1. FUNDAMENTALS
(H. Kobus)

1.1 Introduction

Since Leonardo da Vinci (1452 - 1519) stated as a basic premise to "remember when discoursing on the flow of water to adduce first experience and then reason", the experiment has attained a leading and continuously growing role in fluid mechanics research. Like many other disciplines in engineering, fluid mechanics has experienced a rapid and remarkable development during the 18th and 19th century. However, civil engineering practice remained unaffected by this development for a long time. It is a prime merit of some German engineering scientists at the beginning of this century that they developed experimental methods for the solution of hydraulic engineering problems and that they succeeded in convincing the profession of the usefulness and validity of this approach.

The first river hydraulics laboratory as a permanent experimental installation was founded in 1898 by Hubert Engels (1854 - 1945) at the Technische Hochschule in Dresden. Almost at the same time, Theodor Rehbock (1864 - 1950) built a river hydraulics laboratory at the Technische Hochschule in Karlsruhe in 1901, which was considerably enlarged in 1921. Due to the influence of Engels and Rehbock, the use of hydraulic models rapidly gained attention throughout Germany. In 1903 followed the installation of the third laboratory in Berlin, the Königlich-Preussische Versuchsanstalt für Wasserbau und Schiffbau, which gained considerable importance under Hans-Detlef Krey (1866 - 1928).

Parallel to these developments in hydraulic engineering, the fields of experimental fluid mechanics, aerodynamics and flow machinery experienced rapid progress. Ludwig Prandtl (1875 - 1953) was appointed to the Universität Göttingen in 1904 and founded the Kaiser-Wilhelm-Institut für Strömungsforschung, which rapidly attained a world-wide reputation. Laboratories for flow machinery were founded by Hermann Föttinger (1877 - 1945) in Berlin, by Dietrich Thoma (1881 - 1943) in Munich and by Victor Kaplan (1876 - 1934) in Brünn.

The successful example of the first German river hydraulics laboratories found soon attention and imitation outside Germany. Similar institutions were founded 1907 in Leningrad (V.E. Timonoff), 1908 in Toulouse (C. Camichel), 1910 in Padua (E. Scimeni), 1912 in Vienna (F. Schaffernak), 1917 in Stockholm (W. Fellenius), 1927 in Delft (J.Th. Thijsse) and 1928 in Zurich (E. Meyer-Peter). In the USA, also a number of hydraulics laboratories was founded, to the development of which the activities of John R. Freeman have contributed essentially (Rouse and Ince, 1957).

Hydraulic modelling developed rapidly into an engineering tool of general recognition for the solution of hydraulic engineering problems. It was therefore a logical development at the time to strive for an international organization of hydraulic laboratories. As a consequence of efforts by Rehbock and Fellenius, the "International Association of Hydraulic Research" was founded in 1935 by 66 scientists. Of the founding members, 18 came from Germany, and the first congress of the association took place in Berlin in 1937. In the more than 40 years since then, the association has grown into a world-wide organization with a membership of nearly 300 institutions and about 2000 scientists. In the Federal Republic of Germany alone, today 16 hydraulics laboratories and institutes are devoted to the solution of numerous problems in research and practice.

Today, the use of hydraulic models for the solution of engineering problems has become accepted standard procedure in many areas. Apart from the classical problems of hydraulics, a number of new problem areas arise, which require modern measurement techniques and more sophisticated considerations on hydraulic similitude. The aim of the present brochure is to serve as an introduction to the discipline of hydraulic modelling and to convey to the practicing engineer the present state of the art of hydraulic modelling.

1.2 The Notion of a Hydraulic Model

A model in its widest sense is a simplified representation of a subject, state or event (e.g. conceptual model, systems model, etc.). The following groups can be distinguished:

- Similar models, in which all model parameters exhibit a certain relationship to the corresponding parameters in nature, which is determined by one or several model scales, and
- dissimilar models, for which this requirement is not or only partially satisfied (descriptive or qualitative models).

In the following, we shall address exclusively "hydraulic models", which we define as follows:

> "Any physical model for the simulation of flow processes, flow states and events, which concern problems of hydraulic engineering or technical hydromechanics."

Thus the frame is fixed and spans as wide as the notion of hydraulic engineering or technical hydromechanics. In general, hydraulic models are small-scale reproductions of nature in the laboratory, but in some cases a model scale of 1 : 1 is used. In these cases, a representative piece of nature is reconstructed in the laboratory, where flow processes and their effects can be investigated under controllable boundary conditions. This approach is of importance in those areas in which the translation of results from a small scale model to prototype conditions is questionable.

A certain limitation has to be made with respect to the degree of model abstraction, since in its widest sense experimental research in fluid mechanics could be included entirely under the topic of hydraulic modelling. However, we shall exclude all those experimental investigations in hydraulic laboratories, which are of a basic research character in so far that they do not exhibit a direct relation to hydraulic engineering problems in nature. A decisive criterion of hydraulic modelling can therefore be defined by the fact that the observations made on a small scale model must be transferable to natural conditions or exhibit a direct similarity relationship to problems of hydraulic engineering. However, the transition to experimental research in hydromechanics is not sharply fixed.

Hydraulic model tests usually use water as model fluid, because it is easily available, cheap and simply replaceable and therefore exhibits considerable economic and operational advantages in comparison to other fluids. However, the similarity requirements do not withstand the use of other fluids, of course. This can be advantageous, when the model fluid has more favourable material properties than water. Thus, for example, similarity of sediment transport has been achieved in small-scale models in which the transport of sand in water was simulated by means of the transport of coal dust in glycerin.

Relatively widespread in recent years is the use of air models and sub-critical wind tunnels for hydraulic models. As long as compressibility effects can be neglected, i.e. for wind speeds below 50 m/s, many hydraulic flow configurations (with the exception of cavitation phenomena) can be investigated advantageously in wind tunnels, which allow simpler and less rigid structures and offer highly developed measurement techniques. The main limitation in the use of air models consists in the fact that the reproduction of free surfaces of water bodies is not possible or requires rather considerable efforts. For an undistorted plain water surface, one can either construct a "double model", in which the flow cross section is reflected at the water-line, or else may make use of a false ceiling at the level of the water surface, which must be known from other considerations. These approximations have been used for a number of practical applications, as is shown in Chap. 14.4.

