 

Rotation capacity of prestressed concrete members

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Abstract

A numerical investigation on the behaviour of plastic hinges in prestressed concrete structures is presented. The object of the study is to determine the rotation capacity of prestressed concrete members which is needed to predict the amount of moment redistribution in hyperstatic prestressed concrete structures.

A numerical model has been developed for the analysis of plastic hinges in prestressed concrete structures. It is based on a simply supported beam which simulates the region between two points of zero moment in a continuous beam. A discrete crack model has been applied in the numerical analysis. Realistic constitutive laws of steel, concrete and bond of reinforcing and prestressing steel have been assumed, respectively.

The developed model enables an accurate analysis of the load-deformation response of a statically determinate prestressed concrete beam with bonded or unbonded tendons under monotonic loading throughout all behaviour states up to failure. The parameters influencing the rotation capacity of prestressed concrete beams were studied by means of the numerical model.

Keywords: beam; plastic hinge; prestressed concrete structure; rotation capacity; material laws; bond; slip; stress; strain; crack spacing; load; deformation.

1. Introduction

Due to the nonlinear flexural behaviour of reinforced and prestressed concrete members a moment redistribution takes place in hyperstatic R.C. and P.C. structures. The load reserve of the structure may be fully utilized at the ultimate limit state, if the rotation capacity of the plastic hinge is sufficiently large.

The rotation capacity of R.C. members was systematically investigated in /1/. Considerable work has been devoted to the flexural ductiliy of P.C. sections /2,3/. In the present paper the rotation capacity of P.C. members is studied by means of a recently developed numerical model /4/, in which the different bond characteristics of reinforcing and prestressing steel are considered. With the numerical model the load-deformation behaviour of P.C. members with bonded or unbonded tendons at all loading states, including yielding and ultimate load can be analyzed. The major results of a parametric study are illustrated.

2. Numerical model

2.1. Assumptions

(1) P.C. beam under monotonic (non-repeated, non-reversible) loading (Fig. 1).
(2) Constant depth of prestressing steel along the beam (Fig. 1).
(3) The bond of reinforcing steel is not poorer than that of prestressing steel. Therefore, Bernoulli's hypothesis (plane sections remain plane) is valid for concrete and reinforcing steel (Fig. 3).
(4) Cracks are distributed with a mean crack spacing along the beam (Fig. 1).
(5) The crack width decreases linearly from the bottom of the beam towards the neutral axis (Fig. 2).
(6) Bond-slip relationships of reinforcing and prestressing steel are known.

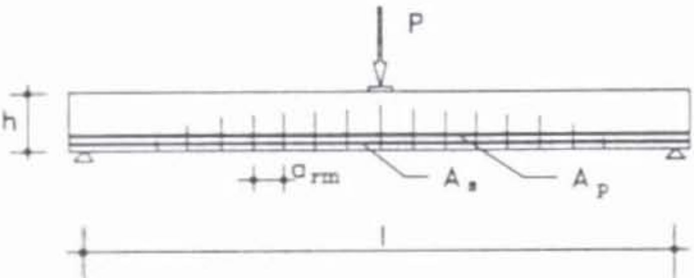


Fig. 1. Simply supported P.C. beam

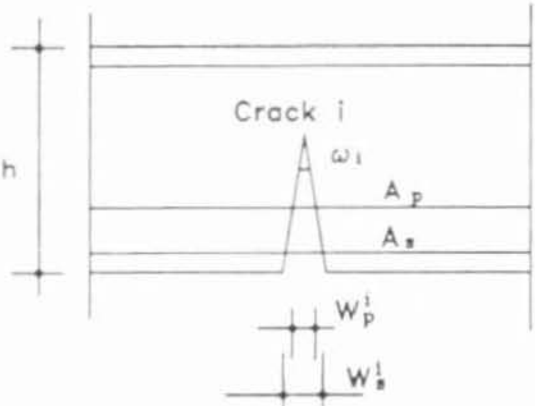


Fig. 2. Crack angel and crack width

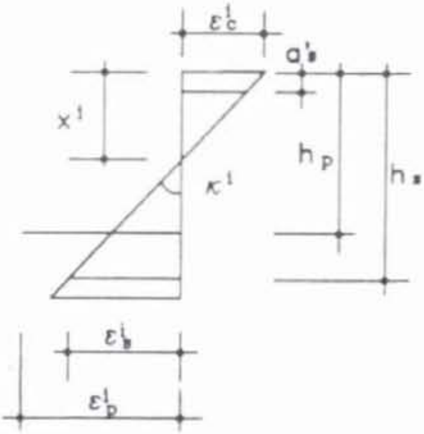


Fig. 3. Strain distribution of cracked cross section

2.2. Material laws

The material constitutive laws adopted for the analysis of the load-deformation behaviour of P.C. members are sumarized in Figs. 4 to 7.

