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Diplomarbeit

Two-Dimensional Bar Charts

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Abstract

Bar charts have been among the most commonly used types of visualizations throughout the years. They usually visualize a set of scalar data points by encoding the values into the length of the bars. The perception of differences of visual elements in length usually works well preattentively in human. Problems emerge when visual similarities present, due to small value differences or in the presence of big extrema, causing the domain to stretch. Comparison of similar lengths has been proven to be more difficult when the distance between them increases. This thesis proposes a new approach to the problems. The concept is to turn the area limited by two side edges of a bar and the two auxiliary lines closest to its value into a grid. The cells of this grid are shaded basing on the ratio of value difference of the bar and the auxiliary lines. The cells can be divided into sub-grids to express finer values. The stacking of cells forms different shapes for different values, which human are good at detecting. The comparison task is thus performed better with this new technique. Further more, by counting and calculating the total value these cells represent, the value estimation process also achieves higher precision.

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2DBC Two-Dimensional Bar Chart 7

CSV Comma-Separated Values 41

D3 Data-Driven Documents 43

DOM Document Object Model 43

DPI dots per inch 27

DTP desktop publishing 25

GUI graphical user interface 40

JSON JavaScript Object Notation 41

SVG Scalable Vector Graphics 39

1 Introduction

Even though most sources considers William Playfair's "*Exports and Imports of Scotland to and from different parts for one Year from Christmas 1780 to Christmas 1781*" graph as the first bar chart in the history, a primitive form of the visualization had been featured as early as in the fourteenth century in the work of Nicole Oresme, "*The Latitude of Forms*." [BR78; CC70; JPowered15; WikiBarChart17]

Since then, bar charts have been among the most commonly used types of graph throughout the years [Rum10]. It's an accessible way to visualize a set of categorical data of one dimensional values for the purpose of comparison. The reason for their popularity lies perhaps in their simplicity. Bar charts can be easily created, from simple sketches with pen and paper to complex computer assisted programs. Nowadays, everybody with elementary education has the knowledge to interpret these kinds of charts efficiently.

For example, viewers can recognize the trends of the data relatively fast with bar charts, especially when compare to raw numbers or data tables. They can also detect pivotal values in a sizable data set at a glance and have reasonably accurate approximations of individual values.

Despite the many strengths, bar charts are far from perfect, just like every other visualization technique and most things in our world. In some scenarios, where the difference between two values is small enough, their representational bars can have indistinguishable appearances. The problem is worse if these bars are positioned far away from each other. The farther the distance between them is, the harder it is for the viewers to compare their values. The auxiliary lines can help eliminate this problem, but not entirely.

Furthermore, text labels aren't usually favorable in bar charts since they can clutter the graph heavily. Therefore, the task of determining the exact values from the chart based purely on visual elements are proven to be not quite simple.

The main objective of this thesis is to offer a new alternative to tackle these limitations of the traditional bar charts.

1.1 The Classic Bar Chart

1.1.1 What Is Bar Chart?

Generally, a bar chart utilizes a two-dimensional Cartesian coordinate system to visualize a set of discrete data. More often than not, this dataset, which consists of nominal or ordinal data points, is represented as a rectangle distributed evenly on the horizontal axis. The heights of the bars are proportionally scaled to the maximum of the vertical axis to denote their data points' values accordingly.

1.1.2 Suitable Data for Bar Chart

Bar charts can be used to display comparative nominal and ordinal data. Nominal data are qualitative information such as nationality, or gender. Ordinal data is basically ranked nominal data such as your satisfaction of a service was poor, fair or good. In addition, the difference between ordinal and nominal is the nominal data's categories can be arranged sequentially in a particular order while it is not viable for ordinal data because there is an obvious sequence for ordinal data. Bar charts are also very helpful in displaying negative values of the dataset by simply position it below or above the horizontal axis. However, datasets with minimal difference of values across categories might have difficulties to accurately display that difference.

1.1.3 Variations

Horizontal vs. Vertical Bar Chart The bar chart can be drawn vertically or horizontally.

Vertical bar charts use the height as a visual metaphor, with the bars being taller meaning that the value is higher. This is a simple yet effective way to represent the data, as it is very intuitive for the human to process the information.

Horizontal bar charts are usually used when the categories have long titles, since horizontal labels are easier to read. Horizontal bar charts can contain a larger amount of data points and are easier to split between pages. If the charts are landscape, there will be more space for the length of the bars, which leads to better comparison performances.

Grouped Bar Chart Grouped bar charts visualize more than one characteristic of the data points as a group of bars. Each separate bar shows a different variate of a category. Bars representing the same variate have the same color for all data points. These bars are always positioned far from each other, which makes the task of comparing them more difficult. Moreover, less data points can be shown since the multiple bars occupy more horizontal space.

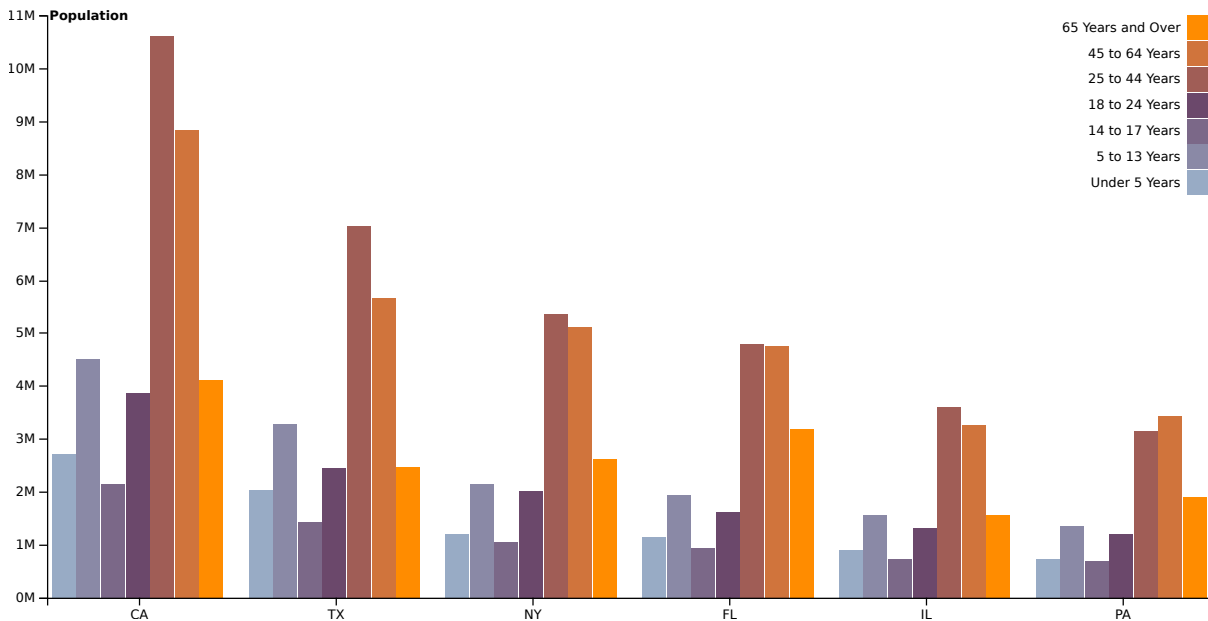


Figure 1.1: Example of grouped bar chart – Demographics of selected U.S. states.
Image source: Mike Bostock [GroupedBC17].

Stacked Bar Chart Stacked bar chart is a space efficient variation of grouped bar chart. Instead of positioning the bars of the same category side by side, they are stacked over each other on a single column. The total height of the column will show the total size of that category. The trade off to the space efficiency is the highly increased difficulty in comparing the sub values.

Other Related Chart Types Line chart and dot plot also use the Cartesian coordinate system. They also rely on the distance between a point and the axes to demonstrate the value of the data point. By using small dots or symbols instead of bars, they are able to visualize bigger datasets than bar charts. Comparison between data points and value estimation are still necessary, but line charts rather focus on the trends, the changes in time series, while dot plot stresses the concentration of the data. Viewers might miss critical data insight due to the visual cluttering in these kinds of charts.