A specific class of hydraulic models is based on the analogy between the laws of fluid mechanics and laws of other areas of physics. The most common examples are found for groundwater flows, where the analogy to the various other fields of application of potential theory (Darcy's law, heat conduction equation, Ohm's law, membrane theory) allows a vast number of analog methods (see Chap. 13). Not less interesting is the analogy between gravity and elasticity waves (Froude- and Mach waves), which permits simulation of supersonic flows in a free-surface-water channel.

Hydraulic modelling belongs to the applications-oriented part of experimental fluid mechanics research, in which hydraulic problems of civil engineering practice are investigated. We have excluded one branch of civil engineering which is increasingly making use of wind and water flume experiments: building aerodynamics. This new branch of modelling is of increasing importance in accordance with the development of modern structural engineering and serves as an aid in solving such problems as wind loads on buildings, structural vibrations caused by wind, effects of smoke stack emissions, aeration and ventilation of buildings or tunnels as well as city climate problems. It may suffice here to say that the fundamentals and experimental methods of hydraulic modelling are directly and immediately applicable also to the field of experimental building aerodynamics.

1.3 Similarity Mechanics

"Similarity" between nature and a model implies geometrical, kinematic and dynamic similarity.

Geometrical similarity of a model is achieved, if all geometrical lengths L_n in nature exhibit a constant ratio to the corresponding lengths L_m in the model. This ratio is called length scale number L_r of the model ($L_r = L_n/L_m$).

Kinematic similarity requires that time-dependent events procede in the model always in such a manner that corresponding time intervals in nature and in the model show a constant ratio (time scale number $t_r = t_n/t_m$).

Dynamic similarity implies that corresponding forces in nature and in the model must show a constant ratio (force scale number $F_r = F_n/F_m$).

Dynamic similarity is necessary in order to ensure that in geometrically similar models time-dependent events occur kinematically similar. This is implied by the relationship between the acting forces and the flow field as expressed in the equations of motion. Therefore, the key requirement for geometrically similar hydraulic models is to ensure dynamic similarity, which is achieved when all acting forces in the model are reproduced at a constant ratio to nature:

$$F_r = \frac{F_{n1}}{F_{m1}} = \frac{F_{n2}}{F_{m2}} = \ldots\ldots = \frac{F_{ni}}{F_{mi}} \tag{1.1}$$

From this follows that the corresponding ratios among the various forces must be the same in model and in nature. With F_r = constant, one obtains

$$\frac{F_{m1}}{F_{m2}} = \left(\frac{F_r}{F_r}\right) \frac{F_{n1}}{F_{n2}} = \frac{F_{n1}}{F_{n2}} \tag{1.2}$$

All similarity laws of fluid mechanics can be derived from the requirement Eq. (1.2): The conventional characteristic numbers in fluid mechanics are defined as ratios of the various types of forces which act on a fluid element.

Consider, for example, the simple case of a fluid element of side length L and of density ρ, subjected to a differential pressure Δp and moving with a velocity v. The inertial reaction of the element (mass times acceleration) can be written as:

$$F_1 = \left(\frac{v^2}{2L}\right) \cdot \left(\rho L^3\right) = \frac{\rho}{2} v^2 L^2 \qquad (1.3)$$

If this inertial reaction is related to the acting pressure force ($\Delta p \cdot L^2$) upon the element, then one obtains the square of the geometrical characteristic number Eu, also called "Euler number":

$$Eu^2 = \frac{v^2}{2\Delta p/\rho} = \frac{\text{inertial reaction}}{\text{pressure force}} \qquad (1.4)$$

The geometrical flow parameter Eu is a dimensionless ratio, which characterizes the relation between the inertial reaction of the fluid and the acting pressure force. Therefore, it is a characteristic number of fluid mechanics in the sense defined above. In incompressible fluids and in the absence of other forces (such as viscosity, gravity, etc.), Eu is exclusively a function of the geometry of the flow boundaries.

It is to be noted here that the geometrical flow number contains only the pressure difference ($\Delta p = p - p_o$) as the driving force of the flow and is therefore independent of the reference pressure p_o. The reference pressure p_o can be chosen to be atmospheric pressure, the base of absolute pressure, or any arbitrary reference value of the system considered. Therefore, whereas the Euler number describes directly the relationship between the acting pressure difference Δp and the velocity field, the calculation of the corresponding pressure fields requires also knowledge of the chosen reference pressure.

In a similar fashion all other fluid mechanics characteristic numbers can be derived. In each case, typical forces upon the fluid element (viscous forces, gravity force, etc.) are put in relationship to the corresponding inertial reaction. However, the same characteristic numbers can be derived directly without intuition and physical interpretation by means of a dimensional analysis.

Dimensional analysis is a most useful tool in experimental fluid mechanics, which allows in a simple and direct manner - implicitly - the formulation of criteria for dynamic similarity. Since all relationships derived by dimensional analysis are independent of the absolute scale, they must be applicable for both small scale models and prototype dimensions. The fundamentals of dimensional analysis and their applications in experimental research in civil engineering are described in (Kobus, 1974). For simple problems, a dimensional analysis leads to the conventional and well-known fluid mechanics characteristic numbers and model laws, as they are described and discussed in the following. For complex flow problems, like for example multi-phase flows, air-water mixtures, sediment transport problems, etc., a dimensional analysis establishes the only promising frame of reference, with the aid of which the complex similarity relationships can be described, necessary simplifications can be identified and quantified, and the choice of experiments can be optimized in such a manner that one obtains from a minimum of experiments a maximum of information.

For the example given above, a dimensional analysis leads equally to the definition of the Euler number or the geometrical flow number Eu. If in a flow field of given geometry, characterized by a reference length L and a reference velocity v, only inertial and pressure forces (Δp) are acting, then there exists only one possible combination of all relevant parameters into a dimensionless number: the geometrical flow number Eu (or any arbitrary power thereof).

$$f(\rho, v, L, \Delta p) = 0 \quad \therefore \quad Eu = \frac{v}{\sqrt{2\Delta p/\rho}} = \text{const.} \tag{1.5}$$

The actual value of this constant number is dependent upon the form of the flow boundaries. The geometrical flow number has many applications in hydraulic engineering in specific definitions. For example, the discharge- or effluent coefficient C_Q for nozzles, orifices or openings is obtained by considering that ($v = Q/A$) as

$$C_Q = \frac{Q}{A\sqrt{2\Delta p/\rho}} = Eu \tag{1.6}$$

The resistance coefficient C_W is defined, with ($\Delta p = F/A$), as

$$C_W = \frac{F}{A \cdot \rho v^2/2} = \frac{1}{Eu^2} \tag{1.7}$$

The frictional loss coefficient λ for pipe and open channel flows can also be interpreted as follows:

$$\lambda = \frac{4r_{hy}}{L} \cdot \frac{h_v}{v^2/2g} = \frac{4r_{hy}}{L} \cdot \frac{\Delta p_v}{\rho v^2/2} = \frac{\text{const.}}{Eu^2} \tag{1.8}$$

