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Isostatic compensation and continental lithospheric thickness

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Abstract. Lateral density inhomogeneities are a general feature of the continental crust and upper mantle. This suggests the generalization of the interpretation of isostatic response functions by introducing internal loads into the conventional model of an elastic plate flexed under a topographic load. With this modification, analyses of continental response functions favour lithospheric thicknesses larger than the average thickness of the crust. These values also conform with independent estimates of the long-term thickness of the continental lithosphere and are therefore more satisfactory than recent estimates of only a few kilometres based on a pure flexure model.

Key words: Isostasy - Lithospheric thickness

Introduction

The Earth's lithosphere may be defined in several ways. Consequently, the meaning of the term is contingent on whether a definition based on chemical, thermal or mechanical arguments is implied. Here we are only concerned with certain mechanical aspects of the continental lithosphere, in particular with its strength and thickness on geologic time scales. The importance of the mechanical lithosphere was first discussed by Barrell (1914), who defined it as the Earth's elastic shell of long-term mechanical strength and suggested the term asthenosphere for its weaker substratum. An obvious method of inferring the mechanical properties of the lithosphere is based on its flexure in response to superimposed loads. For continental areas, the main results have recently been compiled and discussed by Cochran (1980). Depending on the method of interpretation, they may be subdivided into two categories.

(1) If observations of load-induced flexure are available, direct modelling in the spatial domain is possible. This approach has been applied to features of various characteristic loading times such as isolated topographic loads, sedimentary basins or Pleistocene ice sheets. It was pioneered by Walcott (1970).

(2) The overall mechanical properties of the lithosphere for continent-wide areas may be inferred using the response function technique. This method was introduced by Dorman and Lewis (1970) for investigating isostatic compensation in North America.

The basic assumption of the response function technique is that Bouguer gravity anomalies are linearly related to topography and its compensation. A filter may therefore be estimated which, when convolved with topography, reproduces the observed Bouguer anomaly. In the wave number domain, this corresponds to the determination of the isostatic response function, which may then be interpreted in terms of specific compensation models.

Dorman and Lewis (1972) analyzed the North American response function assuming local compensation. Subsequent re-interpretations of the data, however, were based on lithospheric flexure, which is a regional type of compensation (Walcott, 1976; Banks et al., 1977; McNutt, 1980). The flexure model was also applied to interpreting the Australian isostatic response function (McNutt and Parker, 1978).

The basic differences between local and regional compensation have been discussed by several authors (e.g. Garland, 1979) and will not be repeated here. An attractive feature of flexural compensation clearly is that it provides a simple mechanical model of the compensation process. Interpretations of the isostatic response function of continental areas based on this model have, however, usually resulted in lithospheric thicknesses of only a few kilometres. These values were usually explained by stress relaxation within the lithosphere and taken as representing a long-term "effective lithospheric thickness". They are nevertheless significantly below values inferred from direct modelling of individual loads of comparable age, which usually indicated thicknesses between 35 and 60 km (Cochran, 1980).

In an attempt to explain this discrepancy, some authors argued that the flexure model might be inadequate for interpreting the isostatic response function of whole continents (Forsyth, 1979; Garland, 1979, Cochran, 1980). This is because the model implies that compensation is exclusively attained by the depression of crustal material into mantle material due to flexure of the lithospheric plate in response to an external load. But for mountain ranges, whose pronounced topography and gravity signals govern continental response function estimates, downwarping is unlikely to occur. The only flexural effect may in fact be the upwarping in response to the (negative) eroded loads. The non-flexural character of compensation of mountains is also reflected when considering the major geologic processes accompanying orogenic events, such as overthrusting, magmatic intrusions or metamorphism. If, therefore, mountain ranges are compensated today, a substantial portion of this compensation must be caused by chemical or thermal inhomogeneities in the lower crust or upper mantle, which are unrelated to flexure.

Based on these ideas, a quantitative model of composite compensation is developed in the following section. It is tested by applying it to estimates of the isostatic response function of western North America and Australia. As the results show, both data sets allow lithospheric thicknesses in excess of average crustal thicknesses. The consequences of these findings are briefly discussed in the final section.

Composite compensation

The composite compensation model suggested here is based on the reasonable assumption that it is unrealistic to restrict compensation processes to flexural compensation. The incorporation of non-flexural effects into the flexure model presents some difficulties, however, since a simple physical mechanism is lacking.

