

# Transverse Mixing of Stratified Flows in Porous Media

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**SUMMARY** The paper describes a systematic experimental and numerical investigation which explores the transverse vertical mixing across an initially sharp density interface between water containing salt (or high contaminant concentration) and fresh water in a porous medium under nearly horizontal flow conditions. The transversal spreading tendency due to the pore matrix is decreasing with increasing density difference due to gravity action. For a sand of uniform grain size and for a sand with a pronounced grain size distribution, systematic experiments and numerical calculations have been carried out in order to quantify the effects of density differences upon the transverse dispersion coefficient. It was found that in a uniform sand the mixing characteristics are essentially unaffected, whereas the sand mixture exhibits a significant density effect. From an analysis, an approximate relationship for the dispersion coefficient has been derived. The results indicate that density effects on transverse dispersion may be even more pronounced in natural aquifers with smaller velocities and larger inhomogeneities.

## 1 INTRODUCTION

Stably stratified nearly horizontal groundwater flows may be encountered in the vicinity of pollution sources of high concentrations, in regions with salt water - fresh water interfaces, or in the near field of underground cooling water discharges. The question how far density stratifications prevail in porous media flows depends upon the magnitude of the dispersive fluxes across the density interface transverse to the flow direction.

A systematic experimental and numerical investigation has been conducted on the elementary flow configuration of two parallel streams in the horizontal direction with a density difference and an initially sharp interface, as shown in Fig. 1. The

chosen flow configuration resembles the experiments on lateral dispersion by Harleman and Rumer (1963). Under well defined boundary conditions, the transverse dispersive mixing processes at the interface and the influence upon the flow field were studied experimentally. The experiments were performed using fresh water above salt water in a laboratory tank filled with sand with a length of nearly 12 m, a height of 0.50 m and a width of 0.125 m. Vertical salt concentration distributions were measured at a cross section located 11.55 m downstream of the inlet by conductivity measurements.

For varying boundary conditions, test series were run both with a sand A of nearly uniform grain size (mean diameter 0.57 mm) and a sand B with a pronounced grain size distribution (mean diameter 3.5 mm). The height of the tank was chosen such that it was considerably larger than the expected maximum mixing width.