If one considers a fluid element of density ρ with a reference length L and reference velocity v under the influence of viscous forces, then one obtains by dimensional analysis another fluid mechanics characteristic number or force ratio

$$(\rho, v, L, \eta) \longrightarrow \mathrm{Re} \equiv \frac{\rho v L}{\eta} = \frac{\text{inertial reaction}}{\text{viscous force}} \tag{1.9}$$

The Reynolds number Re defined in this manner is one of the most important parameters in hydromechanics. Very small Reynolds numbers characterize by definition flows in which viscous forces dominate and inertial reactions are negligible, as for instance, in seepage flows in the validity range of Darcy's law, or in creeping flow around spheres in the validity range of Stokes' law (Reynolds numbers smaller than one). On the other hand, very high Reynolds numbers characterize flows in which finally the viscous forces become negligibly small in comparison to the inertial reactions, as for instance in fully turbulent pipe or channel flows (see Fig. 1.1).

Corresponding dimensional considerations for a fluid element with inertial reaction under the influence of gravity forces yield as a further characteristic number the force ratio

$$(\rho, v, L, g) \longrightarrow \mathrm{Fr} \equiv \frac{v}{\sqrt{gL}} = \frac{\text{inertial reaction}}{\text{gravity force}} \tag{1.10}$$

The Froude number Fr plays a dominating role in hydraulic modelling and is always of importance whenever the influence of gravity is important, as for instance in all flows with a free surface. Negligible influence of gravity in comparison to inertial reactions corresponds to very large Froude numbers, whereas very small Froude numbers correspond to an overwhelming influence of gravity forces.

The role of the Froude number can be well illustrated by considering the flow in an open channel of depth h and of mean velocity v, and by chosing h as the reference length for the definition of the Froude number, as is customary in open channel hydraulics. The denominator $\sqrt{gh}$ represents in this case the propagational speed of a gravity wave in shallow water, and the Froude number can now be interpreted also as the ratio of the mean flow velocity to this wave propagation speed. Accordingly, open channel flows can be classified into "sub-critical flows" at Froude numbers smaller than one, and "super-critical flows" ($\mathrm{Fr} > 1$), in which gravity waves can no longer be propagated in the upstream direction. Retaining the same Froude number is the most important similarity requirement in modelling open channel flows.

Considering a fluid element with inertial reactions under the influence of forces due to surface tension κ, one obtains as a further characteristic number the Weber number We:

$$(\rho, v, L, \kappa) \longrightarrow \mathrm{We} \equiv \frac{v}{\sqrt{\kappa/(\rho L)}} = \frac{\text{inertial reaction}}{\text{force due to surface tension}} \tag{1.11}$$

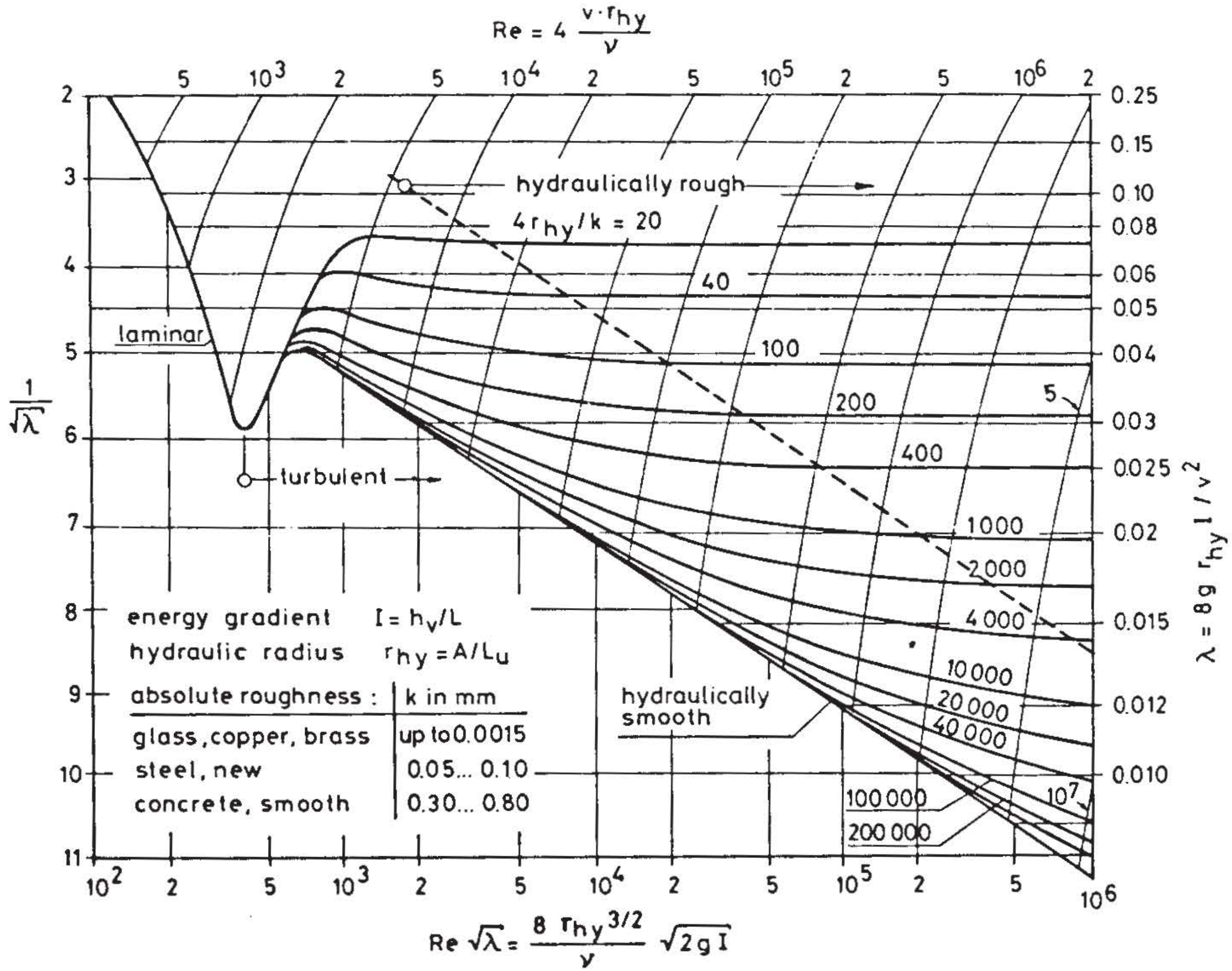

Fig. 1.1: Resistance Diagram for Pipe and Open Channel Flow According to Nikuradse and Moody (Rouse 1950)

Forces due to surface tension can usually be neglected in hydraulic problems in nature (there the Weber numbers are always large), whereas this is not necessarily the case in small scale laboratory models with small water depths.

