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An Engineering Perspective on Nonlinear H_∞ Control

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Abstract

Linear H_∞ control theory is a popular controller design tool in engineering practice. The notion of the H_∞ norm of a linear I/O-system can be extended to the nonlinear case, allowing the formulation of the nonlinear analog of the H_∞ standard problem.
The intension of this paper is threefold:

- we want to give a brief tutorial introduction to the basics of nonlinear H_∞ control theory.
- we want to critically discuss the applicability and usefulness of nonlinear H_∞ theory for practical nonlinear controller design.
- we want to show how nonlinear H_∞ theory can be applied to the problem of synthesizing approximately I/O-linearizing controllers.

The focus is on the question whether meaningful practical control problems for nonlinear systems can be expressed and solved in the nonlinear H_∞ framework. Approximate I/O-linearization of a realistic chemical reactor, that cannot be I/O-linearized exactly, is given.

1 Introduction

Since several years linear H_∞ control theory experiences remarkable popularity in engineering applications. The main reasons for this are the possibility to include robustness considerations explicitly in the design and the fact that meaningful physical performance objectives can be expressed as H_∞ design specifications. During the last couple of years a theory for nonlinear H_∞ minimization has been developed as an extension to the linear theory (e.g. [5, 13, 24, 14]).
The goal of nonlinear H_∞ control is to find a feedback K

$$K: \quad \begin{array}{rcl} \dot{z} & = & \alpha(z) + \beta(z)y \\ u & = & \gamma(z) + \delta(z)y \end{array} \tag{1}$$

for a nonlinear system Σ

$$\Sigma: \quad \begin{array}{rcllll} \dot{x} & = & f(x) & + \; g_1(x)d & + \; g_2(x)u \\ e & = & h_1(x) & + \; k_{11}(x)d & + \; k_{12}(x)u \\ y & = & h_2(x) & + \; k_{21}(x)d & + \; k_{22}(x)u \end{array} \tag{2}$$

with external input d, external output e, measured output y and control input u (Fig. 1) such that (a) the closed loop system is asymptotically stable and (b) the L_2-gain from external input d to external output e for the closed loop Σ_{cl} is smaller than some prescribed number γ:

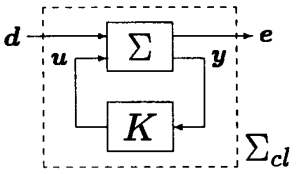

Figure 1: H_∞ standard problem

$$\int_0^T \|e\|^2 dt \leq \gamma^2 \int_0^T \|d\|^2 dt, \qquad \forall d(t), T. \tag{3}$$

By an abuse of notation we will shortly express the L_2-gain condition (3) as

$$\|\Sigma_{cl}\|_\infty \leq \gamma. \tag{4}$$

This problem is called the standard problem of (nonlinear) H_∞ control theory. The setup in linear H_∞ control is the same as discussed, with the restriction that feedback K and system Σ are both linear. Equation (3) means illustratively that the "gain" between the energy of the L_2-input-signal $d(t)$ and the energy of the L_2-output-signal $e(t)$ is smaller or equal to γ for all possible input-signals $d(t)$. Thus this holds also for the worst-case input $d^*(t)$. In the linear setup the smallest γ for which eq.(3) holds is the H_∞ norm of the linear transfer matrix Σ_{cl}, which gives this method its name.

Most practical control problems are of nonlinear nature. Therefore the obvious advantage of nonlinear H_∞ theory over linear H_∞ theory is that system nonlinearities can be taken into account and thus stability and performance can be achieved over a larger operating region.

Virtually all papers dealing with nonlinear H_∞ control start out with a problem description in standard form. However control problems arising in engineering practice are almost never in this form. In practical applications the physical system and desired control objective have to be brought into this form first. Thus every linear or nonlinear H_∞ controller design for a practical control problem consists of three design steps: (i) formulation of the physical control objectives as H_∞ design specification, (ii) transformation of this H_∞ problem into standard form, and (iii) solution of the resulting H_∞ standard problem. For many interesting *linear* control problems there exists good knowledge and rich experience on how to perform the first step. For nonlinear

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problems a simple and systematic approach is however not known. If the problem formulation is done properly, then the transformation to standard form is in most cases not problematic. The third step involves solution of two uncoupled Riccati equations in the linear case, for which reliable numerical tools exist. This third step is however much more involved in the nonlinear case, where nonlinear partial differential equations have to be solved.

