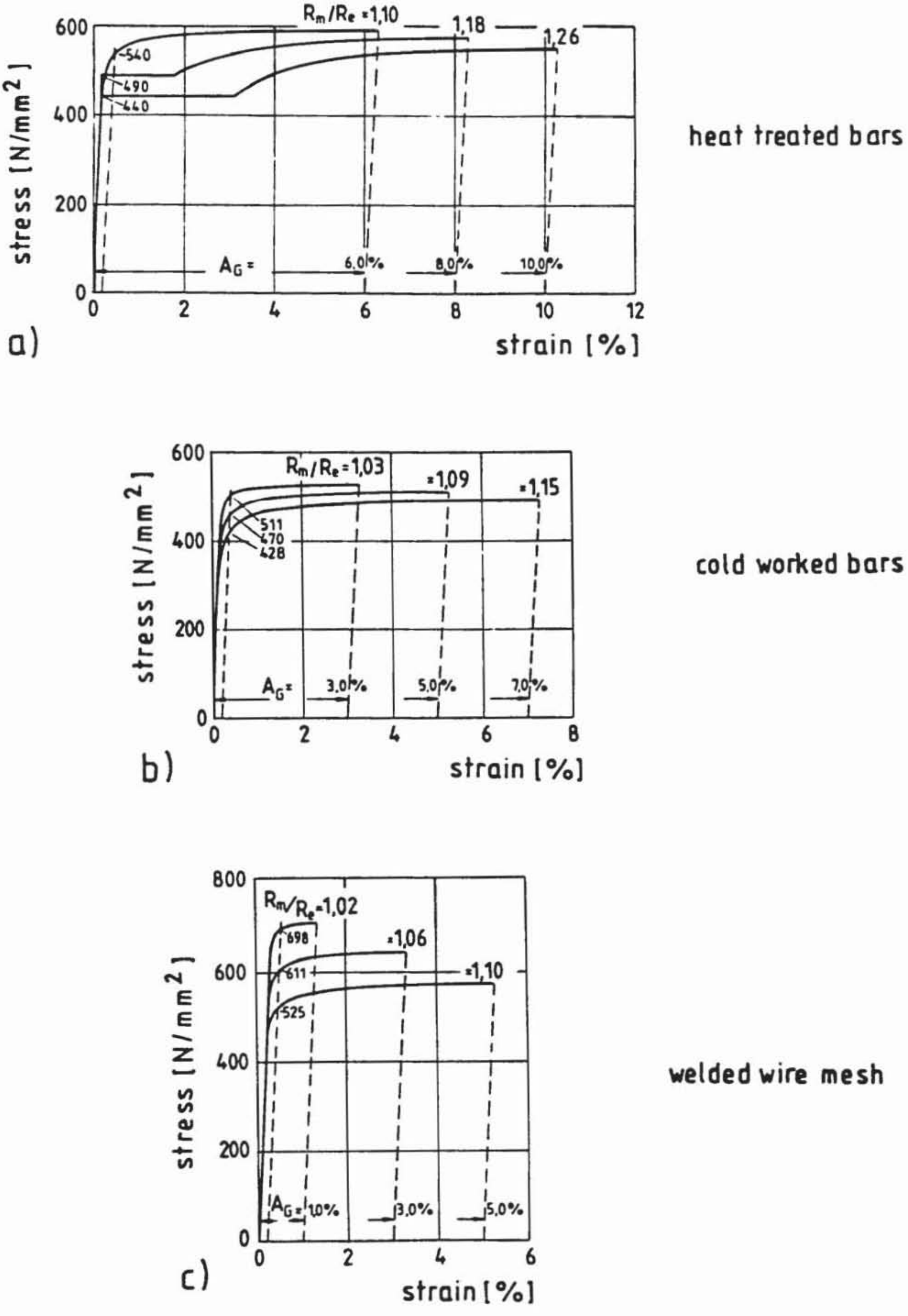


Fig. 2: Stress-strain relationship of reinforcing steel produced in Germany (after /16/)