Another flow parameter characterizing the influence of elastic forces in compressible flows (elasticity modulus E) related to the inertial reaction is the Mach number given by

$$(\rho, v, L, E) \longrightarrow Ma \equiv \frac{v}{\sqrt{E/\rho}} = \frac{\text{inertial reaction}}{\text{force due to elasticity of the fluid}} \qquad (1.12)$$

Considering that in a compressible fluid the denominator $\sqrt{E/\rho}$ represents the speed of propagation of a pressure wave (sound wave), one sees that the Mach number characterizes the ratio of the flow velocity to the speed of propagation of a sound wave. This provides an analogy to the corresponding interpretation of the Froude number, with the aid of which hypersonic flows can be simulated in open channels.

If as a reference the elasticity force of a rigid body is used, one obtains the Cauchy number

$$(\rho, v, L, E_b) \longrightarrow Ca \equiv \frac{v}{\sqrt{E_b/\rho}} = \frac{\text{inertial reaction}}{\text{force due to elasticity of rigid body}} \qquad (1.13)$$

which is of great importance in the treatment of flow-induced vibrations (see Chap. 11).

The densimetric Froude number Fr represents the ratio of inertial forces to the acting gravitational force due to density differences, i.e. buoyancy:

$$Fr_d \equiv \frac{v}{\sqrt{\frac{\Delta\rho}{\rho} g \cdot L}} = \frac{\text{momentum flux}}{\text{buoyancy force}} \qquad (1.14)$$

It can be seen from the definition of Fr_d that the ordinary Froude number Fr corresponds to the special case ($\Delta\rho = \rho$) as it is found in free surface flows. The densimetric Froude number is of importance in the characterization of discharges with buoyancy. The limiting case ($Fr_d \rightarrow \infty$) corresponds to a discharge without buoyancy, driven exclusively by momentum, whereas the other limit ($Fr_d \rightarrow 0$) corresponds to a pure plume. Alternatively, density currents can be also characterized by the Richardson number Ri, which is defined as the ratio of local gradients of density and velocity:

$$Ri = \frac{g}{\rho} \cdot \frac{(\partial\rho/\partial z)}{(\partial u/\partial z)^2} \qquad (1.15)$$

The Richardson number indicates the stability of a stratified flow: large values of the Richardson number correspond to stable stratification. It is to be noted that here also buoyancy- and inertial forces are related to each other, and that with proper choice of the corresponding reference parameters, the Richardson number corresponds to the inverse square of the densimetric Froude number.

There exists a variety of further characteristic numbers of this kind which will be defined in the following chapters as they are needed. All fluid mechanics characteristic numbers given here can also be derived formally from the corresponding equations of motion by transferring these equations into a dimensionless form. Another approach consists in formulating the relevant equations both for the model and for the prototype and then to postulate that all corresponding terms must be in the same ratio to each other (as this was shown above for the force ratios). This requirement leads by inspection of the equations equally to the formulation of the relevant similarity relationships.

In the general case, dynamic similarity requires that all characteristic numbers according to the forces acting must have simultaneously the same values in model and prototype. If, for instance, besides pressure forces and inertial reactions also viscous and gravitational forces are acting, then the geometrical flow number Eu (i.e. for instance a discharge- or resistance coefficient) is dependent not only on the form of the flow boundaries, but also upon the Reynolds- and Froude number:

$$Eu = f\,(\text{shape}, Re, Fr) \tag{1.16}$$

The primary task in determining similarity requirements is therefore the identification of the acting forces which influence the flow process. As soon as these are defined, the similarity laws are found very simply either directly from the fluid mechanics characteristic numbers or with the aid of a dimensional or inspectional analysis.

1.4 Model Laws

1.4.1 Euler Model Law

Flows which are dominated exclusively by pressure forces and inertial reactions and for which viscous and gravitational influences are negligible are characterized by the fact that the geometrical flow parameter Eu is exclusively a function of the shape of the flow boundaries. Therefore, whenever the model is a geometrically similar reproduction of nature, then the geometrical flow parameter assumes a constant value. The magnitude of the Euler number is independent of the absolute values of the model size, the flow velocity, the fluid density or the reference pressure. In this case, there exists no velocity scale; the results can be transferred directly to arbitrary other velocities, dimensions or fluid densities.

Example: Measurements at a nozzle (d = 2 cm) in an air model yield a pressure difference Δp of 4 N/cm^2 at a velocity of 34 m/s. What will be the pressure difference at a geometrically similar water nozzle with a diameter of 10 cm at a velocity of 6 m/s?

For a given geometry there must be

$$(Eu)_m = \frac{v_m}{\sqrt{2\Delta p_m/\rho_m}} = (Eu)_n = \frac{v_n}{\sqrt{2\Delta p_n/\rho_n}} \qquad (1.17)$$

and therefrom

$$\Delta p_n = \Delta p_m \left(\frac{v_n}{v_m}\right)^2 \left(\frac{\rho_n}{\rho_m}\right) = 4 \cdot \left(\frac{6}{45}\right)^2 (800) = 56{,}9\,N/cm^2 \qquad (1.18)$$

This simple example illustrates at the same time that the chosen form of a dimensionless representation (resulting from a dimensional analysis) allows the most simple and comprised representation of results from model experiments and their direct conversion to natural scales.