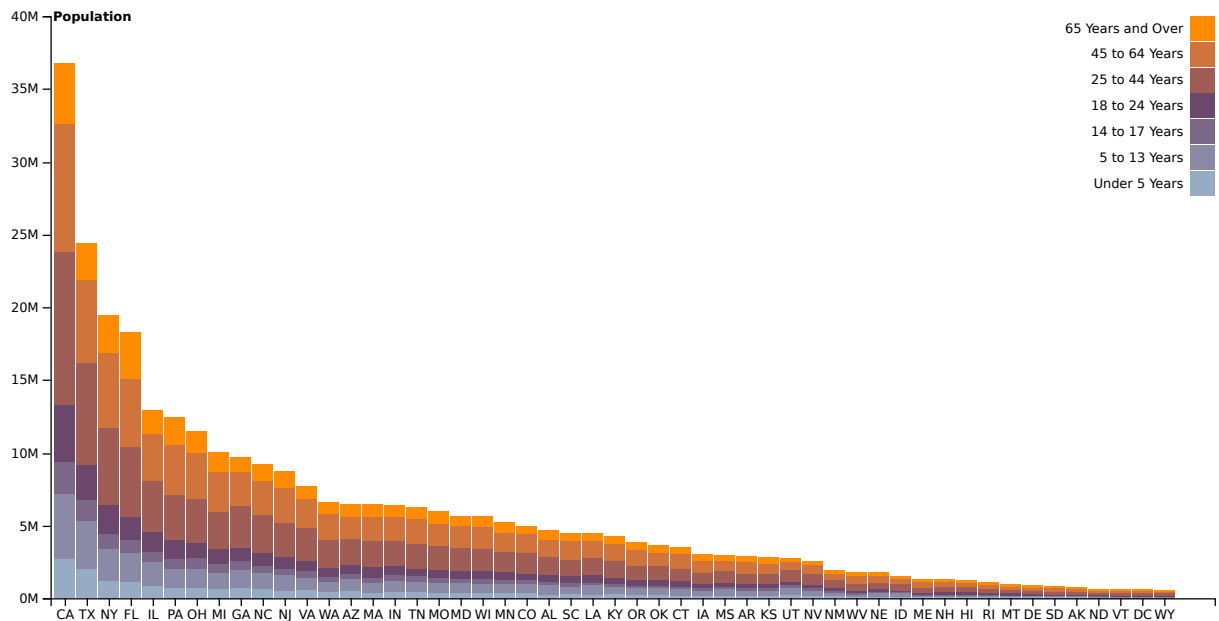


Figure 1.2: Example of stacked bar chart – Demographics of the U.S.A.
Image source: Mike Bostock [StackedBC17].

1.2 Related Works

There has been numerous researches analyzing the physical and psychological aspects of perceiving visual elements related to the bar charts.

O'Reagan, Simons and many others have done extensive works on change blindness, the phenomenon behind the inability to distinguish similar objects when eye movements are involved, and the role of visual attention in retaining details about the objects in memory, which makes comparing them easier as a result [CM+85; CM84; MXW+13; Ore00; ORe01; SA05; SFR00; Sim00; SL97].

DeCarlo [DeC94] discussed shortly how the length judgment process, or proportional judgment in general, is actually comparison with a reference value. Autocorrelation error is bigger when compare to previous length than to a fixed reference value.

Cleveland and McGill [CM84] found out length judgment is relative accurate. It's only ranked after two types of position judgment.

On the other hand, there also has been multiple works on reducing the limitations of bar charts.

The techniques employed by cutting off bar and scale (scale break) reduce the bars' visual similarities by diminishing unused space.

Using logarithmic scale can reduce the trouble comparing between the smaller values, but it is harder to see the proportion of values positioned at different ends of the scale.

1.3 Limitations of the Classic Bar Chart

1.3.1 The Comparison Problem

Visual Similarity

With the traditional bar chart, problems emerge when comparing two or more bars, which have approximately the same length.

The similarities can appear in charts if the value differences are too small, which results in non-discriminable visual elements [SF82]. In some cases, the dataset can contain global extrema that diverge strongly from a group of adjacent values. The differences between some values of this group can become visually insignificant in comparison to the data domain.

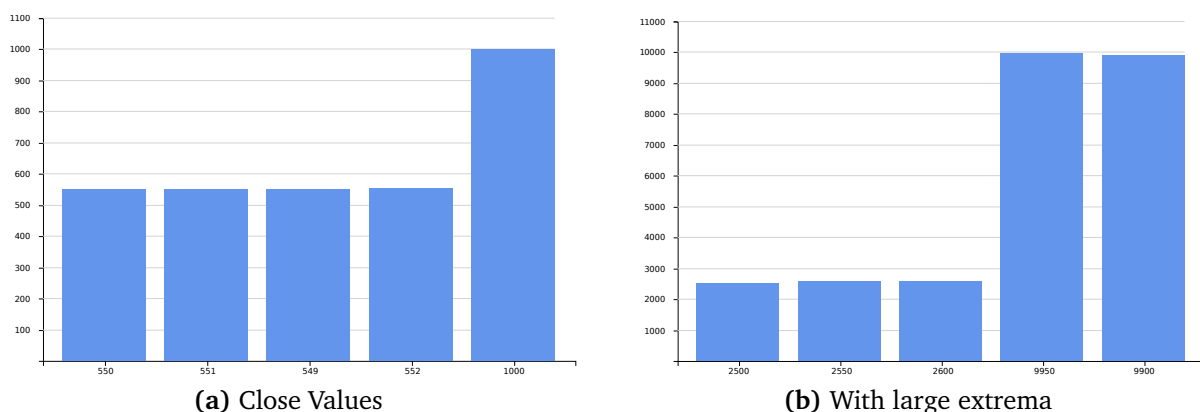


Figure 1.3: Visual similarities between (a) close values and (b) not-so-close values in the presence of big extrema.

Comparison of Distant Bars

In another scenario, the task of comparison is notably hardened if the bars with close values are positioned afar from each other.

A similar problem has often been demonstrated through the well-known puzzle “Spot the Difference,” where two mostly identical images are put side by side. People solve the puzzle by searching for the major details that are different in the two images. Viewers usually need more than a glance to solve the task. Sometimes, the quiz even appears to be unsolvable for some impatient folks.

Change Blindness The inability or poor ability to detect graphical changes when the visual stimuli are disrupted is known as the perceptual phenomenon referred to as “change blindness.” In the case of bar charts, it’s the change or the indifference in length of the bars when the eyes move across the chart and the surrounding shifts.

The knowledge of how humans perceive the world is needed in order to explain this peculiarity. The anatomy of our eyes only allows a limited visual field of 70 and 50 degrees vertically and 60 and 110 degrees horizontally. Changes in the visual field attract the attention, but not all of the changes are noticed [ORe01].

Visual Attention While there are still on going researches about the phenomenon, many studies " believed that change blindness is caused by the failure to store complete visual information in our short-term memory for purposes of comparison, and hence our brains are unable to detect the changes. Psychologists have also found that change blindness is related to visual attention, and changes in locations with low saliency are less likely to be detected. It has also been found that change is more easily detected in a region where there is a significant saliency difference between the image pair. Hence, both saliency and change in saliency are crucial factors in change blindness." [MXW+13]

In case of bar charts with similar columns, the length deviation isn’t considered a saliency and thus visual attention is needed to decipher the values, store them as numerical information and compare them to each other. However, the value estimations are not always accurate. Hence the task can still cause many viewer uncertainties.

2 Two-Dimensional Bar Chart (2DBC)

2.1 The 2DBC Concept

The main idea of 2DBC is to utilize the two-dimensional space at the top of the bars for displaying a more fine-grained accuracy of the data value. This space is defined by the two side edges of the bar and the two auxiliary lines, between which the value lies. The rectangular area will then be divided into a grid. Each cell of this grid represents a certain value depend on the value distance between the two auxiliary lines. Some cells will be colored to account for the difference between the value of the data point and the value of the auxiliary line closer to the root.

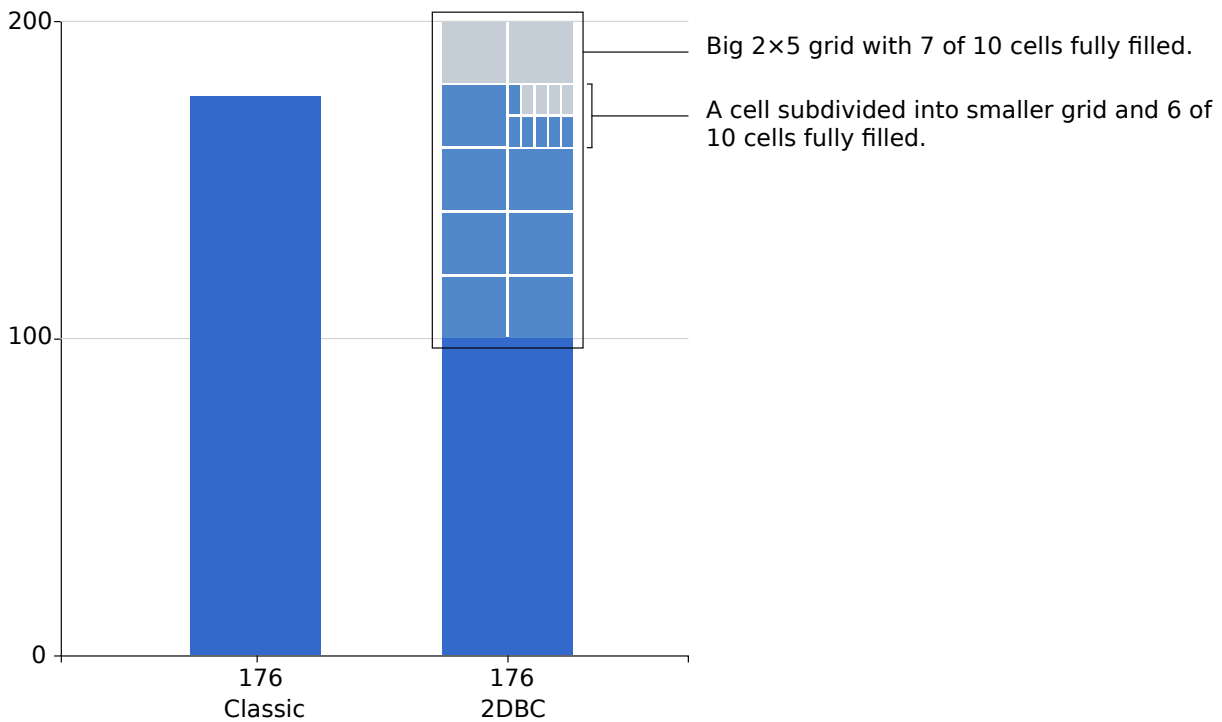


Figure 2.1: The 2DBC grid concept.

