Systematic underestimation of the thrust of vaults among some builders of the 19th century

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ABSTRACT: In building manuals of the 18th and 19th century, we can often read either that vaults don't exert any outward thrust at all, or that their thrust can be easily absorbed by very simple means. Although the theories which are formulated to support these statements are not correct, they have been largely applied even after having been refuted. In case-studies of the works of three builders of vault constructions – in their time highly recognized namely for their technical knowledge, especially in the field of masonry structures and vaults –, some approaches to the problem of thrust will be shown which might appear curious respect to today's knowledge. The understanding of these approaches, however, is first of all useful to explain the structural solutions arising from them. In a more general view, their analysis may be useful to gain a deeper understanding of the problems that arose from the application of the theories by the builders in the 19th century for the realization of their innovative vault constructions.

1 INTRODUCTION

In the design of vaulted structures, the horizontal thrust has been since ever a main concern, and its reduction or even elimination a key issue, for the sake of gaining safety and reducing building costs. About how the thrust of vaults could be even totally avoided, there are several more or less reasonable ideas which appear here and there, in different ages throughout the history of building and theory of structure. One well-known idea, for example, is that of "monolithic" vaults which due to a perfect coherence of the material act as slabs and therefore are presumed not to exert any thrust. We can read about this idea in D'Espie (1754), Moller (1832-34), Breymann's most popular building manual (1849), and much other technical literature.

More sophisticated appears the concept of reducing the horizontal action through the layout of the structure, for instance, in the terms formulated by Rondelet: "The thrust … depends nearly always from the manner in which the vaults are constructed" (1812-14: III-380). In the following, some approaches following this idea shall be discussed.

The following considerations are not aiming to clarify this matter from a structural point of view. Instead, it is more philologically oriented, attempting to identify and describe some conceptual ideas and construction principles which were developed by authors and builders in a certain period, and which have been put in practice. Therefore, it might be a premise to the understanding of particular solutions in the design of some vaulted structures, and their motivation. That is, complementary to the structural analysis and the anamnesis of existing historical structures. To start, we may assist to a harsh quarrel between engineers and architects.

2 A BUILDING FREEZE IN THE WORLD'S HIGHEST BUILDING

The so-called "Mole Antonelliana", a landmark of Turin (Italy), was constructed in the years 1863-1889 (Fig. 1). The fate of the construction was rather turbulent: the original destination as a synagogue was abandoned before it was completed, the owners changed, as did also the purpose of the building several times until now – the only constant use is that of the sight-seeing platform on top of the cupola. The building carries the name of its architect, Alessandro Antonelli (1798-1888). As it was completed, it was the highest building in the world (163.35 m), until it was outmatched in height just some months later by the Tour Eiffel in Paris; but still today it remains the world's highest masonry construction. It is a fascinating example of an extremely light-weight, wide spanned vaulted structure in masonry.
This enormous achievement was due to a construction system developed by Antonelli, which we may call revolutionary. This "Antonellian System" consists in a rational skeleton system made of slender piers, flat arches (so-called "piattabande") as horizontal elements, and very flat and lightweight vaults, and it is developed by transferring the innovations made in iron structures to masonry (Rosso 1977). The systematic use of iron rods makes it possible to consider this the invention of reinforced masonry. Another adoption from contemporary iron structures are the surface structures Antonelli conceived in some structural elements.

In the "Mole", the lower storeys are built in this skeleton system, as well as the great hall. Above this hall, a dome with a square plan spans 27 m – it springs at a height of 45 m and its summit reaches 82 m. On top, a glazed lantern containing sightseeing platforms develops into an amazingly thin and tall spire. The dome is constructed with a double shell and again is designed as a skeleton structure, with meridian and horizontal ribs. The shells are only 12 cm thick, and also the skeleton members are extremely slender.

Both for its structural and architectural concepts, this building must be considered as one of the outstanding masterpieces in the history of architecture.

Figure 1. The "Mole Antonelliana" in Turin, built 1863-89, is still today the world's highest masonry structure (163 m); the great vault spans 27 m.