1.4.2 Reynolds Model Law

In flows with significant viscous effects, the geometrical flow number Eu as well as the Reynolds number Re has to be kept the same in model and prototype. Apart from geometrical similarity, this requires in addition satisfaction of the Reynolds model law:

$$\frac{Re_n}{Re_m} \equiv Re_r = \frac{\rho_r v_r L_r}{\eta_r} = 1 \qquad (1.19)$$

This law is satisfied if the velocity in the model is chosen such that there results

$$v_r = \frac{\eta_r}{\rho_r L_r} \quad \therefore \quad \frac{v_n}{v_m} = \frac{L_m}{L_n} \cdot \frac{\rho_m}{\rho_n} \cdot \frac{\eta_n}{\eta_m} \qquad (1.20)$$

In laboratory experiments making use of the same fluid, there is (ρ_r = 1) and (η_r = 1). In this case, the Reynolds model law reduces to the requirement that the velocity scale must be chosen inversely proportional to the length scale:

$$v_r = \frac{1}{L_r} \therefore \frac{v_n}{v_m} = \frac{L_m}{L_n} \qquad (1.21)$$

This implies that in a small scale model the resulting velocities must be larger than in the prototype. For the derived quantities, the Reynolds model law yields the following relationships:

lengths:	$L_r \equiv \frac{L_n}{L_m}$	
areas:	$A_r = L_r^2$	
velocities:	$v_r = L_r^{-1}$ (for $\rho_r = \eta_r = 1$)	(1.22)
times:	$t_r = \frac{L_r}{v_r} = L_r^2$	
discharges:	$Q_r = v_r \cdot A_r = L_r^1$	

1.4.3 Froude Model Law

For flows under the influence of gravity (e.g. free surface flows) one has to require geometrical similarity and equality of the Froude number Fr in model and prototype. The Froude model law is:

$$Fr_r = \frac{v_r}{\sqrt{g_r L_r}} = 1 \qquad (1.23)$$

This results in the following requirement as for the velocity scale (considering the fact that the gravitational constant g is usually the same in model and prototype, i.e. g_r = 1):

$$v_r = (g_r L_r)^{1/2} = L_r^{1/2} \qquad (1.24)$$

With this relationship one obtains the following scaling rules:

lengths:	$L_r \equiv \frac{L_n}{L_m}$
areas:	$A_r = L_r^2$
velocities:	$v_r = L_r^{1/2}$ (for $g_r = 1$)
times:	$t_r = \frac{L_r}{v_r} = L_r^{1/2}$
discharges:	$Q_r = v_r \cdot A_r = L_r^{5/2}$

(1.25)

1.4.4 Similarity Requirements for Open Channel Flows

In hydraulic engineering usually models with a free water surface are investigated, in which also the influence of viscosity and boundary roughness k is of importance. For open channel flows, a dimensional analysis yields the following similarity relationship:

$$\frac{1}{Eu^2} = \lambda = f\left(\text{shape}; \frac{k}{r_{hy}}; Re; Fr\right) \qquad (1.26)$$

For a geometrically similar model, the geometrical flow parameter (here the loss coefficient) is a function of the relative roughness (k/r_{hy}), the Reynolds number and the Froude number.

The dependence of the friction loss coefficient λ upon the Reynolds number and upon the relative roughness (k/r_{hy}) is given by the Nikuradse-Moody-resistance diagram (Fig. 1.1). This is valid for both model and prototype. Furthermore, open channel flows require similarity according to the Froude criterion.

Simultaneous obedience to the scaling laws of Froude and Reynolds in a small scale model is difficult and requires the use of a model fluid, which satisfies the following relationship:

$$\frac{\eta_r}{\rho_r\, g_r^{1/2}} = L_r^{3/2} \qquad (1.27)$$

If the same fluid is used in model and prototype, then it is obviously not possible to satisfy both the Froude and Reynolds law simultaneously in a small scale model.

In free surface flows, the Froude model law must be followed in order to achieve a geometrically similar reproduction of the water surface (with the exception of stagnant or nearly stagnant water bodies). This means for water models that the Reynolds number of the model is always smaller than that of the prototype:

With $v_r = L_r^{1/2}$ there results $Re_r = L_r^{3/2}$ (1.28)

This relation shows clearly that in a small scale Froudian model viscous forces always have a relatively greater significance than in nature.

This observation is without consequences as long as the flow is in the hydraulically rough region both in nature and in the model, so that a change in the Reynolds number does not cause any change in the frictional loss coefficient (see Fig. 1.1). In river hydraulics, the Reynolds numbers of natural flows can usually be considered to be in the hydraulically rough region (order of magnitude 10^6): This fact is an essential condition for the validity of the conventional discharge equations according to Manning-Gauckler-Strickler or Chezy.

However, often the flow conditions in the small scale model do not fall within the hydraulically rough region. Therefore, the influence of viscosity is not scaled correctly. However, this effect can be compensated by proper choice of a corresponding model roughness, which is not geometrically similar, as is shown schematically in Fig. 1.2: The model roughness is chosen such that one obtains the same frictional loss coefficient λ at the smaller model-Reynolds number as is obtained under prototype conditions. From the relationship

$$\lambda = \frac{h_v}{L} \cdot \frac{8 g r_{hy}}{v^2} = \left(\frac{h_v}{L}\right) \frac{8}{Fr^2} \qquad (1.29)$$

it can be seen that in a Froude model ($Fr_r = 1$) the energy gradient (h_v/L) and hence the surface slope is the same in model and prototype if the frictional loss coefficient is the same in both cases. This requires always that the model must be hydraulically smoother (smaller values of k/r_{hy}) than the prototype. Such a compensation of viscosity and roughness effects can therefore only be realized within a limited range of model scales (see Chap. 2).

By proper choice of the model roughness it is thus possible to simulate correctly the combined effect of viscosity and boundary roughness even in a Froude-scale model, although neither one of the components is simulated correctly by itself. It is therefore possible to obtain proper similarity conditions for water surface and energy gradients in open channel flows. This idea is the essential similarity basis of hydraulic models with gravity and viscosity influences. The adoption to the conditions prevailing in nature ("calibration") is achieved by variation of the model roughness. However, the velocity distributions over the flow cross section are no longer simulated correctly, which can be of importance in investigations of transport and spreading processes (Chap. 4).