The value of the bar can be determined by counting the number of the accented cells. The larger size and the clearly separated cells aid the human perception of value differences.

The remaining value for the last to be filled cell often ends up to be just a fraction of the value of a whole cell. This cell could be proportionally colored base on the ratio of the remainder to the value of the cell. An alternative is to subdivide this last cell into a smaller grid with.

The partition of the remaining cell can be repeated recursively to the limit of the available space or as often as need be.

By turning the top part of the bars into an easily interpretable grid, this visualization technique can help the observers perform the tasks of comparing and determining the values more efficiently and accurately.

2.1.1 Choosing an Appropriate Grid Size

The decision on how to compartmentalize the grid base on the data domain and its characteristics plays a significant role in the users' perception process.

Since the modern human are most familiar with the decimal number system, dividing the grid into ten uniform parts is only logical.

There are also certain numbers, with which human can are perform mental arithmetical calculations more comfortably than others. For example, addition and multiplication with 1, 2, 5 are easier than with 3 or 7. Therefore, cells that hold the value of these numbers and grid that groups an amount of the preferred numbers of cells will help users determine the total value faster.

With that in mind, the following sections present some options for grid size. A grid can use any of these options for the primary, secondary or deeper sub grids. Each axis can be split into the suggested number of parts or a multiple thereof. By all means should the grid be so divided, that the cells are not too small for the average human eyes to distinguish.

10-Based Grid A 10-cell grid, in which each cell is one tenth or 10% of the value space. This is useful in scenarios where the value difference between two adjacent auxiliary lines is a power of ten (i.e. 10^n , even with negative n).

Some possible grid divisions are 2×5 (Figure 2.2a) or 5×2 (Figure 2.2b).

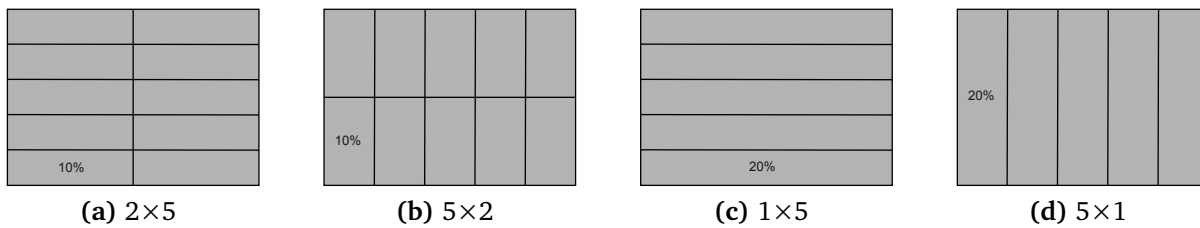


Figure 2.2: 10-based grids and its special 5-based variation.

5-Based Grid 5-Based Grid is a special variation of the 10-based grid. Instead of 10%, each cell holds 20% of the value. This option reduces the number of cells to avoid excessive graphical elements.

Some possible grid divisions are 1×5 (Figure 2.2c) or 5×1 (Figure 2.2d).

Half and Quarter Grids If a grid is divided into two cells, each of those denotes 50% of the total value (Figures 2.3a and 2.3b). Each cell of a quartered grid represents 25% of the total value (Figures 2.3c and 2.3d). These options have minimal number of cells, and thus are effective in situations where space is limited.

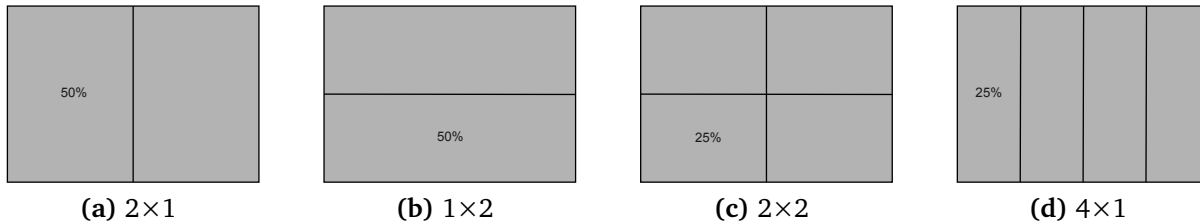


Figure 2.3: Half and quarter cell sized grids.

9-Based Grid As an alternative to using the entire space between two auxiliary lines, we could use only 90% of that space. The gap between the filled space and the next auxiliary line indicates more clearly the difference between 90% and 100%. The used space can be divided into 9 cells, each of which stands for 10% of the whole space value (Figure 2.4).

This approach doesn't work well for data containing values that lies within the gap between 90% and 100%.

10%		
10%		

Figure 2.4: 9-based grid with 3×3 division.

Combination of Different Grid Sizes

Sub-grids should be used in case the value differences are too fine to distinguish with the main grid. Since the space will be divided exponentially, it will get narrow very quickly. For this reason, the sub-grid should have as little number of cells as possible to maximize the area for each cell, if the data allows it. A smaller, minimalistic grid size should be used for the sub-grids.

Different grid sizes can be combined to achieve the grouped cell appearance. A group can help the viewer determine the number of cells faster.

Figure 2.5 demonstrates the usage of multiple grids to group smaller cells together to reduce visual cluttering. Instead of using a 2×10 as the primary grid and 5×1 as the secondary grid, Figure 2.5 shows that a 1×2 grid can be used to group 5 units together, then the 1×5 grid unites each row as a whole unit, the 2×1 halves the unit and finally each cell of the 5×1 grid represents 0.1 unit value. The biggest cell can be quickly comprehended as a group of 5 cells of the second big size.

2.2 Hurdles to Overcome

In this section, we discuss the problems that affect 2DBC and propose some approaches to minimize them.

2.2.1 The Space Insufficiency Problem

Space is the most serious issue with 2DBC. Bar charts aren't ranked high on the space efficiency list. There are usually needs to pack more data into an already limited visualization space. The bars can thus be often very narrow. For traditional bar charts,

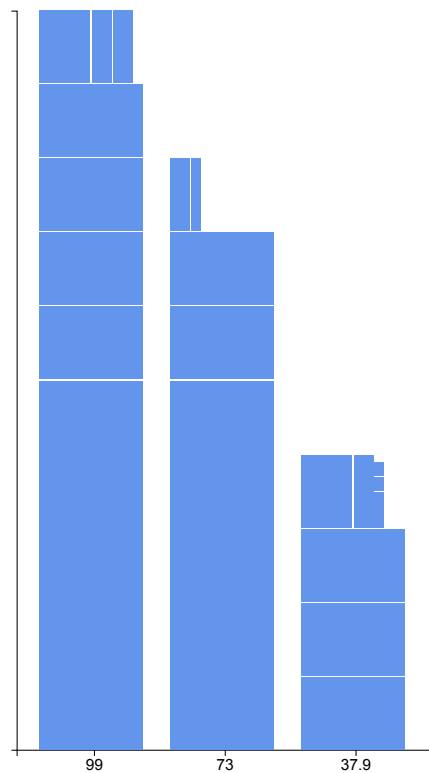


Figure 2.5: Example Grouping Cells

this would not even be a problem, since the focus of the graph is on the information encoded in the bars' length.

However, the space for the grids of 2DBC charts need to be sufficing. Ideally, the viewers must be able to perceive the separated cells well enough to count them. But what is the lower limit of human's vision?

The Smallest Visible Element

The question "How small can a person see with his naked eyes?" is often followed not by an answer, but many more questions. The subject's eyesight, the distance between their eyes and the object, the environment illuminating, the viewing angle, the contrast of the element to its background, etc. all have an effect on how each individual perceives graphics.

However, the centuries-old printing industry has been working on a common standard to ensure the visibility of graphical elements over the years. In print media, absolute measurements are favorable. The point is the smallest unit in typography. It is used to

measure font size, margins and other lengths in printing. Its size has been standardized as $1/72^{\text{nd}}$ of an international inch (about 0.353 mm), also known as the desktop publishing (DTP) point [WikiPoint17].

