



Flying UltraSonic - A new way to measure the wind

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Abstract

Measurements of flow conditions with tall meteorological measurement masts at complex sites are expensive and can only be carried out with great effort. Concepts and new measuring methods are needed to assess these sites.

This work aims to validate the performance of a measuring system based on UAV in complex terrain using on-site measurement. An unmanned aerial vehicle (UAV), more precisely a helicopter, was equipped with a standard 3-D ultrasonic anemometer. This UAV was positioned closed to a meteorological measuring mast and remained stationary at a constant altitude to measure the wind speed components. The data of the UAV were compared with a sensor installed on the measurement mast.

The measurements shows a good agreement with an absolute deviation of 0.004 m s^{-1} and a relative deviation of 0.047% for the horizontal wind speed. In the frequency domain the PSDs of the wind components u, v, w match the theoretical spectrum $f^{-\frac{5}{3}}$ for the inertial subrange very well.

With further improvements, this UAV equipped with a 3-D ultrasonic anemometer could be a very effective measurement tool for atmospheric research.

1 Introduction

The investigation of flow conditions in complex terrain is time consuming and expensive. To capture detailed flow information in complex terrain, high-frequency and high-resolution wind measurements are necessary. Currently, this data is typically obtained using in-situ measurements from meteorological measuring masts (met masts) or from ground-based remote sensing such as lidar or sodar. Ad-hoc measurements using manned and unmanned aircraft or aerial vehicles (UAVs) have also been tried, and smaller UAVs are becoming cheaper, more capable and thus more attractive for such applications.

Installing met masts equipped with in-situ sensors is cost-intensive and increases with the height of the mast, terrain complexity, and instrument quality. Lidar wind profilers have established themselves as an alternative to met masts. This is partly due to comparable costs, easier installation, and the strong correlations that have been demonstrated compared to in-situ measurements from anemometers [1]. For example, one representative study showed correlations with a slope of 0.9558, an offset of 0.1577 m s^{-1} and a R^2 of 0.9984 compared to anemometers on a mast in flat terrain [2]. In complex terrain, however, there are differences between the lidar measurements and the in-situ sensors of 10 % or more, depending on conditions [3]. Another disadvantage of ground-based lidar measurements is the poor reconstruction of turbulence and components of the Reynolds stress tensor [4, 5].

Alternative methods to met mast and lidar to measure the flow conditions are required. The use of unmanned aerial vehicles (e.g. aircraft and helicopters) represents a far more favourable and flexible way of recording flow conditions. These UAVs can be controlled by a pilot (a so-called remotely piloted aircraft system), or fly automatically with the help of an autopilot system that flies the UAV along waypoints on a pre-planned

trajectory. The on-board computers of the test vehicles log the output of the installed sensors (e.g., the measured flow conditions), the site coordinates, the position coordinates of the UAV and other flight parameters. UAVs can be categorised as fixed wing or helicopter aircraft, which have different flight capabilities. The fixed-wing aircraft can, for example, fly through a large area in different flight patterns at high speed. In comparison, the helicopter can fly to a location and then maintain its position. This results in different mission scenarios that leverage these capabilities:

- Fixed wing UAVs could be used for sampling atmospheric conditions for model initiation or validation [e.g., 6]
- Helicopters are potentially ideal for creating ad-hoc or virtual met masts or for providing statistical data on atmospheric conditions at specific locations.

The ability of a helicopter UAV to measure at a fixed point prompted its use in 2013 as part of a German national-funded project in to the measure of wind data in complex terrain using wind lidar (Lidar Complex, funding ID: 0325519). One of the goals of this project was to measure the flow conditions over a 100 m high forested steep escarpment in more detail. However, it was not possible to install a met mast at the site, and the project team searched for possibilities to measure the flow conditions stationary at a fixed position. This challenge motivated us to develop, build and test the "Flying UltraSonic" system in 2013 to measure the flow conditions at that site.

At that time, research into wind measurements by UAVs was in its early stage and the choice of possible carrier systems was limited. There was no multi-copter system available that could satisfy our requirements of sustained hovering flight and with the payload capacity to carry a 3-D ultrasonic anemometer.

For this development an existing helicopter system called the *Autonomous Multipurpose Platform for Airborne Research - AMPAIR* was used. AMPAIR was equipped with a 3-D ultrasonic sensor to measure three wind components (Figure 1). In comparison, a multi-hole probe is often used as a sensor for flow measurements from fixed-wing aircraft [7, 8].

The ultrasonic anemometer has three transmitter-receiver pairs placed at 0°, 120° and 240°. Between the transmitter-receiver pairs there is a predefined distance. The transmitters

emit sound waves and the receivers detect them. The wind speed components u , v , w are calculated from the different running times of the sound waves. The measuring principle should determine the correct velocity component for the translational movements.

The ultrasonic anemometer was installed for this application in a horizontal position. It is designed for vertical mounting. In principle, the position should not make any difference when evaluating the wind speed components. The manufacturer specifies the same measuring resolution ($\pm 0.01 \text{ m s}^{-1}$) for all three wind components.

At the time of the project (2013), the combination of helicopter carrier system and 3-D ultrasonic anemometer had not previously been used for wind measurements. The aim of this experiment was to verify whether it is possible to determine the flow conditions with these alternative measuring methods.

Herein we present the first verification measurements. These were carried out in flat terrain against instruments mounted on a 95 m met mast on a flat site. This allowed us to characterise its performance under homogeneous flow conditions. We present the development process and the results from the validation flights in order to show the potential for such a system.

Since our measurements in 2013, there have been other developments in the field of UAV-based wind measurements, both in the performance of the carrier systems and in the field of sensors. For example, Vasiljević et al. [9] used a commercial hexacopter UAV to carry the telescopes of a wind lidar. The optical unit was on the ground. A detailed validation study using that system has yet to be carried out.

We present our work here with the expectation that our experience will be useful for other researchers. In Section 2, the experiment is described with an introduction of the helicopter system AMPAIR and a description of the location. Section 3 describes the data preparation, and the results are presented in Section 4. We give our conclusions in Section 6.

2 Experiment description

The aim of this experiment was to investigate whether it is possible to measure the flow conditions with an unmanned helicopter. To evaluate the measurements, flow measurements from



Figure 1: Flightsystem AMPAIR with installed boom and ultrasonic anemometer at the nose.

a nearby stationary wind met mast are used. The two different measurement systems are compared with each other using statistical values such as mean value and standard deviation of the measurements, and the data are also examined in the frequency domain. The next section is structured as follows:

In Section 2.1 the used carrier system AMPAIR is introduced. Furthermore, in Section 2.2 the used measurement equipment is described, in the Section 2.3 the site is characterized and in 2.4 the flight trajectory is presented.

2.1 AMPAIR Helicopter

AMPAIR is an electrically-driven helicopter system, which was developed in the research project SOGRO - *Sofortrettung bei Großunfall* [10]. The AMPAIR is shown in Figure 1 and the technical parameters of the UAV are shown in Table 1. Due to the large rotor diameter (2.97 m) and the powerful electric drive, the helicopter has sufficient thrust to be able to carry the sensors used for flow measurements, which are also used on wind met masts.