The paper is structured as follows: In Section 2 open problems in connection with the solution of the suboptimal H_∞ standard problem are briefly discussed. Section 3 gives reasons, why linear H_∞ is so popular in engineering applications and contrasts this with the nonlinear case. Areas are pointed out where, from an engineering point of view, there are still deficiencies in the applicability of this method. In Section 4 we demonstrate an interesting application of nonlinear H_∞ theory to the synthesis of approximately I/O-linearizing controllers. For this problem a systematic formulation of the H_∞ standard problem is given.

2 Comments on the computation of nonlinear H_∞ controllers

As in the linear case the following simplifying assumptions ("standard assumption") are usually assumed to hold for system (2) in order to allow a simple solution of the nonlinear H_∞ problem:

$$\begin{aligned} \boldsymbol{k}_{11}(\boldsymbol{x}) &= 0 \\ \boldsymbol{h}_1^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{k}_{12}(\boldsymbol{x}) &= 0 \\ \boldsymbol{k}_{12}^{\mathrm{T}}\boldsymbol{k}_{12}(\boldsymbol{x}) &= I \end{aligned} \tag{5a}$$

$$\begin{aligned} \boldsymbol{k}_{21}(\boldsymbol{x})\boldsymbol{g}_1^{\mathrm{T}}(\boldsymbol{x}) &= 0 \\ \boldsymbol{k}_{21}(\boldsymbol{x})\boldsymbol{k}_{21}^{\mathrm{T}}(\boldsymbol{x}) &= I \\ \boldsymbol{k}_{22}(\boldsymbol{x}) &= 0. \end{aligned} \tag{5b}$$

We first consider the state-feedback H_∞ control problem for which (5a) is assumed to hold. The optimal state-feedback law that achieves closed loop stability and L_2-gain less than γ is then given by [24]:

$$\boldsymbol{u}(\boldsymbol{x}) = -\frac{1}{2}\boldsymbol{g}_2^{\mathrm{T}}(\boldsymbol{x})V_x^{\mathrm{T}}(\boldsymbol{x}) \tag{6}$$

where V_x is a solution of the Hamilton-Jacobi inequality

$$\begin{aligned} &V_x\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{h}_1^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{h}_1(\boldsymbol{x}) + \\ &\quad V_x\left(\frac{1}{4\gamma^2}\boldsymbol{g}_1(\boldsymbol{x})\boldsymbol{g}_1^{\mathrm{T}}(\boldsymbol{x}) - \frac{1}{4}\boldsymbol{g}_2(\boldsymbol{x})\boldsymbol{g}_2^{\mathrm{T}}(\boldsymbol{x})\right)V_x^{\mathrm{T}} \leq 0. \end{aligned} \tag{7}$$

The solution thus implies to solve the nonlinear partial differential equation (7). A global analytical solution is of course not feasible except for simple problems. In principle most approaches can be applied, that were developed in the sixties and seventies in connection with the analytical and numerical solution of the optimal regulator problem for nonlinear systems (e.g. [4, 23]). Most schemes are nevertheless very involved, except for special cases. Therefore often, a practical application will be prevented due to the lack of a feasible computational procedure. We want to mention one particular approach, that is mostly used in connection with nonlinear H_∞ control, namely Lukes' method [16]. This method is based on a series expansion of the problem and leads to a local approximate solution. For problems with less than ten states an approximate solution up to terms of order ten can be calculated using this method if the linearized problem is not degenerated. The obvious disadvantage is the exclusively local character. A promising new approach is based on integral manifold theory [8].

In the output-feedback case even the character of the solution is not yet fully understood. Necessary conditions [12] as well as sufficient conditions under certain assumptions [25] for the existence of controllers are known. In [14] necessary and sufficient conditions are given (when only smooth solutions are considered), that lead to a pair of partial differential equations of Hamilton-Jacobi type. One of the partial differential equations also contains derivatives with respect to time thus leading to an infinite dimensional controller that needs to be computed on-line.

The state-feedback problem is well understood, but very few meaningful practical control problems can be formulated in an H_∞ state-feedback framework. With the few remarks on the solution of the output-feedback case above, we wanted to indicate the difficulties related to the solution of the much more important output-feedback case.

