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Procedia IUTAM 13 (2015) 34 – 42
Procedia IUTAM
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IUTAM Symposium on "Dynamical Analysis of Multibody Systems with Design Uncertainties"

Inverse fuzzy arithmetic for the quality assessment of substructured models

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Abstract

The dynamical analysis of complex structures often suffers from large computational efforts, so that the application of substructuring methods has gained increasing importance in the last years. Substructuring enables dividing large finite element models and reducing the resulting multiple bodies, yielding a reduction of, in this case, complex eigenvalue calculation time. This method is used to predict the appearance of friction-induced vibrations such as squeal in brake systems. Since the method is very sensitive to changes in parameter values, uncertainties influencing the results are included and identified. As uncertain parameters, standard coupling elements are considered and modeled by so-called fuzzy numbers, which are particularly well suited to represent epistemic uncertainties of modeled physical phenomena. The influence of these uncertainties is transferred to undamped and damped eigenfrequencies of a substructured model by means of direct fuzzy analyses. An inverse fuzzy arithmetical approach is applied to identify the uncertain parameters that optimally cover the undamped reference eigenfrequencies of a non-substructured, full model. If a validity criteria is defined, a positive decision in favor of the most adequate model can be performed.

Keywords: multibody systems; substructuring; model order reduction; friction-induced vibrations; brake systems; complex eigenvalue analysis; uncertain coupling elements; fuzzy arithmetic; inverse approach.

1. Introduction

In recent years, the dynamical analysis of mechanical systems modeled by finite elements (FE) has suffered from large computational costs due to the high number of degrees of freedom (dof) defining the complex geometries, which leads to large data structures that can barely be analyzed in acceptable time. An industrial example for those complex systems are automotive brake systems, where a numerical analysis is required to investigate their undesirable propensity to squeal. The study of this effect of squealing, caused by friction-induced vibrations, is known to be particularly challenging [1]. Although a detailed analysis of the evolution of these vibrations is needed, time-domain simulations prove unaffordable for these complex systems, so that mainly frequency-domain investigations are performed. For

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doi:10.1016/j.piutam.2015.01.011

this purpose, the method of complex eigenvalue analysis is widely used in industry[2]. The obtained results, however, are extremely dependent on the model configuration and very sensitive to the actual parameter values, and so, there is a distinct need for the systematic consideration of uncertainties. When performing uncertainty analyses, the computational complexity of the problem increases, so that the use of reduced, substructured models proves very advantageous.

The substructuring method divides large FE structures into a number of subcomponents of lower dimension. The substructures are then coupled together, and reduced dynamical analyses are performed, allowing faster computations compared to the original analyses. When reproducing the dynamics of these subcomponents, model order reduction methods play a decisive role, and for the case of substructuring the well-established component mode synthesis in combination with a modal representation of the internal dynamics is commonly used[3]. Other reductions are based on more advanced methods, such as frequency-response interpolation methods and Krylov subspaces, which have proven to be well suited if specific frequency ranges are to be emphasized[4].

For the purpose of modeling uncertainties, fuzzy arithmetic is particularly well suited, as it allows the representation of epistemic uncertainties that arise from the modeling procedure due to simplification and idealization[5]. Uncertain model parameters can be modeled by so-called fuzzy numbers, which can be considered as the inputs of the uncertainty analysis of the mechanical model. In the direct fuzzy analysis, the uncertain system outputs are calculated by the use of specific fuzzy arithmetical methods, and a deeper insight into the dynamics of the model in the presence of uncertainties is achieved[6]. The inverse approach, instead, aims at identifying the uncertain model parameters based on the measurements of a reference system and some optimization procedure, so that the resulting fuzzy-valued model outputs optimally cover the reference data[7]. With the identified input model parameters, a fuzzy-parameterized model of the original system is achieved that stands out by exhibiting a simplified structure, e.g. after substructuring, but still is reproducing the original output data. Finally, by evaluating a special model validity criterion for the uncertain model, its appropriateness to reproduce the reference system can be assessed, and thus its quality can be rated. If the criterion is applied to different models, the most adequate model can be selected[8].