The design of the sensor mounting was driven by the need to mount the sensor away from the influence of the rotor downwash. Furthermore, the mounting structure should not influence the flight aerodynamics, and it should prevent the transmission of vibrations to the sensor.

Several concepts for the sensor mount were explored. The design that was implemented used a carbon-fibre beam approximately 2.22 m long, protruding from the nose of the helicopter. This weighed approximately 0.5 kg. The use of carbon-fibre and the beam construction made the mounting structure extremely stiff compared to a single tubular mount.

This mounting should also have advantages with respect to the flow conditions around the

wind sensor. As built, the ultrasonic anemometer is about 1 m outside of the rotor circumference. According to helicopter theory [11], the air flows along a stream tube (control volume) through the rotor. Above the rotor the stream tube has a larger diameter than the rotor; In the plane of the rotor, the diameter is equal to the rotor; Below the rotor the diameter of the stream tube is smaller. This is based on the fact that the mass flux remains the same with higher flow velocity. If the helicopter is in hovering flight (holding position) with headwind, the helicopter must make a slight forward flight to withstand the air speed, i.e. the helicopter is slightly tilted forward. Thus the flow through the rotor is still steered backwards. The lack of influence of the rotor on the ultrasonic has not yet been confirmed experimentally; measurements on the ground do not provide reliable results, because the flow through the rotor cannot flow off freely and flows away sideways in all directions. This effect is not seen during flights away from the ground. It may be possible to estimate the influence of the rotor through experiments in a wind tunnel with defined flow conditions.

Wind measurements were made using a 3-D ultrasonic anemometer (Metek *uSonic-3 Scientific*). These sensors are usually installed vertically on booms extending out from a met mast. Because of vibrations and oscillations occurring during operation the sensor was mounted horizontally on the boom (see Figure 1; the anemometer is the spindle on the left of the figure). The symmetrical arrangement of the ultrasonic anemometer on the longitudinal axis of the helicopter enables a nearly vibration-free operation of the sensor.

The flight time depends on the number of rechargeable battery packs carried. Since the maximum take-off mass is limited, a compromise must be found between the payload (e.g., the number and types of sensors that are carried) and the flight duration (number of batteries). For this

reason, the system is equipped with up to six battery packs but can be flown with fewer. One battery pack contains 11 A h each and has a mass of 4.8 kg. This allows a flight time between 10 and 25 min with a payload of up to 10 kg. The maximum flight time of 25 min is only achieved if the complete payload is used for batteries. During the proof-of-concept flights reported here, the number of batteries was reduced for safety, which further reduced the flight time.

Table 1: Characteristics of the AMPAIR helicopter

Attribute	Value
Maximum take-off weight (MTOW)	46.15 kg
Rotor diameter	2.97 m
Rotation speed	780 rpm to 820 rpm
Powertrain, electrical	10 kW
Payload	up to 9 kg
Power supply	58 V rechargeable lithium-ion polymer battery
Control system	Pilot-in-Command & Computer-in-Command
Positioning system	DGPS
Time of flight (TOF)	max. 25 min
Data acquisition system sampling	10 Hz
Payload system sampling	10 Hz

The AMPAIR is equipped with an autopilot, which allows navigation between waypoints and fully automatic flights. In addition to an inertial sensor system, the helicopter also has an ultrasonic height sensor, which is used to measure low altitudes during landing. A telemetry connection to the system during the measurement flights allows a constant control of the flight and the adjustment of parameters in real time. For the heading of the helicopter a magnetic compass was used. During the experiment it was found out that the position of the magnetic compass was not optimally planned. The magnetic compass was placed as far away as possible from all electrical components and metal parts. Despite this, analysis of the data shows that the compass was affected by the electrical systems on board. The magnitude of the influence on the compass depends on the current in the electrical systems.

2.2 Measurement systems

This study compares data recorded with different measurement systems. These systems

are synchronized with GPS time to ensure that there is no time offset between the different data sources. Data from the wind met mast were collected by a central data acquisition system at a sampling rate of 50 Hz.

Due to its size, the AMPAIR system does not need to be operated with miniaturized sensors, which, compared to commercially available sensor system (e.g., so called FirstClass sensors) show a larger deviation from the beginning. The use of proven and tested sensors that have been on the market for a long time should prevent negative influences from the measuring sensor.

The motion of the UAV will cause the wind speed measured by the ultrasonic anemometer to differ from the wind speed. Therefore, it is necessary to combine multiple data streams to obtain a full understanding of the measurements. These data come from two systems:

- **Payload:** The payload contains the main components of the measuring system. The components are the electronics of the 3-D ultrasonic anemometer, the inertial measurement unit (IMU), the payload computer system electronic of the Differential Global Positioning System (DGPS), and the telemetry unit. The sensor head of the 3-D ultrasonic anemometer is connected to its evaluation electronics in the payload box and allows data recording up to 20 Hz. The AMPAIR is a mobile flying system with six degrees of freedom. In order to transfer the measured wind speed data from the fixed system of the helicopter into the global reference system of the met mast, the attitude is determined with the inertial measurement unit as well as the position via the DGPS. By recording accelerations and rotation rates via the IMU, the helicopter system can be record his own movements. Thus, wind speeds measured by the moving sensor can be corrected.
- **Autopilot:** The autopilot system logs all data of the autopilot system required for flight controller. These can be synchronized with the data of the payload via the GPS time. The position, altitude, acceleration, and heading of the helicopter are all required for the post-processing of the measurement wind data.

2.3 Test site

The experiment was carried out near Grevesmühlen in the district of North-West Mecklenburg, about 17 km south of the Baltic

Sea. The site is shown in detail in Figure 2. At the time of the measurement, three wind turbines were installed at the site and one 95 m high met mast. This is an agricultural area with scattered trees and bushes. Further west, at about 1.3 km distance, there is a small settlement, and further south, at approx. 800 m distance, a densely forested area. The site is a flat site according to the IEC 61400-12 standard [12].

On the met mast there are sensors at different heights for measuring wind speed, wind direction, pressure, and temperature (Table 2). The sensor used for the comparison was the *Thies Ultrasonic Anemometer 3D* at 93.2 m. The wind met mast has an undisturbed sector – which is not affected by the three wind turbines – from 121.83° to 355.17° (according to IEC 61400-12).

Table 2: Summary of the sensors on the met mast.

Type of measurement	Height [m]	Sensor type
Wind speed	95.5 (hub height); 89.2; 59.2; 40	Cup anemometer
Wind speed & direction	93.2	3-D Ultrasonic anemometer
Wind direction	88.3; 39.0	Wind vane
Rel. humidity	92.0	humidity sensor
Temperature	92.0	PT100
Air pressure	5.0	Barometer

The prevailing winds at this site are from the south-west (Figure 3), which is in the free stream sector of the met mast. Wind speeds of up to 14 m s^{-1} and turbulence intensities of up to 20 % dominate (Figure 4). The data for the wind statistics were collected from 04 September 2013 to 18 May 2014.

2.4 Flight phases

The flight consists of several phases carried out on the ground and in the air (Figures 2 and 5). These phases contribute to a safe and repeatable flight.

