B. FINKBEINER, T. ERTL, H. HEROLD, H. RUDER
Lehrstuhl für Theoretische Astrophysik, Universität Tübingen

PARTICLE MOTION IN PULSAR MAGNETOSPHERES

This report discusses some new results we found in studying the trajectories of single charged particles in the vacuum magnetospere of a pulsar using the oblique rotator model (*cf.* H. Herold, T. Ertl and H. Ruder, 1985, Mitt. d. AG, 63, 174, hereafter HER). We believe that investigations of individual particles in the vicinity of the star can be useful for a better understanding of some fundamental problems of pulsar physics, e.g. the global structure of the magnetosphere or the pulsar radiation.

The pulsar is considered as a rotating, homogeneously magnetized sphere of radius a, angular frequency Ω and polar magnetic field strength B_0 with an angle χ between rotational and magnetic axis. Charged particles are accelerated from the surface by the resulting electromagnetic vacuum fields $\boldsymbol{E}$ and $\boldsymbol{B}$ (A. Deutsch, 1955, Ann. d'Ap., 18, 1). The equation of motion to be solved numerically is the Lorentz-Dirac equation which takes into account the effects of radiation damping. In the vicinity of the star where the particle velocities are highly relativistic, $|\boldsymbol{v}| \simeq c$, and the Lorentz-Dirac equation in Landau approximation reads

$$\frac{d\Gamma}{dt} = \boldsymbol{E}\cdot\boldsymbol{v} - D_0\Gamma^2\Big((\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})^2 - (\boldsymbol{E}\cdot\boldsymbol{v})^2\Big) \tag{1}$$

$$\frac{d\boldsymbol{v}}{dt} = \frac{1}{\Gamma}\Big(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B} - \boldsymbol{v}\,(\boldsymbol{E}\cdot\boldsymbol{v})\Big) \tag{2}$$

with the damping constant $D_0 = \Omega e^2/(6\pi\epsilon_0 mc^3\varepsilon^3)$, $\varepsilon = 2mc^2/(B_0 a^2\Omega e)$, and $\Gamma = \varepsilon\gamma$, γ being the relativistic Lorentz factor. $\boldsymbol{v}$ is measured in units of c, t in units of $1/\Omega$, $\boldsymbol{E}$ in units of $cB_0(\Omega a/c)^2/2$ and $\boldsymbol{B}$ in units of $B_0(\Omega a/c)^2/2$. For typical pulsar parameters ($P = 2\pi/\Omega = 0.1\,\mathrm{s}$, $B_0 = 10^8$ T and $a = 10\,\mathrm{km}$) the value of the damping constant for electrons is $|D_0| \sim 10^{14}$. Such high values of D_0 lead to particle trajectories with locally minimized radiation damping. In this case the Lorentz-Dirac equation (1) and (2) can be approximated by a velocity field (HER)

$$\boldsymbol{v} = \frac{1}{\boldsymbol{B}^2+P^2}\left(\boldsymbol{E}\times\boldsymbol{B} + \frac{1}{P}(\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{B} + P\boldsymbol{E}\right) \tag{3}$$

with

$$P^2 = \frac{1}{2}(\boldsymbol{E}^2-\boldsymbol{B}^2) + \frac{1}{2}\left((\boldsymbol{E}^2-\boldsymbol{B}^2)^2 + 4(\boldsymbol{E}\cdot\boldsymbol{B})^2\right)^{\frac{1}{2}} \tag{4}$$

and the sign of P equal to the sign of the charge. In the vicinity of the star, where $|\boldsymbol{E}| \ll |\boldsymbol{B}|$, it is easy to see from (3) and (4) that the motion is dominated by the term

$$\boldsymbol{v}_B = \frac{(\boldsymbol{E}\cdot\boldsymbol{B})}{P(\boldsymbol{B}^2+P^2)}\boldsymbol{B}$$

in equation (3). The particles are predominantly accelerated by the electric field component along $\boldsymbol{B}$ which forces them to follow the magnetic field lines closely. In the limit of $\boldsymbol{E}\cdot\boldsymbol{B} \to \pm 0$ and $|\boldsymbol{E}| < |\boldsymbol{B}|$, however, $\boldsymbol{v}_B$ is discontinuous and changes its value from $\boldsymbol{v}_B$ to $-\boldsymbol{v}_B$. Therefore those particles which are moving along magnetic field lines that are crossing regions where $\boldsymbol{E} \cdot \boldsymbol{B}$ changes its sign, cannot follow the field lines further out, but are captured in these regions. That this consideration is justified, was confirmed by integrating equations (1) and (2) numerically with increasing damping constants D_0. Higher damping constants lead to smaller values of the particles kinetic energies at the $(\boldsymbol{E} \cdot \boldsymbol{B} = 0)$-boundary due to stronger radiation damping. After crossing the boundary the particles are decelerated by the electric field component along $\boldsymbol{B}$. If the kinetic energy at the boundary is not sufficient to overcome the backward force, the particle get stuck in a region more or less close to the boundary. Equation (3) represents the limiting case of this situation with the particles being trapped right at the boundary.