On a large scale, crustal thickening in Alpine-Himalayan mountain ranges is primarily due to crustal shortening as a result of continent-continent collision, whereas in Andean-type mountain ranges it occurs mainly as a result of the addition of basaltic magmas to the crust. On a more regional scale, the complications are perhaps best exemplified by the diversified tectonic setting of the western United States. As discussed by Parkiser (1963), changes in crustal thickness across the boundaries of major tectonic provinces of this region usually bear little relation to changes in altitude but reflect lateral density variations in the upper mantle. Within individual provinces, however, the thickness of the crust tends to vary directly with altitude.

Any type of non-flexural compensation effectively represents a mass deficit which is accompanied by a positive buoyancy force and a negative gravity anomaly. Non-flexural compensating masses may therefore be modelled efficiently as internal loads, which are easily incorporated into existing models of flexural deformation.

Lithospheric flexure has usually been analyzed using thin elastic plate models. Even though thin plate theory is an asymptotic theory, its validity for most problems of geophysical interest has recently been demonstrated by Comer (1983) and Wolf (in press 1984)

The equilibrium equation of a deformed thin elastic plate floating on an inviscid and incompressible substratum is (e.g. Jeffreys, 1976)

$$D\nabla^4 w(\mathbf{r}) + (\rho_2 - \rho_0) g\, w(\mathbf{r}) = q(\mathbf{r}), \tag{1}$$

where $w(\mathbf{r})$ is the downward deflection of the plate and $q(\mathbf{r})$ is the downward traction, both applied at point $\mathbf{r}=(x,y)$ of the horizontal plane. ρ_0 denotes the density of the material infilling depressions of the plate and ρ_2 that of the mantle material below the plate. D is the flexural rigidity of the plate and $D=\mu h^3\,[6(1-\nu)]$, where h, μ and ν denote the thickness, shear modulus and Poisson's ratio of the lithosphere, respectively. As usual, g is the acceleration due to gravity. Taking the two-dimensional Fourier transform of (1) yields

$$\hat{w}(\mathbf{k}) = \frac{\hat{q}(\mathbf{k})}{(\rho_2-\rho_0)g + Dk^4}, \tag{2}$$

where $\mathbf{k}$ is the wave vector and $k=|\mathbf{k}|$. The transform $\hat{q}(\mathbf{k})$ of the traction encompasses a contribution caused by the load and a contribution caused by the associated compensation. Under the assumption that the compensation is linearly related to the superimposed load, we therefore have

$$\hat{q}(\mathbf{k}) = [1-\alpha(\mathbf{k})]\,\rho_0 g \hat{t}(\mathbf{k}). \tag{3}$$

ρ_0 is the density of the topography $t(\mathbf{r})$ overlying the (x,y)-plane and is assumed to be equal to the density of the infill. Here α will be called the inhomogeneity parameter and measures that portion of the compensation which is due to internal loads unrelated to flexure. For the following argument it is sufficient to assume that $\alpha(\mathbf{k})=\text{const}$. Substituting for $\hat{q}(\mathbf{k})$ in (2) yields

$$\hat{w}(\mathbf{k}) = (1-\alpha)\frac{\rho_0 g \hat{t}(\mathbf{k})}{(\rho_2-\rho_0)g + Dk^4}. \tag{4}$$

For $\alpha=1$ we therefore have $w(\mathbf{r})\equiv 0$ and compensation is completely non-flexural.

The transform of the gravity effect of an uneven and possibly non-uniform layer of material was given by Parker (1972). In the linear approximation and for uniform layers the Bouguer anomaly in the (x,y)-plane, of the flexural and non-flexural portions of the compensation, reduces to

$$\Delta\hat{g}(\mathbf{k}) = -2\pi G\{[(\rho_1-\rho_0)+(\rho_2-\rho_1)e^{-kd_1}]\,\hat{w}(\mathbf{k}) \\ +\alpha\rho_0 e^{-kd_2}\hat{t}(\mathbf{k})\} \tag{5}$$