The first phase in the flight is a system check, carried out on the ground. All systems (communication from ground station to helicopter, payload, autopilot, sensors, actuators, etc.) were activated one after the other and checked for proper functioning. This check is very important because of the almost 50 kg weight of the UAV, which could cause considerable damage in the event of loss of control.

The next phase is the take off and approach

to the measurement location. These steps were flown under manual remote control by a pilot. The helicopter was flown near to the previously defined coordinates and altitude and then the control was transferred to the autopilot. The autopilot then flew the AMPAIR to the target position near the met mast.

The helicopter was deliberately flown at a distance from the met mast for safety reasons. First, the owner of the met mast requested that an appropriate distance be maintained to the met mast. Second, the distance allowed space for the UAV autopilot to regain control of the helicopter after an extreme event. In both cases the separation reduces the risk of a collision between the roughly 50 kg-heavy helicopter and the met mast or its guy cables. The average distance between the met mast and the helicopter position was 131.85 m (Figure 5). The measurement position was downwind of the met mast, approximately 89 m in the longitudinal direction (in flow direction) and approximately 97 m in the lateral direction (perpendicular to the flow direction). The autopilot had to adjust continuously to keep position as winds changed. This leads to a small variation in the position of the UAV. The standard deviation of the UAV position signal is 1.696 m and the range between the maximum and minimum distance is $\approx 8.5 \text{ m}$.

After the mission time expired, the pilot took control again and flew the AMPAIR back to the landing site via remote control. Manual control during takeoff and landing was required as the autopilot had not been extensively tested for those phases, and the risk of loss of control was too high. After touchdown, the data was saved and the systems were deactivated.

3 Data analysis

This section describes the steps to compare the measured data of the helicopter with the reference data of the met mast. First, the data from the UAV and met mast are transformed to a common coordinate system (Section 3.1). Then, additional velocities caused by the the helicopter's own motion and the groundspeed of the UAV are removed (Section 3.2).

3.1 Coordinate Transformation

To compare the measured wind speeds from the met mast with the measurements from the



Figure 2: Overview of the test site during this experiment with the three wind turbines (WT1-3) and the met mast (MM), the flight path (dashed cyan line), the measuring location (red dot) and the main wind direction (white arrow).

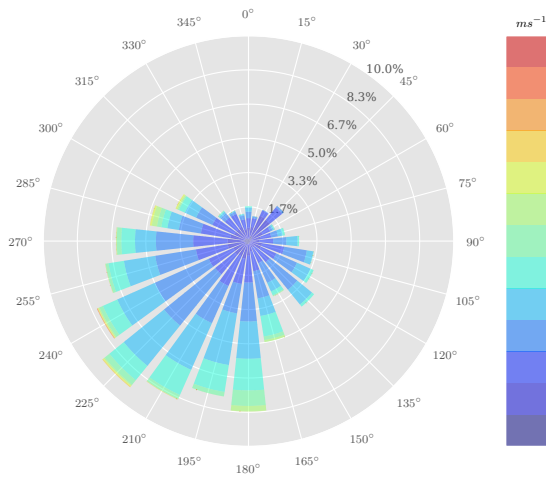


Figure 3: The wind rose of the test site showing the frequency of winds by direction and wind speed

helicopter, the data must be available in a common coordinate system. The coordinate system of the met mast (Global subscript g) is a right-hand coordinate system, the x-axis points to the north, the y-axis to the west and the z-axis to the sky.

The helicopter has two different coordinate systems. One is from the sensor (subscript s), and the other is from the body (body subscript b). Both are right-hand coordinate systems (Figure 6). In the sensor coordinate system the Z_s axis points horizontally to the front, the Y_s and X_s axes are rotated by the mounting angle α to the horizontal plane. The Z_b axes of the body coordinate system point downward, the X_b points forward and the Y_b points right. This is shown schematically in Figure 6b. The sensor coordinate system can be transferred to the body coordinate system with

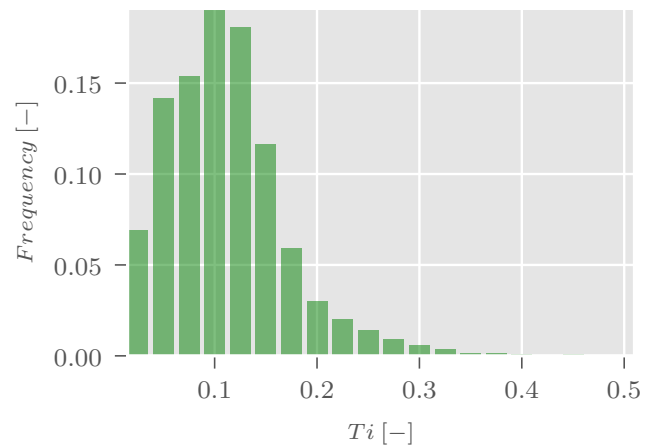


Figure 4: Histogram of the turbulence intensity of the test site.

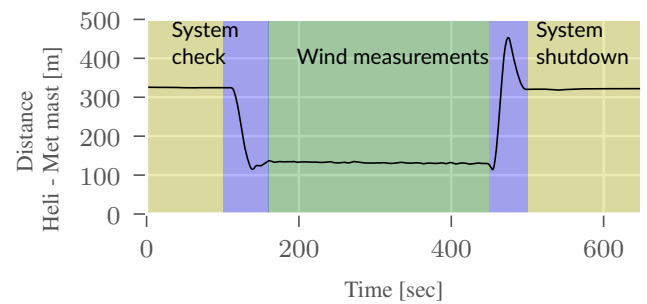


Figure 5: The different phases of the flight, including flight preparation and flight follow-up (yellow), approach and return (purple) and the measurement phase (green).

the fixed mounting angle $\alpha = 30^\circ$. Additionally the axes X_s to $-Z_b$ and the axis Z_a to X_b are swapped, the wind speed components are also swapped accordingly. In the body coordinate system the X_b -axis (roll) shows along the main fuselage, the Z_b -axis (yaw) is parallel to the shaft of the main rotor

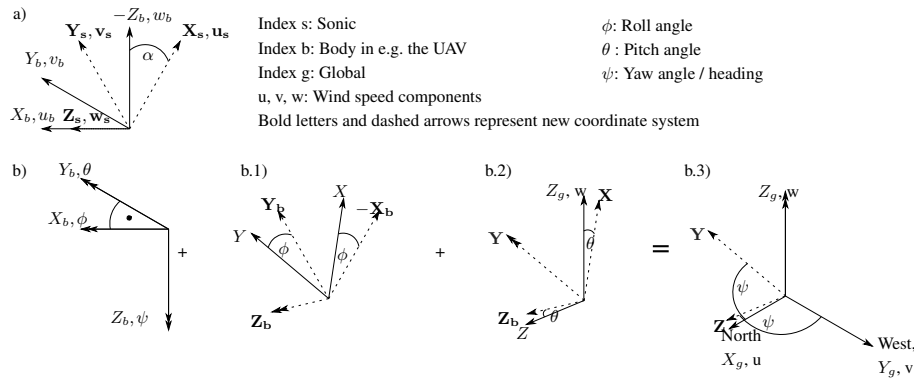


Figure 6: Schematic representation of the different coordinate systems. a) Dependency between sensor and body coordinate system. b) Transformation between the body and global coordinate system

and shows downwards and the Y_b -axis (pitch) accordingly. The $ZY'X''$ convention is used to transform the components in the body coordinate system into the components of the global coordinate system. This means a rotation around the roll, pitch and yaw angles. The following steps are necessary:

- Rotation along the X_b axis with the roll angle ϕ (b.1)
- Rotation along the new Y axis with the pitch angle θ (b.2)
- Rotation along the new Z axis with the yaw angle ψ (b.3)

$$\mathbf{T} = \begin{pmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & -\sin \psi \\ 0 & \sin \psi & \cos \psi \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix}_g = \mathbf{T} \cdot \begin{pmatrix} u \\ v \\ w \end{pmatrix}_b \quad (2)$$

This transformation is shown schematically in Figure 6 b.1 to b.3 and is described mathematically by equations (1) and (2). The wind direction (Equation (3)) and the horizontal wind speed (Equation (4)) can be calculated from the components in the global coordinate system.

$$v_{dir} = \arctan2(v, -u) \quad (3)$$

$$v_{hor} = \sqrt{v^2 + u^2} \quad (4)$$

3.2 Induced velocities

Due to the fluctuating flow conditions, the autopilot must constantly readjust to maintain the predefined position. The self-motion induces additional velocities, which the ultrasonic also records. These induced velocities must be deduced from the recorded wind speed signal. The IMU of the helicopter records the rotation rates $\dot{\phi}$, $\dot{\theta}$, $\dot{\psi}$. These are needed to calculate the induced velocities. The starting point of the calculation is the transformation matrix $\mathbf{T}$ from equation 1:

- With the help of $\mathbf{T}$ multiplied by the direction vector from the center of the ultrasonic anemometer to the center of gravity of the AM-PAIR $\vec{x} = [x, y, z]^T$ the movement of the ultrasonic anemometer in space can be described (Equation 5). The direction vector is $\vec{x}_b = [2.22\text{m}, 0.0\text{m}, -0.08\text{m}]^T$. $\vec{x}_b$ is given in the rotating system. The induced velocities are the time derivative of the equation 5.

$$\dot{\mathbf{X}} = \mathbf{T} \cdot \dot{\vec{x}} \quad (5)$$

- The vector $\vec{x}$ is independent of time, the angles ϕ , θ , ψ are time dependent. The representation of the individual terms of equation 7 is omitted here. The induced velocities calculated from equation 7 are presented in the global coordinate system.

$$\dot{\mathbf{X}}_g = \frac{\partial (\mathbf{T} \cdot \vec{x})}{\partial t} \quad (6)$$

$$= \left(\frac{\partial \mathbf{T}}{\partial \theta} \frac{\partial \theta}{\partial t} + \frac{\partial \mathbf{T}}{\partial \phi} \frac{\partial \phi}{\partial t} + \frac{\partial \mathbf{T}}{\partial \psi} \frac{\partial \psi}{\partial t} \right) \vec{x} \quad (7)$$

- During the measurement phase, the rotation rates vary in the range of $\pm 0.1 \text{ rad s}^{-1}$ for the roll and yaw direction movement. The pitch rotation is above $\pm 0.2 \text{ rad s}^{-1}$ and thus twice as large as the other two rotation rates (Figure 7).

- In Figure 8 the calculated induced velocity components are shown in the global coordinate system. The induced velocities are in the order of $\pm 0.2 \text{ m s}^{-1}$ during the phase of wind measurement (stationary hovering flight). The induced speed has a very small influence ($\leq 0.1 \text{ m s}^{-1}$) on the u component. The v , w components fluctuate to a maximum of $\pm 0.2 \text{ m s}^{-1}$.
- Furthermore, the signal of the ultrasonic anemometer must be corrected by the groundspeed of the aircraft. The groundspeed of the helicopter is determined by the derived position signal (DGPS with a precision of $\pm 10 \text{ cm}$). All three components of the groundspeed (Figure 9) are in the order of $\approx \pm 1 \text{ m s}^{-1}$ during the measurement phase. It would have been optimal if the autopilot of the AMPAIR had been able to keep its position fixed during the measurement phase. However, the dynamic turbulent flow caused changing conditions, forcing the autopilot to readjust. Figure 9 also shows the flight manoeuvres (purple area), where the groundspeed are much higher than during hovering.

Comparing the translational and rotational induced velocities with each other, it is apparent that the translational velocities are larger by a factor of approximately 10. During hovering, the translational motion has a greater influence than the rotation of the UAV.

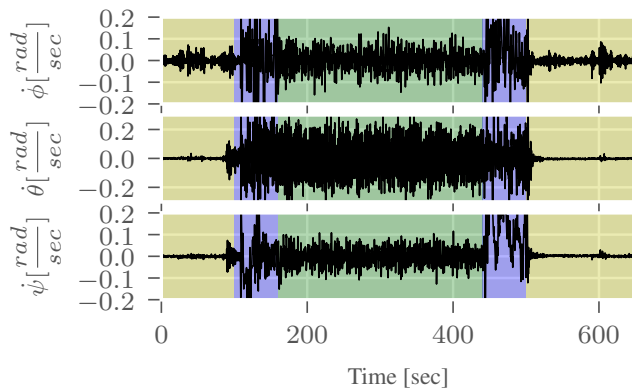


Figure 7: Rotation rate of the three directions (top graph Roll, middle Pitch, lower Yaw) in the body coordinate system.

3.2.1 Effect of rotation on the sonic measurements

As noted in Section 2.1, the orientation and motion of the anemometer is not expected to influence the results. However, this begs the question: what does the ultrasonic anemometer detect when it is rotated around its main axes? Un-

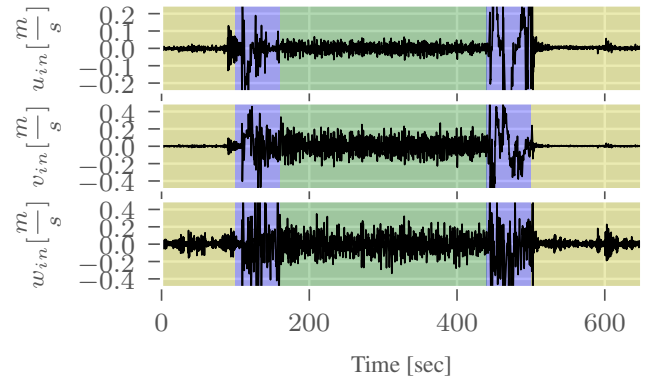


Figure 8: The three components of the induced velocities caused by the rotations due to the motion compensation of the auto-pilot in the global coordinate system.