The safety of this breathtakingly daring structure has been subject to controversial discussions which led to strengthening measures introduced during the 20th century that unfortunately have very much disfigured the building, as it happened also, in a less brutal manner, to the other "opus magnum" of Antonelli’s, the cupola of San Gaudenzio in Novara. But the first serious "attack" to the structural concept of the Mole came already during its construction, in the years 1869-1875.

In that time the dome had been constructed up to about 9 m underneath its vertex, when the building came to a stop. This was first of all due to financial problems of the owner, but in this context also questions about the safety of the structure and the correctness of its design were raised.

During a long polemic public discussion, several expertises on the structural concept were carried out, some of which were directly connected to proposals of alternative roof structures. The whole process has been accurately studied and described (Rosso 1977); in this place we may focus only on the culminating moment of the dispute.

In 1873, the civil engineers Tatti and Clericetti were called to take out an expertise on the structure of the vault. Their approach was to consider the main ribs as independent two-dimensional arches,
which they analyzed by means of graphical statics (Fig. 2); and their conclusion was that the great vault was not sufficiently stable regarding the horizontal thrust, and therefore should be demolished and substituted by an iron structure. This approach to the analysis is disadvantageous as far as it does not take into account the spatial behavior of the structure, which in fact in this case is highly relevant (Nascé et al. 1989).

However, Tatti and Clericetti had used a drawing of the vault which had been supplied by Antonelli himself, but which turned out to differ from the actual structure: it represented a less steep profile of the dome (from an earlier design stage), with only the measures inserted correctly. Therefore, when Antonelli replied, defending his structure, he could use this lever to question the whole expertise, as he did in an open letter, claiming that the radius of curvature of the vault's profile (75 m instead of 32, as shown in the drawing) has enormous consequences for the thrust. And although in fact it is very difficult to establish the radius of the dome on site, the counter-attack by the engineers, saying that Antonelli himself had given them the wrong drawing, was too weak because Antonelli could claim that if they would have taken out a serious analysis rigorously studying the building on site, they would surely have noticed the difference, and that this "huge mistake could have been avoided by just conferring with the author of the structure" (Rosso 1977).

Without a thorough analysis of the documentation, we obviously cannot fully clarify the motivations of Antonelli's shrewd move – the only certain thing is that he flatly refused to present any of his own calculations of the vault. But still, we must assume that Antonelli, regarding the structural design of his daring masterpiece, must have known exactly what he was doing, and therefore may attempt to clarify the theoretical background that could have determined his view of the structural behavior, and his design solution.

Traces could be sought it the current technical knowledge as it was formulated in the contemporary technical literature, and in the way this literature was interpreted. But this might become more clear in another case, some decades earlier, where the same problem had to be treated – namely the possibility of reducing as far as possible the thrust of complex vaulted structures –, and where the designer has exposed his motivations.

3 THE RE-INVENTION OF THE MEDIAEVAL "HALLENKIRCHE" AND ITS INTERPRETATION AS THRUST-ABSORBING CONSTRUCTION SYSTEM

In the context of "Gothic Revival" and the efforts to complete the Cathedral of Cologne as emblematic architectural testimony of the new-born German nation, in the years 1824-31 the architect J. C. v. Lassaulx built a new parish church at Treis, near Koblenz, which was the first Neo-Gothic church in Germany. The architect, who in his time was a prominent scholar in the field of mediaeval architecture, in this project demonstrated the feasibility of Gothic architecture in modern building, and in particular recovered the mediaeval vaulting technique, building the vaults without formwork (Wendland 2003, 2008).

The principal aim of this architect was to develop modern architecture from construction and design principles of mediaeval architecture. He was particularly fond of the scheme of the "hall church", the typology with three naves of equal height which is very common in late-mediaeval German churches – he used this scheme in his own church projects, and also illustrated its advantages in publications as well as 1845 in the Congrès Archéologique at Lille (Wendland 2008). In fact, he considered as a main advantage of this typology that any horizontal thrust from the vaulting system could be avoided.