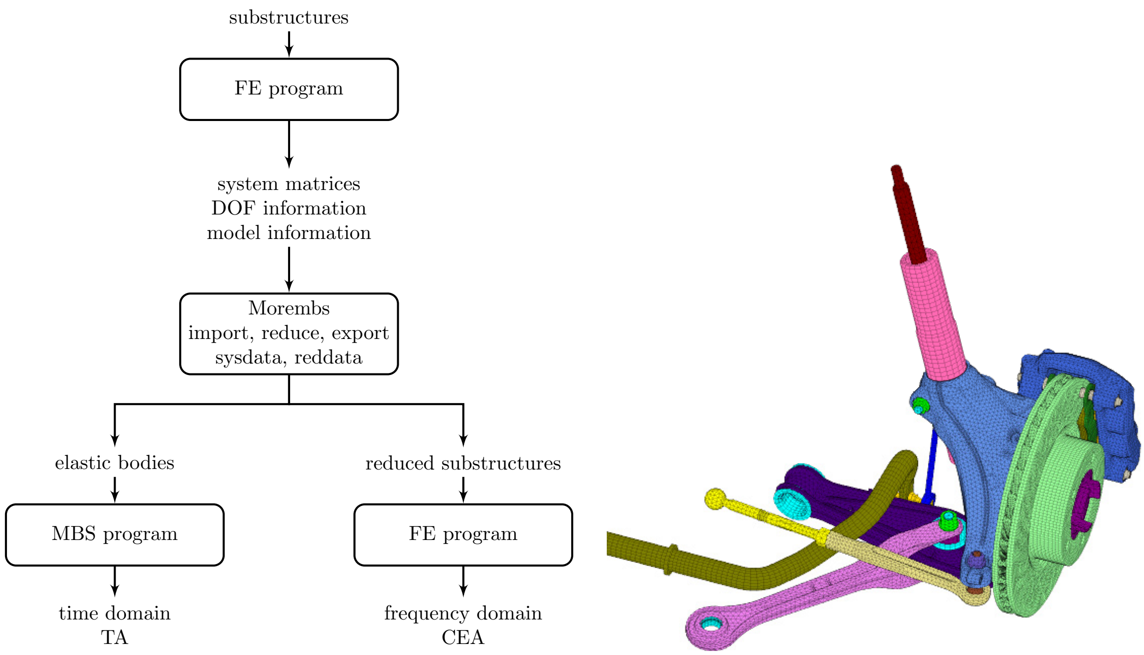
In Section 2, substructuring of large mechanical systems and model order reduction methods are introduced and linked to FE analyses and multibody simulations. The projection methods and the reduction of system dimension are then discussed in Section 3, where also the fundamentals of the method of complex eigenvalue analysis are shown. In Section 4, the fuzzy arithmetical techniques are applied, consisting of a direct fuzzy analysis that serves as a preliminary step, and the inverse approach that subsequently enables the quality assessment of substructured models. Finally, in Section 5, this approach is applied to the example of an automotive brake system.

2. Substructuring of high-dimensional models

In order to reduce the complexity of mechanical systems, full FE models composed of multiple bodies are divided into a number of structures. In this context, reduced structures, known as subcomponents, are coupled to unreduced, top-level structures by means of external dofs at the corresponding interface nodes. This is known as substructuring and leads to a set of hierarchical relations between a so-called top component and the reduced subcomponents or super-elements. Some of these subcomponents are coupled by elements such as springs, dampers or additional nodal masses, which aim at idealizing and simplifying the modeling of joints. In Section 4 these parameters are further investigated.

Reducing a substructure means decreasing the dimension of the system matrices describing the internal dynamics of a component. Of course, this is only admissible as long as the corresponding dynamics is still captured properly or even kept within acceptable error bounds. For this purpose, a wide variety of model order reduction methods based on matrix projection are available in the literature[3,9,10] and are implemented in the preprocessing tool Morembs[11]. For example, the classical implementation of modal reduction has been widely used, but needs a large number of modes so that the high-frequency dynamics of the structure is properly represented. On the contrary, more advanced methods, such as projections onto Krylov subspaces[4], benefit from the interpolation of the frequency response of a substructure and improve the approximation on specific frequency ranges.

Although in this paper substructuring and model order reduction methods are applied to finite element computations, multibody simulations also benefit from them since the widespread and detailed FE modeling of components is



(a) Preprocessing of substructures with Morembs. (b) Full model of a brake system with 1.5 million dofs.

Fig. 1: Preprocessing of substructures.

there used for creating elastic bodies [12]. Both processes are presented in Figure 1a, which shows the classical process chain used to simulate mechanical systems in engineering applications.

As an example of a substructured model, the full brake system in Figure 1b is considered. On the one hand, the system consists of disc-level components which are mounted together or are in the vicinity of the friction-affected disc. These are composed of a hub, anchors, pistons, a caliper, inner and outer pads, and back plates. On the other hand, mounting parts of a quarter vehicle suspension comprise a wheel-carrier, a suspension rod, a stabilizer and longitudinal, transversal, steering and coupling bars. These components are the basis of the substructuring applied to the model, and some of their characteristics are summarized in Table 1.