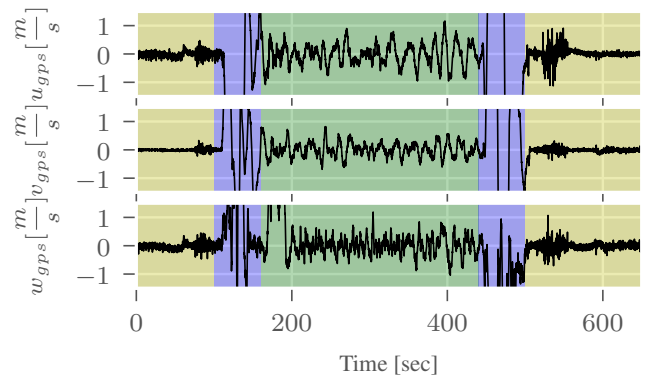


Figure 9: The three components of the induced velocities caused by translational movement due to motion compensation of the auto-pilot in the global coordinate system.

fortunately, no literature was found on this issue. However, considering the order of magnitude of the rotation rates and the resulting induced wind speeds, the possible error appears small during the hovering flight.

3.3 Data processing workflow

Data from the met mast and UAV cannot be compared directly. Instead, several steps are needed to prepare the data and then generate statistics that can be used show how the data compare (Figure 10).

The data of the AMPAIR are recorded with two different systems (AUTOPILOT, PAYLOAD). Both systems have a GPS-disciplined clock, which is used to merge the data in the postprocessing. Both systems are sampling with 10 Hz. The merged data are then subjected to the coordinate transformation and corrected for the induced velocities.

3.3.1 Plausibility testing

The data of the met mast are checked for plausibility. This means the sensors are compared with

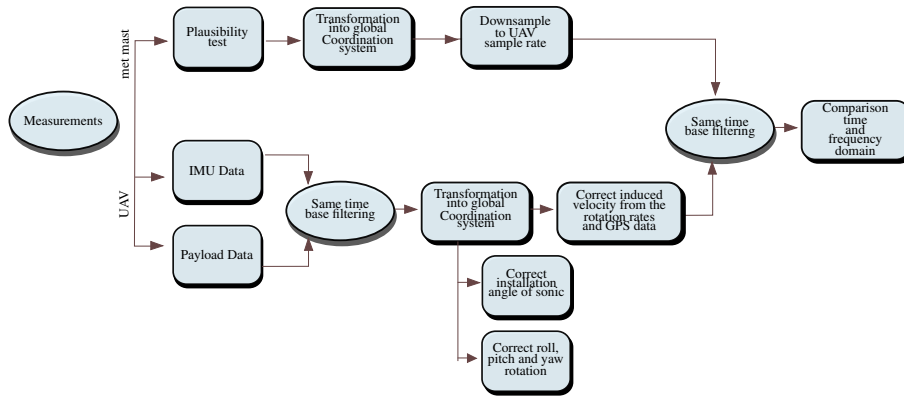


Figure 10: Overview of the data processing workflow, form the measuring to the comparison of the results.

other sensors of the same type and checked for spikes. The sampling rate of the met mast data is 50 Hz.

Finally the data is transformed into the common global coordinate system. The recording of the data of the met mast was continuous and not only during the flight campaign. In order to be able to compare the data with each other, they must be synchronized a second time. This second synchronization includes the time lag between the two positions. For the evaluation only the time period is used at which the AMPAIR has reached its target position and has entered the stationary mode for measurement.

3.3.2 Time resolution

For some evaluations, such as cross correlation or coherence, it is necessary that both signals have the same time resolution. There are two ways to achieve the same time resolution: up-sample the signal with lower frequency or down-sample the signal with higher frequency. When sampling up [e.g. 13], some assumptions must be made, for example that the signal is periodic. Downsampling will cause lost of information in high frequency domain. The evaluation here uses downsampling, as according to the Nyquist-Shannon sampling theorem only frequencies up to half of the lowest sampling rate can be meaningfully compared.

3.3.3 Comparison metrics

A first comparison between the two sensors is made using the mean values ($\bar{x}$) and standard deviations (σ_x).

Turbulence parameters from the devices are compared using the turbulence intensity (Ti), turbulent kinetic energy (TKE), the integral time scale (T_{u^*,v^*,w^*}), and the integral length scale (Λ_{u^*,v^*,w^*}):

$$Ti = \frac{\sigma_{v_{hor}}}{\bar{v}_{hor}} \quad (8)$$

$$TKE = \frac{1}{2} \left(\sigma_u^2 + \sigma_v^2 + \sigma_w^2 \right) \quad (9)$$

$$T_u = \int_0^\infty R_{u'u'}(\tau) d\tau \quad (10)$$

$$\Lambda_u = \bar{u} T_u = \bar{u} \int_0^\infty R_{u'u'}(\tau) d\tau \quad (11)$$

Kundu et al. [14] describe T_{u^*,v^*,w^*} as a scale of time over which u^* , v^* , w^* is highly correlated with itself and measure of memory of the process. The integral of the autocorrelation function $R_{u'u'}$ (u' denotes the fluctuating (turbulence) part of the velocity) in equation 10 is evaluated until the first zero crossing [15, 16].

To calculate the integral time and length scales the wind velocity components are transformed in main flow direction (necessary condition $\bar{v}^* = 0$; $\bar{w}^* = 0$; values in the flow coordinate system are marked with *). The length scales in v^* , w^* direction are evaluated with the wind speed $\bar{u}^*$ in main flow direction.

We also evaluate the time series in relation to each other using the covariance $cov_{x,y}$ and Pearson's correlation coefficient $\rho_{x,y}$:

$$cov_{x,y} = E[x - E[x]] * E[y - E[y]] \quad (12)$$

where $E[x]$ is the expected value of variable X and the (Equation 13)

$$\rho_{x,y} = \frac{cov_{x,y}}{\sigma_x \sigma_y} \quad (13)$$

The Pearson's correlation coefficient is in the range of $[-1, 1]$, where 1 means perfect linear correlation, 0 means no correlation and -1 means perfect negative linear correlation.

Data are also compared in the frequency domain. We use the Power Spectral Density (PSD) [17, 13] for the wind speed components u^* , v^* , w^* and the coherence C_{xy} (Equation 14), where P_{xx} and P_{yy} are the power spectral density and P_{xy} is the cross-spectral density between the two sensors X and Y .

$$C_{xy} = \frac{|P_{xy}|^2}{P_{xx}P_{yy}} \quad (14)$$

4 Results

In this section the results of the comparison are presented. Before the results are looked at in detail, an overview is given of the distribution of the measurement data. Afterwards the results of the analyses in the time domain are considered and then the results in the frequency domain.

Because of safety limitations the helicopter-mounted sensors are approximately 132 m away from the mast during this comparison (longitudinal approx. 89 m; lateral approx. 97 m).

4.1 Distribution of the data

The distribution of the measurement data has a large influence on the statistical quantities such as σ , Ti and TKE . The distribution depends significantly on how long the measurement is taken and whether the time series is stationary. The normalized probability density functions (PDF) of the v_{hor} of the AMPAIR and of the met mast are exemplary presented in Figure 11 and also the theoretical normal distribution is illustrated.

4.1.1 Data sets

To make the influence of the measurement duration on the statistical values visible, the short duration of the hovering flight is compared with a 10 min interval. The 10 min time period is a typical time range in wind energy. The 10 min period also covers the hovering flight. The 10 min period is only calculated for the met mast. The three time series are normalized to $\sigma = 1$ and $\bar{v}_{hor} = 0$, that the PDFs are comparable.

