TOWARDS A SELF-CONSISTENT MODELLING OF PULSAR MAGNETOSPHERES

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Since their discovery almost twenty years ago, the radio pulsars — several hundreds are known today — have been fascinating objects for astronomers and astrophysicists, although (or because) until now only a poor understanding of these objects has been obtained. On some global ideas, namely that a pulsar is a rapidly rotating, strongly magnetized neutron star and that the pulses are produced by some sort of light-house effect, there exists general agreement, but self-consistent models are still lacking (for a review cf. [1]). Since from the energetic point of view the radio emission seems to be only a small part of the total energy balance of a pulsar magnetosphere, all local models for the pulse emission mechanism may be questionable as long as there do not exist any reasonable models for the global magnetospheric structure.

The numerical modelling of the general case of an oblique rotator is a very complicated time-dependent 3-dimensional problem and in its full extent probably outside the capacity of present-day computers. A considerable simplification occurs if one can assume that the essential effects may be understood by modelling the magnetosphere of an aligned rotator (where the rotation axis is parallel to the magnetic axis of the neutron star). This assumption is only reasonable for small obliqueness, since by this approach all electromagnetic wave effects are not taken into account. An advantage, however, is that the unipolar induction, which should be responsible for populating the magnetosphere with charged particles pulled out from the neutron star surface via field emission [2], can be studied in purity.

Thus, the structure of a stationary, axially symmetric pulsar magnetosphere is governed by the following equations. The electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$ can be described (in cylindrical coordinates ρ, φ, z) by the electrostatic potential $\Phi(\rho, z)$, the magnetic flux function $\Psi(\rho, z)$ and the poloidal magnetic field $B_\varphi(\rho, z)$:

$$\mathbf{E} = -\nabla\Phi; \qquad \mathbf{B} = \frac{1}{\rho}\nabla\Psi \times \mathbf{e}_\varphi + B_\varphi \mathbf{e}_\varphi \tag{1}$$

Charge density ρ_e and current density $\mathbf{j}$ determine the electric potential via the Poisson equation and the magnetic field via Ampere's law which read here (all quantities have been made dimensionless by appropriate units):

$$\Delta\Phi = -\rho_e \tag{2a}$$

$$\Big(\Delta - \frac{2}{\rho}\frac{\partial}{\partial\rho}\Big)\Psi = -\rho\, j_\varphi \tag{2b}$$

$$\frac{\partial}{\partial z}(\rho B_\varphi) = -\rho j_\rho \quad , \qquad \frac{\partial}{\partial z}(\rho B_\varphi) = -\rho j_z \tag{2c}$$

The magnetosphere is formed by a collisionless plasma, in which the particles are expected to be extremely relativistic due to the huge electric and magnetic fields (the unipolar induction voltage between pole and equator of the neutron star is typically 10^{17}-10^{18}V). Thus charge and current density are given by integrals of the particle distribution function $f(\mathbf{r}, \mathbf{p})$ over momentum space

$$\rho_e = \int f(\mathbf{r}, \mathbf{p})\, d^3p; \qquad \mathbf{j} = \int f(\mathbf{r}, \mathbf{p})\mathbf{v}\, d^3p, \tag{3}$$

whereas the distribution function is determined by the Vlasov equation

$$\mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \frac{\partial}{\partial \mathbf{p}}\Big[(\mathbf{E} + \mathbf{v}\times\mathbf{B} + {\text{radiation} \atop \text{damping}})f\Big] = 0 \tag{4}$$