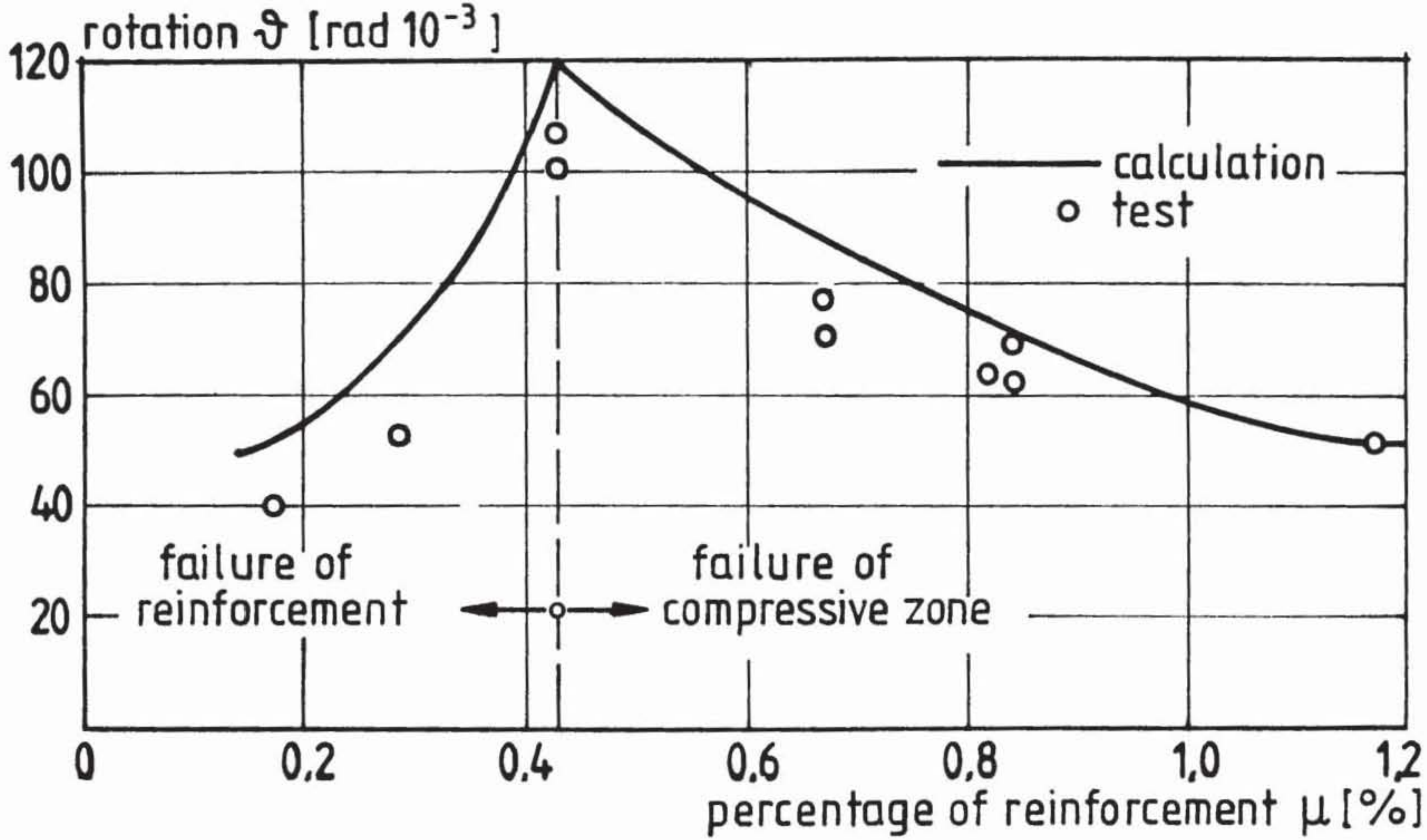


Fig. 3: Influence of the percentage of reinforcement on the rotation capacity of reinforced concrete beams after tests /17/ and calculation /20/