4.1.2 Observation

Differences can be seen between the 10 min interval and the evaluated time interval (Figure 11). The distributions of the AMPAIR and the 10 min interval resemble the normal distribution. This contrasts with the distribution of the met mast for the evaluation period. Especially in the range of -3σ to -2σ the distribution shows larger deviations compared to the normal distribution, also in the region of 0.5σ . There is it more peaked. The evaluated time interval overestimate the statistical parameter like the σ . For the evaluated time interval $\sigma_{v_{hor}} = 1.64 \text{ m s}^{-1}$ is compared to the 10 min period ($\sigma_{v_{hor}} = 1.442 \text{ m s}^{-1}$) 11.6 % higher. The more unstable conditions make it more difficult for the AMPAIR to hover stationary and lead to more control interventions by the autopilot.

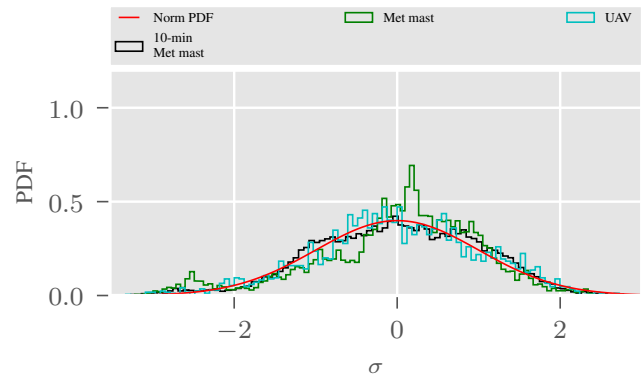


Figure 11: The normalized PDF for the v_{hor} for the 10 min period and the period of the measurements and the theoretical normal distribution.

4.2 Analysis in the time domain

4.2.1 Visual comparison

Figure 12 shows the measured wind velocity components u, v, w in the global coordinate system and the wind direction for the measurement phase. For a first comparison of the time series, the three wind speed components were visually analysed. These curves are not perfectly aligned to each other, but shows a comparable process. Looking at the u component in detail, there are two significant drops in the wind speed at the signal from the met mast. This is not apparent in the data of the AMPAIR. The v component shows a similar behaviour, the velocity of the signal from the met mast drops back several times to almost 0 m s^{-1} . The signal from the AMPAIR fluctuates relatively constantly around -2 m s^{-1} , suggesting slightly different flow conditions. The w component of both sensors fluctuates around 0 m s^{-1} .

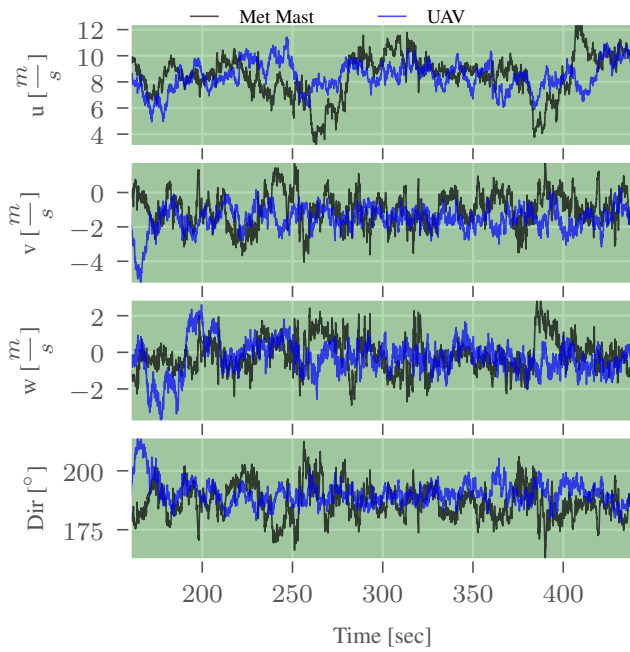


Figure 12: Time series of the wind components u, v, w for both measurements systems during the measurement period.

Although there was a distance of approx. 132 m between the met mast and the UAV, the time series show very similar characteristics. The investigated period represents the wind conditions at this site statistically very well. The mean wind speed and the wind direction are in the region of the most typical wind conditions. The turbulence intensity of 17.1 % for a 10 min period is in the upper range.

4.2.2 Statistical comparison - statistic moments

A comparison of the values from the met mast and AMPIAR system (Table 3) shows no clear trend. The horizontal wind speed agrees very well, but there are some large differences between the wind direction as well as in the wind speed component v measured by the two systems. Similarly, there is no uniform trend to the deviation, where the horizontal wind speed and the u component show the largest deviation.

4.2.3 Statistical comparison - offset corrected

For the further evaluations, the assembly tolerance for the magnetic compass is assumed to be about 1.0° and the disturbance on the caused by the electrical systems is assumed to be about 3.0° . Hence, the compass-heading is corrected by 4.0° . More about this topic in Section 5.

The correction of the compass-heading shows a significant improvement in the agreement between the UAV and met mast (Table 4). The de-

viations of data are reduced significantly, e.g. for the v component the deviation is reduced by half and the u component is nearly the same. Also the difference in the wind direction is reduced.

However, there was no improvement in the standard deviations. All further investigations were performed with the offset-corrected signals.

4.2.4 Statistical comparison - flow properties

The values calculated for turbulence intensity (T_i) and turbulent kinetic energy (TKE) observed by the UAV are lower than the values obtained from the met mast (Table 5).

The result of the integral length scale is differentiated: T_u^* is much larger for the met mast data, but the values T_v^* and T_w^* are smaller. The integral length scale Λ_{u^*,v^*,w^*} behaves in the same way as the integral time scale. The direct comparison of the time series show no linear correlation. The $\rho_{u^*u^*}$ value still has the largest match with 0.25. The other two components range around zero and do not show a linear correlation.

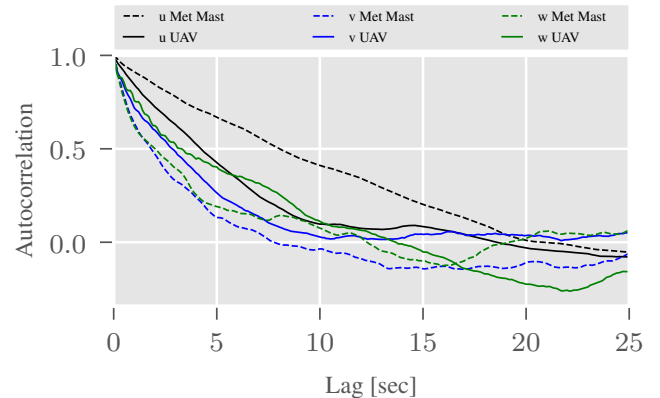


Figure 13: Autocorrelation of wind components u^*, v^*, w^* for the met mast (dashed line) and the UAV (solid line).