Then came the era of computer displays and along with it all new kinds of problems.

Bos explained in his article the reason why pixel is considered the standard unit for the smallest visible element [Bos10]: “The px unit is the magic unit of CSS. It is not related to the current font and usually not related to physical centimeters or inches either. The px unit is defined to be small but visible, and such that a horizontal 1px wide line can be displayed with sharp edges (no anti-aliasing). What is sharp, small and visible depends on the device and the way it is used: do you hold it close to your eyes, like a mobile phone, at arms length, like a computer monitor, or somewhere in between, like an e-book reader? The px is thus not defined as a constant length, but as something that depends on the type of device and its typical use.

To get an idea of the appearance of a px, imagine a CRT computer monitor from the 1990s: the smallest dot it can display measures about $1/100^{\text{th}}$ of an inch (0.25 mm) or a little more. The px unit got its name from those screen pixels.

Nowadays there are devices that could in principle display smaller sharp dots (although you might need a magnifier to see them). But documents from the last century that used px in CSS still look the same, no matter what the device. Printers, especially, can display sharp lines with much smaller details than 1px, but even on printers, a 1px line looks very much the same as it would look on a computer monitor. Devices change, but the px always has the same visual appearance.

In fact, CSS requires that 1px must be exactly $1/96^{\text{th}}$ of an inch in all printed output. CSS considers that printers, unlike screens, do not need to have different sizes for px in order to print sharp lines. In print media, a px thus not only has the same visual appearance from one device to another, but indeed it is measurably the same.

CSS also defines that raster images (such as photos) are, by default, displayed with one image pixel mapping to 1px. A photo with a 600 by 400 resolution will be 600px wide and 400px high. The pixels in the photo thus do not map to pixels of the display device (which may be very small), but map to px units. That makes it possible to exactly align images to other elements of a document, as long as you use px units in your style sheet, and not pt, cm, etc.”

For the high resolution screens that have more physical light emitting dots per pixel, it's possible to display elements smaller than the defined pixel size more clearly. However, for screens with lower resolutions, special rasterisation rendering techniques have to be applied [WikiPixel17]. These techniques are implemented differently on across software

systems and rendering engines. They might lead to unwanted effects, for example aliasing or unrecognizable blurring.

The Gravity of the Space Problem

By dividing the cells into sub-grids, the space runs out exponentially fast. For example, a bar with the width of 30 pixels has the appearance of 1.058 centimeters on a 72 dots per inch (DPI) screen, which is a generous size for a single bar. This width can only be divided by 5 twice before it goes into the subpixel dimension.

Moreover, the gaps between cells also must be accounted for. When the cells are as small as a pixel, the spacing between them would be roughly as big as the cells themselves. That means only approximately half of the space is available for drawing cells.

There are only limited options to use the space efficiently, while ensuring the visibility of the cells and avoiding unwanted rendering effects.

Grid Size Reduction

The obvious solution to the space problem is to use space sparingly. When the grid is divided less, the cells will be larger and easier to recognized. A byproduct of less cells is less separation spaces wasted.

However, by doing so, the accuracy of the value estimation process will also be affected. For example, a 1×5 grid can be used to present a decimal place instead of 2×5 . With any luck, people might be able to distinguish between a whole cell and half scaled one. Once again, the rendering mechanism and the viewing device's resolution decide the matter. There can be no guarantee that the chart can be accurately deciphered on every media.

A 9-based grid as described in sub-section *2.1.1 – 9-Based Grid* can be used in space shortage situations. It can be as small as 5 by 6 pixel: each of the 9 cells occupy a whole pixel, between any three adjacent cells on both axes has to be two lines with the thick of a pixel separating them. [Need one more line of pixel to represent the remaining 10%. The 10% line can be mistakenly perceived as part of the spacing between grids. Colors could be used to solve the problem.]

4-based grid is good for an estimation with the accuracy rounded to 25%.

Omission of Cell Separators

The spacing between cells can be reduced to less than a pixel. The edges of the cells would be blurred on low resolution output devices, but big shapes can still be detected separately. For elements with either width or height around the 1 pixel limit, blurring can make them melt into their surrounding. Thin stripes standing next to each other can be blended into a rectangle that appears to be half transparent.

The spacing between the cells can be omitted if they are colored distinctively.

3 Applications of 2DBC

3.1 Introduction

The causes and difficulties of comparing similar values and estimating values on the traditional bar charts has been discussed in section 1.3 – *Limitations of the Classic Bar Chart*. To best examine more thoroughly the benefits 2DBC has to offer, we picked out some a real world data samples in the following sections.

The Survey We conducted a quick informal survey with a very limited number of just under ten people. Most of them don't have specialized knowledge concerning the field visualization.

The following charts were shown to the participants on different 24 inches computer screens with full HD resolution. The distance between the screens and the observers' eyes varied from subject to subject. The participants were asked to sit at a distance they are comfortable with, which was around 40 to 50 centimeters across the screen. Nobody had any difficulty recognizing the smallest details in the chart, which are thin stripes with the width of 2 pixels.

The participants were asked to compare the values of some close valued columns and to read all the values of the charts.

3.2 Case Study: Exchange Rate Between Euro and U.S. Dollar over the months January and February of 2017

Exchange rates play an important role in foreign trading as well as in many other aspects of the world's economics. For example, a depreciation (devaluation) will benefit exporting businesses, since it makes their goods cheaper in the foreign markets, but it also makes the imported goods or raw materials more expensive. These companies

will then have a choice to sell their products cheaper and to increase the quantity of purchases or to keep the price stable and benefits from a bigger profit margin, provided that the price of productions stay the same [Pettinger13].

The US dollar is commonly used as the base currency for exchange rates. The rates are generally expressed to four decimal places [Investopedia17].

Normally, line charts are used to visualize exchange rates due to their ability to display more data in the same chart space. Trends can also be recognized more easily with line chart. However, reading accurate information, including comparing two similar values, on a tightly packed line chart can sometimes be a challenging task.

The bar chart is more suitable for displaying a limited number of data points, if the chart wants to achieve the goal of providing a closer look on the data.

Figure 3.1 shows the exchange rate of the eurozone's official currency, the euro, to the U.S.A.'s currency, the dollar. The exchange rate in this chart tells us the price of a euro in US dollars on a specific day. One euro in this time period always costs slightly over one dollar. The changes of the exchange rate over time are the main point of interest, not the ratio of the values in comparison to each other. For that reason, these kinds of graphs usually don't start at the zero point on the y-axis.

Classic Bar Chart Figure 3.1a shows the classic version of the case study's data.

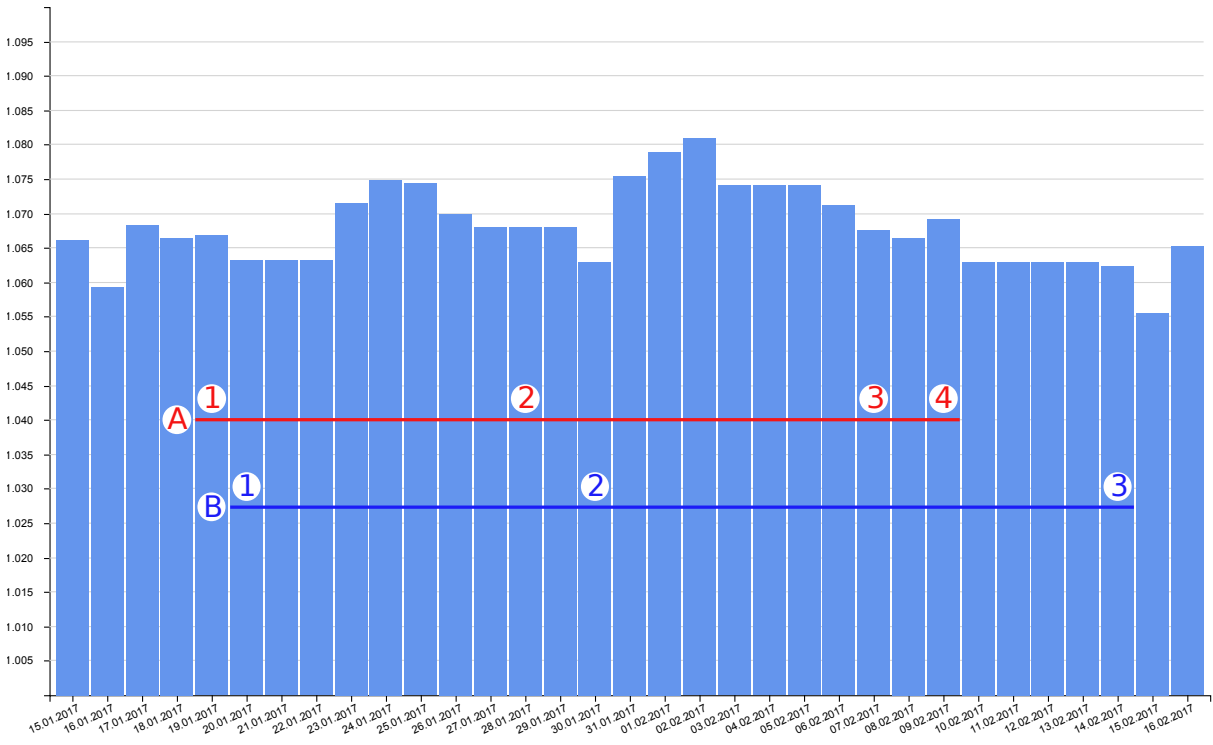
The exchange rate seems to oscillate constantly, but it seems to stand still at four different positions within the month. A closer inspection at the dates tells us that those time always fall on Friday, Saturday and Sunday. A person with little knowledge about how the exchange rate works can easily make the assumption that the price, much the same as the stock price, doesn't change over the weekend.