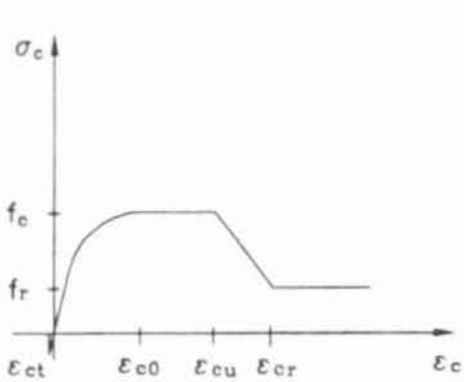


Fig. 4. σ-ϵ law of concrete

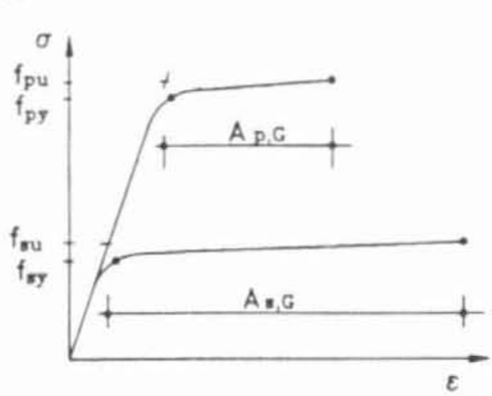


Fig. 5. σ-ϵ laws of steels

Fig. 4 shows the σ_c-ϵ_c relationship for concrete in compression and tension /1,5/. Fig. 5 illustrates the stress-strain curves for reinforcing and prestressing steel /1,5,6/. The curves are definded by a polygone point by point. Hence, they are suitable for any analytical expression with desired accuracy by means of an increasing number of support points. Figs. 6 and 7 show the bond-slip relationships for reinforcing and prestressing steel respectively /1,5/. The bond stresses near cracks are reduced according to /7/.

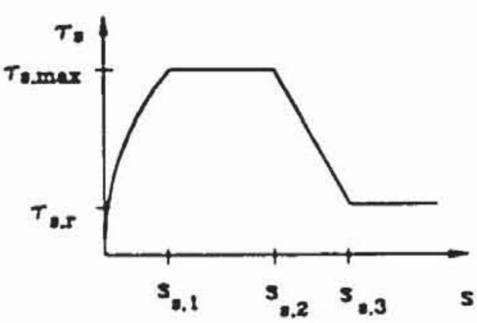


Fig. 6. Bond-slip law of reinforcing steel

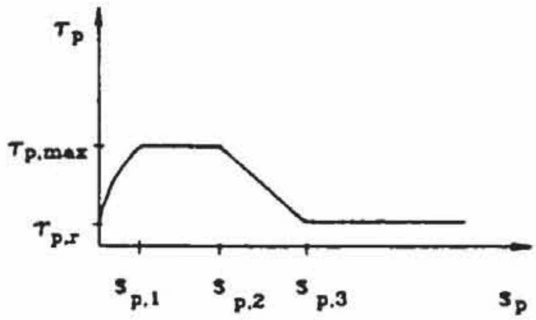


Fig. 7. Bond-slip law of prestressing steel

2.3. Compatibility, equilibrium and boundary conditions

A simply supported P.C. beam (Fig. 1) that simulates the region between two points of zero moment in a continous beam is considered for the analysis of the load-deformation behaviour of P.C. members. The beam is assumed to be prestressed with prestressing tendons as well as to be reinforced with reinforcing steel. The anchoring of the prestressing steel prevents a displacement of the prestressed reinforcement at the beam ends. The reinforcing steel, however, may slip. Therefore, the following boundary conditions are valid:

$$s_p(0)=s_p(l)=0\ , \quad e_s(0)=e_s(l)=0 \tag{1}$$

The elongations of reinforcing and prestressing steel along the beam are related as following:

$$\int_0^l [e_p(x)-e_{pd}(x)]dx=\alpha\int_0^l e_s(x)dx \tag{2}$$

α is a factor which takes into account the different internal lever arms of reinforcing and prestressing steel, respectively, in the cross section. ϵ_{pd} is definded as strain of the prestressing steel at the state of decompression at which the concrete strain at the level of the prestressing steel is $\epsilon_c=0$.

