CREEP AND FATIGUE ANALYSIS OF REINFORCED CONCRETE STRUCTURES



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ABSTRACT

A computional procedure for the creep and fatigue analysis of reinforced concrete structures exposed to flexure is presented. The fatigue loading effects are modified into an equivalent creep analysis. The analysis is based on the finite element method employing beam elements. The material behavior of steel, concrete and bond between reinforcement and concrete are described as realisticly as possible. For the reinforcing steel the stress-strain behavior as measured in tests or given in codes is assumed. For the stress-strain behavior of concrete in compression under static loading, the proposal by Park/Pauley is taken. The influence of sustained or fatigue loading is taken into account by using the isochrone σ-ε-relationship valid for $t > t_0$ or $N > 1$ respectively; whereby the creep coefficients for both type of loadings are taken from MC90. For sustained and fatigue loading the isochrone bond stress-slip relationship is used. The creep coefficients are taken from MC90. The accuracy of the proposed model is checked by comparing the behavior of some test beams under sustained and fatigue loading with the predictions.

Key-words: creep, fatigue loading, reinforced concrete, bond, nonlinear analysis

1 INTRODUCTION

The purpose of this study is to investigate the structural response of reinforced concrete structures which undergoe sustained or fatigue loading and imposed deformations; and to describe it analytically using the combination of known and widely accepted engineering models.

The utilized finite element model is obtained by dividing the beam longitudinally into a series of finite beam elements whose depth is equal to the depth of the beam. Each element cross section is further subdivided into a number of concrete and steel layers. Geometrical nonlinearities are not considered, while the material nonlinearities are taken into account when updating the material properties and stress-strain states for each load increment or iteration.

Also, the purpose of this study was to investigate the effects of fatigue loading on the response of reinforced concrete structures. When considering the influence of fatigue loading on reinforced concrete, it is not only necessary to consider the effects on the concrete and steel, but also on the bond between these materials. The method of analysis is focusing on utilizing the similarities between fatigue and sustained loading, and on solving the behavior under fatigue loading by an analogous analysis under sustained loading.

The presented analysis concerns the loading within the serviceability range only. The loads are supposed to vary in intensity only without changing the direction, and without variation of amplitude, stress levels and frequencies. More details of the analysis are given in /2/.

2 MATERIAL MODEL

2.1 Sustained Loading

2.1.1 Concrete Model

The ascending branch of the uniaxial stress-strain relationship of plain concrete in compression is assumed in form of a quadratic parabola:

$$\frac{\sigma}{f'_c} = 2 \cdot \left(\frac{\varepsilon}{\varepsilon_0}\right) - \left(\frac{\varepsilon}{\varepsilon_0}\right)^2 \tag{1}$$

with $\varepsilon_0 = 0,85 \cdot (f'_c)^{0,246}$ (2)

f'_c = concrete cylinder strength

The behavior under tension loading is idealized by a bi-linear relationship with the direct tensile strength f_{ct} as peak value.

The effects of sustained loads on the ultimate compressive or tensile strength is taken into account according to /8/ or /9/ respectively:

$$f'_c\,(t = \infty) = 0,8 \cdot f'_c\,(t = 0) \tag{3a}$$

$$f_{ct}\,(t = \infty) = 0,8 \cdot f_{ct}\,(t = 0) \tag{3b}$$

The isochrone for t = ∞ is considered based on the assumption that the strains are increasing with time. Eurocode 2 and MC90 incorporate the effects of creep on the concrete behavior in the serviceability limit state ($\sigma_c \leq 0.5\ f'_c$). A linear relationship is assumed between creep and the stress causing the creep. The principle of superposition is assumed to apply for actions occuring at different ages. Using these assumption, the total strain of concrete subjected to an initial loading at time t_0 with a stress σ_0 and subsequent stress variations $\Delta\sigma(t_i)$ at time t_i may calculated according to Eq. (4), which is taken from Eurocode 2:

$$\varepsilon(t,t_0) = \varepsilon_n(t) + \sigma(t_0)\cdot\left[\frac{1}{E_c(t_0)} + \frac{\phi(t,t_0)}{E_c(28)}\right] + \sum\left\{\Delta\sigma(t_i)\cdot\left[\frac{1}{E_c(t_i)} + \frac{\phi(t,t_i)}{E_c(28)}\right]\right\} \qquad (4)$$

where $\varepsilon_n(t)$ = strain caused by imposed deformation
$E_c(t_0)$ = tangent modulus of elasticity at time t_0
$E_c(28)$ = tangent modulus of elasticity at 28 days
ϕ = creep coefficient

The creep coefficient is calculated according to Eurocode 2 (/4/). The values of the creep coefficient are usually between 1.0 and 4.0. When neglecting strains caused by imposed deformations and stress variation, Eq. (4) can be written as

$$\varepsilon(t) = \frac{\sigma(t_0)}{E} \qquad \text{where} \qquad E = \frac{E_c}{1+\Phi} \qquad (4a)$$

2.1.2 Steel Model

A simple bilinear steel model was emloyed in this study. Strain hardening in the reinforcement is not of importance in the present serviceability load analysis. The stress-strain relationship is thus defined by the initial elastic region.

2.1.3 Bond-Slip Model

The behavior of reinforced concrete beams subjected to creep or fatigue loading is greatly influenced by the bond of reinforcing bars. Flexural actions generally cause slip of reinforcement relative to the surrounding concrete. In the finite element analysis, the portion of the flexural beam between vertical cracks and below the neutral axis is modeled by a concentric tensile test specimen in which a tensile force is applied to a bar embedded in a concrete block.

A local bond stress-slip relationship under generalized loading is proposed by Eligehausen et.al. (/3/). This proposal takes into account the bond damage caused by energy dissipated during the loading and unloading processes. For monotonic loading the relationship between bond stress and slip may be computed according to MC90.

Franke was the first to investigate the influence of sustained loads on the behavior of bond between steel and concrete (/5/). For the slip s(t) at time t the following equation is proposed:

$$s(t) = s_0\cdot[1 + k(t)] \qquad (5)$$

where s_0 = slip at $t = 0$

$$k(t) = \left[(1 + 10\cdot t)^a - 1\right] \qquad (5a)$$

a = slope of the creep function in double logarithmic scale
t = load duration in hours

For normal concrete, Franke suggested a = 0.080, and this value associated with Eq. (5a) is incorporated in MC90. The validity of Eq. (5) is restricted to the ascending branch of the bond stress-slip relationship.

2.2 Fatigue Loading

2.2.1 Concrete Model

Fatigue strength is commonly defined as a fraction of the static strength that can be endured repeatedly for a given number of cycles.

Holmen /6/ shows the decrease of the secant modulus with increased number of load repetitions, but does not give an empirical equation to calculate this behavior. The influence of fatigue loading on the decrease of the modulus of elasticity is shown in Fig. 1 with an approximate relationship proposed by Park /7/. The simplified degradation rule is based on the assumption, that the centerline of hysteresis loops always passes through the common point ($-0.5\ \varepsilon_0$; $-f'_c$), see Fig. 1. The modulus of elasticity is computed according to:

$$E = \frac{E_c(t_0)}{1 + \frac{2 \cdot \varepsilon_p}{\varepsilon_0}} \tag{6}$$

in which ε_p = plastic strain
ε_0 = strain related to $\sigma = f'_c$

An empirical expressions for the maximum concrete strains ε_{max} under fatigue loads with constant amplitude is given in /6/. The strains ε_{max} and ε_p shown in Fig. 1 are related by:

$$\varepsilon_p = \frac{\varepsilon_{max} - \frac{1}{2} \cdot \varepsilon_0 \cdot \frac{\sigma_o}{f'_c}}{\frac{\sigma_o}{f'_c} + 1} \tag{7}$$

The strain ε_{max} depends on the ratio N/N_F (N = number of applied cycles, N_F = number of load cycles for fatigue failure). N_F is a function of σ_u/σ_o (σ_u, σ_o = stress under lower and upper load, respectively).