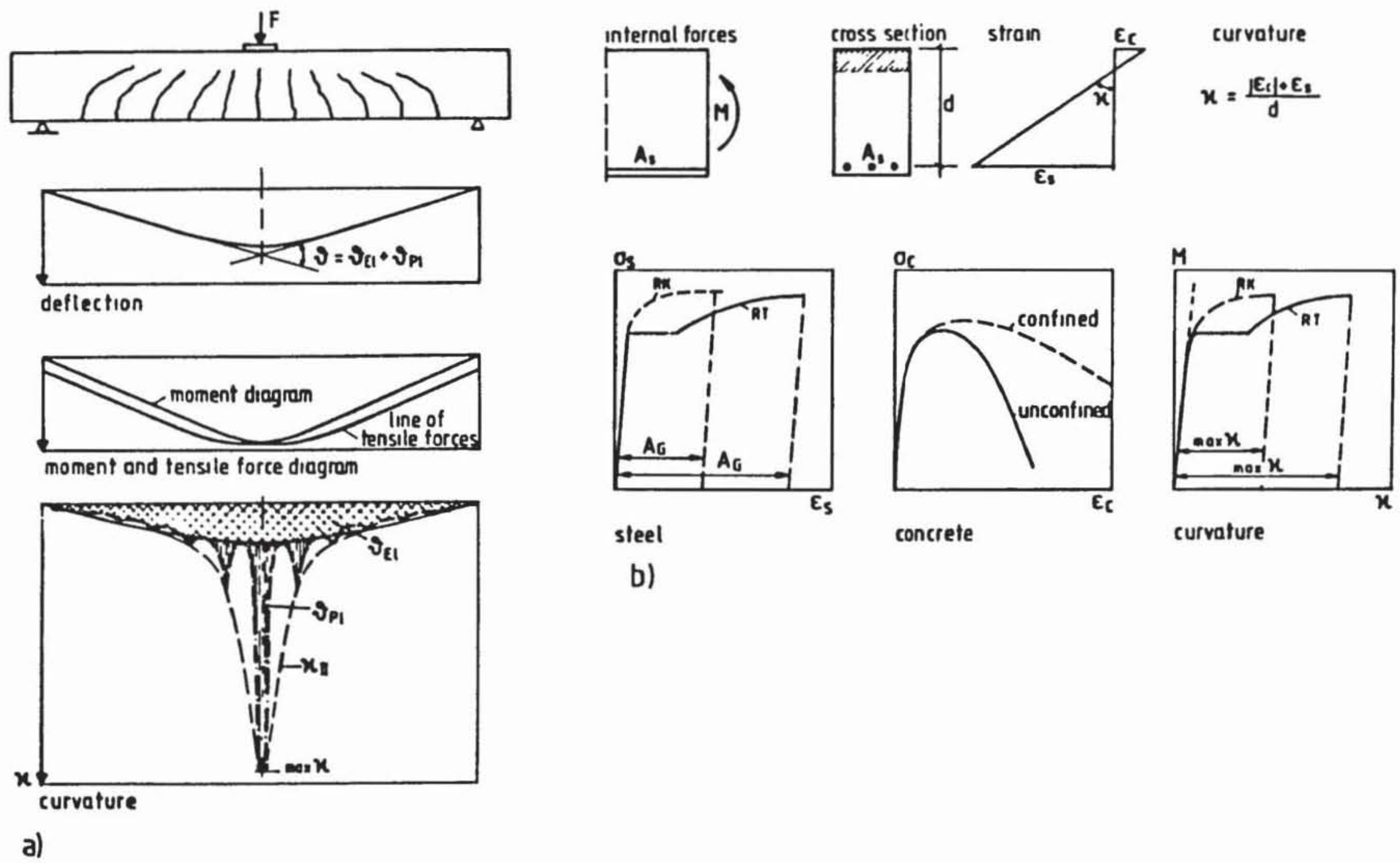


Fig. 4: Mathematical model

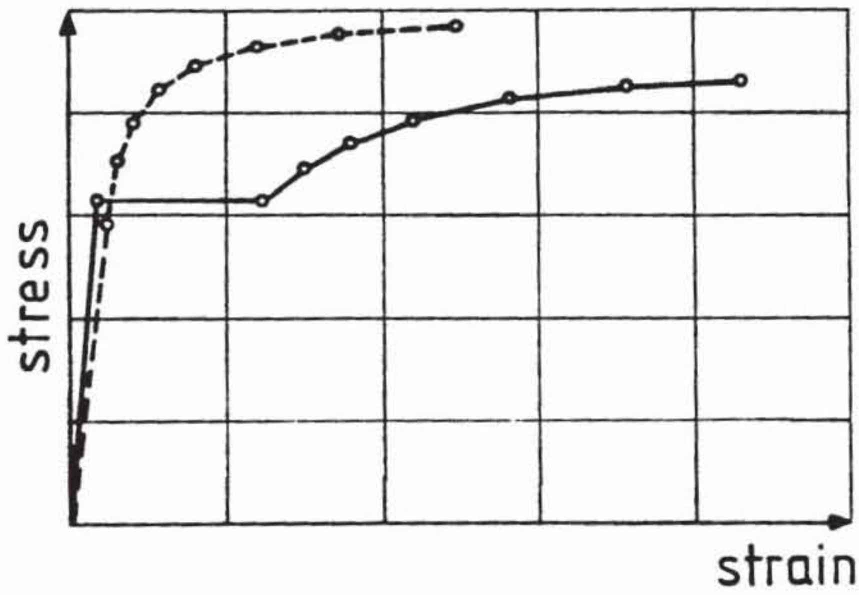


Fig. 5: Polygon defining the stress-strain relationship of the reinforcing steel

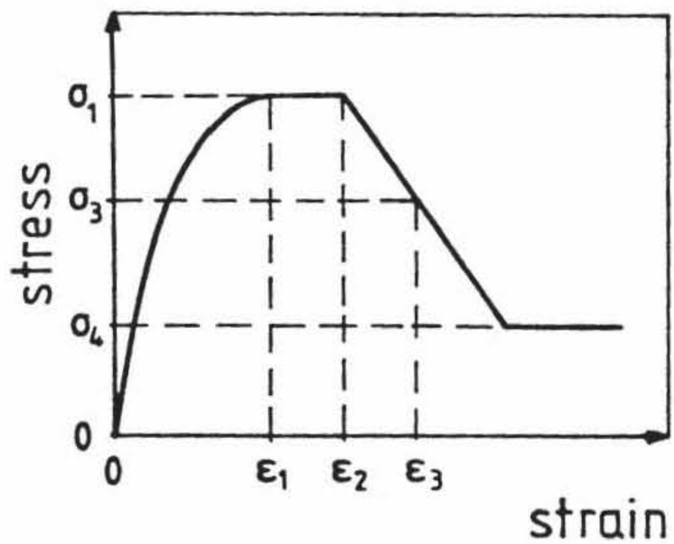


Fig. 6: Stress-strain relationship of concete (after /23/)

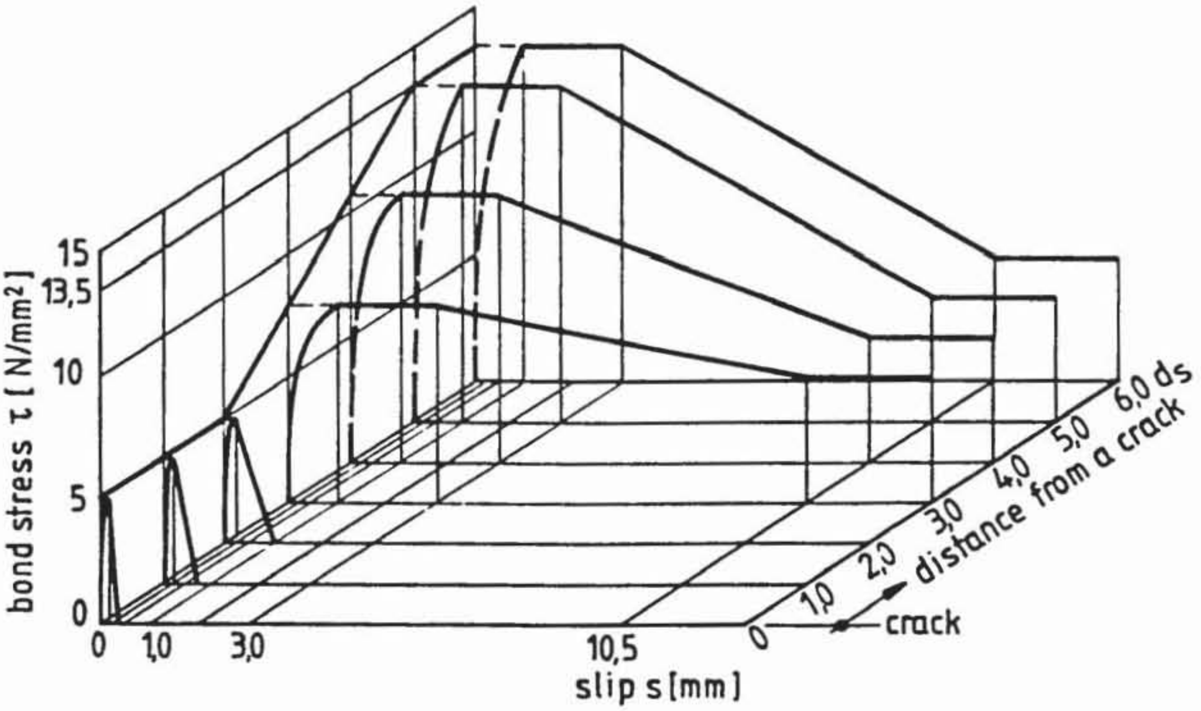


Fig. 7: Bond stress-slip relationships ($f_c' = 30\ N/mm^2$)
(after /24/)