Table 1: Properties of the substructures of a brake model.

component	dofs	nodes	elements	ext. nodes	material
coupling bar	2271	931	432	2	steel
piston rod	9564	3569	1953	3	steel
stabilizer bar	26787	9274	4884	3	steel
steering link	27609	9669	5260	2	steel, aluminum
longitudinal link	58692	20144	10650	2	aluminum
transverse link	87000	29828	15674	3	aluminum
wheel carrier	446385	152688	85180	9	aluminum
all mounting parts	658338	226108	124054	10	steel, aluminum

3. The method of complex eigenvalue analysis

As far as the dynamic analysis of brake systems is concerned, the method of complex eigenvalue analysis (CEA) has become a standard to judge the stability of the steady-state response and to predict the occurrence of friction-induced oscillations [2]. The equations of motion of such a nonlinear system read as

$$\mathbf{M}\,\ddot{\mathbf{q}} + \mathbf{D}\,\dot{\mathbf{q}} + \mathbf{K}\,\mathbf{q} = \mathbf{f}_{\mathrm{nl}}\,(\mathbf{q}, \dot{\mathbf{q}})\,, \tag{1}$$

where $\mathbf{M}$, $\mathbf{D}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices, respectively, $\mathbf{q}$ are the coordinates of the system and $\mathbf{f}_{\mathrm{nl}}$ are the nonlinear forces arising from contact and rotational effects, amongst others. This frequency-domain investigation is based on a linearization around a sliding state $\mathbf{q}_\xi$ that transforms Equation (1) into

$$\mathbf{M}\,\ddot{\mathbf{q}}_\xi + (\mathbf{D} + \Omega\,\mathbf{Y})\;\dot{\mathbf{q}}_\xi + (\mathbf{K} + \mathbf{Q})\;\mathbf{q}_\xi = \mathbf{0}, \tag{2}$$

where $\mathbf{Y}$ and $\mathbf{Q}$ are the computed gyroscopic and circulatory matrices, respectively, and Ω is the rotational velocity of the disc. It is pointed out that these matrices are skew-symmetric ($\mathbf{Y} = -\mathbf{Y}^{\mathrm{T}}$) and antisymmetric ($\mathbf{Q} \neq \mathbf{Q}^{\mathrm{T}}$), respectively, which leads to numerical difficulties when solving eigenvalue problems. In the FE program used in this investigation, namely Permas [13], and in the literature [14], Equation (2) is extended and includes additional damping and stiffness terms that depend on the rotational velocity Ω, leading to

$$\mathbf{M}\,\ddot{\mathbf{q}}_\xi + \left(\mathbf{D} + \left(\frac{1}{\Omega} - 1\right)\,\mathbf{D}_\Omega + \Omega\,\mathbf{Y}\right)\,\dot{\mathbf{q}}_\xi + \left(\mathbf{K} + \left(\Omega^2 - 1\right)\,\mathbf{K}_\Omega + \mathbf{Q}\right)\,\mathbf{q}_\xi = \mathbf{0}. \tag{3}$$

By combining damping and stiffness terms in $\hat{\mathbf{D}}$ and $\hat{\mathbf{K}}$, respectively, and by the use of the ansatz function $\mathbf{q}_\xi = \mathbf{\Phi}_j\, e^{\lambda_j t}$, the quadratic eigenvalue problem (QEVP) of the damped, gyroscopic, circulatory system can be written as

$$\left(\lambda_j^2\,\mathbf{M} + \lambda_j\,\hat{\mathbf{D}} + \hat{\mathbf{K}}\right)\,\mathbf{\Phi}_j = \mathbf{0}, \tag{4}$$

where $\lambda_j = \rho_j + \mathrm{i}\omega_j$ is the j^{th} complex eigenvalue with real part ρ_j and imaginary part ω_j, and $\mathbf{\Phi}_j$ is the j^{th} complex eigenmode. As mentioned before, this QEVP is known as complex to be solved, and thus, system matrices are usually projected into a modal subspace of reduced dimension. For that purpose, system matrices belonging to the undamped, symmetric system are requested and the generalized eigenvalue problem (GEVP)

$$\left(-w_i^2\,\mathbf{M} + \mathbf{K}\right)\,\mathbf{V}_i = \mathbf{0} \tag{5}$$

is calculated, where w_i and $\mathbf{V}_i$ are the i^{th} undamped eigenvalue and eigenmode, respectively. Next, with the projection matrix $\mathbf{V}$, the system matrices in Equation (4) are reduced to $\tilde{\mathbf{X}} = \mathbf{V}^{\mathrm{T}}\mathbf{X}\mathbf{V}$, where $\mathbf{X} = \{\mathbf{M}, \hat{\mathbf{D}}, \hat{\mathbf{K}}\}$, and rewritten as a first-order system, e.g.