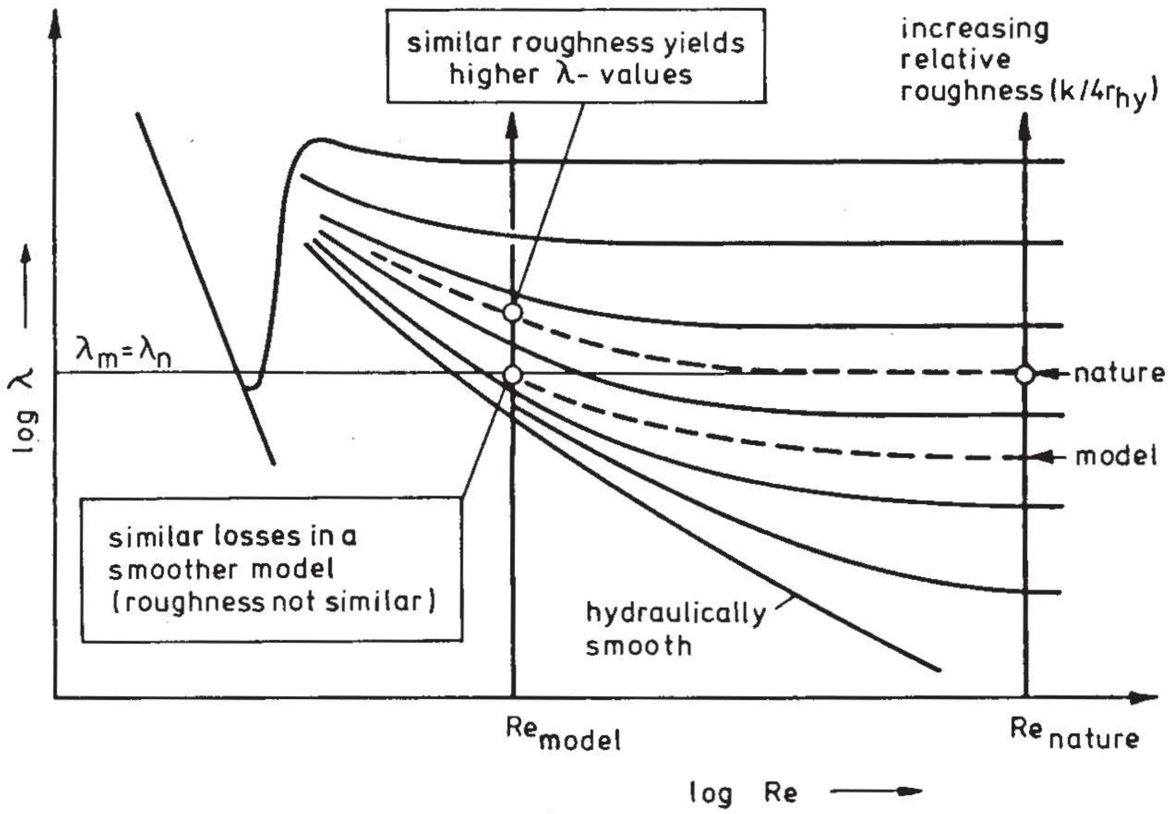

Fig. 1.2: Model Similarity of Energy Losses in Froude Models with Viscous and Roughness Effects

1.4.5 Limitations of Hydraulic Modelling

Hydraulic models must have certain dimensions, which are limited towards the upper end by the prevailing possibilities of the laboratory (space, pumping capacity) and towards the lower end by the similarity conditions.

One lower limit for the model size is given by the improper scaling of viscous effects as described above. From Fig. 1.2 it can be seen that by going to increasingly smaller model dimensions (thus going to smaller Reynolds numbers) finally the resistance curve for hydraulically smooth boundaries would be crossed, which is physically not possible. This dictates certain minimum scales, which conventionally correspond to scale numbers in the order of magnitude up to several hundred. In general, the requirement is that the Reynolds number in the model must always remain large enough to ensure turbulent flow conditions in the model, when the flow in nature is turbulent.

Another limitation is given by the influence of surface tension. Whereas for hydraulic engineering problems the Weber numbers in nature are usually so large that the influence of surface tension can well be neglected, this is not necessarily true in the small scale model. From a comparison of the wave propagation speeds of gravity and capillary waves, one can derive the postulate that a water depth of several centimeters should always be maintained. As a matter of experience, a lower limit of about three centimeters is conventionally used. However, by addition of surface active materials in the model, the surface tension can be drastically reduced and thus the model-Weber number can be increased.

For models of large scale water bodies, the considerations given above frequently require the choice of a smaller scaling number for the vertical lengths than for the horizontal dimensions, i.e. the model is distorted vertically. By doing this, one obtains, for a given horizontal model area, flow cross sections with larger water depths and stronger bottom gradients, which has advantages for flow measurements (see Chap. 2). At the same time, the wall shear stress and therefore the transport capacity of the flow is enlarged, as is often necessary for investigations of sediment transport problems (see Chap. 3). Nevertheless one has always to bear in mind that a model distortion necessarily means deviation from geometrical similarity, so that the model scaling laws apply only approximately to distorted models. The distortion can be compensated by increasing the model roughness such that water levels and discharges - i.e. cross sectional averages - are simulated correctly, but the details of the flow conditions can no longer be modelled correctly. This is shown schematically in Fig. 1.3 for the examples of jet spreading, wave propagation or the flow around a body. These inherent deviations from strict model similarity can lead to drastic differences between the flow conditions appearing in nature and those observed in the model.

The essential limitation for the use of vertically distorted models can be illustrated as follows: In a vertically distorted model, the ratio v_r of the horizontal velocity components is different from the ratio w_r of the vertical velocity components:

$$\frac{v_n}{v_m} \equiv v_r \neq w_r \equiv \frac{w_n}{w_m} \qquad (1.30)$$

This shows immediately that the use of vertically distorted models is only acceptable as long as vertical components can be neglected ($w \approx 0$): i.e. in all those cases in which one can assume with good approximation a hydrostatic pressure distribution in the vertical. For these cases, model laws can be derived which are given in Chapters 2 and 3.

Hydraulic problems are also studied in air models on the basis of the analogy between water and subcritical air flows. This analogy is valid as long as compressibility effects remain negligible in the wind-tunnel. This is the case as long as the Mach number of the wind-tunnel model remains always small in comparison to one. Typical wind-tunnel experiments for hydraulic problems are therefore limited to velocities below 50 m/s.