The more interesting values are marked on the chart as annotated groups.

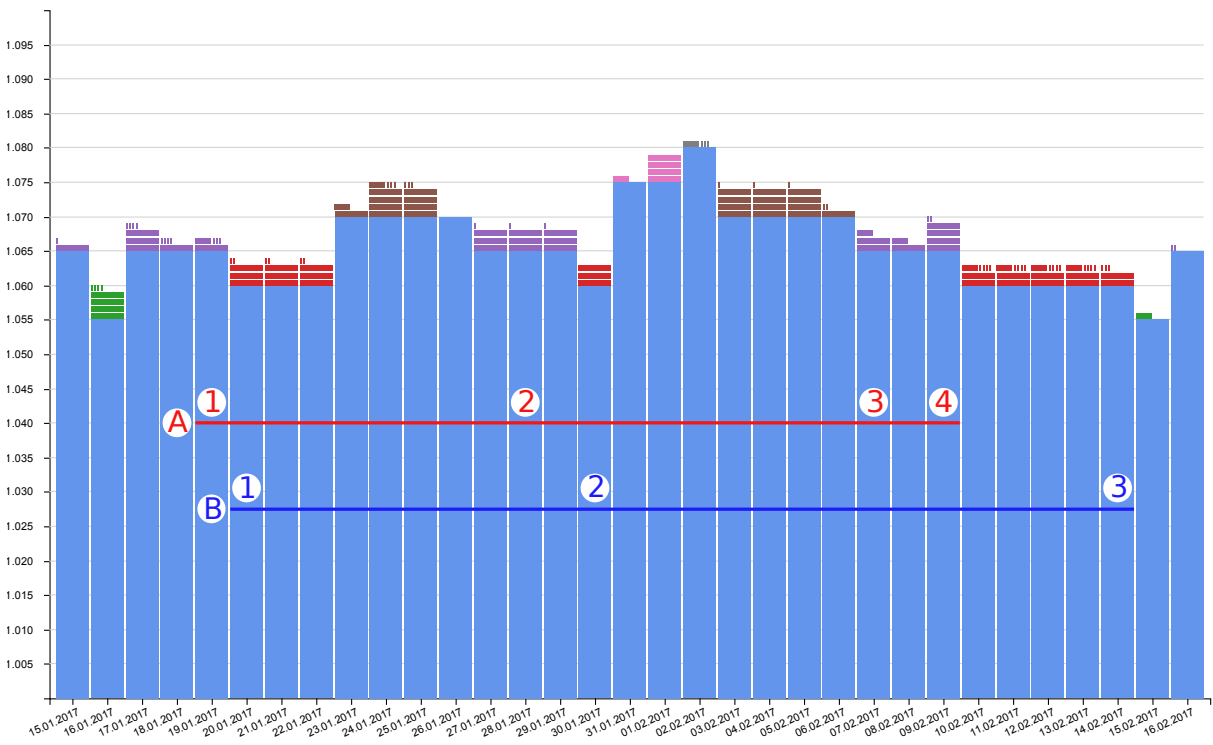
The columns 1, 2 and 3 in group A have very small differences and are difficult to compare because of their distance from each other. With a good resolution of the graph on the screen, which is slightly different from this printed version, most of the surveyed people can recognize that column 2 is slightly higher than columns 1 and 3. A little more than a half of the people could correctly detect that column 4 is higher than column 2, but some of them had to take their time and inspect the bars more closely.

However, when asked to judge the difference of columns 1 and 3, most of them weren't certain about their assumptions. To some surveyees, the gap between column 2 and 3 seem bigger than that between columns 2 and 1, hence the majority of verdicts say that value of 1 is higher than 3. But the data doesn't agree with that statement.

3.2 Case Study: Exchange Rate Between Euro and U.S. Dollar over the months January and February of 2017



(a) Classic Bar Chart



(b) 2D Bar Chart

Figure 3.1: Exchange Rate Euro to U.S. Dollar over the months January and February of 2017. Data source: [TIMF17].

The seemingly bigger gap can be explained as an optical illusion that makes things look bigger than they truly are. Context actually plays a very important role on how we perceive graphical elements. A good example of optical illusion is the widely known Müller-Lyer illusion. The illusion was illustrated by two straight lines of the same length but with different arrow endings, which are perceived as different in size [Mül89].

The situation gets even more extreme with group B. This time, nobody was certain about their answers. The only assessment they can be confident about is that column 4 is shorter than column 3. The majority of participants reluctantly say that other columns are equal in height.

The *law of closure* in the *gestalt theory* states that our brains have the tendency to complete the shape of objects even when parts of them are absent [Wer22]. In this particular case, we are more likely to perceive the upper border of group B's bars as an edge of a rectangle, which means that they should be a straight line and all values are equal.

Once again, the facts don't agree with our intelligent brains.

Does 2DBC perform better? Figure 3.1b is the 2DBC version of the same data as Figure 3.1a's.

The length of the spaces limited by two auxiliary lines has the value of 0.005 dollar or half a cent. The space from the top of each bar to the next auxiliary line is transformed into a grid and filled with different sized rectangles. The long horizontal rectangles that have the same width as the bars denote one unit in the third decimal place, which is equivalent to a tenth of a U.S. dollar cent. The middle long horizontal rectangles represent half of that value, which is 0.0005 dollar. Each of the thin stripes represents a unit in the fourth decimal place, or a hundredth of a cent.

Even without that knowledge from the legend, we can easily see that the first column of group A has one full length rectangle, while the third column has 2. The number of wide rectangles of columns 2 and 4 are respectively 3 and 4. Everyone managed to rank the values of those bars considerably fast and correctly.

As for the equal weekend values, for example the values on the dates 20th, 21st and 22nd of January, people observed that they are identical almost instantly. They all have three tiers of horizontal rectangles and two vertical stripes on top. By quantifying the top area of the bars into countable sets of geometries, the process of comparing bars by height, which relies entirely on inaccurate and sometimes even deceptive visual approximation, has changed into the task of counting and comparing numbers, both of which have higher precision.

The inequality was also swiftly detected. The columns in group B all have different count of rectangles and stripes, making them easy to discriminate. The different stacking of basic geometries helps form diverse shapes, which the human brain is good at detecting preattentively [Hea+07].

Reading the value of the columns took a slightly more time. The participant have to first figure out how much value the rectangles and the stripes hold. Nevertheless, they everyone were able to read all four decimal numbers of each value, which were not possible with the normal chart. The reaction time decreased as participants spent more time with the grids.

3.3 Case Study: Formula 1 Lap Time

Formula 1 is the world's highest class of single-seater open-wheeled auto racing competitions. Its races first started in 1947 and have become the standard for the World Championships in its format since the year 1983.

Formula 1 is essentially managed by 3 entities: The Federation Internationale' de Automobile - FIA, the Formula One Management - FOM and the F1 Strategy Group [Thakur15].

Formula 1 races take place in Grand Prix events all around the world. One Season includes a minimum of 8 and a maximum of 21 races. The total laps in a Grand Prix are measured by the least amount of laps that are needed on a track to reach at least 305 kilometers. The Monaco Grand Prix is the only exception due to the lower average speed that is able to be driven on this track. The maximum race duration is two hours total racing time. Independent of how long the drivers are actually racing, a Grand Prix event will end after 4 hours' time from the beginning of the event.

In order to qualify for a Grand Prix, drivers must have a FIA Super License and must take part in the qualification rounds the day before the official race. This will decide the start position of each racer, prior to the race. Generally the qualifications will be decided in three sessions. The first session being 20 minutes, the teams with the longest lap times deciding the end-positions. The second session being 15 minutes, deciding the seven middle positions. The last session consisting of 10 minutes deciding the top 10 positions. Any racer that fails to reach a time (after penalties, etc.) that is within 107% of the best qualifier's time, will be disqualified with the option to apply for an exemption, should they have proven a better time in the free run period.