In addition to the computational difficulties the stringent assumptions (5) on the generalized plant (2) lead to serious problems. It turns out that the so-called 'standard' assumptions (5) are in no way standard in real life applications. In the linear setup assumptions (5) can always be satisfied for example by various 'loop-shifting' transformations [22]. Assumption $\boldsymbol{k}_{22} = 0$ can be assured simply by applying the change of variables

$$\boldsymbol{y}_{new} = \boldsymbol{y} - \boldsymbol{k}_{22}\boldsymbol{u} \tag{8}$$

to the original plant. The other transformations, especially the one to assure $\boldsymbol{k}_{11} = 0$ are more involved, but nevertheless there are numerically stable algorithms to perform them. In the nonlinear setup there is yet no simple equivalent to the linear 'loop-shifting' transformations. Some assumptions (e.g. $\boldsymbol{k}_{12}^T\boldsymbol{k}_{12} = I$) can be satisfied by a simple input transformation. Also $\boldsymbol{k}_{11} \neq 0$ constitutes no severe problem as a general solution can still be written down. However the case $\boldsymbol{k}_{22} \neq 0$ is as yet unsolved. The trivial change of variables (8), used in the linear case, fails because $\boldsymbol{k}_{22}$ is state-dependent in the nonlinear case.

3 Formulation of practical control problems as H_∞ standard problems

In this section we examine whether meaningful nonlinear control problems can be formulated in the H_∞ framework and compare the linear and the nonlinear case.

Two important design objectives can be expressed as H_∞ specifications in the linear case: robust stability with respect to unstructured uncertainties and nominal performance with respect to a certain performance definition. The possibility to express conditions for robust stability as a bound on the H_∞ norm of certain transfer matrices is the main reason for choosing the H_∞ framework for linear control problems.

A typical and often used unstructured uncertainty definition for linear problems is the following: The real plant G_r is assumed to consist of a series connection of a linear nominal plant G_n and a linear uncertain part (Figure 2):

$$G_r(s) = \big(I + W_u(s)\Delta_m(s)\big)G_n(s), \quad \|\Delta_m\|_\infty \le 1. \quad (9)$$

Transfer matrix Δ_m in eq.(9) can be an arbitrary lin-

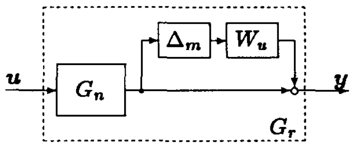

Figure 2: Multiplicative output uncertainty as example for an unstructured uncertainty description.

ear dynamical system of appropriate dimension, with the only assumption that its H_∞ norm is bounded. Knowledge about the frequency dependent size of the uncertainty can be embodied in the so-called *uncertainty weight* W_u. This special type of unstructured uncertainty is called *linear multiplicative output uncertainty.* The term "unstructured" refers to the fact, that only *one* perturbation Δ is assumed at *one* location in the control loop, and that this perturbation Δ may be an arbitrary norm bounded dynamical system. Necessary and sufficient conditions for robust stability of the closed loop are given in the following theorem [7]:

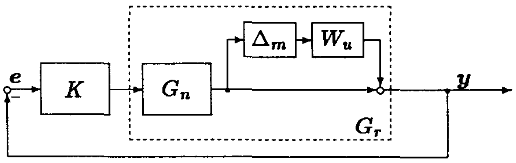

Figure 3: Closed loop with uncertain real plant G_r and controller K.

Theorem 1 *Under certain (nonrestricting) assumptions [7] the closed loop system in Figure 3 is asymptotically stable for all stable perturbations Δ_m satisfying $\|\Delta_m\|_\infty < 1$ if and only if the nominal closed loop is asymptotically stable and*

$$\|W_u \cdot G_nK(I + G_nK)^{-1}\|_\infty \le 1. \quad (10)$$

Obviously, condition (10) can be brought in the form of the linear H_∞ standard problem easily. The same type of theorem can be given for other "locations" of the uncertainty Δ, like for instance multiplicative input uncertainty or additive uncertainty [7]. We refer to different definitions of the uncertainty description (i.e. definitions where the assumed location of perturbation Δ is at different places in the system) as different "types" of unstructured uncertainty.

In practical applications it is usually not known which type of unstructured uncertainty is present in the system. For linear systems it is however possible to express stability with respect to one type of uncertainty by a robustness condition on another type. This is potentially very conservative, meaning that for example robustness with respect to a large uncertainty at the system input is needed in order to achieve robustness with respect to a small uncertainty at the system output. The larger the condition number of system G_n, the more conservative the robust stability condition will be. For practical applications this is usually still a sensitive thing to do: You can make your best guess where the main uncertainty has to be expected. If, by a robustness condition of type (10), robust stability is assured for the assumed uncertainties, the closed loop also exhibits robustness with respect to other possible locations.