The velocity $\mathbf{v}$ is given by $\mathbf{v} = \mathbf{p}/\sqrt{(\varepsilon^2 + \mathbf{p}^2)}$, where the dimensionless parameter ε is defined by $\varepsilon = 2mc^2/(eB_0a^2\Omega)$, i.e. by the ratio between rest mass energy and unipolar induction energy (B_0 is the frozen-in magnetic field, a is the radius, Ω is the angular velocity of the neutron star). Because typical values for ε are extremely small ($\varepsilon \sim -10^{-12}$ for electrons, $\varepsilon \sim 10^{-9}$ for protons), one can conclude that the particles gain relativistic energies with Lorentz factors $\gamma = (1 - v^2)^{-1/2}$ of the order of $1/|\varepsilon|$. Therefore, the quantity $\Gamma = \varepsilon\gamma$ should be of order 1, at least if the radiation reaction during phases of acceleration can be neglected. This is not the case, however, as can be seen by studying the trajectories of particles in realistic pulsar fields. The equations of motion, i.e. the Lorentz-Dirac equation in the Landau approximation (cf. [3]), can be written as (provided that $|\varepsilon| \ll 1$)

$$\dot{\mathbf{v}} = \frac{1}{\Gamma}\mathbf{F}, \qquad \dot{\Gamma} = \mathbf{E}\cdot\mathbf{v} - D_0\Gamma^2\mathbf{F}^2 \tag{5}$$

with $\mathbf{F} = \mathbf{E} + \mathbf{v}\times\mathbf{B} - \mathbf{v}(\mathbf{E}\cdot\mathbf{v})$, by numerically integrating these equations, one can decide on the role of radiation damping. We have performed such integrations, but also without considering the explicit solutions of (5), it is understandable that the radiation reaction is very important, since the dimensionless damping constant $D_0 = e^2/(6\pi\epsilon_0)\,\Omega/(mc^3\varepsilon^3)$ takes — for typical pulsar parameters — the values $D_0 \sim -10^{14}$ for electrons, and $D_0 \sim 10$ for protons. Thus, at least for the electrons, the radiation reaction force dominates the particle motion. An example for the integration of eqs.(5) is shown in fig 1, where corresponding trajectories without and with radiation damping are compared.

Large values of $|D_0|$ imply that the factor of D_0 in (5) remains always very small; this leads us to the observation,

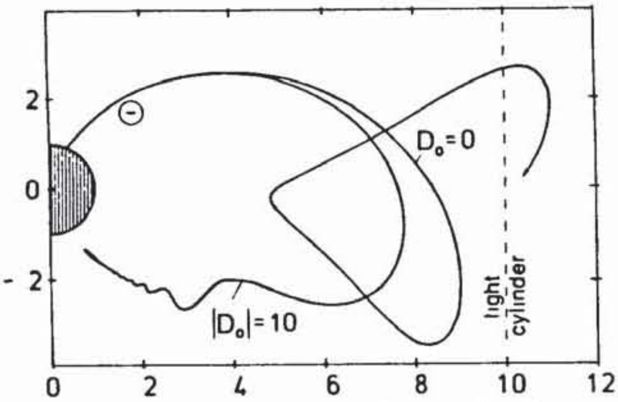

Fig.1: Comparison of an undamped ($D_0 = 0$) and a radiatively damped ($|D_0| = 10$) particle trajectory in a magnetic dipole and an electric monopole + quadrupole field. The radiation reaction leads to the capture of the particle in the region $\mathbf{E} \cdot \mathbf{B} \approx 0$.

which has been confirmed numerically, that during the motion the condition

$$\mathbf{F} = \mathbf{E} + \mathbf{v} \times \mathbf{B} - \mathbf{v}(\mathbf{E} \cdot \mathbf{v}) \approx 0 \tag{6}$$

is fulfilled, which means that the radiation reaction is locally minimized along the trajectory. Fquation (6) is a condition for the velocity and yields, for given E- and B-fields (cf. [4]),

$$\mathbf{v} = \frac{1}{\mathbf{B}^2 + P^2}\Big[\mathbf{E} \times \mathbf{B} + \frac{1}{P}(\mathbf{E} \cdot \mathbf{B})\mathbf{B} + P\mathbf{E}\Big] \tag{7a}$$

with

$$P^2 = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) + \frac{1}{2}\Big[(\mathbf{E}^2 - \mathbf{B}^2)^2 + 4(\mathbf{E} \cdot \mathbf{B})^2\Big]^{\frac{1}{2}} \tag{7b}$$

i e we get a local velocity field $\mathbf{v} = \mathbf{v}(\mathbf{E}, \mathbf{B})$ and thus a fluid-like picture for the particle motions in the magnetosphere.

