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EFFECT OF BUOYANCY ON THE FLOW AND TEMPERATURE FIELD NEAR INJECTION WELLS

(Subject B.d.)

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SYNOPSIS

The injection of heated water into a ground water stream of constant velocity is studied. The heated water is discharged through a fully penetrating injection well into a confined aquifer of constant thickness. For steady state conditions and under the assumption that heat conduction and dispersion in the aquifer and through the confining layers are negligible in comparison to the convective heat transport, it is shown that the flow and temperature field is dependent on the relative infiltration rate and on the relative buoyancy of well discharge. First results of a steady-state three-dimensional numerical model are given.

RESUME

L' injection d'eau chauffèe dans un écoulement d' eau souterraine à vitesse constante est étudié. L'eau chauffée dans un aquifère phréatique d' épaisseur constante á l' aide d' un puits d'injection pénétrant toute l'épaisseur. Dans un état stationnaire ou suppose que la conduction de chaleur et la dispersion dans l' aquifère et à trovers l' imperméable couche superficielle peuvent être négliquées on comparaison du transport convective de chaleur. En ce cas il est montré que la distribution de la vitesse et de la température dépend du débit relative d' infiltration et de la sous-pression hydrostatique relative de l' eau chauffée. Premiers résultats d' un modèle numérique tridimensionelle sont présentés.

1. STATEMENT OF PROBLEM

In densely populated areas, the accumulation of cooling water discharge into aquifers can cause considerable temperature changes [1] . Such deviations from the natural temperature of the ground water and the soil may cause ecological changes or problems when the water is used for supply purposes.

In the present paper, the flow and temperature field in the vicinity of a heated water discharge is studied by means of numerical methods. The influence of the various relevant parameters upon the flow and temperature field is investigated systematically under simplified conditions. Figure 1 shows the flow configuration of the study. Warm water is discharged through a fully penetrating injection well into a confined aquifer of constant thickness m containing a ground water stream of constant velocity u_o . The warm water discharge Q has a temperature difference $\Delta\theta$ above the ambient ground water temperature, which effects both the density and the viscosity of the fluid. The primary purpose of the study is to determine the resulting flow and temperature field under steady conditions, i.e. after very long injection times.

2. DIMENSIONAL ANALYSIS

For an isolated confined aquifer, in which the heat transport through the upper and lower confining layers is zero, a dimensional analysis yields five parameters, which characterise the flow and temperature field:

flow field
temperature field

$$= f\left(\frac{Q}{u_o m}, \frac{\Delta\rho g}{u_o \frac{\rho_o \nu}{k_o}}, \frac{\nu_1}{\nu_o}, \frac{\Delta\rho}{\rho_o} \cdot \frac{g \frac{k_o}{\nu_o} m}{D}, \frac{n \beta c_w}{\rho' c'} \right) \qquad (1)$$

These parameters can be interpreted as follows:

$\frac{Q}{u_o m^2}$ relative infiltration rate.

$\frac{\Delta\rho g}{u_o \frac{\rho_o \nu}{k_o}}$ relative buoyancy, i.e. ratio between the buoyancy force due to the temperature-dependent density differences and frictional force due to the porous matrix.

$\frac{\Delta\rho}{\rho} \frac{\frac{k_o}{\nu_o} m}{D}$ Rayleigh - number: ratio between buoyancy-induced convective and dispersive vertical transport.

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[1] Balke, K.-D.: "Der thermische Einfluß besiedelter Gebiete auf das Grundwasser dargestellt am Beispiel der Stadt Köln". gwf-Wasser/abwasser 115 (1974), H. 3, pp. 117-124.

$\frac{n_e \rho_o c_w}{\rho' c'}$ relative specific heat: ratio between the heat storage capacity of the water and of the water-grain mixture. This corresponds to the ratio of the celerity of a heat front to the transport velocity of a neutral tracer.

$\frac{\nu_1}{\nu_o}$ ratio of kinematic viscosities. This ratio corresponds to the inverse of the Darcy-coefficient k_f of the warm water flow and the reference flow.

For a given reference temperature, there exists a unique relationship between the temperature-dependent density variation $\Delta\rho$ and viscosity variation $\Delta\nu$. The following considerations are based on a reference temperature of 1o°C, which is typical for central European climate conditions. In this case the influence of this parameter is implicitly described by the buoyancy parameter.