Based on the small gain theorem similar results can be shown for the nonlinear case. Nonlinear unstructured uncertainties can be defined in an analogous way as in the linear case with G_n, W_u being nonlinear I/O-operators and Δ_m being an arbitrary nonlinear I/O-system with (nonlinear) L_2-gain smaller than one. A sufficient, but *not* necessary condition for robust stability can also be given (e.g. [2]):

Theorem 2 *Under certain assumptions [2] the nonlinear closed loop system as in Figure 3 with nonlinear I/O-operators G_n, K, W_u and Δ_m, is asymptotically stable for all stable nonlinear perturbations Δ_m satisfying the L_2-gain condition $\|\Delta_m\|_\infty < 1$ if the nominal closed loop is asymptotically stable and*

$$\|W_u \cdot G_nK(I + G_nK)^{-1}\|_\infty \le 1. \quad (11)$$

The same type of robustness theorem can also be proven for nonlinear additive uncertainties, nonlinear multiplicative input uncertainties, etc. Like for linear control problems, in practical applications the actual uncertainty will not be of the type described. But for nonlinear systems different locations of the unstructured uncertainty cannot be shifted to other locations in general.

Example: We consider the simple nonlinear nominal system

$$\begin{aligned} y &= \sin x \\ \dot{x} &= -x + u \end{aligned} \quad (12)$$

and its linearization around the steady state $x_s = 0$

$$\begin{aligned} y &= x \\ \dot{x} &= -x + u. \end{aligned} \quad (13)$$

Both the linear as well as the nonlinear system have an L_2-gain of one. If we assume a multiplicative uncertainty with $W_u = I$ and $||\Delta_m||_\infty < 1$ for linear system (13), then it is easy to show that robustness w.r.t. this multiplicative uncertainty also guarantees robustness w.r.t. additive uncertainties satisfying $||\Delta_a||_\infty < 1$. For the nonlinear system robustness w.r.t. multiplicative output uncertainty $||\Delta_m||_\infty < 1$ does not give rise to *any* robustness to additive uncertainties. This can be seen immediately by the following argument: The output of nominal system (12) is always constrained to lie between minus one and plus one. A multiplicative output uncertainty can never lead to a change of sign between the real and nominal output y. If we consider additive uncertainties with $||\Delta_a||_\infty < \epsilon$, then for any $\epsilon > 0$ there exists always an input function $u(t)$ large enough and an uncertainty Δ_a^* satisfying $||\Delta_a^*||_\infty < \epsilon$ such that the sign of the nominal and real output differ. Therefore this system cannot tolerate any additive uncertainty. $\lhd$

This example demonstrates that a robustness guarantee with respect to one unstructured uncertainty does not permit any statements about the amount of robustness concerning other possible types of uncertainties in the nonlinear case. Moreover condition (11) by itself can be arbitrary conservative as it is only a sufficient condition in the nonlinear case, whereas it is also necessary in the linear case.

A certain robustness can however always be expected. Any linear or nonlinear H_∞ state-feedback law guarantees an "infinite gain margin and 50% gain reduction tolerance" to static input uncertainties [9]. Furthermore if feedback (6) renders the nominal closed loop exponentially stable, then solution V of eq. (7) is a Lyapunov-function for the closed loop and thus guarantees stability also for the closed loop with $f = f_n + \Delta f$ and $g = g_n + \Delta g$ etc. where Δg, Δf have to satisfy some Lipschitz condition.

We now turn our attention briefly to the formulation of performance specifications in the H_∞ framework. In the linear case two different approaches to accomplish satisfying performance are used in practice. Although both are somewhat similar, a clear distinction can be made from an application point of view. The most common approach is the so-called *loop-shaping design*. In loop shaping design important characteristic quantities of the closed loop, reflecting major closed loop properties, are iteratively formed in order to achieve the desired performance. There is usually a very limited number of those characteristic quantities that are of prime importance in a specific practical application. This is especially the case for single-input/single-output systems. An often used special case is the shaping of the singular values of the sensitivity and complementary sensitivity function [7] over frequency. With some experience it is most often possible to "form" the singular values of these linear transfer matrices in few iterations so as to satisfy the performance specifications. Finding a compromise between conflicting specifications is transparently possible.