Based on these results, the task to determine a self-consistent solution is simpler than before, but still difficult due to the great non-linearity of the problem. Our approach is based on the idea to start from the vacuum solution and to fill up the magnetosphere with the particles which are pulled out from the neutron star surface. This is not a real time-dependent calculation, since we assume that the electric field is always described by an electrostatic potential, but we need the time-dependent continuity equation and therefore $\dot{\mathbf{E}}$ cannot be omitted in Ampere's law. Thus the eqs.(2c) have to be replaced by

$$\Big(\Delta - \frac{2}{\rho}\frac{\partial}{\partial \rho}\Big)(\rho B_\varphi) = \rho\Big(\frac{\partial j_z}{\partial \rho} - \frac{\partial j_\rho}{\partial z}\Big). \tag{8}$$

In summary, we solve at each time step the Poisson equation (2a) and the elliptic equation (2b) with Dirichlet boundary conditions and the equation (8) with the von Neumann boundary condition $\partial(\rho B_\varphi)/\partial r = \rho j_\vartheta$ on the star's surface. A simple emission law is assumed (namely $j \propto E_\parallel$), and for the continuity equation, which the particle density is determined from, an explicit discretization in time (with 2-dimensional Flux Corrected Transport [5]) is used. The spatial discretization is made in the spherical coordinates (r, ϑ). actually we have an equidistant grid for $\bar{r} = (r - r_0)/(r + c_0)$ ($r_0 = \Omega a/c$, c_0 can be chosen suitably) and $\mu = \cos\theta$. The three elliptic equations are solved by successive over-relaxation (SOR) in a vectorizable checkerboard scheme.

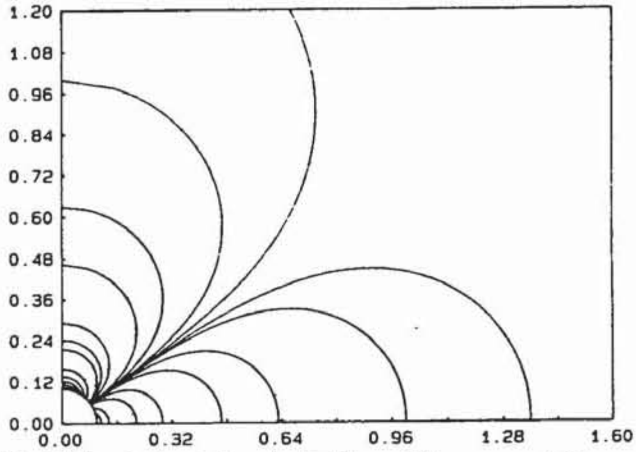

Fig.2: The electrostatic potential lines of the vacuum field configuration of an uncharged homogeneously magnetized neutron star (our starting point).

Fig.3: The electrostatic potential lines at the time $t = 2\pi/\Omega$ for a small emission rate of the surface. The charge in the magnetosphere has already deformed the vacuum electric field to some extent.

Starting from the vacuum fields — magnetic dipole + electric quadrupole (cf. fig.2) —, at the beginning we assumed the current coming out of the star to be relatively small. In the first phases of the time integration, only negative particles are emitted leading to a noticeable change of the toroidal magnetic field B_φ; at this stage the electric field is altered only slightly. Later on, the developing charge in the magnetosphere gradually deforms the electrostatic potential in such a way that positive particles can be pulled out from the star. The electrostatic potential at that stage (after one revolution) is shown in fig.3. Afterwards the density of positive particles increases, especially near the equator, but one cannot conclude yet that a corotating zone is forming, since the evolution of the magnetospheric charge distribution is far from being completed.

In subsequent runs, we tested what is happening if the surface emission rate is enhanced to a more realistic value. This leads, of course, to stronger effects in a shorter time, but conclusive statements about the results cannot be made yet.

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