In the above considerations the time-dependence of the heat exchange between water and porous matrix was not considered. Its shown in chapter 4, that for natural ground water flows the temperature of the porous matrix can be put equal to the temperature of the surrounding water with very good approximation.

For a steady-state flow temperature field and with simplifying assumption that the diffusive and dispersive heat transport can be neglected in comparison to the convective transport, the problem is reduced to two parameters:

$$\begin{matrix}\text{flow field} \\ \text{temperature field}\end{matrix} = f\left(\frac{Q}{u_o m}, \frac{\Delta\rho g}{u_o \frac{\rho_o \nu}{k_o}}\right) \qquad (2)$$

Thus, the resulting flow field is completely characterised by the relative discharge and the relative buoyancy. In Figure 2, the expected flow configurations are shown in this two-parameter field for the limiting conditions of either predominant or negligible relative discharge or buoyancy flux.

3. BASIC EQUATIONS OF THE FLOW AND TEMPERATURE FIELD

The flow field under steady conditions is described, by use of the continuity equation and Darcy's law by the following relationship [2] :

$$\frac{1}{\rho}\left[\nabla \frac{k_o}{\nu} \nabla p - g \frac{\partial}{\partial z}\left(\frac{k_o}{\nu}\rho\right)\right] = 0 \qquad (3)$$

By considering the heat energy budget of an elementary volume, one obtains the equation for the heat transport in a porous medium [3] :

$$\frac{\rho_w c_w}{\rho' c'} \vec{v}_f \nabla\theta - \nabla D \nabla\theta + \frac{\partial\theta}{\partial t} = 0 \qquad (4)$$

The first term in this equation describes the convective transport, the second gives the dispersive transport and heat conduction, and the last represents the heat storage. If the dispersive heat transport can be neglected, then eq. (4) is reduced to the form

$$\frac{\rho_w c_w}{\rho' c'} \vec{v}_f \nabla\theta + \frac{\partial\theta}{\partial t} = 0 \qquad (5)$$

4. ONE DIMENSIONAL HEAT EXCHANGE CALCULATIONS

Eqs. (4) and (5) are valid only under the condition, that the temperature of the water and of the porous matrix are equal at all points, which implies that the heat exchange between porous matrix occurs sufficiently fast. For a slow heat transfer between water and porous matrix, one has to expect an additional mixing effect. In order to check the magnitude of this effect, a one-dimensional numerical model was constructed which considers convective heat transport as well as heat exchange rate between porous matrix and water. For this purpose, separate heat budgets must be formulated for the water and for the porous matrix, which leads to the following equations for the time dependent temperature of each phase:

water:
$$\frac{\vec{v}_f}{n}\frac{\partial\theta_w}{\partial x} + \frac{\partial\theta_w}{\partial t} + \frac{\alpha}{n\rho_w c_w}(\theta_w - \theta_k) = 0$$

porous matrix:
$$\frac{\alpha}{(1-n)\rho_k c_k}(\theta_w - \theta_k) = \frac{\partial\theta_k}{\partial t}.$$

Both equations are coupled by the heat exchange coefficient α^* (W/m^3 K), which defines the heat transfer rate between the water and the porous matrix and corresponds to the product of the classical heat transfer coefficient α [W/m^2 K] and the specific inner surface area [m^2/m^3] . Eqs. (6) and (7) have been solved simultaneously for the following initial and boundary conditions:

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[2] de Wiest, J.M. (Ed.): "Flow through Porous Media", Academic Press, New York and London, 1969.

$$\theta_w(x,t=0)=0 \qquad v_f(x,t)=v_o$$
$$\theta_k(x,t=0)=0 \qquad \theta_w(x=0,t)=\Delta\theta_o \sin(\frac{t}{T}) \qquad (6)$$

Figure 3 shows the result of these calculations in dimensionless form, expressed in terms of the relative heat transfer coefficient as parameter. It is evident that the heat exchange is irrelevant for the temperature distribution as long as the heat exchange coefficient has values larger than $= 5\cdot 1o^3$. In this range, the calculated temperature distribution is invariant and identical with the solution of the equation (5). For natural ground water flows, the heat exchange coefficients exhibit values which are considerably larger: for a typical wave length of the sinusoidal temperature variation of 1 day to 1 year and a typical effective grain diameter of 1 mm, one obtains heat exchange parameters in the order of magnitude of $2\cdot 1o^4$ to $7\cdot 1o^6$.