4.3 Analysis in the frequency domain

To compare the data in the frequency domain, the PSD is calculated. It should be mentioned again that the sampling frequencies of the two measuring systems differ (UAV: 10 Hz; met mast: 50 Hz) and for that the met mast signals are down sampled to 10 Hz. The PSD was determined with $NFFT = 1024$ (NFFT: The number of data points used in each block for the FFT) data points. Furthermore, the time series were corrected by the respective mean value. In Figure 14 the PSDs are shown in double-logarithmic scale. Additionally the theoretical spectrum for the inertial subrange ($\sim f^{-5/3}$) [18].

Table 3: Summary statistics from the measurement phase for the UAV and met mast data.

Parameter	Met mast	AMPAIR	Deviation	Deviation Met mast [%]
$\bar{u}$ [m s ⁻¹]	8.48	8.28	0.20	2.35
σ_u [m s ⁻¹]	1.64	1.10	0.54	32.80
$\bar{v}$ [m s ⁻¹]	-0.97	-2.11	1.14	118.20
σ_v [m s ⁻¹]	0.97	0.70	0.27	27.17
$\bar{w}$ [m s ⁻¹]	-0.18	-0.30	0.12	67.61
σ_w [m s ⁻¹]	0.90	0.93	0.03	2.89
$\bar{v}_{hor}$ [m s ⁻¹]	8.58	8.58	0.00	0.04
$\sigma_{v_{hor}}$ [m s ⁻¹]	1.63	1.07	0.56	34.17
$\bar{v}_{dir}$ [°]	186.61	194.46	7.85	4.21

Table 4: Summary statistics from the measurement phase for the UAV and met mast data with direction offset corrections.

Parameter	Met mast	AMPAIR	Deviation	Deviation Met mast [%]
$\bar{u}$ [m s ⁻¹]	8.48	8.41	0.07	0.83
σ_u [m s ⁻¹]	1.64	1.10	0.54	33.05
$\bar{v}$ [m s ⁻¹]	-0.97	-1.53	0.56	58.64
σ_v [m s ⁻¹]	0.97	0.72	0.25	26.29
$\bar{w}$ [m s ⁻¹]	-0.18	-0.30	0.12	67.61
σ_w [m s ⁻¹]	0.90	0.90	0.00	0.44
$\bar{v}_{hor}$ [m s ⁻¹]	8.58	8.58	0.00	0.05
$\sigma_{v_{hor}}$ [m s ⁻¹]	1.63	1.07	0.56	34.19
$\bar{v}_{dir}$ [°]	186.61	190.47	3.86	2.07

Table 5: Summary turbulence statistics from the measurement phase for the UAV and met mast data.

Parameter	Met mast	AMPAIR	Deviation	Deviation Met mast [%]
Ti [-]	0.19	0.13	0.06	34.21
TKE [m ² s ⁻²]	1.05	0.80	0.25	23.81
T_u^* [s]	8.86	5.16	3.7	41.76
T_v^* [s]	2.41	4.53	2.12	87.97
T_w^* [s]	3.11	4.60	1.49	47.91
Λ_u^* [m]	75.51	44.15	31.36	41.53
Λ_v^* [m]	20.52	38.74	18.22	88.79
Λ_w^* [m]	26.52	39.34	12.82	48.34

Table 6: The covariance and the Pearson coefficient

Parameters	Quality
cov_{u^*,u^*} [m ² s ⁻²]	0.44
cov_{v^*,v^*} [m ² s ⁻²]	-0.06
cov_{w^*,w^*} [m ² s ⁻²]	0.03
ρ_{u^*,u^*}	0.25
ρ_{v^*,v^*}	-0.08
ρ_{w^*,w^*}	0.04

The spectrum of the u component follows the theoretical spectrum very closely, whereas the spectrum of the v component of the UAV shows a significantly smaller energy component in the frequency range ≤ 0.3 Hz compared to the met mast. The components v, w of the UAV have a spike at 2.5 Hz, which do not occur in the met mast data.

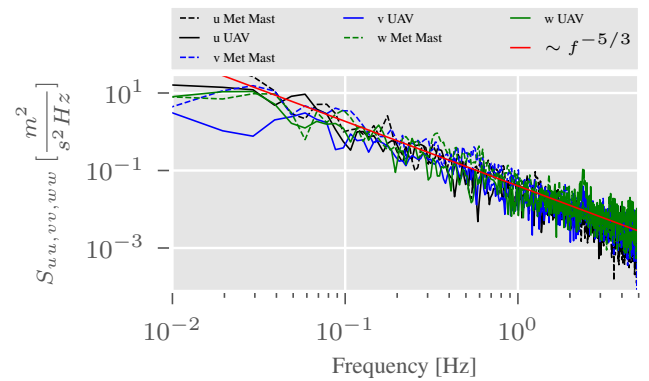


Figure 14: Power spectral density of wind components u^*, v^*, w^* for the met mast (dashed line) and the UAV (solid line).

In order to investigate the dependencies of

the time series of the wind speed components u^* , v^* , w^* in the frequency domain, the coherence (Figure 15) was calculated. The measured coherence shows no significant correlation between the sensors.

The coherence model of Pielke and Panofsky [19] was also used for comparison. Pielke and Panofsky proposed $coh(f) = \exp(-\frac{a \cdot f \cdot D}{\bar{u}})$, where f is the frequency, $\bar{u}$ is the mean wind speed, D is the distance between the measuring points, and a is a decay parameter of order of 10. In this case we set $D = 89$ m for the longitudinal distance. We set $a = 10$ after Kristensen [20] and $a = 1$, which is closer to that found by Simley and Pao [21]; Simley and Pao [21] uses a modified model with a similar structure of terms, but found $a \in [0.2, 5]$ depending on Ti and atmospheric stability. The model predicts no meaningful coherence at this separation distance and in this frequency range.

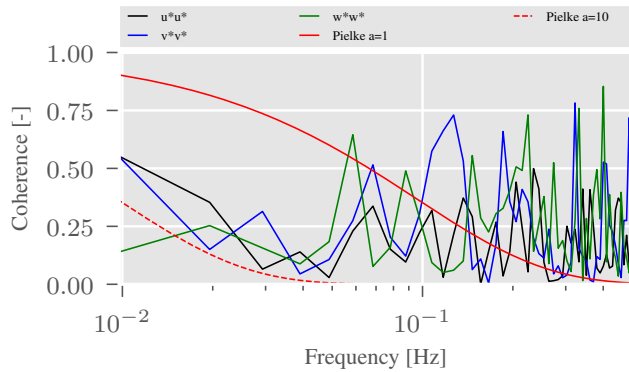


Figure 15: Coherence of wind components u^* , v^* , w^* between the met mast and the UAV and the Pielke model (red lines).

5 Discussion

The total period of UAV measurements presented here is approximately seven minutes (e.g. Figure 12), and the UAV was approximately 132 m (longitudinal approx. 89 m; lateral approx. 97 m) from the mast in wind speeds around 8.5 m s^{-1} (10 s to 15 s propagation time at these wind speeds). During this time the mean wind speed measured by the UAV and met tower were within 0.05 % of each other (Table 3). Other mean statistics were within a few percent, while some of the higher order statistics showed higher deviation. Also, the PDF (Figure 11), autocorrelation (Figure 13), and frequency content of the two sets of measurements is very similar (Figure 14). This agreement suggests that the two systems were measuring the same flow and are capable of resolving the same frequency content.