Lassaulx' approach, well explained in his own publications, is partly based on stabilizing the arcades by additional load, partly by reducing the weight of the vaults using extremely light masonry material, and partly by reducing the thrust through the shape and construction manner of the vaults. For the last, he frequently refers to Rondelet's 'Art de Bâtir' (he used the 5th edition, 1812-14), a book he calls "outstanding" and the use of which he combines with his recovery of mediaeval architecture.

The loading of the pillars, with the aim of stabilizing them to the asymmetric horizontal load
from vaults of different spans, occurs through heavy walls above the vaults supporting the roof structure. The vault masonry consists of blocks cut from an extremely light volcanic material which Lassaulx had asserted in experiments to be sufficiently strong.

These items are supposed to provide additional security, as Lassaulx believes that his vaulted structure will not exert any horizontal thrust at all:

1. The shape of the vault profile (the pointed Gothic arch) is very advantageous: Lassaulx refers to Rondelet saying that this type of arch doesn't exert any thrust at all.

2. In general, the thrust of vaults is usually overestimated: according to his own observations, in many cases a crack is visible in the summit of the vaults of church naves, which is supposed to indicate that there is no pressure in the summit, excluding therefore the possibility of horizontal forces. Therefore, the vaults can be considered as corbelling from both clerestories to the center. In this statement, Lassaulx seems to connect his observations with one of Rondelet's kinematic schemes (Fig. 7, right). In consequence, the buttresses of gothic churches, as Lassaulx says, serve to stabilize the high slender perimeter walls perforated by larges windows, and not to withstand any vault's action.

3. The direction of the courses in the vault masonry has important influence on the thrust – we can find this statement in Lassaulx' publications for the case of barrel vaults, but the analysis of the masonry texture in Treis revealed that he applied this idea also to cross-vaults in Gothic style (Wendland 2008).

4. In the particular case of the "hall-church", Lassaulx describes a mechanism that excludes any horizontal action, as the thrust of the vault over the central nave would only push upwards the summits of the vaults in the lateral naves, and therefore not be transmitted to the perimeter walls (Lassaulx 1835).

Some of these statements are directly taken from Rondelet's 'Art de Bâtir' (1812-14), which Lassaulx as many of his contemporaries considered a reliable source on structural design – it may therefore be interesting to follow this trace further. But even more interesting appear some other statements, which can be considered as original theoretic position developed upon Rondelet, especially as they have become rather popular in the following decades.

A PECULIAR INTERPRETATION OF THE PRINCIPLE OF STRATEGIC LOADING

More scope of discussing the structural concept is in another "hall church" built by Lassaulx in Vallendar, also near Koblenz (Germany) in the years 1837-41, this time in a style orientated to Romanesque.

The vaulting system consists of domical vaults between semicircular arches that confine the bays of the three naves, all springing on the same level. The span of the central nave is 10.36 m, the summit reaches 16 m. The pillars are 0.96 m thick, the perimeter walls 1.26 m, and there are no buttresses.

Only two items of the structural design should be discussed here. One is the shape of the arches that carry the roof structure and load the columns, for the aim of stabilizing the latter like in Treis. The other one is a sketch showing an earlier design stage, explaining how, according to Lassaulx, the thrust of the vault could be avoided.

The arches over the top of the vaults have the shape of the catenary (Fig. 5). This is exceptional in the work of Lassaulx's, who normally traced arches with circle segments, according to mediaeval design principles, or in some rare cases with other geometrically defined curves. On the other hand it is puzzling as this shape of the arches is not correct if we suppose that shape-optimization was the aim: the thrust line and therefore the ideal shape of arches cut in a wall is not the catenary, but a parabola.
This choice again becomes understandable if we remind that Lassaulx extensively used Rondelet's building manual, where we can read about catenary arches that this curve can be employed with great advantage in arches with large spans, and those that carry heavy load; and that Rondelet himself employed them "with success" in the great arches that support the circular colonnade of the Panthéon Français and for the great vault between the two shells of the dome (Rondelet 1812-14, III-139).