Substitution of ε_p from Eq. (7) in Eq. (6) yields:

$$E = \frac{E_c}{1 + \Phi} \tag{8}$$

in which

$$\Phi = \frac{2 \cdot \frac{\varepsilon_{max}}{\varepsilon_0} - \frac{\sigma_o}{f'_c}}{\frac{\sigma_o}{f'_c} + 1} \tag{9}$$

The form of Eq. (8) resamples the expression for the effective modulus of elasticity in a creep analysis, with a creep coefficient ϕ instead of the coefficient Φ (compare Eq. (4a)). The fatigue creep coefficient Φ is defined for $N \leq 0.8\ N_F$. It increases with decreasing ratio σ_u/σ_o and increasing ratio σ_o/f'_c and takes values between $0 < \Phi \leq 2$.

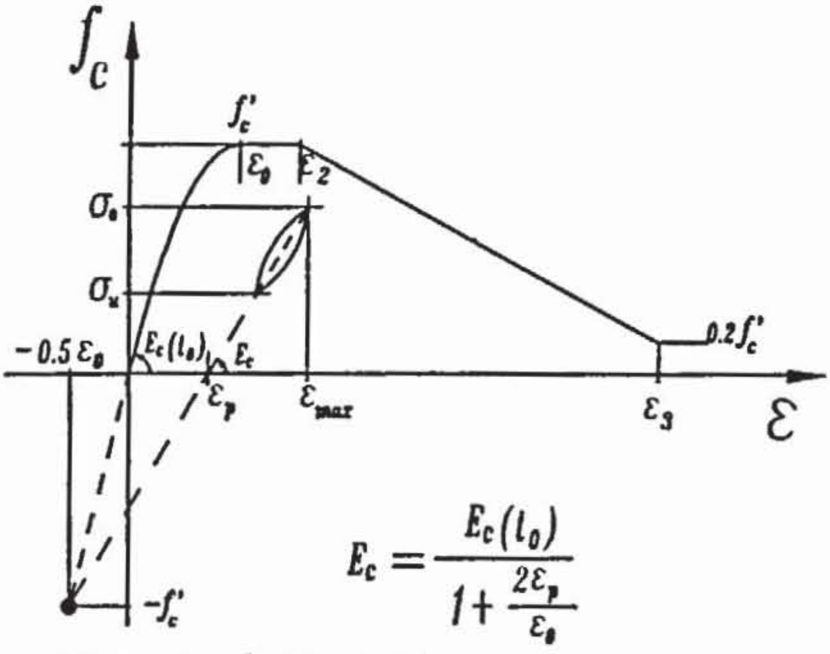

Fig. 1: Simplified degradation rule after Park /7/

2.2.2 Bond Stress-Slip Relationship

The principle of the bond stress-slip relationship due to short-term and sustained loading has been shown in section 2.1.3. For a repeated loading Eq. (5) applies when k(t) is replaced by k(N):

$$k(N) = \left[(1+N)^{0,107} - 1\right] \tag{10}$$

with N = number of load cycles

The validity of Eq. (10) is restricted to the ascending branch of the bond-slip relationship and for values of τ_{max} smaller than the fatigue strength of bond.

2.2.3 Analogous Creep Analysis

The computional procedure for the creep analysis with consideration of the tension stiffening due to bond between steel and concrete is given in the previous section. The degradation of the modulus of elasticity in Eq. (8) is the same as for the effective modulus of elasticity for the creep analysis (compare Eq. (4a)).

In the serviceability limit state, a linear creep function is assumed. The employment of the previous Eq. (4) for the fatigue analysis is possible by substituting the creep coefficient ϕ by the factor Φ, valid for fatigue loading, and by consideration of the mean stress σ_m.