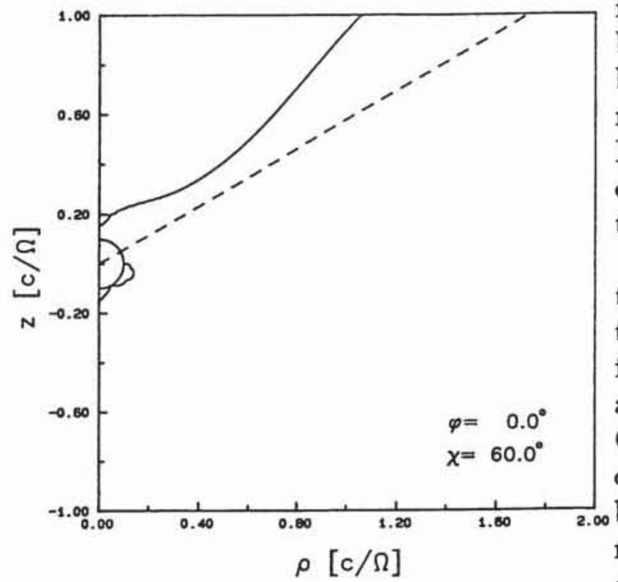

Fig. 1 $(\boldsymbol{E} \cdot \boldsymbol{B} = 0)$-surface for an oblique rotator with $\chi = 60°$ and $a = 0.1c/\Omega$ in the half plane $\varphi = 0°$ containing the magnetic axis (dashed line). The rotational axis is directed along the z-axis; the surface region above the rotational equator is open allowing electrons to escape from the star

In the case of an orthogonal rotator the $(\boldsymbol{E} \cdot \boldsymbol{B} = 0)$-boundary is a surface entirely enclosing the star, so that in the framework of our model no particles are able to leave the vicinity of the pulsar. Changing to the more realistic case of an oblique rotator, the surface of the star can be subdivided by the rotational and the magnetic equator into four separate regions. Two of them are again totally surrounded by an $(\boldsymbol{E} \cdot \boldsymbol{B} = 0)$-surface so that particles starting from those regions are trapped in the vicinity of the star. The regions around the magnetic poles, however, are not closed (Fig. 1), opening up the possibility of trajectories leading outward. As is shown in Fig. 2 the field geometry indeed results in trajectories leading into the wave zone. Investigating systematically the open regions, we found that the starting points for particles reaching the wave zone are not situated symmetrically around the magnetic poles, but are shifted towards the direction of rotation and towards the rotational equator. We feel that this is an important new aspect for the treatment of the oblique rotator.

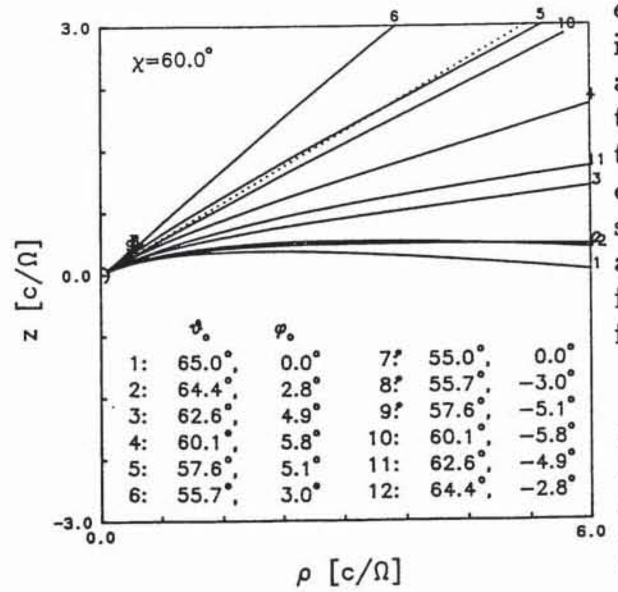

Fig. 2 12 electron trajectories for the above example of an oblique rotator with $\chi = 60°$ and $a = 0.1c/\Omega$ projected onto an arbitrary plane $\varphi = \text{const}$. The starting points are situated symmetrically around the magnetic north pole