The compatibility conditions between concrete and reinforcing steel as well as prestressing steel can be expressed as:

$$\frac{ds_s(x)}{dx}=e_s(x)-e_c(x)\ , \quad \frac{ds_p(x)}{dx}=e_p(x)-e_{pd}(x)-e_c(x) \tag{3}$$

For an element between two cracks the following expressions result from integrating equation (3), when the concrete strains are neglected (Fig. 8).

$$s_s(x)=\int_0^x e_s(x)dx+s_s^l\ , \quad s_p(x)=\int_0^x [e_p(x)-e_{pd}(x)]dx+s_p^l \tag{4}$$

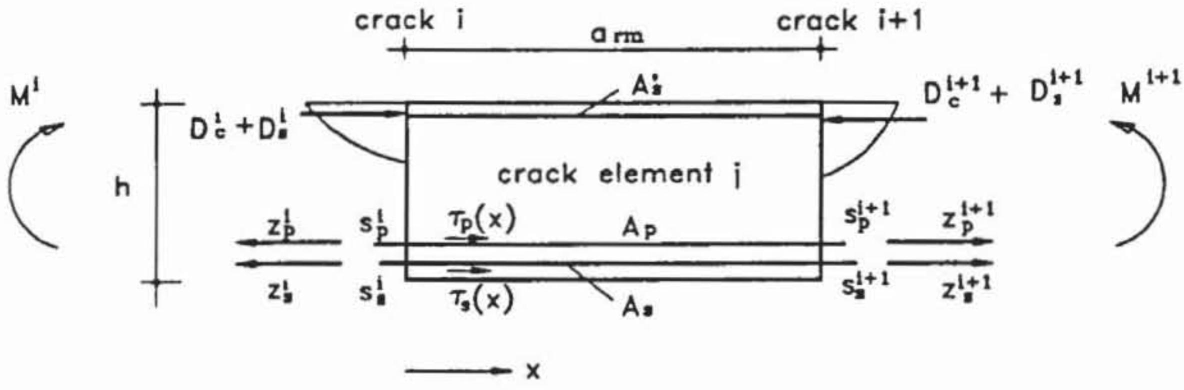


Fig. 8. Forces and displacements on crack element

By substituting equation (4) in equation (2) and considering the boundary conditions according to equation (1), the compatibility of elongations of reinforcing and prestressing steel may be transformed to the compatibility of crack widths at the height of the reinforcing and prestressing steel (Figs. 2 and 3).

For bonded prestressing tendons equation (5) is valid:

$$\sum_{i=1}^{N} w_p^i = \sum_{i=1}^{N} \alpha^i w_s^i , \quad w_p^i = \alpha^i w_s^i \tag{5}$$

For unbonded prestressing tendons equation (6) is valid:

$$(e_p - e_{pd}) l = \sum_{i=1}^{N} \alpha^i w_s^i \tag{6}$$

with

$$w_s^i = s_{sl}^i + s_{sr}^i , \quad w_p^i = s_{pl}^i + s_{pr}^i , \quad \alpha^i = \frac{h_p - x^i}{h_s - x^i} \tag{7}$$

$s_{sl}^i, s_{sr}^i, s_{pl}^i, s_{pr}^i$ denote the actual slip of reinforcing and prestressing steel at the left or right crack face, respectively. For a beam under symmetric loading the slips of reinforcing and prestressing steel in the crack at midspan are related according to equation (8)

$$s_{sl}^m = s_{sr}^m , \quad s_{pl}^m = s_{pr}^m , \quad s_{pr}^m = \alpha^m s_{sr}^m \tag{8}$$

The equilibrium equations for reinforcing and prestressing steel in the tensioned region of the cross section can be expressed as following:

Between two cracks (Fig. 8)

$$\sigma_s(x) = \frac{U_s}{A_s} \int_0^x \tau_s(x) dx + \sigma_s^i , \quad \sigma_p(x) = \frac{U_p}{A_p} \int_0^x \tau_p(x) dx + \sigma_p^i \tag{9}$$

In the cracks (Figs. 2, 3 and 8)

$$\sum H = 0 : \ Z_p^i + Z_s^i - D_c^i - D_s^i = 0 , \quad \sum M = 0 : \ Z_p^i h_p + Z_s^i h_s - D_c^i h_c - D_s^i a_s' + M^i = 0 \tag{10}$$

By substituting x_1^i as curvature of the cross section κ^i, and x_2^i as strain of the reinforcing steel ϵ_s' equation (10) may be translated to the following form:

$$H(x_1^i,x_2^i)=0\ , \quad M(x_1^i,x_2^i)=0 \tag{11}$$

The strain distribution of the cracked cross section can be determined by solving equation (11) by iteration.