A severe limitation of model similarity is always reached whenever cavitation

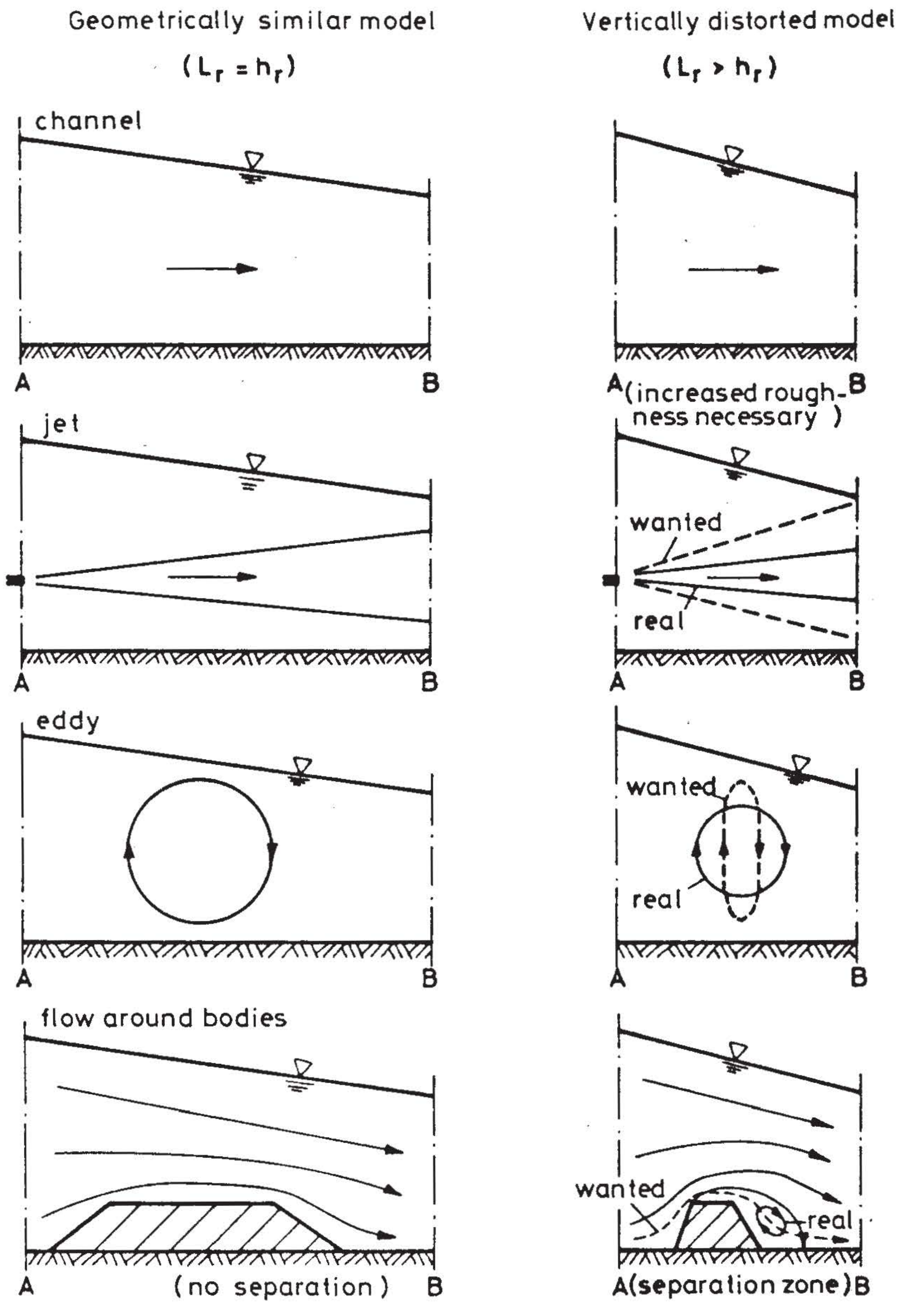

Fig. 1.3: Influence of Vertical Model Distortion upon the Flow Conditions (According to a Personal Communication from G. Abraham, Delft)

effects are observed. Whenever the local pressure in a flow field drops at the local temperature to the boiling point, then water vapour bubbles (cavitation bubbles) are formed, which can cause drastic changes of the flow field (see Chapters 11 and 14.3). The respective boiling point of water provides an absolute limit, which influences the relative velocity- and pressure distributions in model and prototype differently and not to scale. A characteristic flow parameter which relates the boiling pressure p_d to the parameters of the flow is the cavitation number, also called Thoma number:

$$Ka \equiv \frac{p_o - p_d}{\rho_w v^2 / 2} \qquad (1.31)$$

Effects of cavitation are simulated in the model correctly provided that the cavitation number has the same value in both cases. This requires experimental setups in which the reference pressure p_o can be varied - so-called "cavitation tunnels" (see Chapters 11 and 14.3).

1.5 Distinction from Numerical Models

In recent years, numerical models have been applied in some areas of technical hydromechanics with great success (groundwater flows, pipe flows, water levels in open channels, simple jet configurations). In view of the rapid development of computer technology, the question has to be raised whether in future numerical models can play the role of the hydraulic experiment, and how the areas of advantageous application of either type of model can be distinguished.

A comparison of hydraulic and numerical models shows at first glance that both types of model have very much in common (Fig. 1.4). Each must be preceded by a conceptional phase, in which the physical relationships are identified which are to be simulated by the model. The effort in constructing a hydraulic laboratory model is comparable to the effort of working out a solution scheme for the numerical model. Both methods must make use of certain simplifications and approximations and have to be adapted to the real situation in nature - in the one case by adapting the empirical coefficients, in the other by changing the model roughness. The main and principal difference between the two methods consists in the fact, however, that a numerical model requires the formulation of equations which describe the flow field, whereas it is sufficient for the hydraulic model to identify the acting forces and from these to formulate similarity parameters.

If one is faced with the decision to solve a problem either by means of a hydraulic model or else with the aid of a numerical model, one has to consider a variety of aspects as criteria in the decision process (Fig. 1.5). The con-

STEP	HYDRAULIC MODEL	NUMERICAL MODEL
1	Definition of the problem, identification of the essential acting forces	
2	Formulation of similarity requirements	Formulation of set of equations
3	Formulation of boundary conditions	
4	Construction of a model	Development of a numerical solution scheme
5	"Adaption": calibration of the model (variation of roughness or the like)	(variation of coefficients)
6	Measurement ⟶ Solution	Calculation ⟶ Solution
7	Optimization of the solution according to problem formulation (model geometry variations)	(variation of input data)
8	Transfer of results from model to nature and examination by field measurements	

Fig. 1.4: Application of Hydraulic and Numerical Models to Hydraulic Engineering Problems

sideration of principal limitations may exclude a priori the one or the other type of model for certain problems. It is furthermore of great importance for the decision which degree of accuracy or resolution is required from the model. Also essential is the question of simplicity and economics of the models, i.e. time and cost considerations. The greater flexibility of numerical models is often compensated by the more convincing intuitive power of the hydraulic model. For the credibility of a model it is important to know on the one hand, which experiences are already available with similar types of models, and on the other hand to know the extent of possible feedback between nature and model. It is of crucial importance to know how well and reliably a model can be verified by means of prototype data. Of decisive importance is finally the expected prognostic capability of the model. All these criteria must be considered anew for every new application; no ready-made generally applicable recipes can be offered for the decision process.