In the nonlinear case equivalent characteristic quantities reflecting the main properties of the closed loop, are not so easy to define. Due to the nonlinear nature, restriction to a *limited number* of quantities, that are meaningful for many problems, is not realistic. Therefore it is not so easy for the design engineer to gain experience in how certain "shapes" can be achieved, and to understand the connection between those "shapes" and the closed loop behavior. Furthermore in the linear case the frequency dependency of these quantities allows a clear physical interpretation. This is not the case any more for nonlinear systems. Therefore a straightforward extension of linear loop-shaping for nonlinear systems is not easily possible.

It is however possible to extend the second approach to linear H_∞ control [6] to nonlinear systems. In contrast to loop-shaping designs, the performance to be achieved is *quantified* by an H_∞ criterion in this approach. Because of lack of space we do not want to give any further details here. Applications of this design approach are for example given in [3, 1].

In this section we tried to show that nonlinear H_∞ is not the sum of all the advantageous properties of linear H_∞ theory plus the possibility to consider nonlinearities. At least at present nonlinear H_∞ is not an "all purpose" design tool as linear H_∞. For each practical control problem a sensitive setup considering robustness and performance has to be found. This is not always possible, but for many meaningful practical problems a solution can be found.

Exemplary we describe a systematic approach to the control of a large class of nonlinear systems in the next section.

4 Application of nonlinear H_∞ theory to approximate I/O-linearization of nonlinear systems

In this section we briefly consider the problem of approximately linearizing the I/O-behavior of a nonlinear system. This problem has attracted considerable attention in the engineering community during the last years. We will show that this problem can be solved by considering an appropriate nonlinear H_∞ control problem. In this context a systematic formulation of the H_∞ standard problem can be given.

The underlying idea is to formulate the approximate I/O-linearization problem as the (almost) disturbance decoupling problem shown in Figure 4: We want to find a compensator K such that the compensated nonlinear system has the same I/O-behavior as the linear reference system G, i.e. the output z has to be small for all possible inputs w. The problem of exact disturbance decoupling is by now well understood [11, 21]. For the almost disturbance decoupling problem several approaches are known [18, 19]. Here we consider the H_∞ almost disturbance decoupling problem [20] where a feedback is sought that attenuates the effect of L_2-inputs w on the output e to an arbitrary degree of ac-

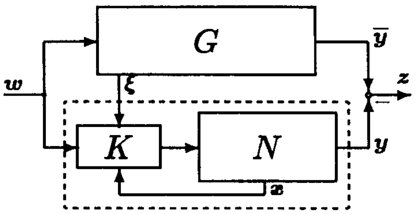

Figure 4: Configuration for approximate I/O-linearization

curacy. We make two further simplifications in order to be able to apply standard nonlinear H_∞ theory: We only require that the L_2-induced norm be smaller than some bound γ, plus we include a term related to the control effort in the criterion. The problem we want to solve is thus: *Find a state-feedback law*

$$u = \alpha(x, \xi, w) \tag{14}$$

so that the following inequality is satisfied for all $w(t) \in L_2$:

$$\int_0^T (\|z\|^2 + \epsilon\|u\|^2)dt \leq \gamma^2 \int_0^T \|w\|^2 dt, \quad \forall w(t), T. \tag{15}$$

Through inclusion of the term $\epsilon\|u\|^2$ we try to find a compromise between linearity and control effort. From a practical point of view this is a very sensible and necessary thing to do because we do not want to pay an increasing linearity by excessive input moves. The case $\epsilon = 0$ gives rise to a singular H_∞ problem for which only preliminary results are known [17]. A further advantage of considering the case $\epsilon \neq 0$ is that systems can be approximately linearized for which an exact I/O-linearization is not possible. This is also demonstrated with the example below. For systems that can be exactly I/O-linearized, the almost disturbance decoupling problem can be solved up to an arbitrary degree of accuracy. In connection with nonlinear model matching a similar approach was also followed in [10].