2.4. Determination of rotation

The rotation of the beam may be determined by integrating the curvatures of all crack elements (Fig. 9).

$$\theta=\sum_{i=1}^{N}\Big[\int_0^{a_{rm}}\kappa(x)dx\Big]^i=\sum_{i=1}^{N}\Big[\int_0^{a_{rm}}\frac{\varepsilon_s(x)}{z'(x)}dx\Big]^i \tag{12}$$

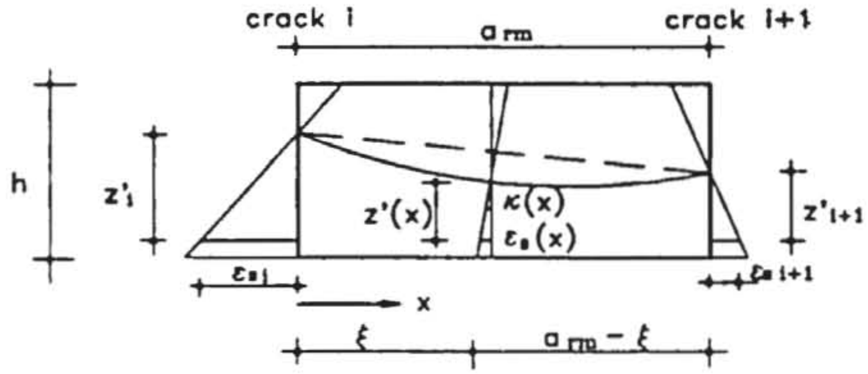


Fig. 9. Strain distribution of cross section between cracks

If $z'(x)$ is varying monotonically between the cracks, the rotation of a crack element may be expressed as /8/:

$$\int_0^{a_{rm}}\frac{\varepsilon_s(x)}{z'(x)}dx=\frac{1}{z'(0)}\int_0^{\xi}\varepsilon_s(x)dx+\frac{1}{z'(a_{rm})}\int_{\xi}^{a_{rm}}\varepsilon_s(x)dx \qquad \text{with } 0\le\xi\le a_{rm} \tag{13}$$

Applying the expressions

$$z'(0)=z'_i\ ,\quad z'(a_{rm})=z'_{i+1}\ ,\quad \int_0^{\xi}\varepsilon_s(x)dx=s_{sr}^i\ ,\quad \int_{\xi}^{a_{rm}}\varepsilon_s(x)dx=s_{sl}^{i+1} \tag{14}$$

equation (12) may be formulated as:

$$\theta=\sum_{i=1}^{N}\frac{s_{sl}^i+s_{sr}^i}{z'_i}=\sum_{i=1}^{N}\frac{w_s^i}{h_s-x^i}=\sum_{i=1}^{N}\omega^i \tag{15}$$

According to equation (15) the rotation results from the sum of crack angels ω^i. This is in accordance with the definition given by Bachmann /9/.

2.5. Computational features

A general computer program has been developed in order to solve the equations iteratively for symmetrically loaded beams at all loading stages. Any symmetric concrete section with one layer each of reinforcing and prestressing steel in the tension zone of the cross section can be analyzed. The computations are done under deformation control by controlling the strain of concrete, reinforcing- or prestressing steel at midspan, respectively, so that the deformations of the beam under peak load can be obtained.

3. Verification of the numerical model with test results

The analytical model is suitable for the analysis of P.C. members with bonded or unbonded tendons as well as for the analysis of R.C. members. Various test results of P.C. and R.C. beams /10,11,12,13,14,/ were analyzed.

Figs. 10 and 11 show a comparison of calculated curvatures at peak load and calculated load-deflection relationship with test data /10/. The calculated results are in good agreement with values measured in the experiment.