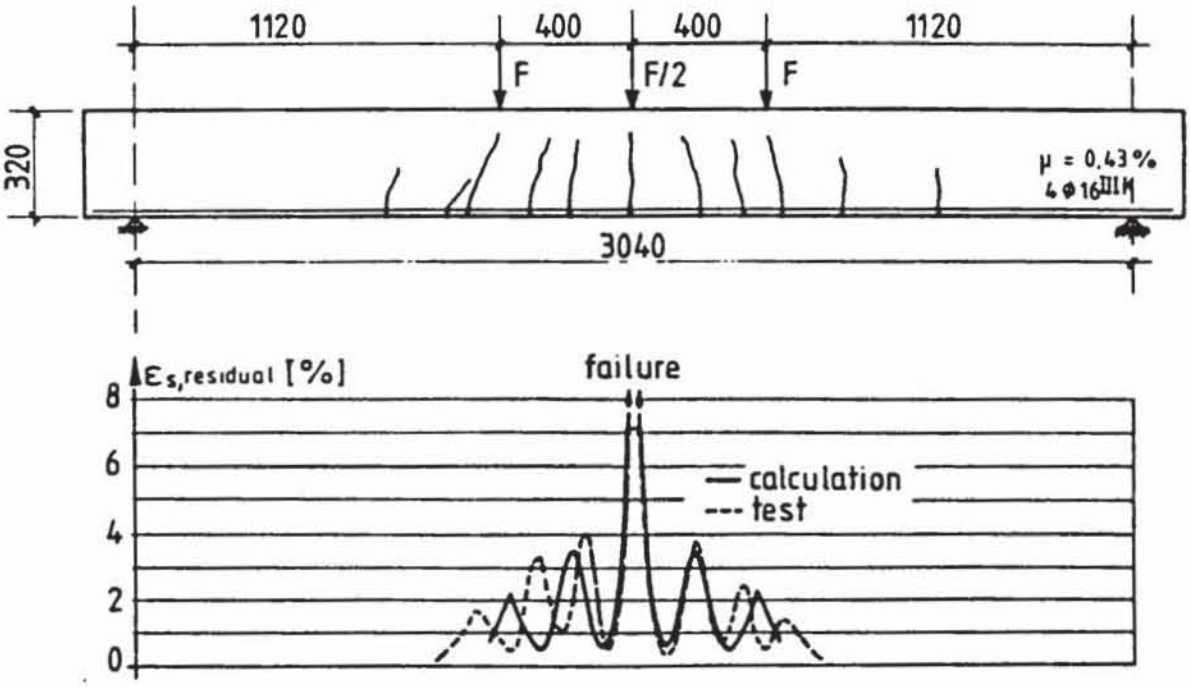


Fig. 8: Residual steel strain in the area of a plastic hinge
according to test /17/ and calculation

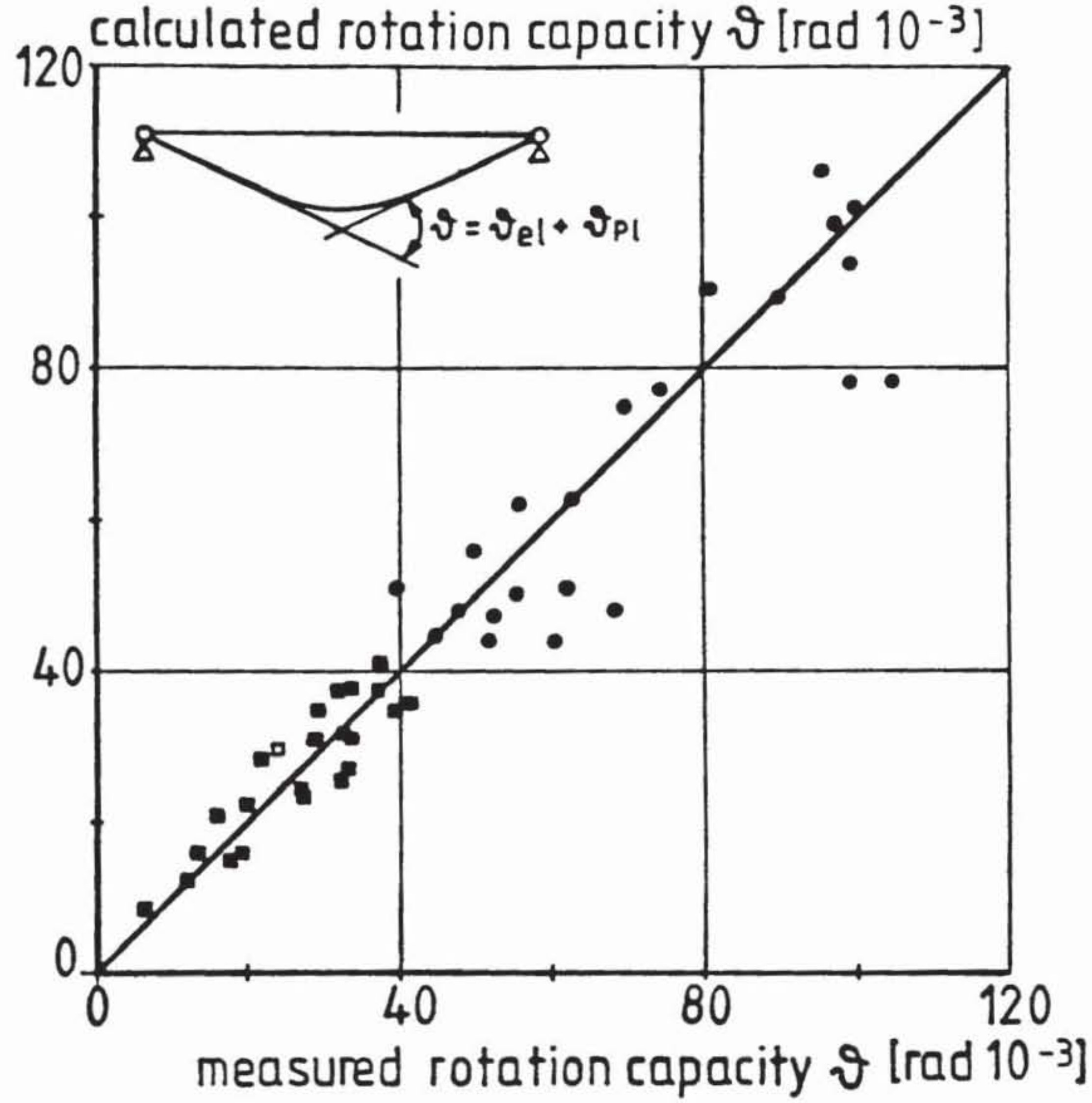


Fig. 9: Rotation capacity of reinforced concrete hinges according to calculation and experiment

