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Parametric modelling of nonlinearities by covariance analysis

W. Schiehlen & R. Krause
Institute B of Mechanics, University of Stuttgart, Germany

ABSTRACT: An integrated approach for the identification of mechanical systems with nonlinear force elements is shown. The method is split in two parts. The first part based on the covariance method uses measured data from the system and a linear model to proof which nonlinearities have an influence on the dynamical behavior to get a nonlinear model. In the second part, the parameter of this nonlinear model will are identified using measured covariances of the system for different excitation levels.

1 INTRODUCTION

Modeling dynamic systems requires assumptions and idealizations resulting in mathematical equations which determine the dynamic behavior of the model. In general, such models include characteristic parameters which determine the behavior of the model. For some of these parameters with physical meaning, like mass or geometrical data, values can be found by direct measurement. Others are part of idealized physical laws, e.g. stiffness and damping elements, and cannot be determined directly. It is the task of identification methods to evaluate these parameters indirectly by comparing the dynamic behavior of both the dynamic system and the model.

The covariance method was developed for parameter identification of linear dynamic systems subject to stochastic excitation (Weber and Schiehlen 1983, Kallenbach 1987). The parameters of the system can be identified using second moments of the system's response as well as the response of a linear filter added to the system's measurement devices.

The proposed paper shows an extension of the covariance method for the detection of nonlinear damping and stiffness elements and the parameter identification of these nonlinear forces. In both cases the second moments of the nonlinear system will be investigated in respect to different excitation levels, because one excitation level is only sufficient for linear systems.

2 DETECTION OF NONLINEAR FORCE ELEMENTS

In the first step, for the detection of nonlinearities of a mechanical system is chosen a linear time-invariant model

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector. Matrices $\mathbf{A}$ and $\mathbf{B}$ comprise known and unknown parameters of the model. The measured data of the excitation $\mathbf{u}(t)$ and the response $\mathbf{x}(t)$ of the technical system are considered as excitation and response of the linear model. The stochastic excitation $\mathbf{u}(t)$ is supposed to be an ergodic, Gaussian and stationary process which can be described by a time-invariant form filter

$$\dot{\mathbf{u}} = \mathbf{K}\mathbf{u} + \mathbf{L}\mathbf{w} \tag{2}$$

where $\mathbf{w}$ is a Gaussian white noise with zero mean. For detection or identification purposes the input and output of the technical system has to be passed through an l- dimensional linear filter

$$\dot{\mathbf{y}} = \mathbf{F}\mathbf{y} + \mathbf{H}\mathbf{x} + \mathbf{G}\mathbf{u} \tag{3}$$

with known matrices $\mathbf{F}$, $\mathbf{H}$ and $\mathbf{G}$, and $l > m + n$, see Fig. 1.

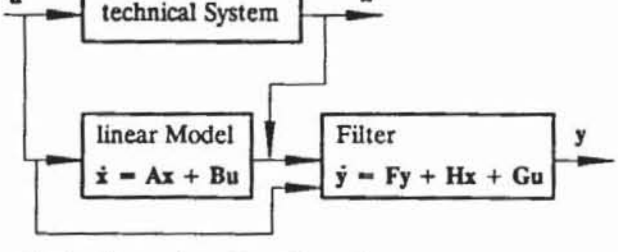

Fig.1. Detection of nonlinearities.

Using (1), (3) and the measured data, stationarity of $\mathbf{C}_{yx} = E\{\mathbf{y}\mathbf{x}^T\}$ will yield algebraic relations for the system parameters:

$$\frac{d}{dt} C_{yx} = C_{yx} A^T + C_{yu} B^T +$$

$$F C_{yx} + H C_{xx} + G C_{ux} \stackrel{!}{=} 0 \quad (4)$$

After comprising all unknowns of **A** and **B** in a parameter vector **p** and collecting all non-trivial equations of (4) we end up with an overdetermined set of linear algebraic equations

$$Cp = c \quad (5)$$

for identifying the unknown parameters. These parameters may be estimated in the sense of least squares by premultiplying (5) with C^T and solving the equations for **p** :

$$p = \left(C^T C\right)^{-1} C^T c \ . \quad (6)$$

If the system is linear, the parameters are independent of the excitation intensity C_{uu}. But in the case if the system is nonlinear, the parameters will change in respect to the excitation intensity and kind of nonlinearity. Some applications with nonlinear force elements like prestress, back lash, cubic stiffness and coulomb friction show characteristic parameter curves (El-Dessouki 1990). Depending on the excitation level, only the parameter of the force elements change the values, which replace the nonlinear elements. All parameter of linear force elements from the system keep constant by the detection of nonlinearities. Problems occur only if a nonlinear force element keeps two parts with 'inverse' behavior like prestress and back lash.

3 PARAMETER IDENTIFICATION OF SYSTEMS WITH NONLINEAR FORCE ELEMENTS

After the detection of nonlinearities it is possible to define a nonlinear model

$$\dot{x} = f(x, u, p) = \sum_{i=1}^{q} p_i \, \bar{f}_i(x, u) \quad (7)$$

with the model vector $f(x, u, p)$ and parameter vector $p \in \mathbb{R}^q$. The parameter must be linear combination in respect to all other variables for the following parameter identification. In this case an extended model is defined

$$\dot{z} = \sum_{i=1}^{q} p_i \, g_i(z, u_1 \dots u_r)$$

$$\text{with} \quad z = \begin{bmatrix} x_1 \\ \vdots \\ x_r \end{bmatrix}, \quad g_i = \begin{bmatrix} \bar{f}_{i,1}(x_1, u_1) \\ \vdots \\ \bar{f}_{i,r}(x_r, u_r) \end{bmatrix} \quad (8)$$

where r is the maximum number of parameters to describe a stiffness or damping element of the nonlinear model. Important is here, that the r-different excitation levels are chosen carefully to have a different influence of the nonlinearities. Using (8), (3) and the measured data leads to the extended covariance analysis

$$\frac{d}{dt} C_{yz} =$$

$$\sum_{i=1}^{q} p_i \, C_{yg_i} + F \, C_{yz} + H \, C_{zz} + G \, C_{uz} \stackrel{!}{=} 0 . \quad (9)$$

Analogous to solve (4) in respect to the parameters eq. (9) is an overdetermined set of equation which can solved as shown in (5) and (6). Applications to mechanic systems up to 4 degree of freedom and up to 7 nonlinearities show good results (El-Dessouki 1990).

4 CONCLUSION

It is shown that the covariance analysis is a strong tool for detection of nonlinear force elements. The identified parameter curves in respect to the excitation are characteristic for the above mentioned nonlinearities, also by multiple nonlinearities in one force element. The parameter identification is also very accurate as long as the nonlinearities have an influence on the dynamic behavior. Nonlinear force elements, which have no influence on the interesting excitation level can not be identified. The extension to other time independend nonlinear forces seems obvious, further investigations are necessary to decide about time varying nonlinearities.

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