The most important role in the decision making process is played undoubtedly by the limiting factors of either type of model. The limitations given in Fig. 1.6 inherent to hydraulic and numerical models show that the limiting factors

Decision criteria
Principal limiting factors
Required accuracy
Simplicity
Cost and time requirements
Flexibility
Intuitive power
Credibility
Feedback to nature (calibration possibilities)
Prognostic capabilities

Fig. 1.5: Hydraulic or Numerical Model

are of entirely different nature in the two cases. Hydraulic models are limited on the one hand by model size, by the discharge and the energy head of the flow, i.e. by laboratory space and pumping capacity, the extension of which is usually mainly a matter of cost. The other principal limitation is given by the similarity laws, which must be followed in the hydraulic model. The essential limitation for the application of the hydraulic model experiment is the fact that only a limited number of processes can be simulated to scale. This limitation does not exist in numerical models. Here the limitations are given by storage capacity and computational speed, which in future can certainly be considerably increased. The decisive limitation is here the fact that for the majority of flow processes of interest in hydraulic engineering no closed system of equations can be formulated. This has the severe consequence that at the present state of research one can not attribute general prognostic capabilities to numerical models for turbulent flows, but must see these limited to certain classes of problems and otherwise see them at present to be rather of reproducing character.

Furthermore, there exist a number of practical limitations. Due to the fact that hydraulic models are usually tested in a laboratory with the same fluid as in nature, i.e. water, there result certain requirements as to the minimum model scale due to the model laws. With these requirements and with the maximum feasible model size, one obtains limits for the extent of an area which can be modeled correctly in a hydraulic laboratory. As a hint, the maximum area for scale models can be taken to be of the order of about 10 km in nature, for vertically distorted models about 100 km. On the other hand, numerical models experience limitations due to the simplifications in the equations and due to the availability of empirical coefficients. Another practical limitation is furthermore given by the resolution of the model, which is determined by the choice of the grid size for the solution scheme. This means that the numerical model is limited in space towards the lower end of the scale, whereas the hydraulic model is limited towards the upper end (maximum extension). Therefore numerical models are usually more suitable for simulation of large scale flow processes, whereas the hydraulic model is more suitable for investigations of local flow configurations. Such a separation of the areas of application has been applied successfully already in some areas: For instance in the calculation of pipe networks, where the entire system is calculated by a numerical model, whereas local loss coefficients or discharge coefficients are still determined from hydraulic model tests.

HYDRAULIC MODEL	NUMERICAL MODEL
Principal limitations	
Model size (laboratory)	Storage capacity
Discharge (pumping capacity)	Computational speed
Energy head (pumping capacity)	Incomplete set of equations:
Model laws	Turbulence hypothesis
Practical limitations	
Minimum model scale	In simplified set of equations:
(Surface tension, viscosity,	- accuracy of assumed relationships
roughness)	- availability of coefficients
Model size	Space and time resolution
(upper limitation)	(lower limitation)
Measuring methods and data	Numerical stability and
collection	convergence of the solution scheme
Availability of boundary- and	Availability of boundary- and
initial conditions	initial conditions

Fig. 1.6: Limiting Factors

1.6 Classification of Hydraulic Modelling

A classification of hydraulic modelling can follow generally the type of water body or structure to be modeled. This reflects the basic structure of this book.

Furthermore, the various types of models can be grouped according to the similarity laws which they must obey. For instance, river and tidal models with a fixed bed must always follow Froude's model law, whereas viscous and roughness effects must be compensated in the manner described above (Chapters 2 and 6). For river and tidal models with a movable bed, the similarity requirements for sediment transport must be satisfied in addition (Chapters 3 and 7), and the simulation of transport processes requires furthermore the proper reproduction of density currents, turbulent dispersion processes and reactions and exchange processes (Chapters 4 and 8).

Lake and reservoir models are generally characterized by small flow velocities and a practically undisturbed water surface. It is therefore justified in many cases to neglect correct simulation according to Froude's law. However, in lakes and reservoirs density currents are often dominating, which are characterized by the densimetric Froude number Fr_d (Chap. 5).

Type of model	Hydraulic model	Numerical model
River and tidal models with fixed bed	Local problems, complex geometry	Large scale problems, simple geometry
River and tidal models with movable bed	Bed load transport, erosion and deposition problems	Suspended load transport (bed load transport for very simple geometry)
River and tidal models for transport processes	Near-field problems	Far-field problems
Lake and reservoir models	Detailed questions, fundamental experiments	Mainly used
Harbour and coastal models	Mainly used	Wave pattern for simple geometry
Models of hydraulic structures:		
- discharge characteristics	Complex geometry	Simple geometry
- energy dissipation	Complex geometry	Simple geometry
- erosion	Necessary	
- flow forces	Complex geometry	Simple geometry
- vibrations	Necessary	
- cavitation	Necessary	
Pipe flow models	Local problems, complex geometry (sediment transport)	Mainly used
Groundwater models	Detailed questions	Mainly used

Fig. 1.7: The Role of Hydraulic and Numerical Models

In harbour and coastal models usually wave patterns and wave forces are investigated, which require simulation according to Froude's model law. On the other hand, viscous effects and therefore the Reynolds number requirement can often safely be neglected (Chap. 9).

In models of hydraulic structures, the influence of viscosity and roughness can usually be neglected because of the relatively short length of the model. For structures with free surface flows, therefore, Froude's model law is important (Chap. 10), whereas flows in closed systems and pipelines (where gravity exerts no influence upon the flow) follow the simple Euler similarity criterion (Chap. 12). For the experimental determination of flow forces, flow-induced vibrations and cavitation phenomena, a number of additional similarity requirements have to be satisfied (Chap. 11).

The flow velocities in groundwater models are usually so small that inertial reactions can be safely neglected. In creeping flows ($Re < 1$), the flow field is determined by the ratio of viscous and gravity forces, whereas the conventional fluid mechanics characteristic numbers are of no significance (Chap. 13).

The various types of models are listed in Fig. 1.7. Furthermore, the corresponding areas of application of hydraulic and numerical models are indicated, although by necessity in a grossly simplifying fashion. A detailed discussion is contained in the corresponding chapters of the book.