This means, that Rondelet does not reflect the coincidence of arch curve and thrust line, but states this arch shape as absolute, recommending it for uses where this shape in reality is not optimal (Trautz 1998; Wendland 2008). Hence, not just the occurrence of a catenary arc h, but its particular use, i.e. the manner of its application and the context within the structure, proves the relation of the design to Rondelet's treatise. In this case, Lassaulx uses catenary arches according to Rondelet where they have to carry heavy load.

The sketch that illustrates a description of the structural concept of the vault shows a half cross-section of the nave (it belongs to an early stage of the design, 1834) with its masonry texture and some construction details (Fig. 6). Regarding the masonry texture, we note that the joints of the arches are radial only in the upper portion, starting from the 45° inclined radial joint, and underneath the courses are horizontal. Further, we note an iron chain running across the side aisle.

It is surprising to see an iron chain across the rather narrow side aisle, but no anchor for the much wider central aisle, and, in any case in this location above the arch it would not be very effective to contrast the thrust of the vault. But we can understand both the purpose of this chain, and the peculiar masonry pattern, in the light of considerations Lassaulx made upon the role of the lower parts of the vaults for the overall mechanical behaviour (e.g. Lassaulx 1846). According to this, first the lower portion of the vault would not exert any thrust if built with horizontal courses. Second, these parts can contrast the thrust, if they are built as monolithic volumes, because by tending to turn inwards they counter-balance the thrust of the central part. "Thus, the thrust is reduced in double manner, by reducing the size of the actual vault, and by transforming the pushing squinches in counter-acting weights". Apparently, these massive volumes are supposed to be tied together by the iron anchors. These considerations again lead us to Rondelet, although they are further developed by Lassaulx himself and his colleague and friend Georg Moller.

Rondelet's exhaustive building treatise "L'Art de Bâtir" was first published in 1802, a thoroughly revised appeared in 1830; through numerous further editions as well as translations it became the most popular building manual in the 19th Century in Europe. The theory on the mechanical behaviour of vaults contained in this book also became very successful for its own and was adapted by many other authors of technical literature. This theory is based partly on La Hire and the successive scientific achievements (Rondelet makes broad reference to these sources), partly on studies on models performed by the author himself. Rondelet's theory has been recognized as being questionable and even dangerous (e.g. Huerta 2004), we may suppose that the serious flaws are due to the intention of bringing two different approaches together and the attempt of treating a enormously complex matter in simple terms, apt for a manual of general use. In any case, we shall not attempt in this place to clarify the background of this theory, but to outline its main contents, and to interpret this text regarding its reception. Rondelet confirms in his approach the principal role of the "fracture joint" which La Hire used in his model of the mechanical behaviour of arches (Benvenuto 1991), and which is the joint inclined 45° - we have already seen this feature in Lassaulx's sketch (Fig. 6).

As outcome of exhausting "analytic" deductions Rondelet reduces the mechanical behaviour of arches to the balance of the turning moments between the main elements: the abutment solids and the lower and upper halves of the arch, the latter
separated by the 45° inclined joint. The upper solids are characterized as the "pushing part" of the vault, while the lower solids due to their tendency of turning inwards is the "resisting part"; hence, equilibrium is obtained if these parts are in balance.

A central statement is that if the rotation of these parts is counterbalanced, then the thrust of the arch is zero. This is demonstrated in particular by the model of an arch with additional loads applied in the position of the fracture joint (Fig. 7 left): this load helps to counterbalance the action of the "pushing" part of the vault and to obtain equilibrium. The consequence that in this case also the thrust disappears automatically, is obviously wrong: we know that if the sum of the moments is zero, there are still horizontal forces at the support. But this problem passed unobserved; as a matter of fact, the experimental models built by Rondelet were not apt to detect the horizontal forces at their base.