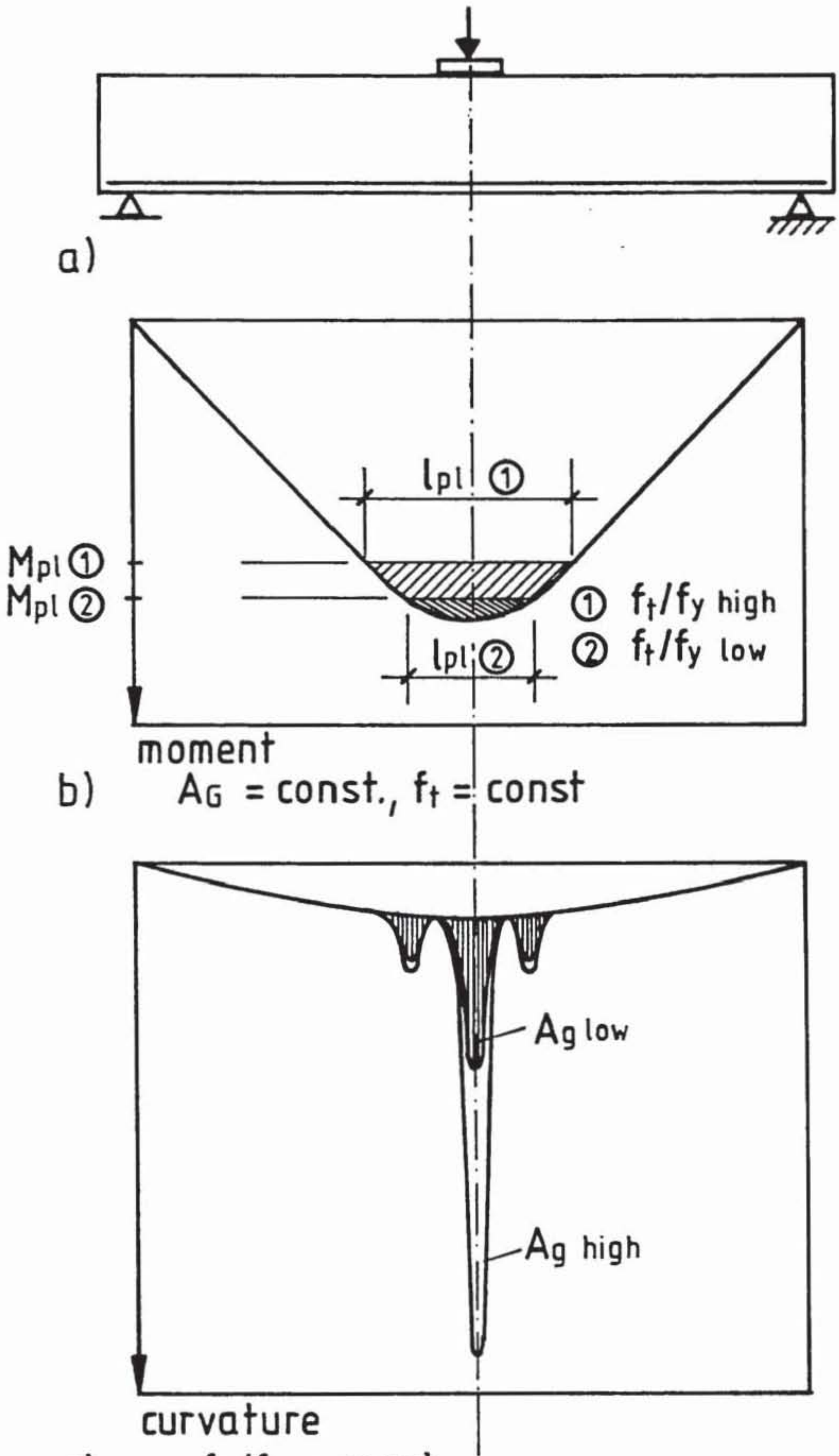


Fig. 10: Influence of stress-strain relationship of reinforcing steel on the rotation capacity of plastic hinges (schematically)