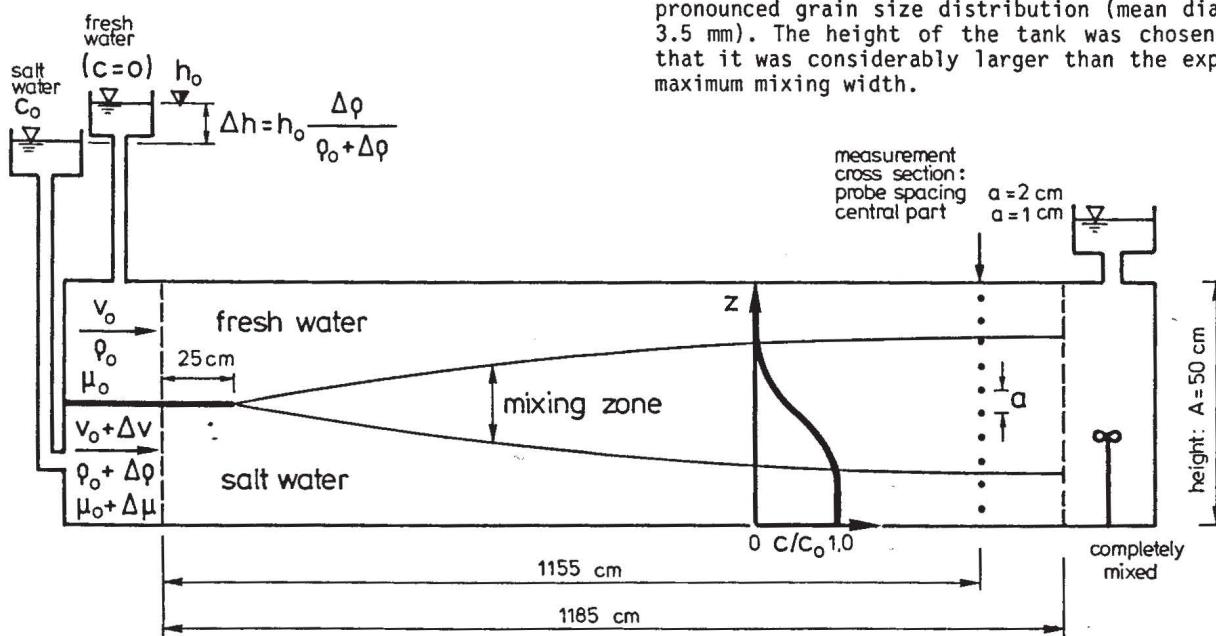


Figure 1 Flow configuration

The spreading tendency due to the porous matrix is comparatively large for the tracer case (no density differences) and is decreasing with increasing density difference due to the action of gravity. Since the salt content changes both the density and the viscosity of the water, the Darcy coefficient  $k_f$  (m/s) is also changing for a given permeability, which has a marked influence upon the flow pattern in the tank.

In the equations of motion, the convective transport as described by Darcy's law in its general form implicitly contains the effect of changing fluid properties (density and viscosity). On the other hand, the dispersive transport as described by Fick's law does not reflect the effects of varying fluid properties. The basic aim of this study is therefore to explore qualitatively and quantitatively the effects of density differences upon the transverse dispersion coefficient in stably stratified groundwater flows.

## 2 BASIC EQUATIONS

For sufficiently small Reynolds numbers, the flow in a saturated porous medium is described by Darcy's law in its general form (Bear, 1972)

$$\vec{v}_f = -\frac{k_0}{\mu} (\text{grad } p - \rho \vec{g}) \quad (1)$$

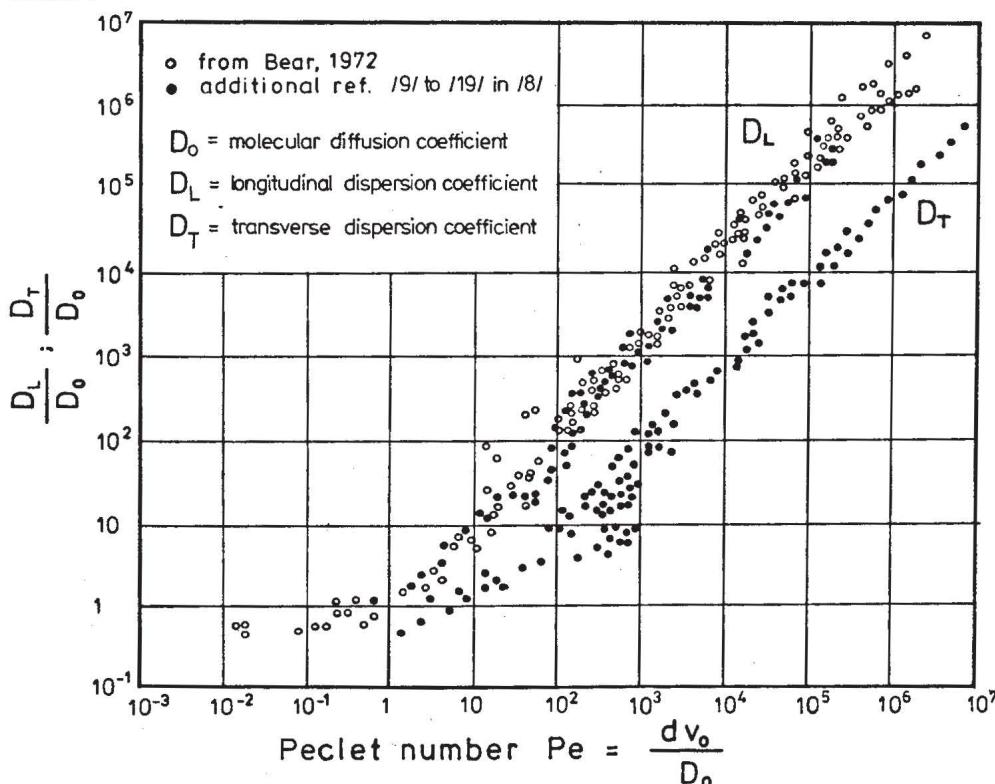
and by the mass conservation law for an element fixed in space, which is given for steady conditions by

$$\text{div}(\varphi \vec{v}_f) = 0 \quad (2)$$

The transport equation, described in general form by Bachmat and Bear (1964), is given for the investigated flow configuration in the following form

$$\text{div} \left( c \frac{\vec{v}_f}{n_e} \right) - \text{div}(D \text{grad } c) = 0 \quad (3)$$

In this equation,  $D$  represents the dispersion tensor.