3 COMPARISON WITH TEST RESULTS

3.1 Beams under Sustained Loading

Stevens (/11/) performed tests on 24 pairs of beams with constant dimensions, as shown in Fig. 2, but with different reinforcement. The beams were loaded at about the third points so that the central zone was subjected to a uniform bending moment.

Material properties and loads are listed in Table 1. In order to study the influence of the environment, the beams, denoted A-M, were produced in two sets of 12 identical pairs. One set was tested in the laboratory and the other outside. The present numerical method is employed in the analysis of beams of type J, K, L and M which were all reinforced by high strength (f_y = 550 N/mm²) deformed bars. The characteristics of the beams are listed in Table 1. In his tests, Stevens measured the midspan deflection relative to the ends of the constant moment zone. Each of the results is the average value from a pair of beams. The input data for the calculation are also listed in Table 1.

Beam	Reinforcement	Load kN	f'_c N/mm²	Time in Days	$k(t)$[1]	$\phi(t)$[2]
J	top: 2 ø10	55.2	29.6	28	-	-
	bottom: 2 ø22			390	1.48	2.50
	cover: 25 mm			760	1.63	2.76
				10^5	2.89	3.18
K	top: 2 ø10	28.0	36.7	28	-	-
	bottom: 2 ø16			390	1.48	2.24
	cover: 25 mm			760	1.63	2.48
				10^5	2.89	2.86
L	top: 2 ø10	51.2	27.1	28	-	-
	bottom: 2 ø22			390	1.48	2.61
	cover: 51 mm			760	1.63	2.88
				10^5	2.89	3.33
M	top: 2 ø10	26.2	29.9	28	-	-
	bottom: 2 ø16			390	1.48	2.49
	cover: 51 mm			760	1.63	2.74
				10^5	2.89	3.17

1) after Eq. (5a)
2) after /1/

Table 1: Input data for the calculation

Fig. 3 shows the measured and calculated deflections of the beams. Note that the deflections given by Stevens for the time t = ∞ are extrapolated and not measured.