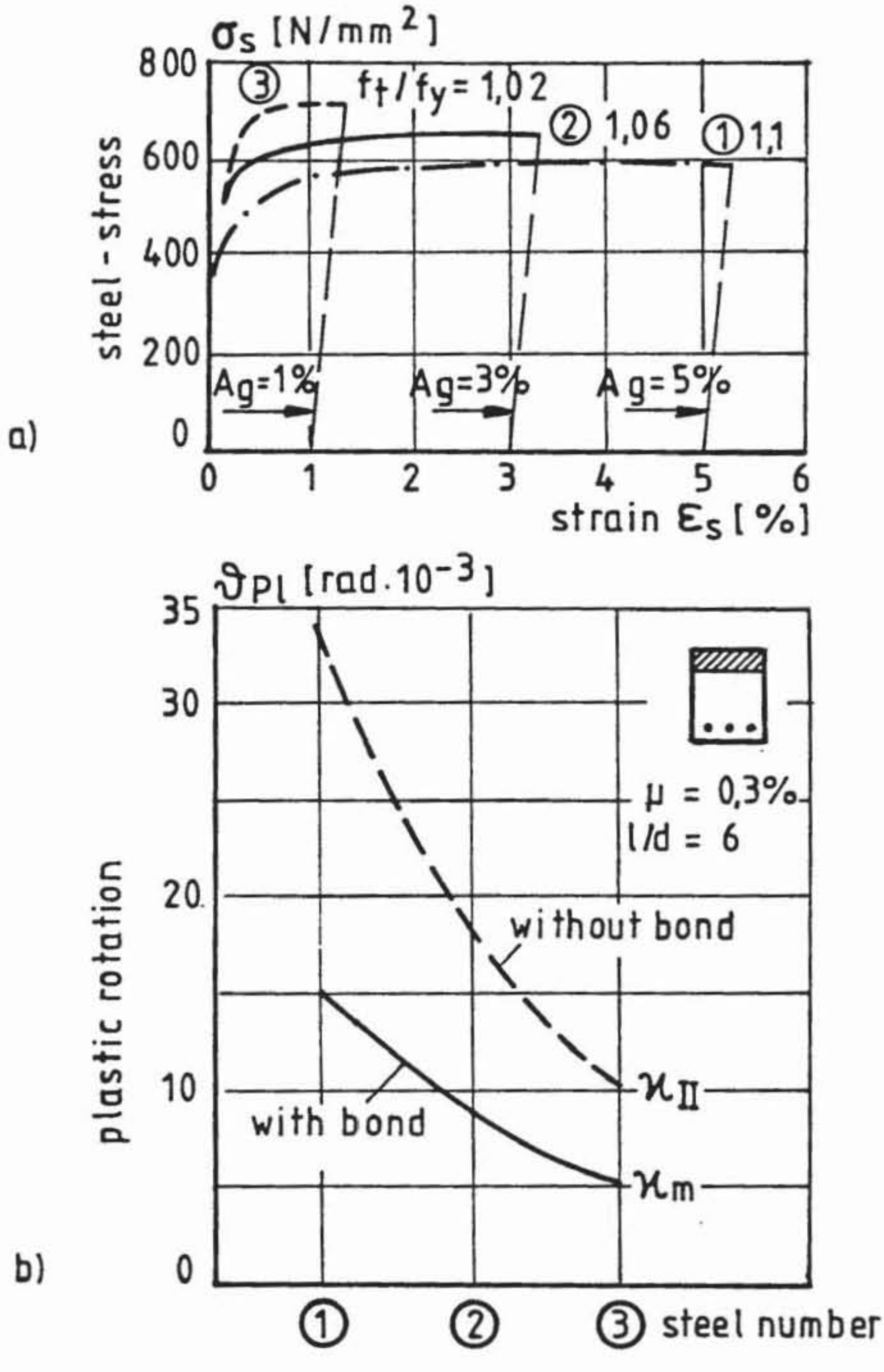


Fig. 11: Influence of stress-strain diagram on the plastic rotation capacity

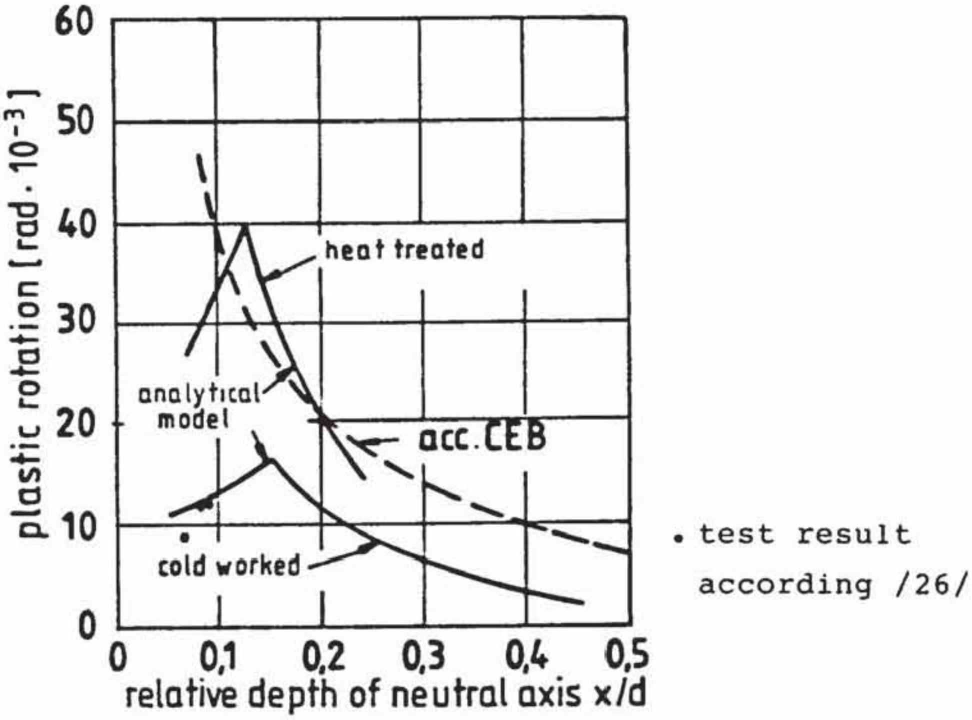


Fig. 12: Plastic rotation capacity according to CEB and analytical model for different types of reinforcement

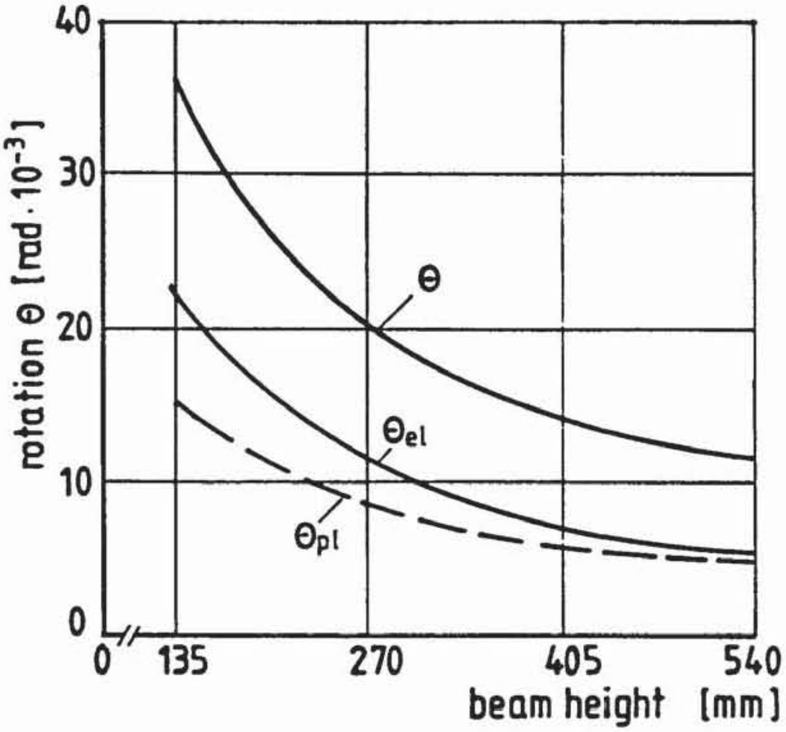
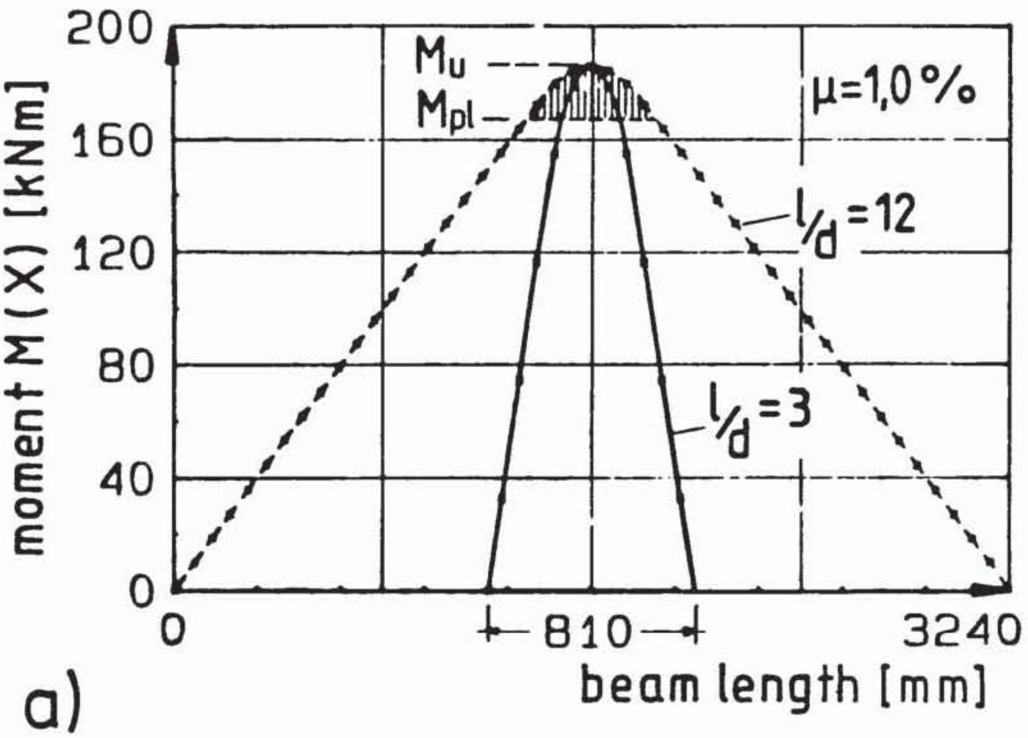