If the fluid properties  $\varphi$  and  $\mu$  are given as functions of the concentration  $c$ , then this set of three equations can be solved for the three unknowns velocity  $v_f$ , pressure  $p$  and concentration  $c$  for given boundary conditions.

A numerical model for solving this set of differential equations has been developed, using a finite difference scheme. In solving for the flow field, an equivalent fresh water head (Frind, 1982) has been introduced. The main characteristic of the transport model is a weighting function for the differences according to Spalding (1972). The numerical model has been tested against analytical solutions. It allows calculations of the flow and concentration distributions for the investigated configuration under arbitrary boundary conditions.

The dispersion due to the porous matrix has been investigated theoretically and experimentally by many authors (Bear, 1972; Fried, Combarous 1971). From a dimensional analysis, one obtains the following relationship for the dispersion coefficients in a homogeneous, isotropic medium of uniform grain size  $d$ :

$$\frac{D_{L,T}}{D_0} = f_{1,2} \left( \frac{v_0 d}{D_0}, \frac{D_0}{v} \right) \quad (4)$$

Literature data from numerous experimental investigations of this relationship for uniform sands are shown in Fig. 2. These relationships indicate that the dispersion coefficient is primarily dependent upon the Peclet number. The parameter  $D/v$  (inverse Schmidt number), which characterizes the relative role of molecular diffusion, is of secondary importance only and can be neglected for Peclet numbers above 1 to 10.

The functional relationship Eq.(4) can be alternatively also expressed in terms of a Reynolds number instead of the Peclet number, with the definition  $(Re = v_0 d / \nu)$ . As long as our considerations are restricted to salt water - fresh water mixing, the inverse Schmidt number remains approximately constant at  $10^{-3}$ , and hence the Reynolds number will be simply a multiple of the corresponding Peclet number.

If the range of extremely small velocities is excluded, then the observed dispersion coefficients can be described for ( $Pe > 10$ ) with good approximation by

$$\frac{D_{L,T}}{D_0} = \alpha_{1,2} \left( \frac{v_0 d}{D_0} \right)^{\beta_{1,2}} \quad (5)$$

with the exponents  $\beta_1$  and  $\beta_2$  being very close to one (Bear, 1972).

Figure 2 Dispersion coefficients: Experimental data

### 3 EFFECT OF STRATIFICATION UPON TRANSVERSE DISPERSION

Essentially, a density stratification introduces additional buoyancy forces due to the density differences in the flow. According to Josselin de Jong (1969), the resulting vertical velocity component due to buoyancy because of density differences between an elementary volume and its surroundings is given by

$$v_s \propto k_f \frac{\Delta\varphi}{\varphi} = k_f \frac{(\partial\varphi/\partial z) \Delta z}{\varphi} \quad (6)$$

The second expression represents the velocity for an elementary volume in a linearly stably stratified surrounding if it is shifted in vertical direction by a length  $\Delta z$ . In a uniform sand,  $\Delta z$  can be chosen as the main grain diameter  $d$ . If this induced velocity is related to the mean flow velocity ( $v_0 = -k_f I$ ), then a dimensionless parameter is obtained which describes the relative importance of the density stratification and hence has to be added as a third independent parameter to the dimensional relationship given by Eq.(4). Alternatively, the dispersion coefficient under stratified conditions can be directly related to the tracer case as given by Eq.(4) and Fig. 2 in the following form:

$$\frac{D_T(c)}{D_{T0}} = f \left( \frac{(\partial\varphi/\partial z) \Delta z}{\varphi I} \right) \quad (7)$$

In order to find the functional form of this relationship, Rinnert (1983) has developed the following conceptional model:

On the microscopic scale, the pores are represented by a network of cylindrical pipes in various directions, which macroscopically exhibit homogeneous and isotropic characteristics. Expressing the laminar flow through a pore of a given inclination angle  $\psi$  and then integrating the vertical flow components over all inclination angles results in an expression for the mean vertical flow. Assuming that the transverse dispersion coefficients can be considered proportional to the corresponding vertical flow rates, one arrives at the relationship (Spitz, Kobus, 1984)

$$\frac{D_T(c)}{D_{T0}} = \left( 1 - \frac{(\partial\varphi/\partial z) \Delta z}{\varphi I} \right)^{-1} \quad (8)$$

This expression contains the ratio of gravity effects to flow resistance effects as defined in Eq.(7). It is seen, that an increase in density difference causes a decrease in the dispersion coefficient, and at a given density difference this decrease is the more pronounced the smaller the flow velocity and hence the piezometric head gradient  $I$  is.