Example: Approximate I/O-Linearization of a nonlinear CSTR by nonlinear H_∞ minimization

As an application of approximate I/O-linearization we consider the production of cyclopentenol from cyclopentadiene in a CSTR with cooling jacket. The reaction mechanism is described by

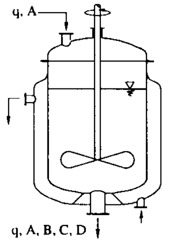

Figure 5: Nonlinear CSTR

$$\begin{gathered} A \xrightarrow{k_1} B \xrightarrow{k_2} C \\ 2A \xrightarrow{k_3} D \end{gathered} \tag{16}$$

with A denoting the initial reactant (cyclopentadiene), B the desired product (cyclopentenol) and C, D unwanted by-products. The mathematical model (four nonlinear differential equations) used to describe the process stems from the mass balances of A and B and from the energy balances of reactor and cooling jacket [15]. The reactor inlet flow q is the control input u. The output y is the product concentration c_B. The corresponding I/O-behaviour displays strong nonlinearity as can be seen from the step responses in Figure 6:

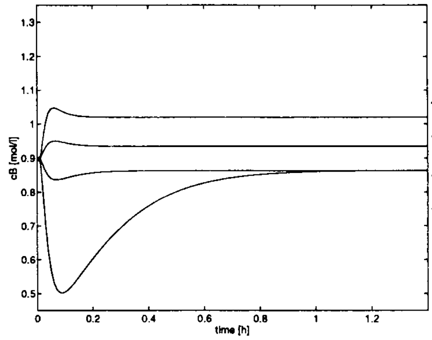

Figure 6: Step responses with different step sizes of the uncompensated system

Quadrupling the input step size does not lead to an output of four times the size; a step with opposite sign leads to a qualitatively different output function. This system has unstable zero dynamics at the operating point considered [15]. Therefore an exact I/O-linearization with internal stability is not possible. Following the procedure proposed above, we calculate a feedback that approximately linearizes the reactor. Choosing the Jacobian linearization of the reactor model as reference system, and solving the Hamilton-Jacobi equation connected with the resulting nonlinear state feedback H_∞ problem (with $\gamma = 0.01$ and $\epsilon = 10^{-5}$) up to fourth order terms by Lukes' method [16], yields an (approximate) solution to the proposed linearization problem. Indeed the step responses of the compensated system to the new input w display a significantly more linear behavior than the uncompensated system (Fig. 7). Especially the different qualitative behavior for input steps with opposite sign displayed by the uncompensated reactor is suppressed quite well. Nevertheless the compensated system still remains nonlinear. This is due to the unstable zero dynamics, that prohibits an exact I/O-linearization. A linear controller design based on the compensated system leads to a much improved performance as compared to a linear controller design for the uncompensated system [1].

5 Conclusions

In this paper we tried to give an engineering perspective on the applicability of nonlinear H_∞ theory to practical control problems. Three areas can be made out at present, where there are still major deficiencies as compared to the popular linear case. The first area concerns important open problems in the theoret-

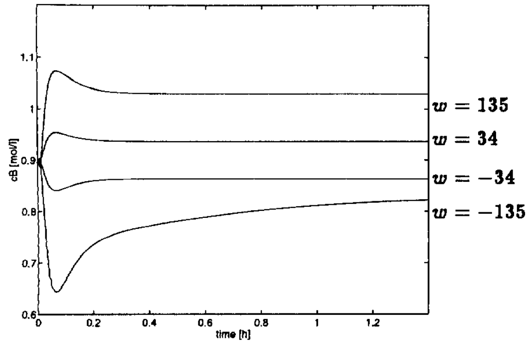

Figure 7: Step responses with different step sizes of the compensated system

ical development. From an application point of view, especially open problems in connection with the output-feedback case, the restrictive standard assumptions, the problem at optimality, and the singular problem are of main interest. The second area is related to the computational solution of the standard problem. At the moment few practical solutions are known. Because of the rapid development of computing power, there is however the hope that a numerical (on-line) solution of the Hamilton-Jacobi equations is feasible. The third problem area concerns the formulation of meaningful nonlinear H_∞ problems. For linear systems there is a wealth of practical problems that can be formulated in the H_∞ framework. For nonlinear systems this is not the case any more. We tried to point this out by exemplary discussing issues related to robustness towards unstructured uncertainty. It is clear that for nonlinear systems a generally valid design procedure cannot be expected. However for specific practical problems, nonlinear H_∞ theory can be useful with respect to achieving robustness and desired performance. We tried to demonstrate this with an application of nonlinear H_∞ theory to the problem of approximately linearizing the I/O-behavior of nonlinear systems. With this approach systems can be approximately I/O-linearized that cannot be linearized exactly.

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