In a next step, this concept is even more simplified, and not to the better: the equilibrium is supposed to be assessed by simply subtracting the horizontal projection of the "pushing" and the "resisting" parts of the vault. For the analysis of composed vaults, the line of the 45° fracture joint is projected in plan, and the surfaces are subtracted. According to this, a cross vault exerts a lot of thrust because the projection of the lower portions is relatively small, but cloister vaults and semispherical domes are supposed not to need any abutment larger than the thickness of their shell, because the surface in plan (!) of the "resisting" part is larger than that of the "pushing" part (Fig. 8).

Regardless to the obvious mistakes and also the problematic relation to the scientific development of mechanics, this theory was widely accepted and used throughout the century. For instance, it is also summarized in the later editions of Breymann's building manuals.

The German architect Georg Moller (1784-1852) published in 1832-34 a series of small volumes on building construction, which were declared as supplements to Rondelet's treatise in the context of its German translation. In this publication, Moller exemplifies some principles of construction and structural design mainly on his own works. In particular, we find considerations about vaults "that exert only vertical pressure" which are derived from Rondelet and are very close to the ideas of Lassaulx we mentioned before – it is most likely to believe that Moller and Lassaulx developed this concept in collaboration, as they had very close contact. Moller also discusses a case of "monolithic" vault where a perfect coherence is supposed to be obtained by a special masonry fabric, but most interesting is his idea of a sail vault (a truncated dome on a square plan) which because of its manner of construction is supposed not to exert any thrust (Fig. 9).

The design of the vault is based on the idea (which Moller shares with Lassaulx and many other authors) that the pressure of a vault acts principally perpendicular to the masonry courses. Therefore, he builds the squinches up to the quarter of the semicircular section (according to the 45° joint in Rondelet's models) as massive volumes with horizontal courses. And he explains that the position of the center of gravity of these volumes would counterbalance the thrust of the central part of the vault; "the tendency of the squinches towards the inside will be compensated by the effort of the central calotte to push outwards; and therefore one must suppose that the entire vault acts only in vertical direction" (Moller 1832-34, v.4).

Basically, this is an attempt of applying the principle of what we may call "strategic loading", which is the original connotation of the French term "tas-de-charge": in some cases it can be advantageous to apply additional loads to particular parts of the vault or the abutment in order to obtain a more beneficial thrust line or a constant pressure on the buttressing system (a fascinating example is the Cathedral of Palma; Roca, 2004). This principle is surely correct and useful. But again, the mistake is to suppose the absence of horizontal forces.
the 19th century, we can state that the contents of the approach was already obsolete at the beginning of manual until 1903. If we remind that this theoretical journal in 1865, it remained part of Breymann's of thrust-free sail vaults was disproved in a technical literature are completely detaching from scientific progress.

Moller's construction is on one hand extremely interesting as it confirms and definitely proves our interpretation of Lassaulx's structural concept (Fig. 6). But beyond that, it turns out to be of high practical relevance, because it was included (the drawings as well as long quotations of the text) into Breymann's building manual, published for the first time in 1849 and in several enlarged and modernized editions until 1903, which was very much used in Germany and had also considerable impact in other European countries – and although Moller's concept of thrust-free sail vaults was disproved in a technical journal in 1865, it remained part of Breymann's manual until 1903. If we remind that this theoretical approach was already obsolete at the beginning of the 19th century, we can state that the contents of the technical literature are completely detaching from the scientific progress.

In the light of the theory developed by Lassaulx and Moller on the base of Rondelet, we can also hazard a guess why in the new construction of the high nave of the Cathedral of Cologne (starting 1844, the vaults of the central aisle were built in the 1860's), which was one of the most daring vault construction of its time, which seems that no calculation of the thrust was taken out. As a matter of fact, these vaults present the same peculiar masonry pattern as in those built by Lassaulx in the near-by Treis: the courses are horizontal, instead of running parallel to the ridge or perpendicular to the cross-ribs as usual. As Lassaulx published his design including the technical details and his ideas about the structural behaviour, and beyond that we know that he was also personally engaged in the efforts of completing the Cathedral of Cologne, it is at least probable that the builders thought that by using this particular layout of the vault masonry they could avoid any thrust. If this is true, it is understandable that they considered superfluous to elaborate a calculation.