At the end of each Grand Prix race the top ten drivers to cross the finish line will receive points ranging from 25 points for the 1st Place position, down to a single point for the

3 Applications of 2DBC

10th Place. The final results of each Grand Prix will be summed up at the end of the season and the driver with the most points is crowned the World Champion of Formula 1 racing [Formel117; WikiFormulaOne17].

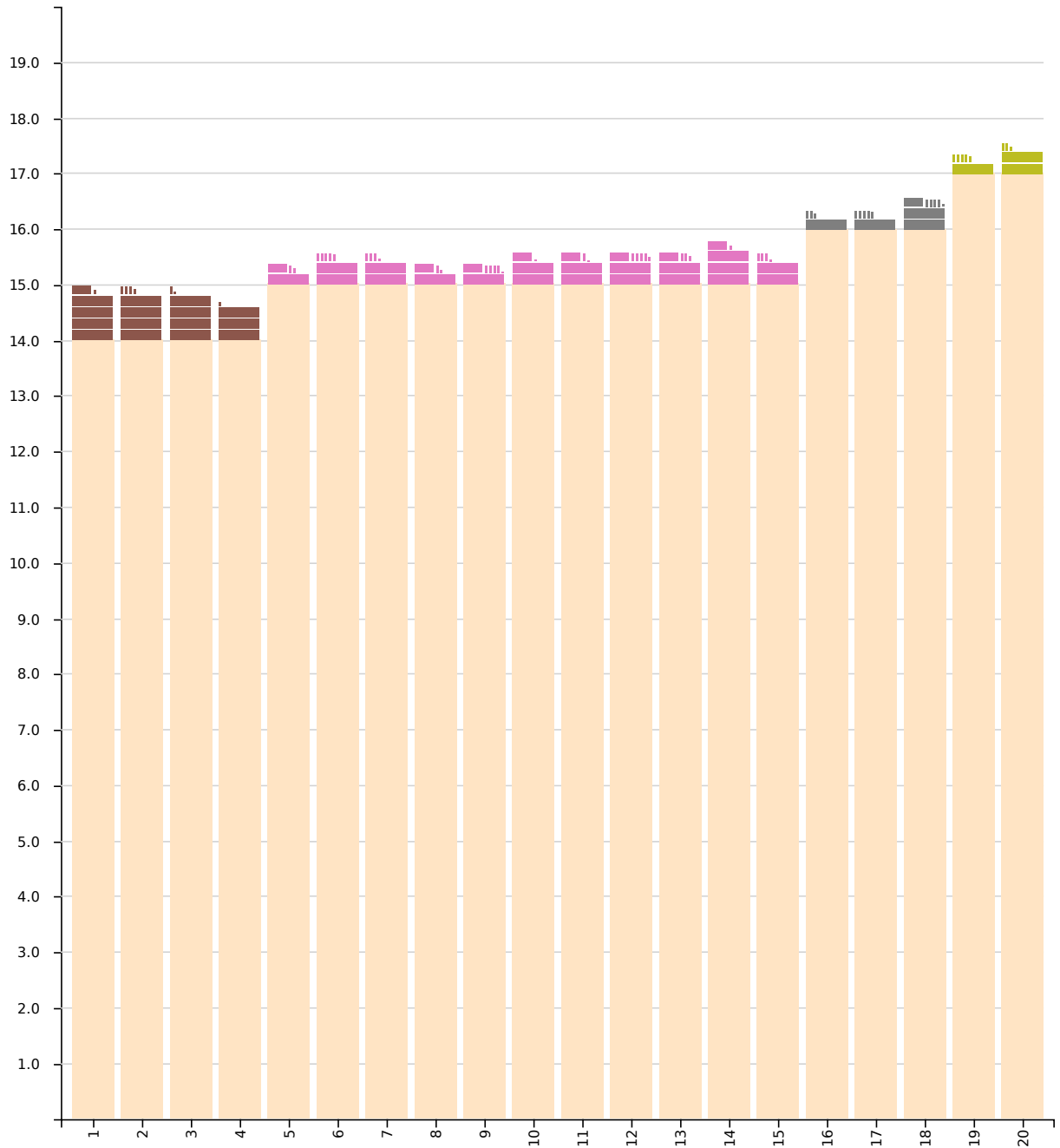


Figure 3.2: Results of the first qualifying round of the Monaco Grand Prix 2016. Data source: [WikiFormulaOne11].

The time shown on the chart is the best lap time of the qualifying round of the 2016 Monaco Grand Prix. The scale is cut off at 1 minute. It only shows the number of seconds after the first minute. Results are measured with milisecond precision, which means the values have 3 decimal places. The difference between winning and loosing could be as small as hundreds of miliseconds.

Case Study: Temperature Change There is a close connection between global temperature and climate change. Small variances in temperature can have a large impact on sea level rise. For every 0.4 degree Celsius increase in the global temperature, the sea level increases sharply by 5 centimeters in a low-estimated climate change scenario; or could go up to 11 centimeters in a high-estimated scenario [MWC+05]. In order to achieve a precise analysis, bar chart presentation would be the ideal tool in clarifying even a minimal figure.

Anomalies vs. Average Temperature While conducting studies about climate change, temperature anomalies are favored more compared to absolute temperature. Variation between the current and an average, or baseline, temperature defines a temperature anomaly. Commonly, 30 or more years of temperature data would be averaging to arrive at the baseline value. If the observed temperature is warmer than the baseline, it will be proved by a positive anomaly. On the contrary, if the observed temperature is cooler than the baseline, a negative anomaly will be produced. There are several factors that can affect the average calculation of absolute temperature data such as station location or elevation. For example, higher elevations is usually cooler than lower elevations, and urban areas is normally warmer than rural areas. Nonetheless, these aspects have less impact on anomalies. For instance, an area's temperature during summer may be cooler than average, both at a mountain top and in a nearby valley. However, their respective absolute temperatures could be really different.

When stations are added, removed, or missing from the monitoring network, problems could arise. In these situations, anomalies could help reduce the obstacles. As shown on the diagram, there are absolute temperatures for five neighboring stations as lines, with the 2008 anomalies as symbols on the right. When compared to the absolute temperatures, all the anomalies fit into a tiny range. If one station was removed from the data, the average anomaly would not change drastically. However, depending on which station omitted from the record, the overall average temperature could change notably. For example, if data of the coolest station, Mt. Mitchell, was removed from the graph, the average absolute temperature would become greatly warmer. Yet, the change in average anomaly would not be affected much due to the similarity between its anomaly and those neighboring stations [NOAAAnomaly17].