$$\left(\lambda_j \begin{bmatrix} \tilde{\mathbf{M}} & \tilde{\mathbf{0}} \\ \tilde{\mathbf{0}} & \tilde{\mathbf{I}} \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{D}} & \tilde{\mathbf{K}} \\ -\tilde{\mathbf{I}} & -\tilde{\mathbf{0}} \end{bmatrix}\right) \begin{bmatrix} \tilde{\mathbf{\Phi}}_j \\ \lambda_j\,\tilde{\mathbf{\Phi}}_j \end{bmatrix} = \mathbf{0}. \tag{6}$$

Finally, the reduced, first-order GEVP in Equation (6) is solved by the use of an FE solver or dedicated numerical libraries. The described method is implemented in most commercial FE software packages and consists of four basic steps. First, in order to calculate the contact forces, a nonlinear static analysis is performed. Second, displacements due to the rotation of the disc are calculated by means of a quasi-static analysis. Next, eigenmodes of the damped system are extracted by means of a vibration analysis, and finally, complex eigenvalues and eigenmodes are calculated for different rotating velocities by a so-called modal rotating analysis.

If any of the resulting eigenvalues λ_j has a positive real part $\rho_j > 0$, the sliding state $\mathbf{q}_\xi$ and its steady-state response are classified as unstable at the corresponding critical frequency ω_j, indicating locally unstable vibration behavior. This local but not necessarily global unstable behavior serves as an indication of the occurrence of friction-induced vibrations.

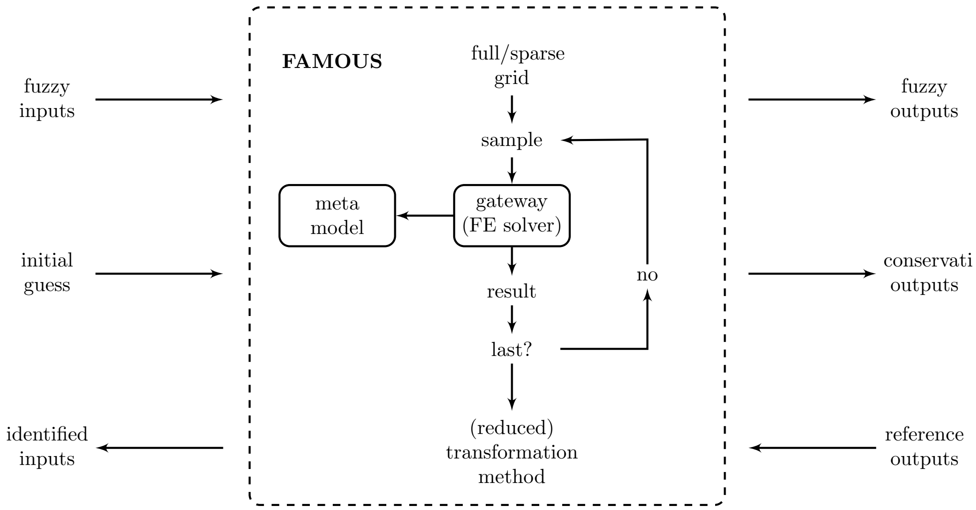


Fig. 2: Uncertainty analysis with FAMOUS.

4. Fuzzy arithmetic and uncertainty analysis

In the following, the standard coupling elements introduced in Section 2 are considered as uncertain and are modeled by so-called fuzzy numbers, which are particularly well suited for representing epistemic uncertainties, such as vagueness and lack of information in the modeling procedure[5]. These diffuse numbers can be interpreted as nested intervals, ranging from a worst-case scenario in case of maximum uncertainty to a crisp nominal value in case of complete certainty. By the use of fuzzy-valued model parameters, well-selected crisp parameter combinations are chosen and several model evaluations are performed. In this way, the nominal results of conventional, crisp-valued model simulations are extended to the inclusion of uncertainties, and valuable conclusions on the overall effect as well as on the individual influence of uncertain parameters on the output of the model can be drawn[15]. As computational complexity increases considerably with the number of uncertain parameters, reduction of the calculation time becomes a matter of major interest, and therefore, substructuring and model order reduction methods have been introduced to CEA calculations in Section 2.