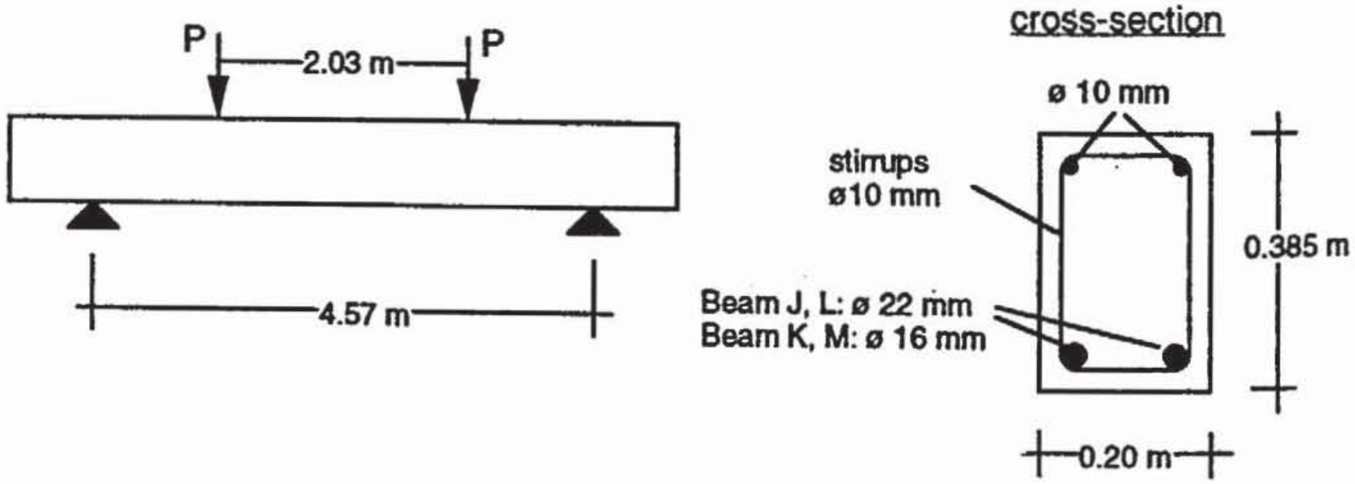

Fig. 2: Beams tested by Stevens, /11/

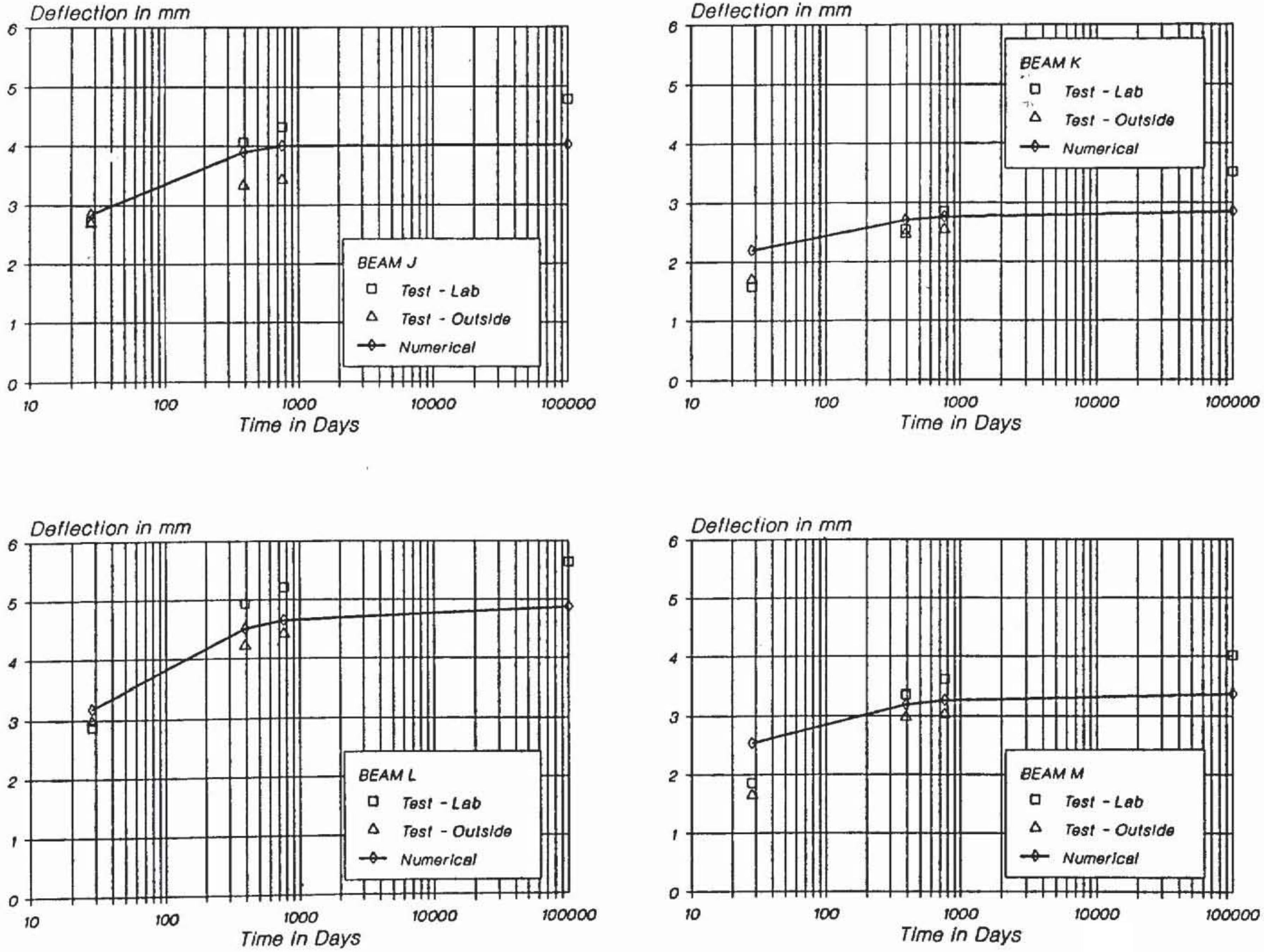

Fig. 3: Measured and calculated deflections

3.2 Beam under Fatigue Loading

The presented analysis is illustrated and verified for the T-beam shown in Fig. 4, tested by Soretz (/10/). Four concentrated loads which variied between $P_u = 15$ kN and $P_o = 200$ kN are acting at equal spacings. The dead load is assumed as 21 kN/m. The reinforcement consists of 9 ø25 ribbed bars. The shear reinforcement is composed of rectangular two-legged stirrups ø10 at e = 0.14 m spacing. The concrete cylinder strength is $f'_c = 32.8$ N/mm^2.