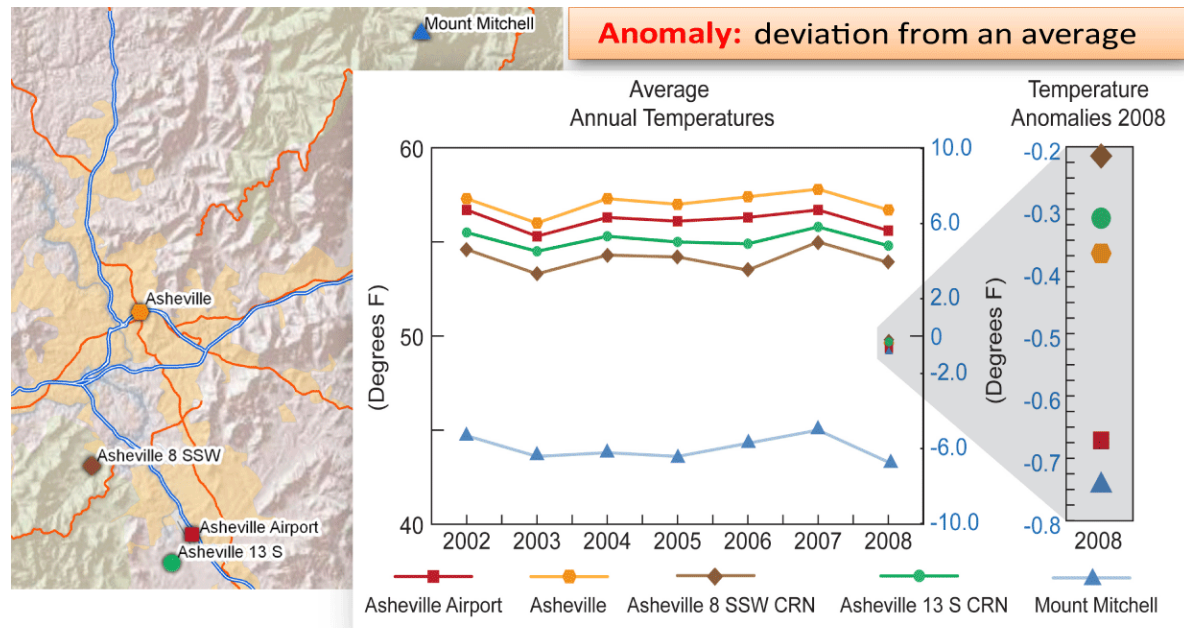
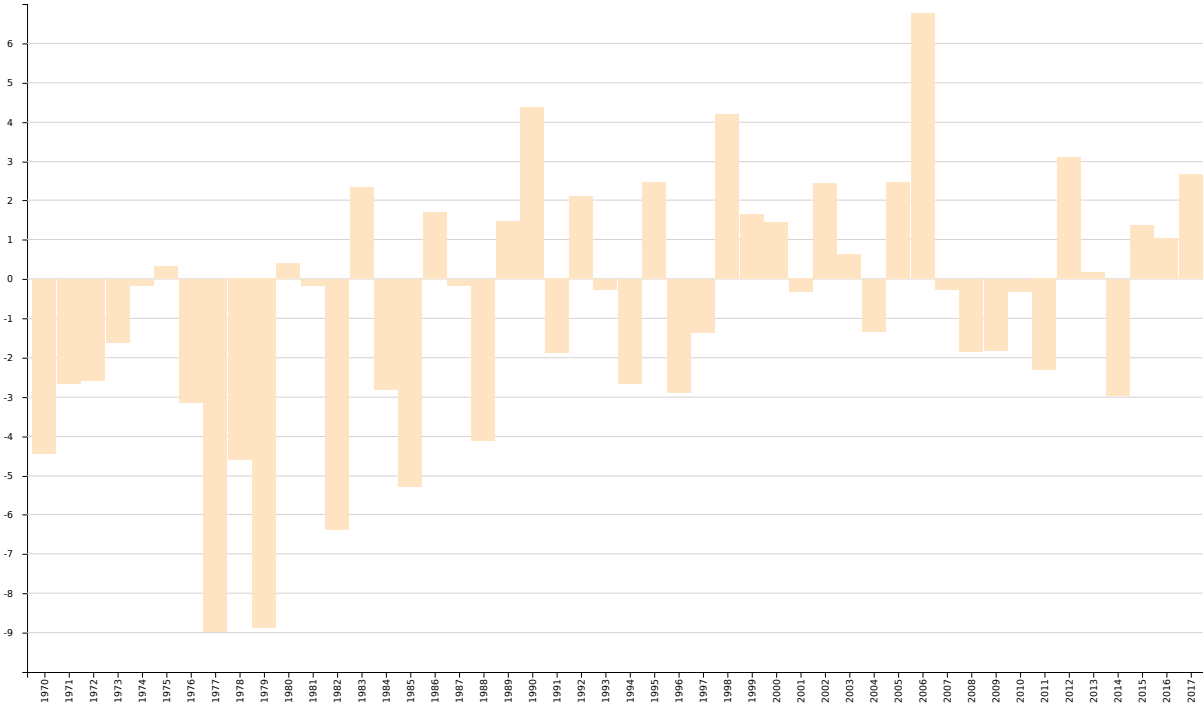
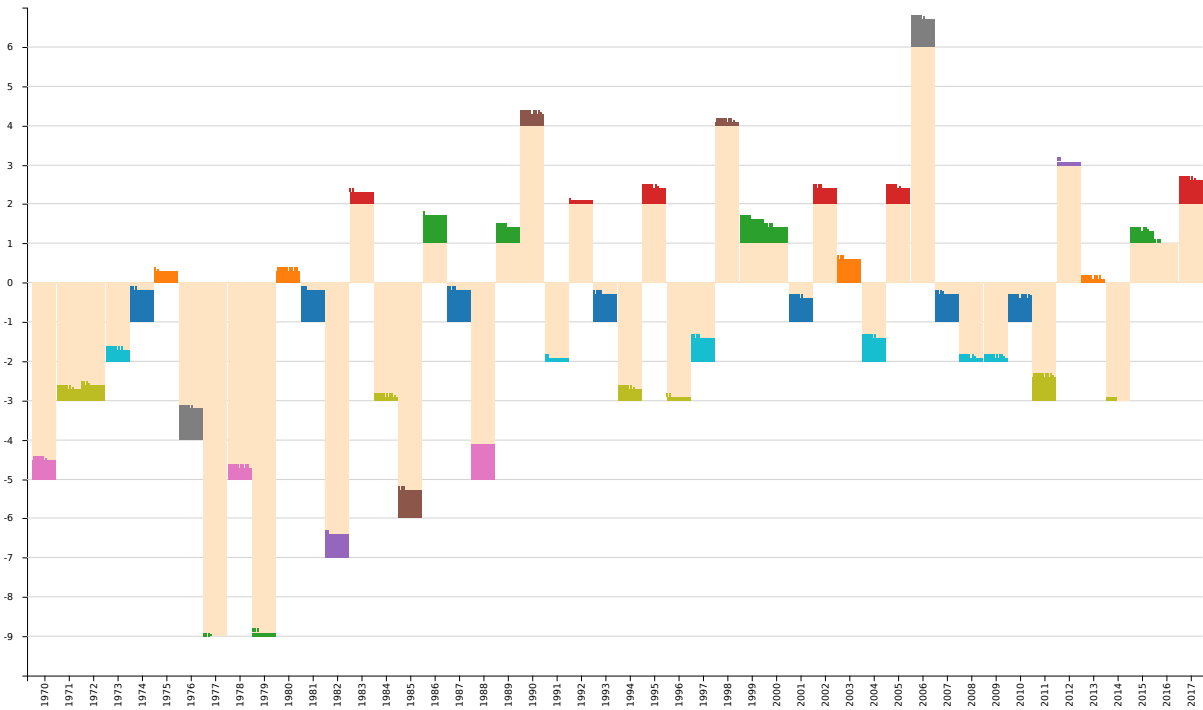


Figure 3.3: Average Annual Temperature and temperature anomaly 2008. Source: National Centers for Environmental Information [NOAAAnomaly17].

3.3 Case Study: Formula 1 Lap Time



(a) Classic Bar Chart



(b) 2D Bar Chart

Figure 3.4: Contiguous U.S. January Minimum Temperature Anomaly. Data source: National Centers for Environmental Information [NOAATempIndex17].

4 The Web Application

4.1 Introduction

A web application was developed to accompany this thesis. Its goal was to provide a platform to demonstrate 2DBC through data samples, while enabling users to easily experiment on their own data and different grid setups.

A web 2.0 application is fully capable and well-suited for this purpose. Nowadays, web sites can be accessed from most computing devices across many platforms. Web apps are becoming more and more powerful thanks to the fast advancement of modern browsers. They support scripting that can generate vector and raster 2D graphics on the fly.

The application is built using standard HTML5 technologies: HTML, CSS, JavaScript and the XML-based vector graphic format Scalable Vector Graphics (SVG). Therefore, it requires very little setup and should be able to run on any modern web browser.

The website is able to run as a local web page by simply open the file `index.html` or dragging it into a browser window. However, due to security measures taken by the web browsers, local web pages aren't allow to read files from the user's computer without their explicit consent through interaction. Data and chart files have to be upload manually through the web interface.

If the web app is hosted on a web server, it can be set up to visualize a predefined set of data. Users can also provide a link to their data files, should their browser allow cross origin HTTP requests.

The primary application logic was implemented with the renowned JavaScript-based visualization library Data-Driven Documents, which is usually shortened as D3.js or simply D3. The mobile friendly user interface is finished with the help of Semantic UI and jQuery [jQuery17].

The end result is an SVG image, which is displayed on the web interface and can be saved in SVG or PNG format. It can also be manipulated directly, for example to change the border radius of the bars or cells, which wasn't originally implemented.

The application is published under the MIT License. The latest version of the application is available on the project's GitHub repository at <https://github.com/d-huynh/2DBarChart>.

4.2 Features

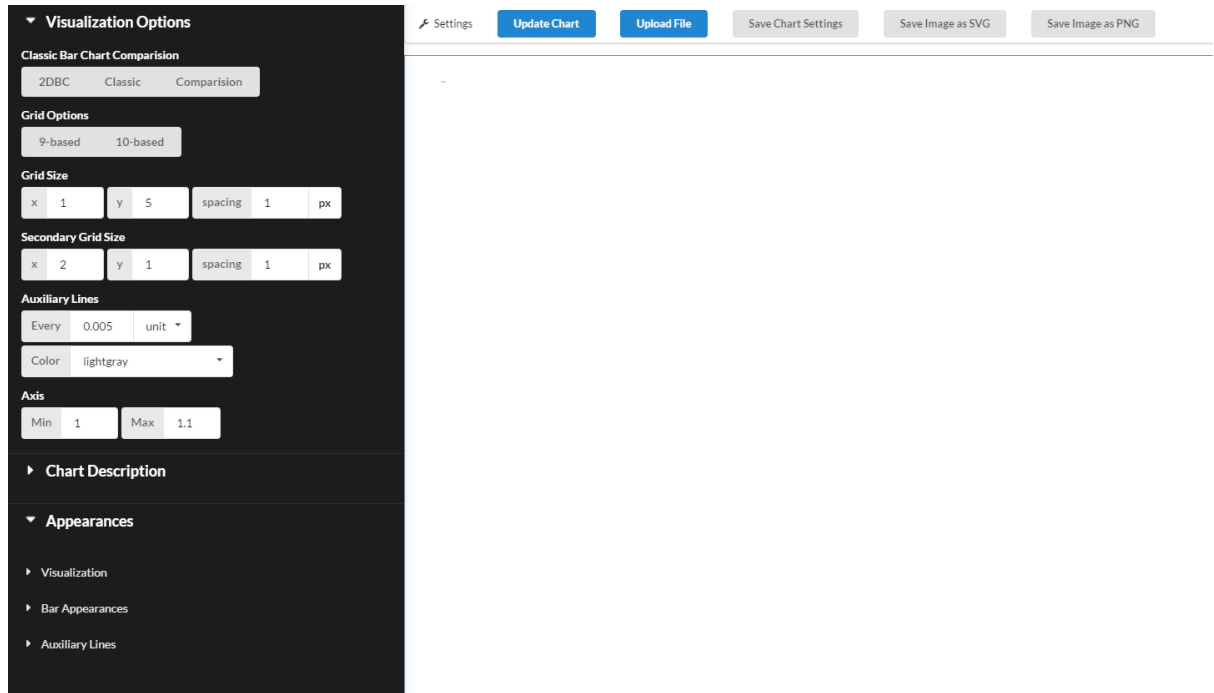


Figure 4.1: The user interface of the web application.