Fuzzy arithmetic is successfully applied, for example, in multibody dynamics[6,16] and in the method of complex eigenvalue analysis described in Section 3. As an example, for the brake model depicted in Figure 1b, four nodal masses of interest, m_1, m_2, m_3 and m_4, are considered as uncertain and modeled by triangular fuzzy numbers with lower and upper bounds of ±10% with respect to their nominal values. For the definition, the simulation, and the evaluation of the fuzzy-parameterized model, the software package FAMOUS[17] (Fuzzy Arithmetical Modeling Of Uncertain Systems) is used and successfully coupled to the FE program Permas, as depicted in the flowchart diagram in Figure 2. As a result, absolute and relative influences of uncertainties on the outputs are determined, and the nominal results of the Campbell diagrams for the critical frequencies are extended to the inclusion of different certainty levels, reflected as contour lines for the damping ratio in Figure 3.

4.1. Basic concept of quality assessment using inverse fuzzy arithmetic

Based on the direct approach, inverse fuzzy arithmetic can be viewed as the backwards implementation of fuzzy arithmetic in order to identify the uncertain parameters, i.e. the fuzzy inputs, that optimally cover measurement and reference data after performing an uncertainty analysis[7]. In a first phase, some output signal of interest y is observed and measured along an independent variable x, such as time or frequency, see Figure 4a. Then, by analyzing the input sources z_i, the observed, real phenomenon is modeled. For this purpose, some simplifications and idealizations that lead to mathematical models of the form $y(x)$ are made, which are then capable of explaining the nature of the system depending on the input parameters. Next, the parameters governing each mathematical model have to

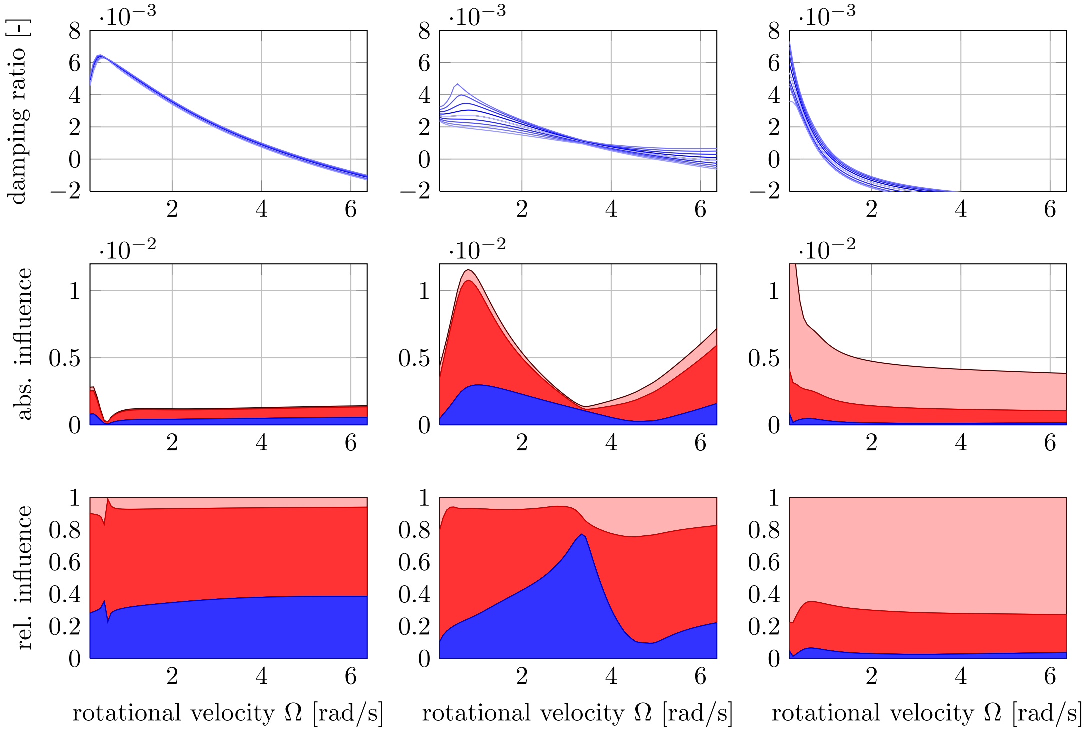
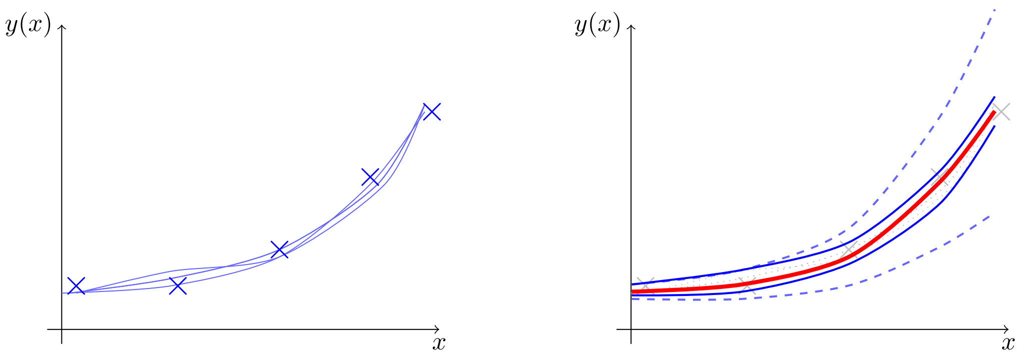