It is to be noted, that one obtains a significant dumping of the temperature amplitudes for small values of the heat exchange parameter, whereas for extremely small values and finally for $\varphi = 0$ (which corresponds to the case of no heat transfer, i.e. thermally isolated porous matrix) the temperature curve becomes again sinusoidal with the initial amplitude, but with a considerably lager wave length. This limiting case corresponds to the temperature curve for convective transport in a free water body.

5. SIMPLIFIED THREE-DIMENSIONAL STEADY - STATE MODEL

On the basis of equation 5 for the description of the temperature field, a first simplified steady - state finite - difference model was constructed. Since under the given conditions the temperature along a stream line is constant, one can separate the flow regions of the injection water and of the cold ground water, respectively. For each of these regions, the pressure and velocity field can be calculated separately, once the position of the interface is known. This position is determined by iteration for the condition that the pressure on either side of the interface must be equal; with numerical position corrections according to the calculated pressure differences. This procedure has the tremendous advantage in comparison to the direct solution of equation 5 that it avoids numerical dispersion.

The purpose of this model is to investigate the quantitative influence of the relative discharge and the relative buoyancy upon the flow and temperature field, and also to serve

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[3] Stallmann, R.V.: "Computation of ground water velocity from temperature data", U.S. Geol. Surv. Water Supply Pap., 1544 H, pp. 36-46, 196o.

as a reference for further models, which consider also heat conduction and dispersion. Figure 4 shows a first result of test runs, which yield the position of the interface for a given set of parameters. It is seen that buoyancy effects are quite pronounced even for small temperature differences.

6. CONCLUSIONS

The injection of heated water into the ground water stream in an isolated confined aquifer can be described by five parameters. It is shown, that for natural ground water flows it can be safely assumed that the porous matrix and the surrounding water exhibit the same temperature at any time. For steady conditions, i.e. after very long injection times, the resulting flow fields is seen to depend upon the relative infiltration rate and the relative buoyancy force. A first simplified finite difference model in three dimensions allows calculation of the flow and temperature field under neglegtion of heat conduction and dispersion effects.

List of Symbols

$c';c_k;c_w$	specific heat of the water-grain mixture; grain, water [J kg^{-1} K^{-1}]
D	dispersion coefficient (mechanical dispersion and heat conduction) [m^2/s]
g	acceleration due to gravity [ms^{-2}]
k	specific permeability [m^2]
k_f	Darcy-permeability [ms^{-1}]
m	thickness of the aquifer [m]
n	porosity [-]
p	pressure [Nm^{-2}]
Q	warm water infiltration rate [m^3 s^{-1}]
Ra	Rayleigh-number [-]
t ,T	time; wave length of a sinusoidal temperature [s]
u_o	reference velocity [m s^{-1}]
v_f	filter velocity [m s^{-1}]
α	heat transfer coefficient [Wm^{-2} K^{-1}]
α^*	heat exchange coefficient between water and porous matrix [Wm-3 K^{-1}]
$\theta',\theta_k,\theta_w$	temperature of water-grain mixture; grain; water [K]
ν,ν_0,ν_1	kinematic viscosity of water, cold water, warm water [m^2 s^{-1}]
$\rho',\rho_0,\rho_k,\rho_w$	density of the water-grain mixture, cold water, grain, water [kg m^{-3}]
$\Delta\rho$	difference in density of cold and warm water [kg m^{-3}]