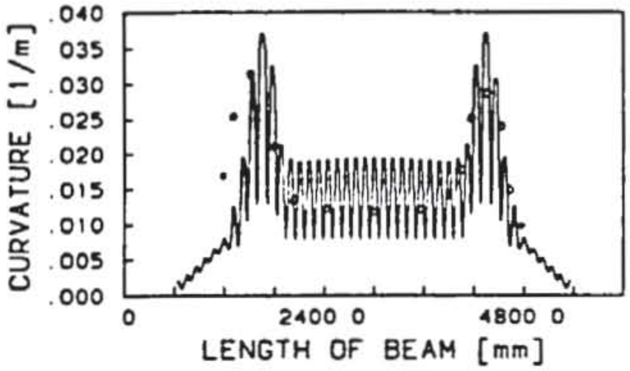


Fig. 10. Curvature distribution under peak load

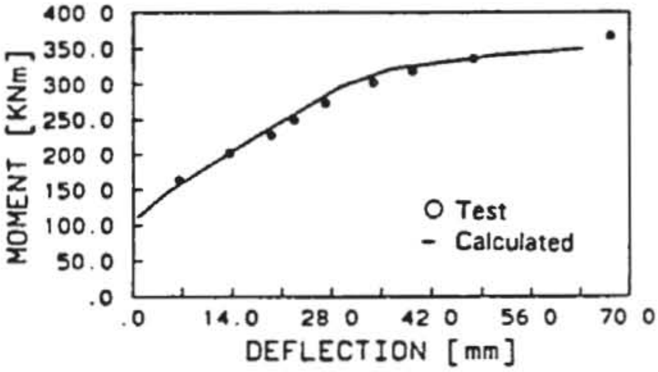
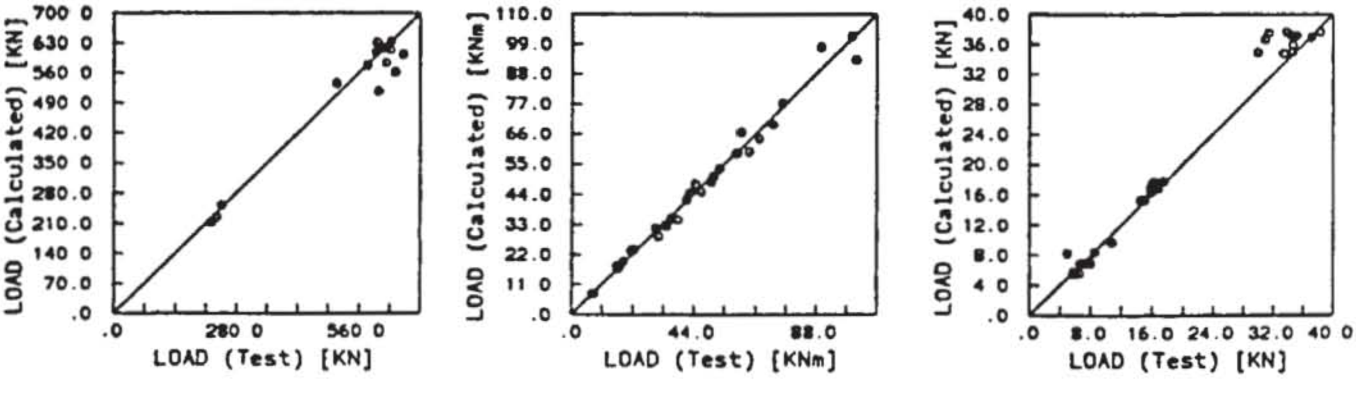