Fig. 3: Uncertainty analysis using four fuzzy input parameters m_1 (dark blue), m_2 (light blue), m_3 (dark red) and m_4 (light red).



(a) Measurements (cross) or reference data (solid line). (b) Nominal (red) and fuzzy (blue) outputs of a model.

Fig. 4: Basic idea of inverse fuzzy arithmetic with respect to the outputs of a mechanical system.

be determined. In the conventional identification procedure, crisp-valued parameters are determined by a best-fit optimization of the available measurement data, and by evaluating each mathematical model with its corresponding crisp-valued parameters, different crisp outputs are simulated and predicted. However, these outputs do usually not match properly the measured data due to mathematical models being simplifications or idealizations of reality. Thus, the need of considering the uncertainties involved in each modeling process arises.

In the proposed inverse fuzzy arithmetical approach, an initial guess on the fuzzy input parameters is made. As shown at the beginning of Section 4, with these initial fuzzy model parameters a preliminary forward simulation is carried out, and fuzzy outputs for the simplified model are calculated, see the dashed blue lines in Figure 4b. In

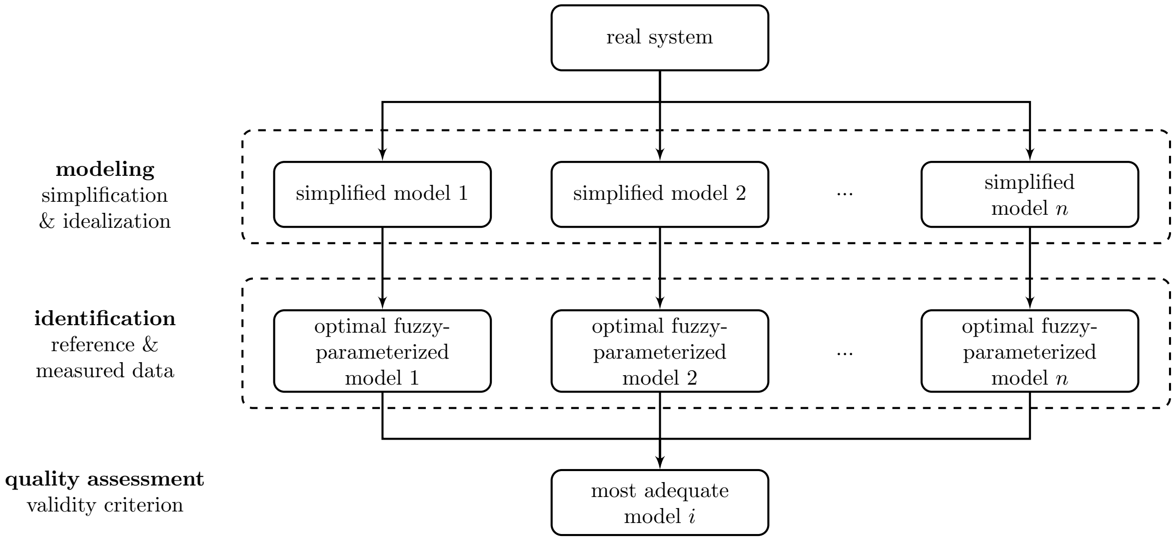


Fig. 5: Quality assessment based on inverse fuzzy arithmetic.

this way, the nominal outputs of the system are complemented by uncertainty levels, and the effect of considering uncertainties is observed. However, these initial outputs do not optimally cover the curves measured at the beginning of the design phase, and the uncertainties modeled by the initial guess are classified as too conservative. Therefore, the shapes of the initial fuzzy numbers are optimized by means of a back-propagation procedure[7], and updated optimal fuzzy parameters are identified. With the identified parameters, a forward simulation is performed and measurement data is optimally covered by the re-simulated fuzzy outputs, see the solid blue lines in Figure 4b. By following this approach, optimal uncertain parameters for each idealized model can be identified. Each set of identified fuzzy numbers represents the amount of parametric uncertainty to effectively cover the measurement data for the given idealized models.

