1.2b BEWEGUNGSGLEICHUNGEN, MEHRKÖRPERSYSTEME

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Closing Conditions and Reaction Forces of Multibody Systems

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Over the years a considerable effort has been focused on the development of methods and computational algorithms for mathematical modelling and simulation of multibody systems [1]. The methods are mainly concerned with effective formulation and numerical treatment of the dynamical equations of a general multibody system,

$$\boldsymbol{M}(y, t)\,\ddot{y} + \boldsymbol{k}(\dot{y}, y, t) = \boldsymbol{q}(\dot{y}, y, t),$$

where y is an n-vector of independent, generalized coordinates; M is an $n \times n$ inertia matrix; the n-vector k represents the gyroscopic forces; the n-vector q expresses the applied (and/or control) forces on the system; t is the time; and n is the number of degrees of freedom. The above formulation is easy understandable for open-loop multibody systems with chain and tree topology. In many technological processes, however, the multibody systems come in contact with environment and/or closed-loops are designed to improve their abilities. This leads to additional closing conditions (constraints) which have to be considered with the dynamical equations. Most common approaches to the problem, that is by introducing the Lagrange multipliers or by using a range of projection methods to eliminate the reaction forces from the dynamical equations, often lead to numerical instabilities and computational inefficiency.

In this note attention is drawn to advantages that may arise in the dynamic analysis of constrained multibody systems by applying special algorithms for dynamics formulation and inverse kinematics developed in the field of robotics.

The property of the chain topology of a manipulator allows to introduce a special recursive formalism resulting in explicitly resolved dynamical equations, for details see [2–4], i.e.

$$\ddot{y} = M^{-1}(q - k) \,. \tag{1}$$

As stated in [3], the recursive relations for M^{-1} are rather complex and the numerical inversion of the mass matrix M, especially for small systems, may be competitive or even superior. This resolved form of dynamical equations, however, is useful in the dynamic analysis of constrained systems and allows one to avoid some multiplication/inversion matrix operations as compared with dealing with the dynamical equations in standard form, $M\ddot{y} + k = q$.

The m closing conditions may be introduced either implicitly, $\varphi(y) = 0$, or explicitly, $y_c = y_c(z)$, where the $f = n - m$ independent coordinates from y are conserved in z, and y_c denotes the remaining, dependent coordinates. Evidently, the two formulations of closing conditions are equivalent, nevertheless, $y_c(z)$ is usually more difficult to derive than $\varphi(y) = 0$. The explicit formulation of closing conditions may then be obtained by using procedures of inverse kinematics analysis of manipulators, see e.g. [5–7], which results in recursive relations for $y_c = y_c(z)$. Moreover, the inverse kinematics formalisms supply the user with recursive formulations for velocities and accelerations, i.e. $\dot{y}_c = I_c\dot{z}$ and $\ddot{y}_c = I_c\ddot{z} + \xi$, where $I_c(z) = \partial y_c/\partial z$ and $\xi(\dot{z}, z) = \dot{I}_c\dot{z}$.

Applying the explicit formulation of closing conditions, the dynamic equations of the constrained motion read as

$$\ddot{y} = \begin{bmatrix} \ddot{z} \\ I_c\ddot{z} + \xi \end{bmatrix} = M^{-1}(q - k + Q_c\lambda) \,, \tag{2}$$

where $Q_c^{\mathrm{T}} = [-I_c, E]$ (E is the $m \times m$ identity matrix); and λ is the m-vector of Lagrange multipliers or the generalized reaction forces, respectively. Then, the term $Q_c\lambda$ denotes the reaction forces projected in the directions of y. In order to determine them explicitly, from $-I_c\ddot{z} + \ddot{y}_c - \xi = Q_c^{\mathrm{T}}\ddot{y} - \xi = 0$, after substituting (2) for $\ddot{y}$, the multipliers λ can be determined from the equations of reactions, i.e.

$$N_c\lambda = -Q_cM^{-1}(q - k) + \xi \,, \tag{3}$$

where $N_c = Q_c^{\mathrm{T}}M^{-1}Q_c$ is the $m \times m$ invertible reaction matrix. Denoting $G = M^{-1}$, it is easy to find that $N_c = I_cG_{zz}I_c^{\mathrm{T}} - G_{yz}I_c - (G_{yz}I_c)^{\mathrm{T}} + G_{yy}$, where G_{zz}, G_{yz} and G_{yy} are appropriate block submatrices of G. Introducing $y_c = y_c(z)$ and $\dot{y}_c = I_c(z)\,\dot{z}$, (3) can be symbolically rewritten as

$$\lambda = \lambda(\dot{z}, z, t) \,. \tag{4}$$

The first f equations of (2) represent the equations of motion of the constrained multibody system, i.e.

$$\ddot{z} = G_z(q - k) + (-G_{zz}I_c^{\mathrm{T}} + G_{zy})\,\lambda = \ddot{z}(\dot{z}, t) \,, \tag{5}$$

where $G_z = [G_{zz}, G_{yz}]$; $G_{zy} = G_{yz}^{\mathrm{T}}$; and λ is determined from (3). Note that the first term of (5) represents the unchanged recursively resolved equations of the open-loop system, whereas the second term expresses the contribution of closure reactions.

The crucial features of the reported approach can be summarized as follows: (1) The analysis is carried out in a minimal set of arbitrarily chosen independent coordinates and the minimal-order equations of motion (5) are to be integrated. – (2) Reaction forces due to the closing conditions are determined at every instant of motion. – (3) In principle, the dynamic solution of (5) is released from the problem of violation of the constraints. – (4) Dynamical equations of the open-loop system with recursively inverted inertia matrix, cf. (1), and recursive relations coming from inverse kinematics, $y_c = y_c(z)$, $\dot{y}_c = I_c\dot{z}$ and $\ddot{y}_c = I_c\ddot{z} + \xi$, are used in the reported formulation. Hence, the approach is rather computer-oriented. – (5) For a given choice of independent coordinates z from y, the problem of singularity may appear. This may require redefining z and appropriate rearranging the subsequent formulation.

Efficiency and attractiveness of the reported approach as compared with the other methods for dynamic analysis of constrained multibody systems is an open question. The advantages of this method are the minimal possible dimension of final governing equations, explicit determination of reaction forces, and the absence of the problem of constraint violation. On the other hand, the recursive inversion of inertia matrix and inverse kinematics formulation may require a considerable amount of labour for deriving appropriate, usually complex relations, see e.g. [3–7]. This may decrease the computational efficiency of the method. However, the conjecture of the authors is, assuming some skill of the user in inverse kinematics methods, that the reported approach for many practical applications is superior to many other well-known methods for analysing constrained multibody systems. This has been already shown for a crank-slider mechanism and a few-bar linkage. The research project is going on.

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