Here ρ_1 denotes the crustal density, d_1 the crustal thickness and d_2 the depth of non-flexural compensation. G is Newton's gravitational constant. Substituting for $\hat{w}(\mathbf{k})$ finally yields

$$\phi(k) = \frac{\Delta\hat{g}(\mathbf{k})}{\hat{t}(\mathbf{k})} = -2\pi G\rho_0 \\ \cdot\left[(1-\alpha)g\frac{(\rho_1-\rho_0)+(\rho_2-\rho_1)e^{-kd_1}}{(\rho_2-\rho_0)g+Dk^4} + \alpha e^{-kd_2}\right]. \tag{6}$$

This is the isostatic response function of the composite compensation model. We find, for $d_1=d_2$,

$$\lim_{k\to 0}\phi(k) = -2\pi G\rho_0, \tag{7}$$

which is the gravity effect of a uniform layer of unit thickness. If only non-flexural compensation is permitted, (6) simplifies to

$$\lim_{\alpha\to 1}\phi(k) = -2\pi G\rho_0 e^{-kd_2} \tag{8}$$

This represents the amplitude of the gravity effect of a harmonic density sheet of unit thickness at depth d_2 (linear Airy compensation).

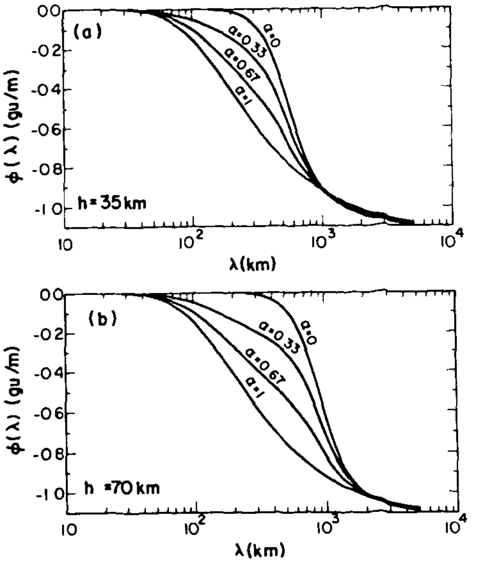

Fig. 1a and b. Theoretical response functions for inhomogeneity parameters $\alpha=0.00$, 0.33, 0.67, 1.00 for lithospheric thickness **a** $h=35$ km and **b** $h=70$ km, respectively. The other parameters are $\mu=0.3\cdot10^{11}$ Nm^{-2}, $\nu=0.25$, $\rho_0=2{,}670$ kg m^{-3}, $\rho_1=2{,}840$ kg m^{-3}, $\rho_2=3{,}270$ kg m^{-3}, $d_1=d_2=32$ km

Results

Figure 1 shows theoretical response functions $\phi(\lambda)=\phi(2\pi/k)$ for two lithospheric thicknesses and several degrees of non-flexural compensation. The other parameters (see Fig. 1) are fairly characteristic of normal continental crust and upper mantle and will serve as convenient standards in the following. As can be seen, the transition from local compensation ($\alpha=1$) to flexural compensation ($\alpha=0$) is accompanied by a substantial decrease in the magnitude of the normalized Bouguer anomaly. The effect is more pronounced for large lithospheric thicknesses. This is to be expected, since uncompensated topography, i.e. infinite flexural rigidity, is associated, by definition, with zero Bouguer anomalies.

In Fig. 2 the observed isostatic response function for western North America (McNutt, 1980) is shown together with two theoretical curves for different lithospheric thicknesses. If the inhomogeneity parameter α is between 0.90 and 0.95, thicknesses between 35 and 70 km are clearly compatible with the data. From what is known about the tectonic history of the western United States, internal loads must be significant there, and lithospheric thicknesses of the order of the thickness of the crust or larger are therefore reasonable.

McNutt's (1980) estimate of less than 5 km for the thickness of the western North American lithosphere was based on the conventional flexure model. Clearly, almost any lithospheric thickness will fit the data, if these indicate conditions close to local compensation. For large plate thicknesses this requires a high degree of non-flexural compensation. If on the other hand, only flexural compensation is permitted, the thickness of the lithosphere must be sufficiently small, such that the flexural model approximates local compensation. Since the major density contrast responsible for the Bouguer anomaly is close to the depth of the Moho, lithospheric thicknesses of only a few kilometres, however, indicate internal inconsistencies of the flexure model.