Fig. 13: Influence of beam height on rotation capacity



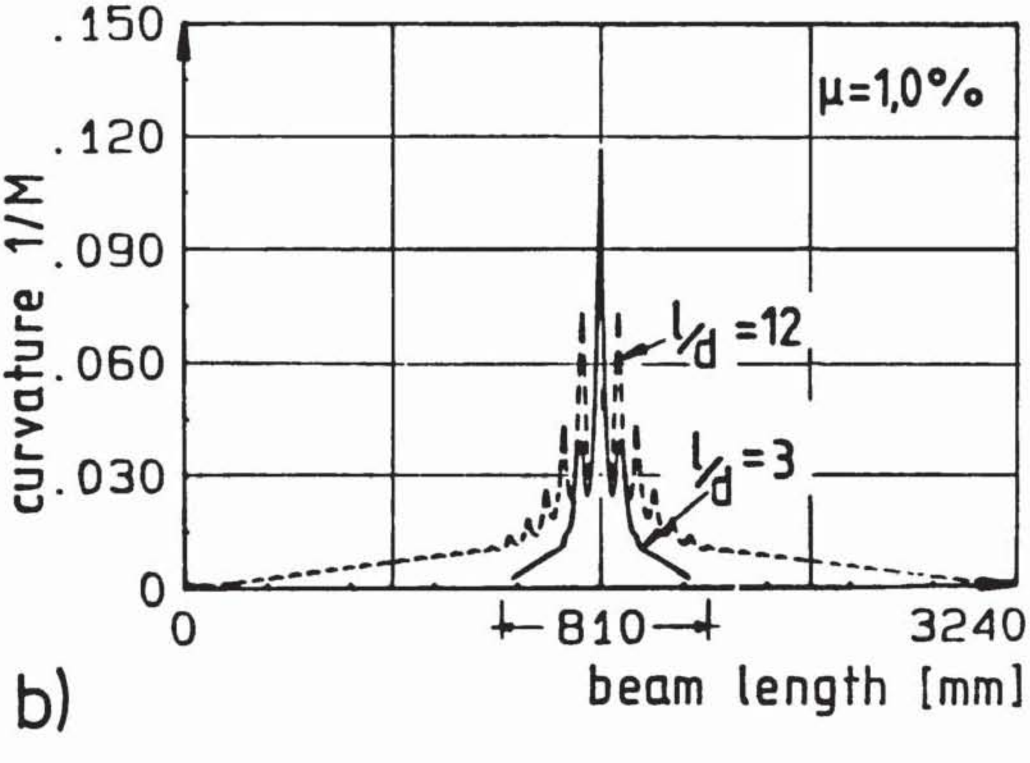


Fig. 14: Distribution of moment and section curvature along beam for different slenderness ratios l/d

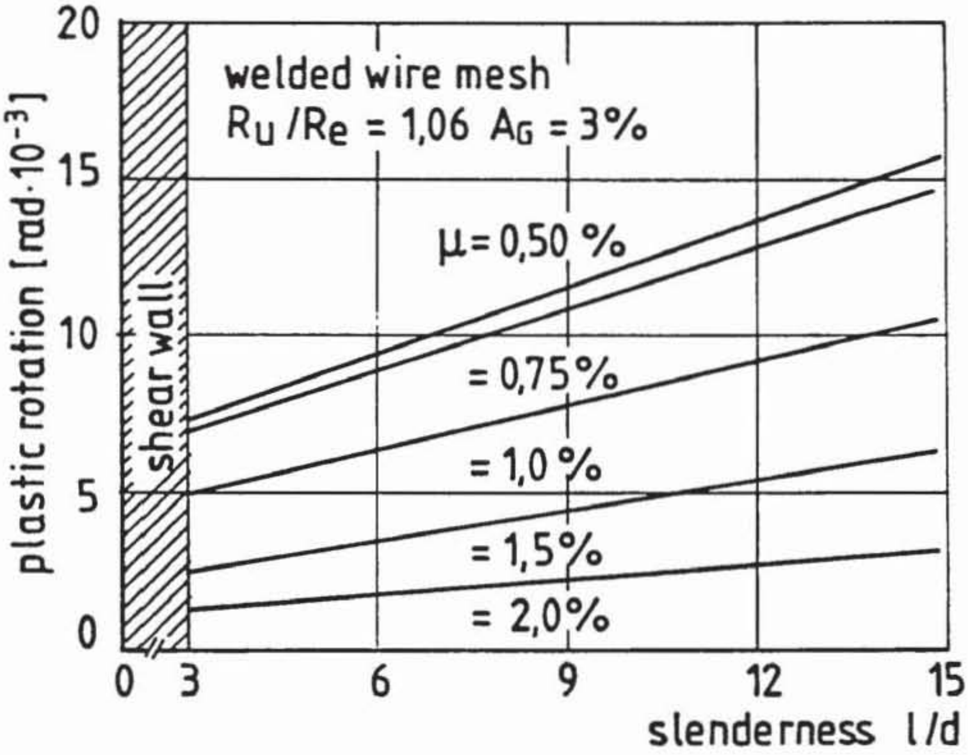


Fig. 15: Plastic rotation capacity as a function of the slenderness l/d according to calculation