Looking at the order of magnitude of the various factors in this relationship makes clear that a density gradient has only a small effect on lateral dispersion in a uniform grain sand. This is also evident from a numerical calculation and from the experiments. However, in nonuniform porous media or in natural aquifers with large scale inhomogeneities, it is not sufficient to refer to the mean grain diameter as the only length scale. In all such cases of undetermined reference length, one can use as a first approximation a global relationship of the form

$$\frac{D_T(c)}{D_{T0}} = \left( 1 + \frac{\Delta\varphi}{\varphi I} \right)^{-1} \quad (9)$$

	Sand A	Sand B
$d_{50}$ in mm	0,57	3,5
$\alpha_T$ in mm	$3 \times 10^{-2}$	1,35
$\alpha_L$ in mm	1	200
$\alpha_L/\alpha_T$	66	148
$n_e$	0,398	0,4
$k_f$ in m/s	$2,1 \times 10^{-3}$	$5,1 \times 10^{-3}$

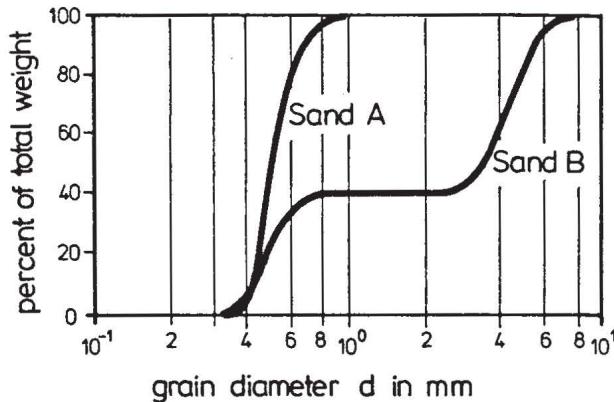


Figure 3 Characteristics of the sand material

### 4 EXPERIMENTAL RESULTS

The characteristics of the two types of sand used in the experiments are given in Fig. 3, which shows that sand A has a nearly uniform size distribution, whereas sand B is markedly graded.

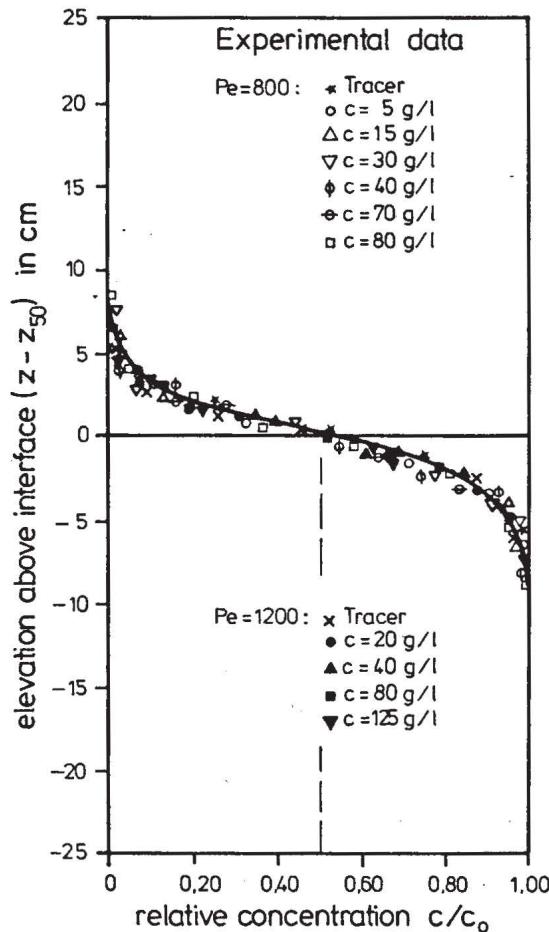


Figure 4 Concentration profiles for the uniform sand A

Concentration measurements were performed by extracting probes of 1 ml at different elevations in the vertical measurement cross section through small tubes of 1 mm in diameter and then measuring the probe conductivity.