The principal analysis is obtained according to the equivalent fatigue loading analysis described in section 2.2.3. The mean concentrated forces $P_m = 0.5\,(P_u + P_o) = 107.5$ kN and distributed load q = 21 kN/m are acting as a sustained load. In the analysis the analogous creep and slip coefficients as listed in Table 2 were employed.

Number of Load cycles	k(N)	Φ
$N = 2$	0.1	0.1
$N = 5\cdot10^5$	3.1	1.2
$N = 10^6$	3.4	1.5

Table 2: Creep coefficients for fatigue loading, f = 5 Hz, $f'_c = 32.8$ N/mm^2

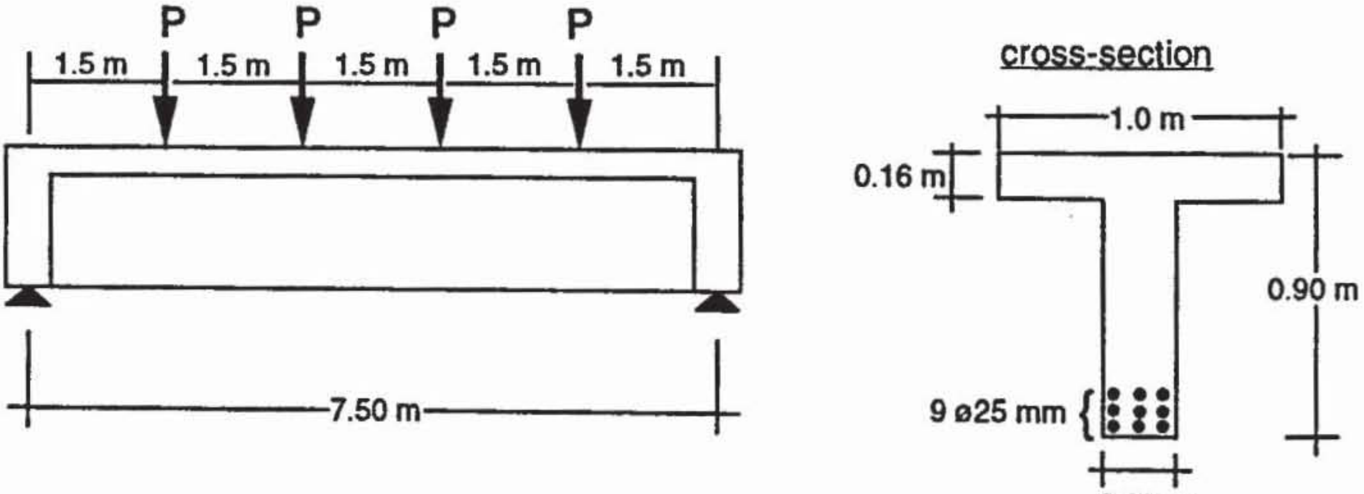

Fig. 4: Beam tested by Soretz /10/

The deflected shapes of the beam for N = 2, $5\cdot10^5$ and 10^6 load cycles are plotted in Fig. 5. It can be seen, that the numerically obtained deflections agree rather well with the values measured in the experiment.