4.2.1 Charting

The application gives users the possibilities to create, compare and export traditional and 2DBC bar charts.

These actions are currently supported through the graphical user interface (GUI):

- Choose the distance between auxiliary lines base on y-axis unit or percentage of the scale.
- Set the minimum and maximum values of the y-axis.

- Define the size for main and subgrids by specifying the number of cells the grids to be divided into horizontally and vertically. The vertical and horizontal spacing between the cells can also be declared separately. Users can define up to five levels of grids through the GUI. Deeper subgrids can be defined by editing the setting file or the preference object.

These aesthetic characteristics of the chart can also be changed:

- The size of the chart.
- The thickness of the bars and the auxiliary lines.
- The distance between the bars.
- Color of the bars and auxiliary lines.
- The chart's title and subtitle, the description for the axes, the font and font size for these texts.
- The number format for the y-axis.
- The orientation for the labels of the x and y axes.

4.2.2 Data Upload and Chart Export

The users have the option to use their data in Comma-Separated Values (CSV) or JavaScript Object Notation (JSON) format for charting. Data files can be chosen from the Upload menu button or simply dragged into the website. Data files will not be actually uploaded to any web server, since they are read solely on the user's device through the HTML5's File API.

CSV is the main file format supported by the app. They are easy to create and edit, thanks to being stored as plain text. There are numerous free software tools available for major platforms. They assist in not only editing the files as strictly text, but also as table data.

The web application can work with minimal CSV files, which must have a column titled `value` and the values on separate lines. However, to make use of other features of the app, the CSV can contain extra information about the value as showed in Listing 4.1.

Users can choose the delimiter for their CSV files after opening them on the web app.

Even though JSON files are a little more complicated to edit, they are better suited for storing metadata along with the dataset, for example whether the `value` column is of

Listing 4.1 CSV data file format

value *required* – The value of the data point.

key The unique identifier for the value. If this column is missing, a key will automatically assigned for each value. This column also acts as the label on the x-axis, in case the label column is not present.

label The representative label for the value on the x-axis.

bar-color The color of the bar for this value.

property	description
{	
data	: an array containing the data points
axisMin	: the minimum value for the y-axis
axisMax	: the maximum value for the y-axis
}	

Table 4.1: JSON data file format

type integer, float or date and time. The application Currently supports the metadata fields listed in Table 4.1.

If the users wish to save a copy of the chart settings, they can download it and use the upload feature to recreate their chart at a later point in time. The generated preference files contain the dataset and all the settings of the chart in JSON format.

4.3 Libraries

4.3.1 D3

Data-Driven Documents [D3js15] is an open source JavaScript library specialized in visualizing data using web standards such as HTML, SVG and CSS. By emphasizing on these standards, it ensures compatibility across modern browsers. The generated graphical elements can be easily inspected through the browser’s native development tools, making them accessible for debugging purposes and further experimentation.

The library is well-known and well-received among visualization enthusiasts and professionals alike. It is capable of creating complex visualizations of large datasets, while

its flexibility allows transformations of the graphics as the data changes. It is therefore good for animation and interactive visualization.

The data-driven approach to visualization binds the data with the HTML and SVG DOM (Document Object Model) elements. Since the data's attributes are bound dynamically to the Document Object Model (DOM) elements', Data-Driven Documents (D3) will update the elements when the data's attributes are updated, thus saving developers the effort of writing update codes.

D3 also offers useful functionalities for laying out data. For example, our web app uses the provided linear scaling function to map a value to a coordinate on screen; the axes drawing and number formatting functions; the delimiter separated values files analysing function and the accompanied color schemes.

4.3.2 Semantic UI

Semantic UI [SemanticUI17] is an open source JavaScript library for creating rich and responsive web user interface.

Being relatively new in comparison to other frameworks, Semantic UI offers a bundle of viable components with polished appearance and pleasant user experience.

With the natural language descriptive syntax, hence the name Semantic, its best feature is perhaps the short learning curve. For example, containers with the class `ui three buttons` or `ui large green button` or `ui compact dropdown` will produce exactly what they mean in English.

Our web app uses Semantic to swiftly developed a simple user interface with a main menu, a settings sidebar and an dialog. The deciding reason for us to have chosen Semantic was the search dropdown component, which lets user choose a predefined item in the list or enter a new value themselves. Our web app uses this for the font and color inputs.

The disadvantages of Semantic are its poor documentation for niche cases, minor bugs that might be deal breakers for some projects, but more importantly, its small team of contributors and slow release process.

4.4 The Architecture of the Web Application

The application is open sourced and was designed with the openness in mind. More specifically, the data and the charting objects don't have any private members. While

this practice goes against software engineering's design guidelines and thus is not recommended for production, it allows every part of the code to be directly accessed, changed, replaced or extended. This practice suits the experimental nature of the project. For example, you can change the coloring function on the fly with the browser's developer tools, or replace the grid division algorithm without knowing exactly what the original algorithm does.

The Prefs object stores the settings of the chart. The DATA object contains data processing functions. The TwoDimensionalGrid object has the grid division logic. The VIS object contains the functions for mapping data to visual elements. All these main application files can be found under the folder `/js/app/`.

5 Conclusion and Future Works

5.1 Conclusion

In conclusion, the participant in our small informal survey have given positive feedback about the new technique of visualizing bar charts. They were able to compare the close values faster and with more confidence than with classic bar charts, even when the columns in comparison don't stand close to each other. The performances on the value estimation task yield results with higher accuracy. Depends on the grid division and the data, the number of exactly correct answers are very high, which isn't quite possible with the traditional bar charts.

Naturally, due to the space insufficient problem, the high precision is only applied to a certain extent. In scenarios where the partially filled cells are unavoidable, viewers still have to resolve to magnitude estimation of lengths. However, the error margins are would still be smaller than with the classic bar charts.

On the other hand, this new kind of bar chart isn't as intuitive as the traditional one. People need time to get used to the grid concept. Viewers also have to determine the values each different sized cell represents, before they can calculate the exact values of the bars.

The grids also deprive the simplicity of the bar charts and might cause visual cluttering. Another setback of 2DBC is the difficulty in noticing the trends of the data.

5.2 Future Works

There are several possible applications for this new kind of chart. For example, we could apply the concept to negative values, grouped and stacked bar charts.

The grids for the negative values would work similarly.

2DBC is highly suitable for the grouped bar charts, since the to be compared bars are distributed evenly throughout the chart. Because the bars already use so many different colors, the grids should use less colors to minimize the visual cluttering.

For the stacked bar charts, the partitions of interest could be distant not only on the x-axis but also on the y-axis. The space between the two auxiliary that is transformed into a grid would serve up to two bar segments. Viewers can compare the values of two segments of different bars by determine the exact values of them and compare those to each other. The process would me more complicated, but it should perform much better than the normal stacked bar charts.

Another aspect 2DBC could improve is maximizing the differences of grid shapes by stacking the cells in more varying sequences. Strong visual contrast should help human brains pick up the differences at a preattentive level. For example, alternatively shading the cells between left and right or skipping every other cell can produce strongly distinctive shapes and patterns for each value steps. Varying cell colors could also be used to create patterns without encoding any magnitude data.

Another idea worth exploring is experimentation with grid sizes and cell stacking, so that they give the viewers an impression of a slope and help them recognize the trends of the dataset more easily.

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All links were last visited on March 30th, 2017.

Declaration

I hereby declare that the work presented in this thesis is entirely my own and that I did not use any other sources and references than the listed ones. I have marked all direct or indirect statements from other sources contained therein as quotations. Neither this work nor significant parts of it were part of another examination procedure. I have not published this work in whole or in part before. The electronic copy is consistent with all submitted copies.

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