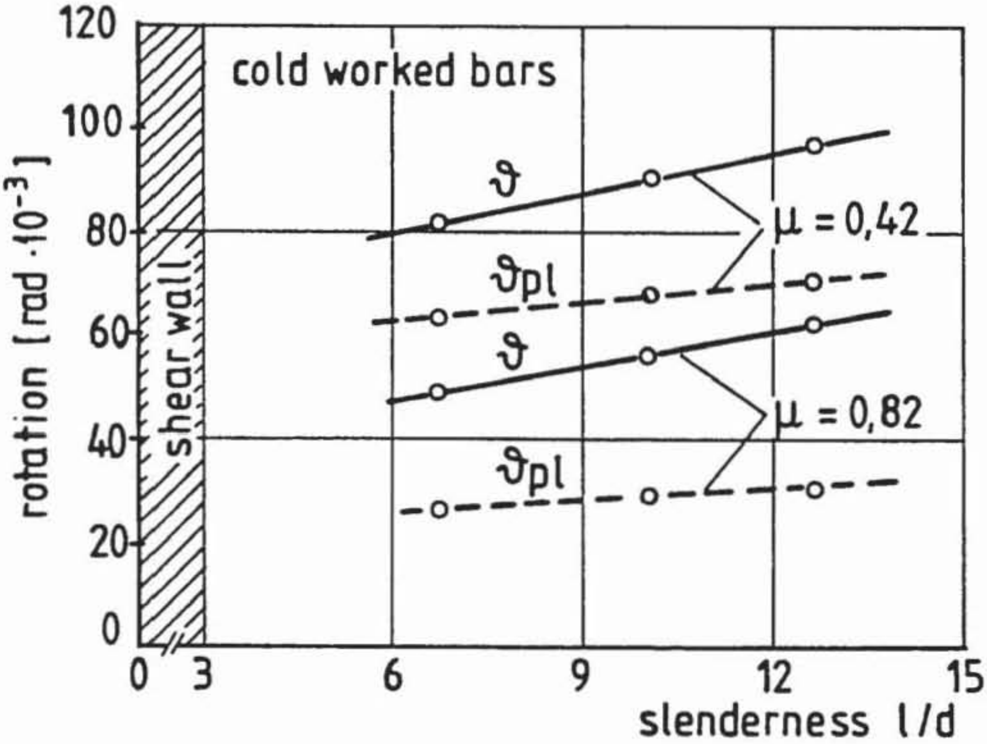


Fig. 16: Rotation capacity as a function of the beam slenderness l/d according to tests /17/
= total rotation capacity
= plastic rotation capacity

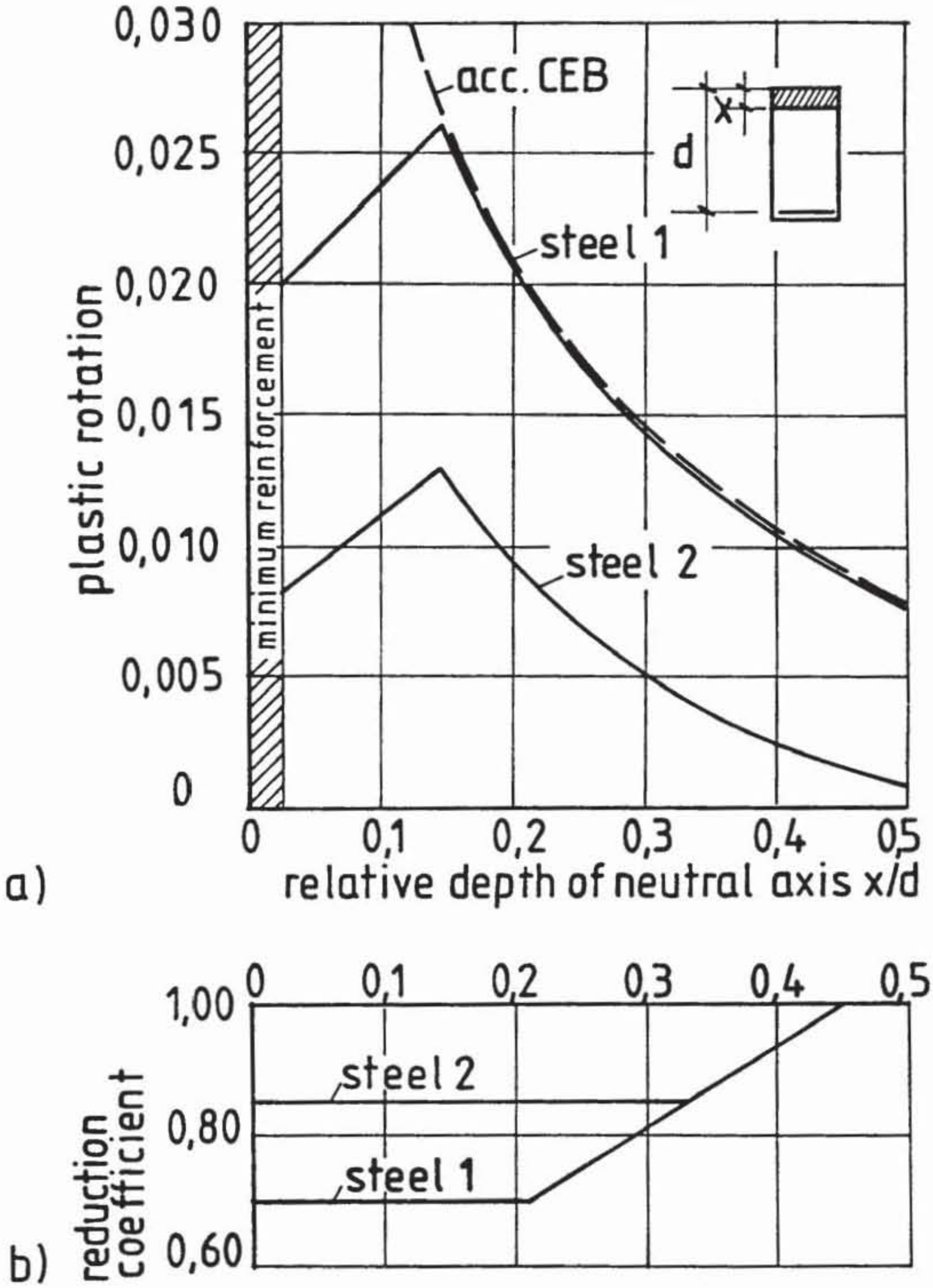


Fig. 17: Proposal for the allowable plastic rotation capacity and the allowable degree of moment redistribution

CHAPTER 7

The text in the following chapter should be considered as:

Text as in MC78	Text-proposal from Commission	Text-proposal from reporter	Text-proposal from MCRG	Memo
				*

CHAPTER 8

The text in the following chapter should be considered as:

Text as in MC78	Text-proposal from Commission	Text-proposal from reporter	Text-proposal from MCRG	Memo
	*			

CHAPTER 9

The text in the following chapter should be considered as:

Text as in MC78	Text-proposal from Commission	Text-proposal from reporter	Text-proposal from MCRG	Memo
		*		

CHAPTER 7 " DETERMINATION OF THE LOAD EFFECTS "

The revision of this chapter is foreseen by PC II after the discussion of chapters 8 and 9 to be held at the Plenary Session in Treviso.