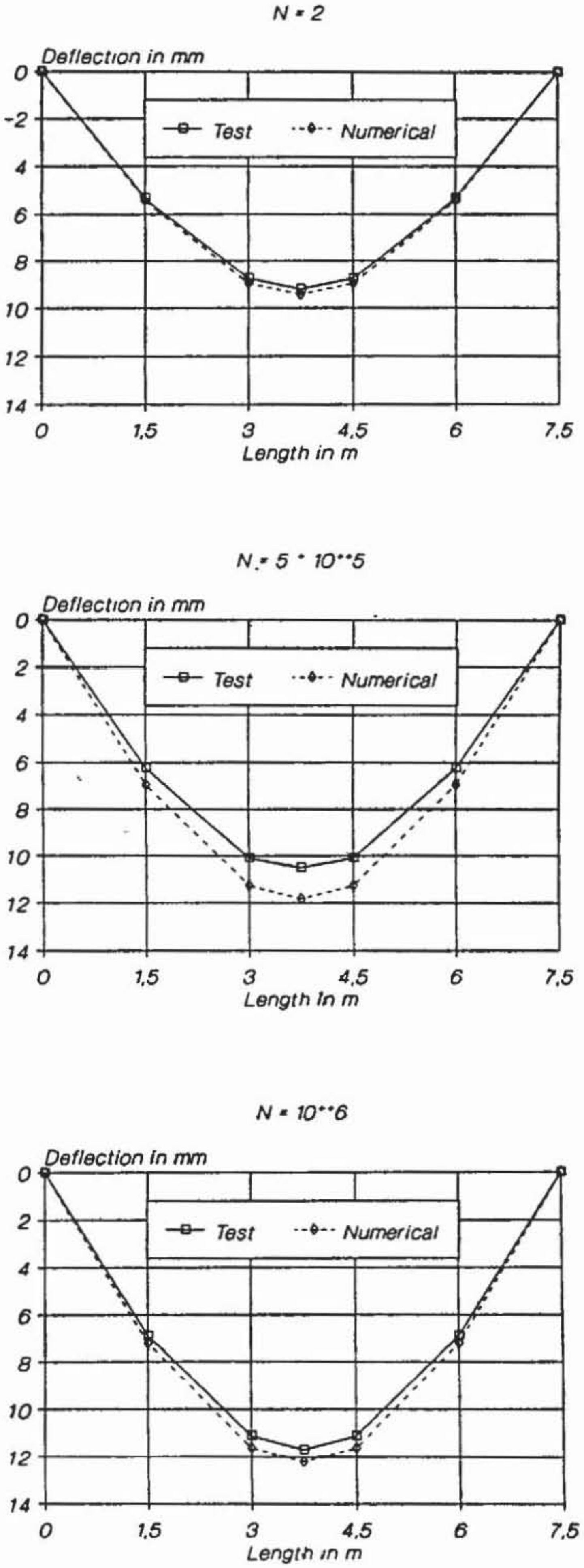

Fig. 5: Measured and calculated deflections along the beam length

4 CONCLUSIONS

A computionally efficient finite element model for the analysis of reinforced concrete beams under creep and fatigue loading is presented. The model is based on the nonlinear constitutive relationships of concrete, steel and bond between steel and concrete. It is verified by an numerical analysis of reinforced concrete beams under sustained loading tested by Stevens (/11/) and a beam under fatigue loading tested by Soretz (/10/). However, further studies are necessary to assess the accuracy of the proposed model. The described method may be used for a sensitive study of the major parameters (e.g. material properties, beam dimensions, reinforcement ratio and bond behavior of the bars) influencing the response of beams under sustained and fatigue loading.

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