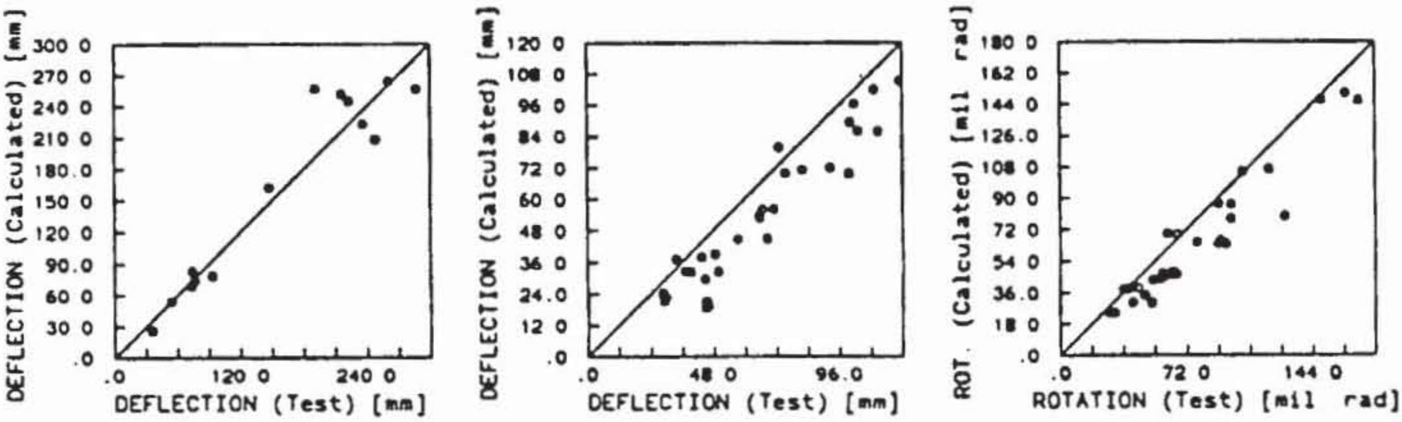
Fig. 11. Moment deflection relationship

In Figs 12a to 12c the calculated ultimate loads are plotted as a function of the values measured in tests with P.C. beams with bonded or unbonded tendons as well as with R.C. members. The accuracy between measured and calculated values is sufficient. Figs. 13a to 13c show plots of the calculated deflections or rotations as a function of deflections or rotations, respectively, observed in tests. Fig. 13 indicates that the calculated deformations are somewhat smaller than the measured values.



(a) with bonded tendons (b) with unbonded tendons (c) R.C. members

Fig. 12. Comparison of calculated loads with test data



(a) with bonded tendons (b) with unbonded tendons (c) R.C. members

Fig. 13. Comparison of calculated deformations with test data

4. Rotation capacity of prestressed concrete members

4.1. Definition of elastic and plastic rotation

The elastic and plastic rotation are determined according to CEB (Fig.14) /15/.

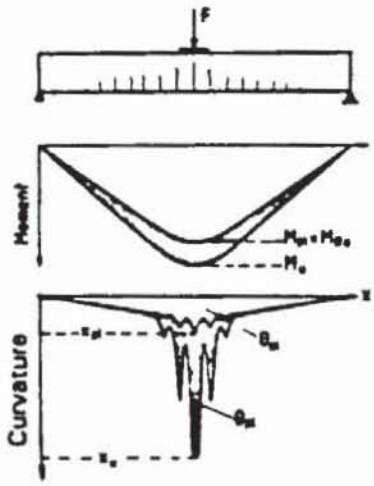


Fig. 14. Definition of plastic rotation

The elastic and plastic rotation respectively are defined as follows.

- Elastic rotation Θ_{el}

The elastic rotation is equal to the integration of curvatures along the beam under yielding load (load under which either the reinforcing or the prestressing steel reaches its yield stress).

- Total rotation Θ

The total rotation is the rotation under peak load. This is defined by three failure criteria: reinforcing or prestressing steel reaches its tension strength, or the compressed concrete reaches its crushing strain ϵ_{cu}.

- Plastic rotation Θ_{pl}

The plastic rotation is the difference between total and elastic rotation.

4.2. Parametric study

A parametric study has been conducted in order to assess the influence of the basic parameters governing the rotation capacity of P.C. members.

The numerical constants of material laws as well as the geometry of the concrete members are identified in table 1 by reference to the material laws in section 2.2.

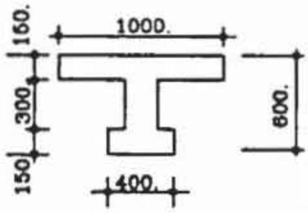
Material laws		Geometry
Concrete	$f_c = 42.5$ N/mm² $\epsilon_{co} = -2.0‰$ $\epsilon_{cu} = -5.0‰$	$l = 5.0$m $d = 0.6$m $\lambda = 0.51$ $\beta = 0.55$ $\xi = 0.5$
Reinforcing Steel	$f_{sy} = 500$ N/mm² $f_{su} = 550$ N/mm² $A_{s,G} = 5.0\%$	
Prestressing Steel	$f_{py} = 1080$ N/mm² $f_{pu} = 1230$ N/mm² $A_{p,G} = 4.0\%$	
Reinforcing Steel Bond	$\tau_{s,max} = 12.0$ N/mm²	
Prestressing Steel Bond	$\tau_{p,max} = 6.0$ N/mm²	

Table 1. Material laws and geometry of beams for parametric study

The parameters influencing the rotation capacity of P.C. members may be expressed as material, geometrical and loading parameters, respectively.