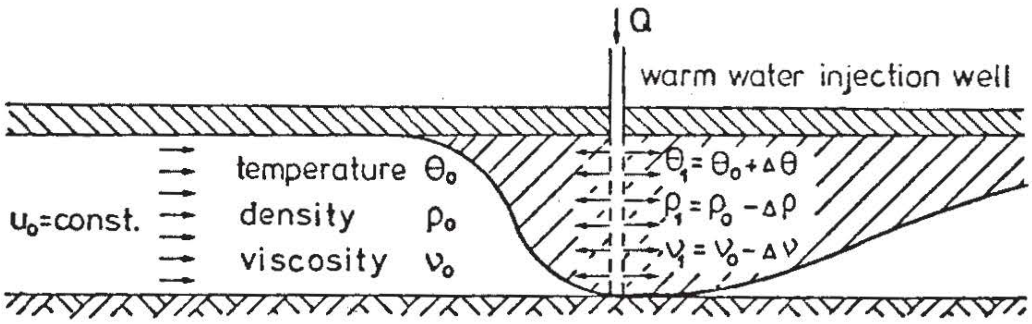

Fig. 1: flow configuration
configuration d' écoulement

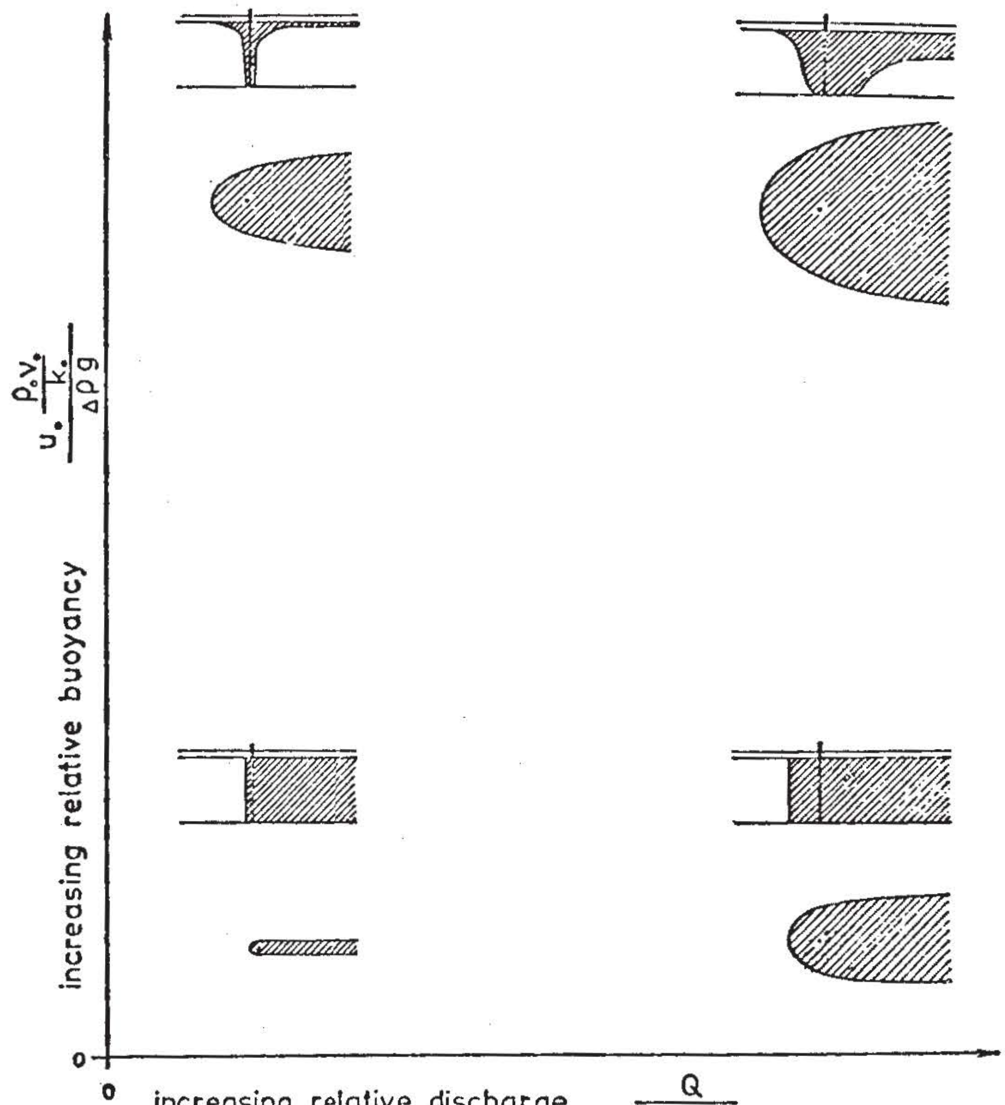

Fig. 2: flow pattern as a function of relative discharge and buoyancy
configuration d' écoulement dépendant du débit et de la sous-pression hydrostatique relative de l'eau chauffée