In this way, the quality of simplified models can be assessed, and objective statements on their individual validity can be made. As presented in Figure 5, in the modeling step of a real system, different simplifications and idealizations can be made, resulting in n different idealized models. In the identification step, the above-mentioned back-propagation procedure is performed, and based on the measurements or reference data the uncertainty ranges of the input parameters are identified. By this, n optimal fuzzy-parameterized models can be achieved. If a validity criteria is defined, the total relative uncertainty Λ can be calculated for each optimal fuzzy-parameterized model and a selection in favor of the most adequate model i is done. It has to be pointed out that these models may or may not be dependent on the same input parameters and even the number of uncertain parameters may be different. Regarding the initial guess, its nominal values need not necessarily match the crisp-values of the best-fitted parameters. Last but not least important, material parameters are known to be aleatoric variables and are usually not considered as modeling uncertainties. In a classical uncertainty analysis based on fuzzy arithmetic, however, such parameters can be included in order to calculate their absolute and relative influence on the output.

5. Example: analysis of a brake system

In this section, the process chain in Figure 5 is applied to the example of a substructured brake system. Thereby, one model order reduction method introduced in Section 2, namely classical modal reduction, is adopted, and the undamped frequencies of the projection procedure of the CEA, see Equation (5), are defined as outputs. As fuzzy input parameters to be optimized, four spring elements are considered, namely k_p at the pad-caliper interface, and k_{c_1}, k_{c_2} and k_{c_3} at the three fixation directions of the caliper. As initial guess, triangular fuzzy numbers with $\pm 10\%$

lower and upper bounds are defined, whose nominal values are $k_p = 200\text{N/m}$, $k_{c_1} = 10^6\text{N/m}$, $k_{c_2} = 1\text{N/m}$ and $k_{c_3} = 10^6\text{N/m}$, respectively.

In the first step, i.e. the direct uncertainty analysis, fuzzy outputs of the first 20 undamped frequencies and their absolute and relative influences are investigated similar to Figure 3. For this purpose, fuzzy input parameters and gateways to the FE solver, which, for each sample, extract the undamped frequencies as results, have to be defined in the software package FAMOUS, see Figure 2. Next, a meta model is created by using either full or sparse grids. Meta models are advantageous since they enable direct model evaluations without continuously running time-consuming gateways. In this way, the fuzzy outputs and absolute and relative influences of the example are calculated, yielding, among others, negligible influence of k_{c_2} for almost all undamped frequencies. Therefore, it is stated that deviations in the parameter do not significantly alter output signals. In the following, k_{c_2} is included in the inverse calculation, however, it could be omitted so that the parameter space is reduced in further uncertainty analyses.

Next, the identification procedure for obtaining optimal fuzzy input parameters k_p, k_{c_1}, k_{c_2} and k_{c_3} is performed, leading to the nominals values and lower and upper bounds of the stiffness parameters shown in Table 2. Based on the identification process, valuable conclusions can be drawn. For stiffness k_{c_3}, for example, the optimization delivers a crisp number of the value 10^6N/m. Therefore, the parameter is no longer considered as uncertain, and in the re-simulation the number of evaluations is reduced. Similarly, k_{c_1} results in a one-sided fuzzy number whose left deviation has been reduced from -10% to -4%. Although k_p is also obtained as a one-sided fuzzy number, it has to be pointed out that the new upper bound strongly oversteps the initial upper bound. This result delivers valuable information since it presents a hard-to-identify uncertain parameter at that side of the nominal value. The same conclusion can be achieved for the stiffness k_{c_2}, which exhibits the previous characteristics at both sides. As a consequence, the optimization yields a non-identifiable parameter that cannot be optimized. This is in accordance with the preliminary direct analysis, where the influence of the parameter is observed to be negligible.