Initial salt concentrations ranged from zero to maximum values of 146 g/l. The piezometric head gradient was varied between 0.3 and 0.03, which corresponds to a range of filter velocities of  $1.5 \cdot 10^{-2}$  to  $0.8 \cdot 10^3$  m/s.

Fig. 4 shows the measured concentration profiles for the uniform sand A. In the experiments, the elevation of the "interface" as given by the 50% value varied by about + 5 cm due to the experimental boundary conditions and to the change of Darcy coefficient  $k_f$  with the fluid properties  $\varphi$  and  $\mu$ . This lateral shift of the profiles was eliminated in Fig. 4 by referring all measurements to their corresponding reference 50% value.

The experiments in Fig. 4 show no significant differences in the width of the mixing zone. This holds true for the experiments at a Peclet number of  $Pe = 1200$  as well as additional experiments at a lower velocity ( $Pe = 800$ ).

The nonuniform sand B showed a marked decrease of the mixing width with increasing concentrations (Fig. 5). It is evident that these effects are more pronounced in the experimental series at smaller velocities ( $Pe = 1450$ ) than at the larger velocities ( $Pe = 2900$ ).

In order to verify Eq.(9), the transverse dispersion coefficient is determined for each salt concentration by relating the experimentally determined concentration profile to the numerical

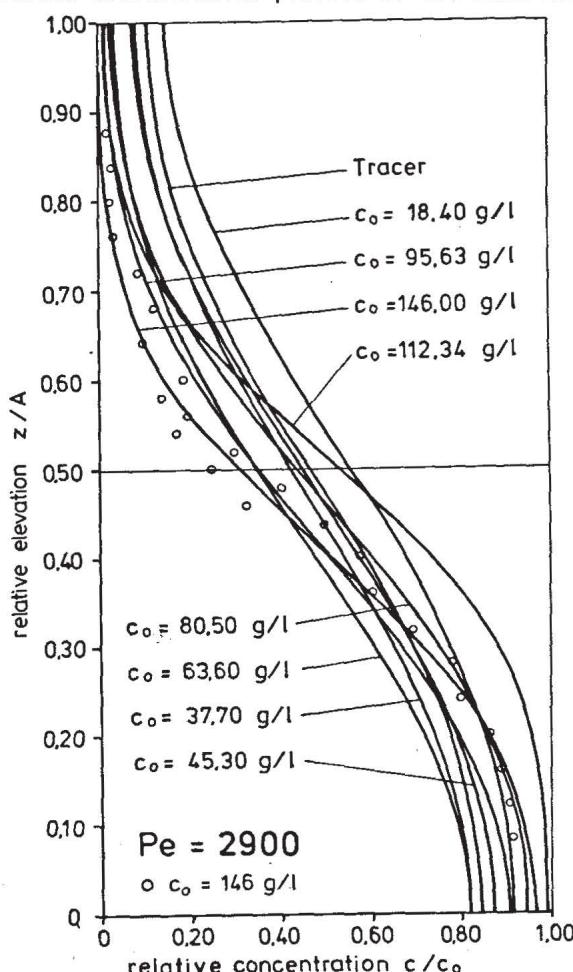


Figure 5 Concentration profiles for the nonuniform sand B

solution of the differential equations with the experimental boundary conditions. It could be shown in this way that the transverse dispersion coefficient can be determined in good approximation by relating the experimental data to the analytical solution for the tracer case as given by Harleman and Rumer (1963). The concentration profiles always show a normal distribution, which can be described by the distance between the 16% and 84% values corresponding to twice the standard deviation  $\sigma$ , which is related to the dispersion coefficient by

$$\sigma = \sqrt{2D_T s/v_0} \quad (10)$$

It follows therefore that the ratio of the dispersion coefficients with and without density difference can be derived directly from the concentration profiles according to