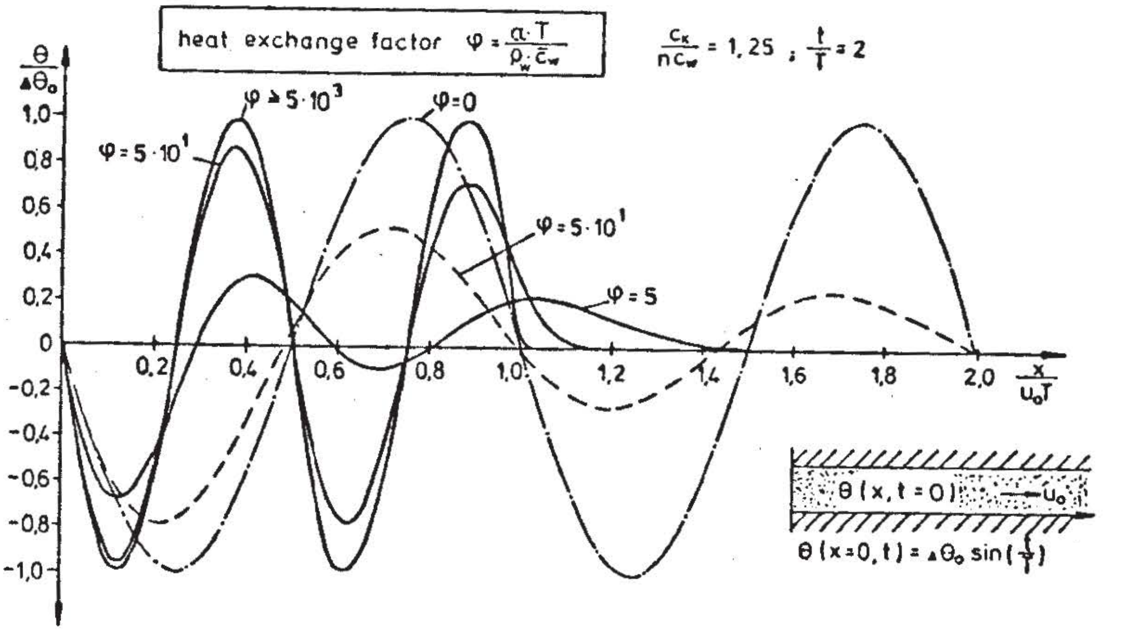

Fig. 3: temperature curves for various heat exchange factors
graphique de temperature pour les divers facteurs d' échange de chaleur

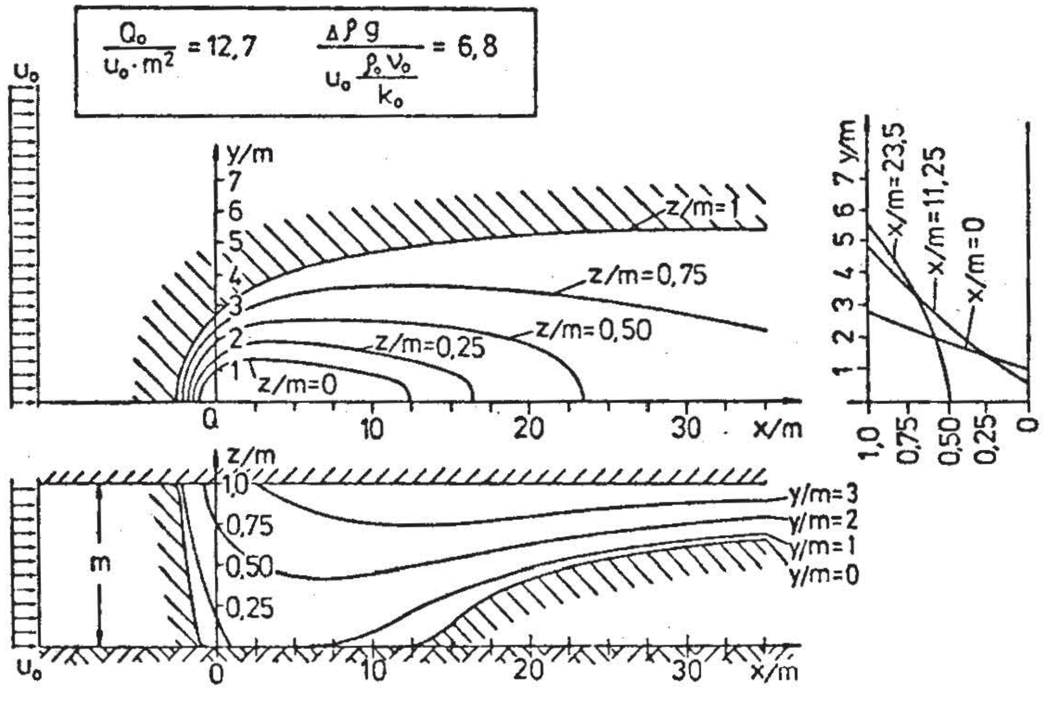

Fig. 4: position of interface for example situation
position de l' interface dans une situation exemplaire