• The material parameters are: σ-ϵ laws of concrete, reinforcing and prestressing steel; bond-slip relationships of reinforcing and prestressing steel.

• The geometrical parameters are: percentage of tension and compression reinforcement, degree of prestressing, diameter of reinforcing and prestressing steel, slenderness of the beam and mean crack spacing.

• The loading parameters are: prestressing ratio, moment/shear ratio and load distribution over the beam.

Some influences on the rotation capacity of P.C. members are presented in the following. Further studies are described elsewhere /4/.

Influence of ductility of prestressing steel on the rotation capacity

The ductility of prestressing steel are descriped by yielding stress f_{py}, the ratio f_{pu}/f_{py} and the elongation at maximum load $A_{p,G}$.

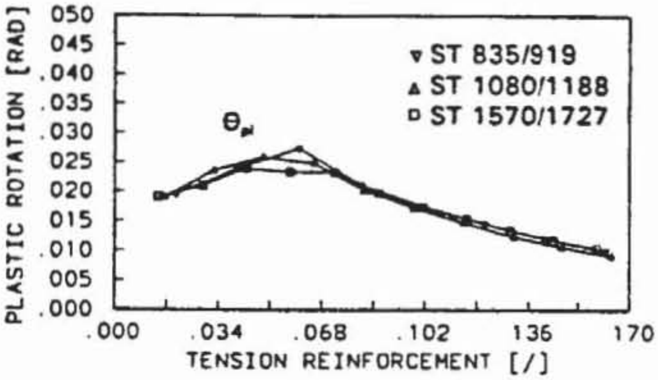


Fig. 15. Pl. rotation versus f_{py}

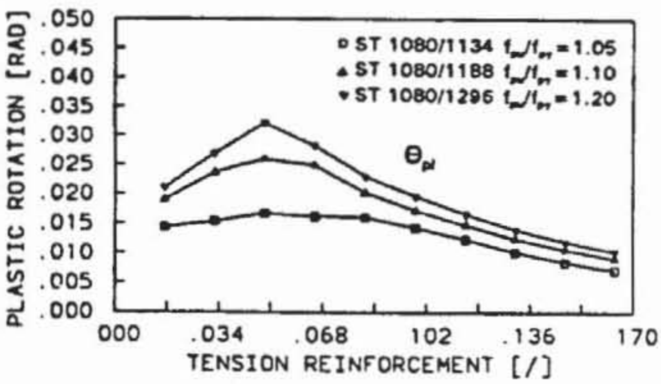


Fig. 16. Pl. rotation versus ratio f_{pu}/f_{py}

Figs. 15 to 17 show the plastic rotation capacity of a prestressed concrete beam as a function of the mechanical reinforcement ratio $\omega = (f_{py} \cdot A_p + f_{sy} \cdot A_s)/b \cdot d \cdot f_c$. Parameter is the type of steel. The Figures are valid for a degree of prestressing $\lambda = f_{py} \cdot A_p / (f_{py} \cdot A_p + f_{sy} \cdot A_s) = 0.51$, a prestressing ratio $\beta = f_{p0}/f_{pu} = 0.55$ and a bond ratio $\xi = \tau_{pm}/\tau_{sm} = 0.5$. It can be seen from Fig. 15 that the strength of the prestressing steel has little influence on the rotation capacity, if f_{pu}/f_{py} and $A_{p,G}$ remain constant. However, the ratio f_{pu}/f_{py} and the margin $A_{p,G}$ have a significant influence on the rotation capacity of P.C. members.