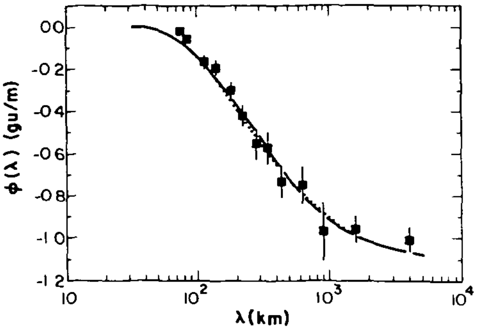

Fig. 2. Observed isostatic response function for western North America after McNutt (1980). Best fitting theoretical response function (in a least squares sense) for $h=35$ km requires $\alpha=0.91$ (solid); $h=70$ km requires $\alpha=0.95$ (dotted). The other parameters are given in the caption of Fig. 1

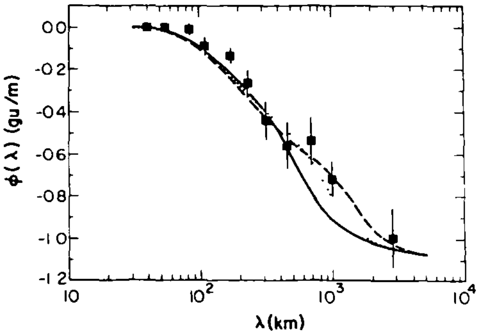

Fig. 3. Observed isostatic response function for Australia after McNutt and Parker (1978). Best fitting theoretical response function (in a least squares sense) for $h=35$ km requires $\alpha=0.63$ (solid); $h=70$ km requires $\alpha=0.66$ (dotted), $h=140$ km requires $\alpha=0.72$ (dashed). The other parameters are given in the caption of Fig. 1

Figure 3 shows a re-interpretation of the isostatic response function for Australia, on the basis of which McNutt and Parker (1978) and Cochran (1980) inferred lithospheric thicknesses of approximately 5 km using the conventional flexure model. If the inhomogeneity parameter α is about 0.65, the data are again consistent with lithospheric thicknesses larger than 35 km. The fit in fact improves further for lithospheric thicknesses in excess of 70 km. This may, however, not be significant, since, for simplicity, the degree of non-flexural compensation has been assumed to be independent of wavelength here.

Conclusions

The discrepancy between estimates of continental lithospheric thickness based on the response function approach and estimates based on conventional modelling of individual loads is obviously due to the inadequacy of the conventional flexure model when analyzing response function estimates for continent-wide areas. If proper allowance is made for internal loads in the theoretical model, lithospheric thicknesses in excess of the normal crustal thickness are, however, clearly consistent with the data. On the other hand, if the observed response function indicates conditions close to local compensation, our improved model is not very sensitive to lithospheric thickness. This is demonstrated in Fig. 2, where thicknesses of both 35 km and 70 km satisfy the data exceedingly well. The response function analysis therefore does not seem to be very suitable for imposing tight bounds on the thickness of the lithosphere.

More promising for that purpose are truly external loads, such as sediments or glacial loads. For Pleistocene ice sheets the characteristic time of the loading event is only of the order of 10^4 a. Post-glacial rebound data have therefore been mainly used for inferring the viscosity stratification of the mantle (e.g. Peltier, 1982). Initial attempts have, however, been made to constrain lithospheric thickness from records of load-induced deformations in the peripheral regions of Pleistocene ice sheets (Peltier, in press 1984).

Non-flexural compensation may also have some bearing on the thickness of the oceanic lithosphere. Suyenaga (1979) presented strong evidence that the warping of the Moho below Hawaii might not be completely of flexural origin but rather be partly due to chemical inhomogeneities below the volcanic island. If this is true, chemical inhomogeneities related to topography must be a fairly general feature of the oceanic lithosphere. So far, the isostatic response function associated with oceanic topography has only been interpreted using the conventional flexure model (e.g. McKenzie and Bowin, 1976; Watts, 1978; Cochran, 1979). The incorporation of non-flexural effects into the model may therefore also require an increase in the thickness of the mechanical lithosphere below oceans. This may explain in part the large differences between the thickness of oceanic lithosphere inferred seismically and that obtained from studies of the flexure associated with seamounts and guyots.

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