Table 2: Nominal values and lower and upper bounds of the input parameters.

	initial uncertain input parameter			identified uncertain input parameter		
parameter	nominal value	lower bound	upper bound	nominal value	lower bound	upper bound
k_p	$+2.00 \cdot 10^2$	$+1.80 \cdot 10^2$	$+2.20 \cdot 10^{2*}$	$+2.00 \cdot 10^{2*}$	$+2.00 \cdot 10^{2*}$	$+1.44 \cdot 10^4$
k_{c_1}	$+1.00 \cdot 10^6$	$+0.90 \cdot 10^6$	$+1.10 \cdot 10^6$	$+1.00 \cdot 10^{6*}$	$+0.96 \cdot 10^{6*}$	$+1.00 \cdot 10^{6*}$
k_{c_2}	$+1.00 \cdot 10^0$	$+0.90 \cdot 10^{0*}$	$+1.10 \cdot 10^{0*}$	$+1.20 \cdot 10^{0*}$	$-6.89 \cdot 10^5$	$+9.18 \cdot 10^4$
k_{c_3}	$+1.00 \cdot 10^6$	$+0.90 \cdot 10^6$	$+1.10 \cdot 10^6$	$+1.00 \cdot 10^{6*}$	$+1.00 \cdot 10^{6*}$	$+1.00 \cdot 10^{6*}$

To conclude, a re-simulation of the direct uncertainty analysis is performed with the updated nominal values and the “less uncertain” bounds marked by an asterisk ($*$) in Table 2, resulting in the fuzzy outputs covering optimally the reference data. As regards model quality, for the relative validity of the identified input model parameters and the re-simulated fuzzy outputs a value of $\Lambda_{\text{modal}} = 9.83 \cdot 10^{-13}$ is obtained.

As performed for modal reduction, substructuring with Krylov projection matrices is used in conjunction with the previous procedure. Regarding the reduction method, the frequency response of the substructured system is approximated using three frequency shifts of second order at 500Hz, 1500Hz and 2500Hz. For the sake of brevity, no numerical results are presented and only some important characteristics with respect to the modal reduction are summarized. In the preliminary direct fuzzy analysis, for example, relative influences are distributed similar to the modal reduction. The parameter identification delivers different optimal stiffness parameters that result in an overall less imperfect fuzzy analysis, as attested by a better since lower relative validity of $\Lambda_{\text{Krylov}} = 3.86 \cdot 10^{-16}$. As shown in the flowchart of Figure 5 and based on the relative validities Λ_{modal} and Λ_{Krylov}, the substructured model reduced with Krylov subspaces is considered to be the most adequate model with respect to the considered uncertainties in the stiffness parameters.

6. Summary

In this paper, fuzzy arithmetic is applied to the uncertainty analysis of an industrial brake system. In order to face the complex geometry of the mechanical system, the preprocessing tool Morembs is used and substructuring is performed. This leads to a reduced, substructured model whose dynamical analyses result computationally more efficient. Thereby, the importance of model order reduction methods and the parallelism to elastic multibody simulations is pointed out. The proposed substructured brake model enables accounting for uncertainties in model parameters, such as spring or nodal mass elements at the coupling interfaces. These elements are known to influence the method of complex eigenvalue analysis, the standard method used to investigate friction-induced vibrations in brake systems. As regards the method, the nonlinear equations of motion and the assumed linearization that lead to the different types of eigenvalue problems are explained. Together with the calculation of the complex eigenvalues, the equations focus on the relation of the undamped frequencies with the performed subspace projection and proposed substructuring methods. Next, by the use of triangular fuzzy numbers implemented in the software package FAMOUS, epistemic uncertainties in the coupling parameters are represented and their influence is simulated. As an example, a direct fuzzy analysis shows how uncertainties in mass model parameters influence the damped, complex eigenvalues of a Campbell diagram that depends on the rotational velocity of the disc. Last, the basic concept of inverse fuzzy arithmetic for identifying uncertain model parameters is described in scope of quality assessment for a number of uncertain, substructured models. For a second example, an initial guess is optimized for two different substructuring strategies based on modal reduction and Krylov subspaces, respectively. With the identified uncertain model parameters, the two uncertainty analyses are updated and their outputs are recalculated. In this way, relative validities Λ are determined and an objective election in favor of the substructured model reduced with Krylov subspaces is done.

In future research, the inverse fuzzy arithmetic will be extended to high-dimensional substructured systems where the choice of the model order reduction method plays a more decisive role. For that purpose, a wider spectrum of reduction method have to be considered. Time-domain investigations and multibody systems are also intended to benefit from the inverse approach, by allowing an appropriate election in favor of the model that optimally covers reference or measurement data under the consideration of modeling uncertainties.

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