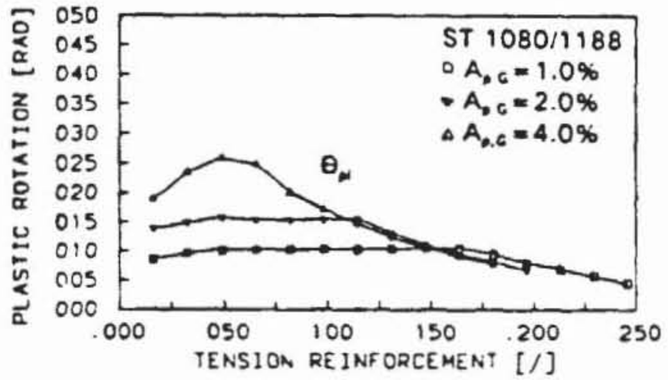


Fig. 17. Pl. rotation versus margin $A_{p,G}$

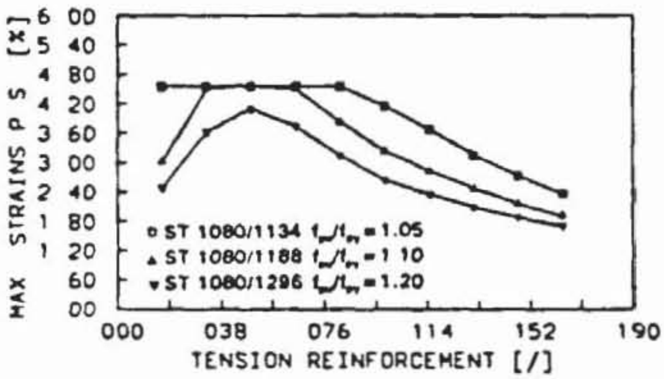


Fig. 18. ϵ_p under peak load versus f_{pu}/f_{py}

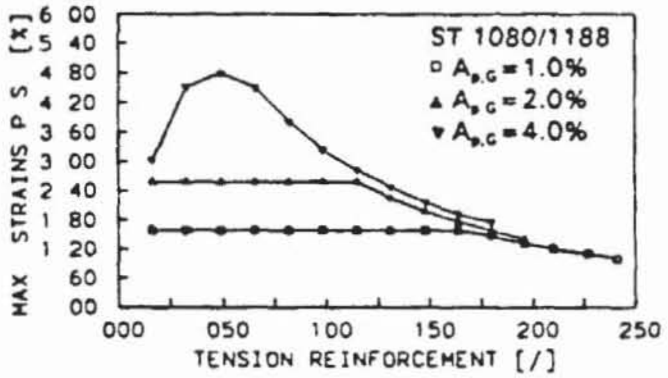


Fig. 19. ϵ_p under peak load versus $A_{p,G}$

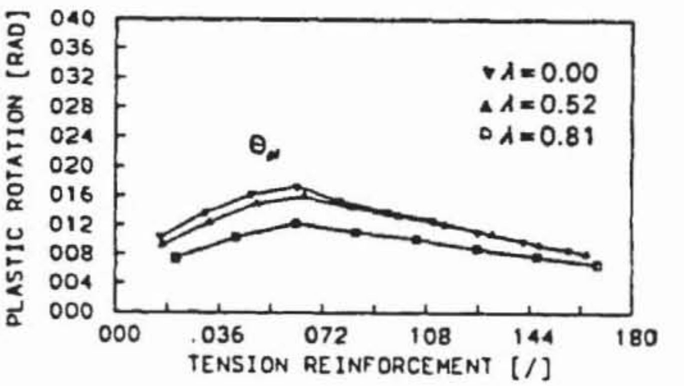


Fig. 20. Comparison of pl. rotations (PC and RC)

Figs. 18 and 19 show the maximum strains of prestressing steel at peak load, indicating that the prestressing steel with a lower ratio f_{pu}/f_{py} and a lower value for $A_{p,G}$ reaches its tension strength for low values of ω, thus determining the rotation capacity, while the more ductile prestressing steel is not fully utilized.

4.3. Comparison of the rotation capacity of P.C. members and R.C. members

Fig. 20 shows a plastic rotation capacity of R.C. members ($\lambda = 0.0$) and P.C. members ($\lambda = 0.51$ and $\lambda = 0.81$ respectively) as a function of the reinforcement ratio ω. While the plastic rotation of reinforced and prestressed concrete beams with $\lambda \leq 0.5$ is not much different, the rotation capacity decreases significantly for heigher degrees of prestressing.

5. Conclusions

A numerical model was developed for studying the rotation capacity of prestressed concrete members. The rotation capacity is needed to assess the possible amount of moment redistribution in hyperstatic prestressed concrete structures. The numerical model accepts any type of constitutive material laws, so that all the parameters affecting the rotation capacity of prestressed concrete members can be studied. The predicted results agree sufficiently well for practical purposes with test data.

According to the parametric study the rotation capacity of prestressed concrete beams is influenced by the ductility of the prestressing steel in much the same way as reinforced concrete beams by the ductility of the reinforcing steel. For constant values of the mechanical reinforcement ratio the rotation capacity decreases with increasing degree of prestressing.

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