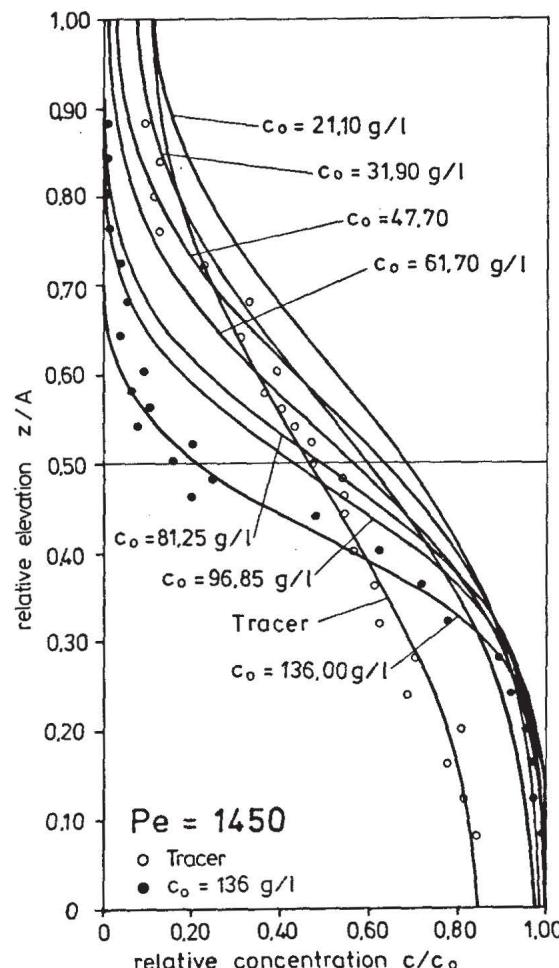
$$\frac{D_T(c)}{D_{T_0}} = \left( \frac{\sigma(c)}{\sigma_0} \right)^2 \quad (11)$$

Fig. 6 shows that the main characteristic of the functional relationship is properly described by Eq.(9) although there are differences evident at lower velocities, which are due to the simplifications.

## 5 CONCLUSIONS

The present investigations yield a number of conclusions with regard to the effect of density differences upon transverse mixing:

- In stably stratified flows, density differences affect a symmetrical reduction of the mixing width at the interface.
- The observed density effects are more pronounced in regions of smaller flow velocities.
- For salt water - fresh water interfaces, density



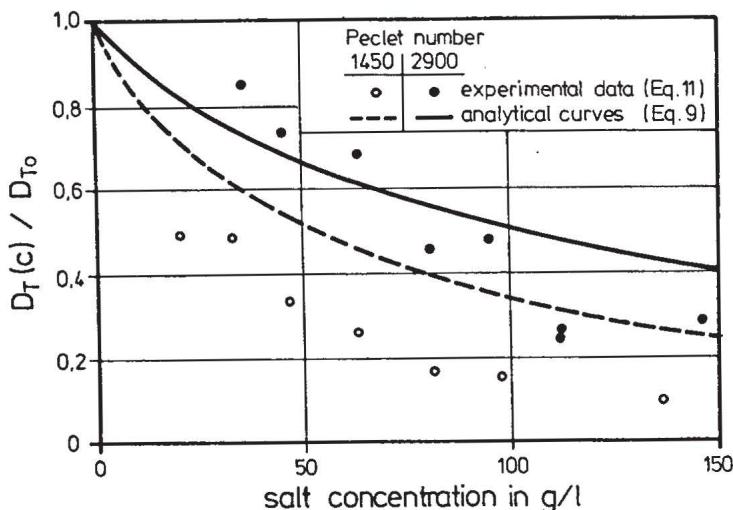


Figure 6 Comparison between analytical and experimental dispersion coefficients

differences are invariably linked with viscosity differences. Viscosity differences mainly affect the position of the interface (50% line), which is moved downward as compared to the tracer case. In uniform sands, the downward displacement of the interface due to viscosity differences is generally negligible.

- For mechanical dispersion in the porous matrix of a uniform sand, the changes due to density differences remain within a few percent. The flow around the individual grain shows marked effects only in situations with extremely large local density gradients.
- Laboratory experiments with a sand mixture, which come more closely to the conditions of a natural aquifer, show a significant density effect. The mixing intensity is decreasing with increasing salt concentration. For salt concentrations of 100 g/l, the mixing width is reduced to about one half.
- These investigations indicate that the effects of density differences on transverse dispersion may be even more pronounced in natural aquifers with larger inhomogeneities.
- For engineering purposes, the global reduction in mixing width can be estimated with the help of Eq.(9). However, the interaction of inhomogeneities and density differences needs further research.

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