5 AN ATTEMPT OF "PHILOLOGICAL" INTERPRETATION OF STRUCTURE

Regarding the "Mole Antonelliana" discussed above, a far more demanding vault construction than even the Cathedral of Cologne, we are convinced that such a "philological" analysis as sketched in the other examples can clarify Antonelli's structural concept and the motivations of his design in detail. That is, on the base of the current technical literature, and, equally important, on the current interpretation of this literature. Anticipating the outcome of such an analysis, in this case we may concentrate just on one little trace: the "parabolic arches" that carry part of the load of the external shell of the great vault, and the external colonnade (n.2 in Fig. 2). These arches (in reality, their shape is traced with several arcs) had serious problems due to their asymmetric load (Rosso 1977).

In the case of Lassaulx's arches in Vallendar (Fig. 5), we identified the catenary arches as a trace leading to Rondelet, because of the way they were used. If we remind Rondelet's lines about the use of catenary arches, and that he himself used them in the "great arches that support the circular colonnade under the dome" of the Panthéon, we find that this exactly matches to our case: Antonelli put the "parabolic arches" in the same place where Rondelet says he used catenary arches, and therefore we may suppose that Rondelet's treatise is a source for the design of this structure. Here we may consider secondary whether the arches are actually parabolic or catenary, in any case their shape is not corresponding to the thrust line according to their loading: it is again the context in which this element is applied that gives way to interpretation. According to Rondelet, the vault of the 'Mole' would surely not exert any thrust, first of all because it consists only of the "resisting part" which tends inwards. The design of the construction and its detailing, in fact, confirms this approach.

The three builders we have called were all highly interested in structure, very well informed of the technical publications of their time, and had an extremely good intuition about how their structures worked. Therefore, remains the question why they attached to a theory that is obviously to the least too simplistic, instead of recurring to the state of the art in mechanical sciences?

One reason could be that the scientific texts required an extremely high degree of mathematical knowledge, which made it difficult to adapt them to particular problems. The other problem is that Mechanics tried to describe the most elementary cases, mainly plane arches, but did not deal with the complex shapes of usual vault typologies – this was pointed out already by an early 19th century author (Friderici 1800). Hence, these builders found themselves left alone by the scientists.
It must be said that Lassaulx's vaults have performed well; the one in Treis has survived earthquakes and even a fire in the roof. Moller's sail vault is sound because the well-bound masonry in the lower portion can take the hoop stresses. And Antonelli was surely right believing in the spatial performance of his vaults, and later developed a remarkable understanding of the performance of complex masonry structures: in his dome at Novara, he was probably the first ever to design a structure exactly according to the flow of forces.

6 CONCLUSIONS

The examples discussed above show how design solutions in historical structures are far from being arbitrary, or developed ad hoc, but that the design is well founded on theory and technical rules. Therefore, peculiar solutions may be interpreted and understood in regard to the applied theory and rules, and the usual ways of their adaptation – by a philological approach as it has been sketched. Since the turn of the 18th to 19th century, these rules were formulated in the technical literature, an innovation linked to the beginning of the education of architects, engineers and craftsmen in public institutions. Apart from the well-studied scientific literature on mechanics, there is the "theoretical-practical" literature treating also the topic of structural behavior and the principles of structural design. This corpus of publications was decisive in the realization of high-rise, wide-spanned, prominent structures, although it is only poorly connected with the scientific progress of its time.

Therefore, for understanding particular design choices, a critical study of this literature can be extremely helpful, complementary to the anamnesis, diagnosis and structural modelling of the building in question on the base of today's knowledge. And due to the high practical relevance, such studies are also complementary to those on the history of mechanics. In this field of study, much work still remains to be done.

As a further conclusion, we may remark that the problem of linking practical design guidelines to scientific innovation is of course not particular to the 19th century. Design guidelines such as building codes, and also the content of teaching, must be as clear as possible, simple in its application, and generally applicable. Here is a principle difference from the way and the dynamics scientifically founded knowledge develops; still, this gap must be kept narrow, which is possible only by a constant effort, through close communication.

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