The deviation of higher order statistics (e.g. in Table 3) and poor covariance (Table 6) suggests that the two systems were measuring the same flow, but that the time series is simply not long enough to get better statistical agreement, given the distance and the length- and time scales prevalent in the atmosphere. This hypothesis is supported by the low covariance of the measurements (Table 6).

The frequency spectrum agrees very well with the theoretical spectrum except for the peaks and the gap. The first assumption was, that these spikes were caused by the motion of the UAV (induced velocities), but these spikes are not visible in the PSDs of the roll, pitch and yaw rotation rates and also not in the PSDs of the induced velocities. Also possible are oscillations, which are coupled in by the structure of the AMPAIR. However, it has not yet been possible to investigate this further. It cannot be excluded that these are interferences of the rotor speed. The rotor speed ranges is between 780 rpm to 820 rpm. Reasons for the gap could be probably the short measuring time and the poor resolution of the low frequencies.

The poor correlations of the time series of the two measuring systems in the time and frequency domain (Pearson, coherence) are no surprise. The studies of Saranyasoonorn et al. [22] and Simley and Pao [21] show similarly bad matches of the coherence at spatial (lateral and longitudinal) offset of the measurement points. Saranyasoonorn et al. [22] used data from sonic anemometers which are installed on measurements masts with lateral offset and Simley and Pao [21] used LES-Simulations. The results of the two studies can be summarised as follows: Coherence decreases with distance. Furthermore, atmospheric stability and mean wind speed have an influence on the coherence. Changes in lateral distance have a greater influence than in longitudinal direction. Especially the results of Saranyasoonorn et al. [22] show almost no coherence between the sensors at already small distances in lateral direction (38 m). In the presented experiment, there is a superposition of both with a large distance and a relatively short measurement time (< 10 min). Both factors have a direct influence on the result of the coherence. Therefore the results are not good, but are not unexpected.

Influence of the rotor on the flow measurement

How to recognize the influence of the rotor on the flow measurement? With this selected horizontal installation position of the ultrasonic anemometer, the influence of the rotor should be visible in the data of the w component. On the one hand by a larger negative average value, because the rotor accelerates the flow in the opposite direction to the coordinate system. On the other hand, by a larger fluctuation due to the periodic acceleration by the rotor. The mean value of the w component is slightly below zero and a little less than the value of the measuring mast. The difference of the standing deviations is zero. Based on this results, it is assumed, that the rotor has no influence on the flow measurement. The deviation of the mean value is attributed to small inaccuracies in the position angles.

Influence and error of the magnetic compass

The results in Table 3 are inconclusive before the offset correction. The $\bar{v}_{hor}$ match well, but the $\bar{v}$ components and the $\bar{v}_{dir}$ show clear differences between the measuring systems. This is supposed to be resulted from the measurement error of the compass heading. The accuracy of the magnetic compass has a significant influence on the correspondence of the flow components.

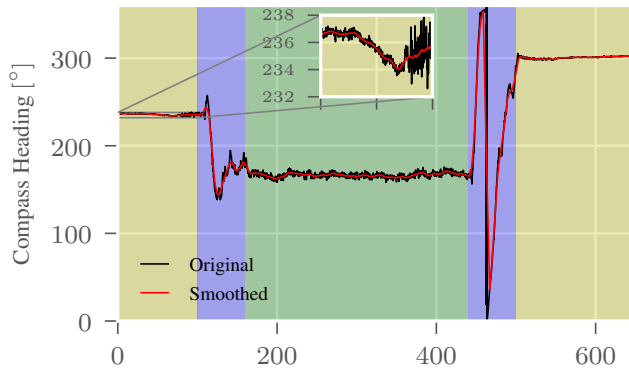


Figure 16: The magnetic compass heading during the flight time and the "System Check" phase in detail.

During the "system check" phase the AMPAIR is motionless on the ground and the different electrical systems are activated one after the other. As shown in the scaled detailed cut out in Figure 16, the magnetic compass changes its value by about -3° . After the rotors are turned on, the signal of the compass changes its characteristics and the amplitude increases. The reverse behaviour can also be observed during the "system shutdown" phase, where the amplitude decreases and the signal increases again by 3° . During the hovering

phase, only a very broad signal is visible. It is not possible to determine whether an additional offset has occurred, due to the UAV now being in operation mode and consuming more power. The signal of the magnetic compass was smoothed using the Savitzky-Golay [23] algorithm.

In this case the misalignment of the compass is caused by the electrical fields of the individual components. The manufacturer of the magnetic compass does not specify the accuracy of its product, but the handbook provides installation instructions. The sensor should be

- Not near ferrous metals
- Far away from electric motors, including those found in servos and in pan/tilt cameras

for the installation. However, this is very difficult when space is limited. Furthermore, assembly tolerances during installation can still occur. The installation of the sensors was carried out with the greatest care, however, it is assumed that the tolerances during the assembly could be around 2° .

6 Conclusions

In the previous sections, an experiment was presented in which an unmanned aerial vehicle equipped with a standard 3-D ultrasonic anemometer measured the flow components in stationary hovering flight. The comparison with a 3-D ultrasonic anemometer installed on a 95 m high met mast shows very good agreement of the horizontal wind speed with a relative deviation of 0.05 % at mean wind speed of approx. 8.58 m s^{-1} . For the wind component u the differences are also very small (0.83 %). However, the v, w components show larger deviations. For v the deviation is $\approx 0.56 \text{ m s}^{-1}$ and for $w \approx 0.12 \text{ m s}^{-1}$.

The higher moments, the covariance and the coherence show no correlation. This is due to the atmospheric conditions which change the turbulent structures over the large longitudinal and lateral distance [22, 21].

However, further improvements to the AMPAIR are needed. Sampling with 10 Hz is below the capability of the installed ultrasonic anemometer (20 Hz). Furthermore, the effective measuring time must be increased. This is made possible by carrying more battery. However, the additional weight has a negative effect on the flight duration. A doubling of the battery capacity does not mean a doubling of the flight time. Another possibility to increase the flight duration would be to con-

vert the electric propulsion system to a combustion engine.

A reliable system is needed for determine the compass heading, which can avoid the influence of the electrical board systems. Maybe the alignment of the UAV can be realized by DGPS, which is independent of the electromagnetic field. The length of the boom seems to be sufficient. There is no evidence in the data of rotor influences on the measurements, for example through increased w' values.

At the current state of the art, hovering UAVs are not yet suitable to replace stationary measurements with met masts over a long-term measurement. The primary problem will be the permanent power supply of the UAV. For short-term measurements (< 1 h), hovering UAVs have great potential and can become an important tool in atmospheric research. Especially in rough terrain, e.g. over a wooded escarpment, they can become an important instrument for atmospheric research. The comparatively short observation times are sufficient for the validation of flow simulations, or can provide important information for input variables of flow simulations. Furthermore, measurements at higher altitudes (> 200 m) could supplement shorter met masts.

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